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A Comparative Study of Fractional Partial Differential Equations with the Help of Yang Transform

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Abstract: In applied sciences and engineering, partial differential equations (PDE) of integer and non-integer order play a crucial role. It can be challenging to determine these equations' exact solutions. As a result, developing numerical approaches to obtain precise numerical solutions to these kinds of differential equations takes time. The homotopy perturbation transform method (HPTM) and Yang transform decomposition method (YTDM) are the subjects of several recent findings that we describe. These techniques work well for fractional calculus applications. We also examine fractional differential equations' precise and approximative solutions. The Caputo derivative is employed because it enables the inclusion of traditional initial and boundary conditions in the formulation of the issue. This has major implications for complicated problems. The paper lists the important characteristics of the YTDM and HPTM. Our research has numerous applications in the disciplines of science and engineering and might be seen as a substitute for current methods.

Keywords: fractional differential equations; Adomian decomposition method; homotopy perturbation method; Caputo operator; Yang transform

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1. Introduction

Numerous researchers have been researching fractional calculus for a long time. This is a continuous process, and one can see how new methods and mechanisms emerge within the field of fractional calculus studies, allowing for the development of significant, difficult insights and previously unrecognized connections between many branches of physics. Scientists' interest in non-local field theories has recently increased. There is a solid argument for these late developments; it will help us solve high-energy and particle physics issues that, up to now, could only be solved using local field theories. Due to their non-locality quality, fractional derivatives have demonstrated their ability to describe a number of phenomena connected to memory and after effects [1,2]. Such processes frequently occur in biological systems, physical processes, and cosmic issues. For instance, [3–5], fractional rheological models were used to test the low applied force frequencies. It became necessary to clarify the model solutions that account for these phenomena as a result. To attain these goals, several analytical methodologies are offered. Since the fractional derivative generalizes the classical derivative to an arbitrary order, all of these ways are actually just extensions of the methods now in use to handle the integer case models [6–8].

The fundamental reason for studying numerical techniques for fractional differential equations is the growing popularity of fractional derivative models within the larger scientific community. In the subject of nonlinear science, which has been utilized to describe the issues in numerous fields, including quantum physics, image processing, ecology and economic systems, and epidemiology, the nonlinear PDEs have emerged as a major topic [9–11]. PDEs are widely employed in several physical applications, including magnetic resonance imaging, dispersing and propagation of waves, magnetohydrodynamic movement through pipes, computational fluid dynamics, acoustic transmission, traffic, and phenomena of supersonic and turbulence flow. The references in [12] can be used to obtain more information. PDEs are employed in population modeling, medical imaging, ensuring that healing tissues receive the appropriate amount of oxygen, electrically signaling of nerves, and other applications [13–15]. A very accurate estimate of the number of COVID-19 patients has validated the prevalence of PDE [16,17]. The shape of COVID-19 can be modeled using PDEs, as shown in [18]. However, the fractional PDE is more accurate than the integer-order partial differential equation for several challenging issues in these domains. Therefore, it is essential to develop numerical solutions for fractional PDEs. Thus, when studying differential equations, and more specifically when studying equations from the mathematics of finance, symmetry analysis is a great subject to study [19–21]. Symmetry is key to nature, but it is absent from the majority of observations of the natural world. The occurrence of spontaneous symmetry-breaking is a potent method for masking symmetry. Two different types of symmetries are finite and infinitesimal. Discrete or continuous finite symmetries are possible. Space is continuously changed, although parity and temporal reversal are discrete natural symmetries. Patterns have always captivated mathematicians. In the seventeenth century, classifications of spatial and planar patterns represented a considerable advance. Unfortunately, accurate solutions of non-linear fractional differential equations have proven to be challenging.

Due to the significance of numerically solving fractional PDEs (FPDEs) in science and engineering, many renowned scholars have made contributions in this field and some strong numerical algorithms have been presented. The methods for investigating approximate solutions of fractional differential equations have been studied in many different ways, including the Yang transform decomposition method for the Noyes–Field model for the time-fractional Fisher's equation [22] and the time-fractional Belousov–Zhabotinsky reaction [23], Elzaki homotopy perturbation method for fractional-order regularized long-wave models [24], natural transform decomposition method for the solution of Kersten–Krasil'shchik coupled KdV-mKdV systems [25], fractional Gardner and Cahn–Hilliard equations [26], the q-homotopy analysis transform method fractional Kundu–Eckhaus equation and fractional massive Thirring model [27], the residual power series method for fractional foam drainage equation [28], time-fractional schrödinger equations in one-dimensional space [29], variational iteration transform method for the fractional-order Boussinesq equation [30], fractional-order Newell–Whitehead–Segel equations [31], the first-integral method to study the Burgers–Korteweg–de Vries equation [32], optimal homotopy asymptotic method for the solutions of fractional order heat- and wave-like partial differential equations [33], and many more [34–39].

In light of their widespread use and applicability, there is a clear flaw in the numerical techniques that are currently available for approximating solutions to FPDEs. The current research was motivated by the need for a general method that may be applied to issues involving linear, nonlinear, homogeneous, non-homogeneous, and multivariable FPDEs without requiring significant modifications. Numerous academics have recently looked into the numerical solutions of fractional PDEs, which has significantly advanced the study of nonlinear PDEs. Numerical approaches may, however, generally have significant drawbacks, including limited precision, mesh generation, transformations, stability, convergence, and difficulties applying to complicated geometries.

Two unique methodologies, known as the Yang transform decomposition method and the homotopy perturbation transform method, are described in this research. The Yang transform (YT), which was introduced by Xiao-Jun Yang, can be used to resolve a variety of differential equations with constant coefficients. Adomian has created a numerical method for resolving functional equations since the 1980s [40,41]. He offered the result as an infinite series that typically leads to a precise solution. The homotopy perturbation method (HPM) [42], first proposed by He in 1998 and later developed and improved by He [43,44], leads to a very rapid convergence of the solution series; in the majority of cases, only one iteration results in a high accuracy of the solution, making it a useful and practical mathematical tool for nonlinear equations.

The way in which our work will be displayed is as follows: The history of the natural transform method and definitions of fractional derivatives are first provided in Section 2. The applications model of FDEs employing the suggested methods are covered in Sections 3 and 4. We resolve fractional FDEs in Section 5. Section 6 concludes with our final observations.

2. Preliminaries

This section describes the properties of fractional derivatives and a few essential details concerning the Yang transform.

Definition 1. *The fractional derivative in terms of Caputo is as follows [45,46]:*

$$D_{\psi}^{\varrho}\mathbb{W}(\xi,\psi) = \frac{1}{\Gamma(k-\varrho)}\int_0^{\psi}(\psi-\varrho)^{k-\varrho-1}\mathbb{W}^{(k)}(\xi,\jmath)d\jmath, \quad k-1<\varrho\leq k, \quad k\in N. \tag{1}$$

Definition 2. *The YT is represented as [47,48]:*

$$Y\{\mathbb{W}(\psi)\} = M(u) = \int_0^{\infty} e^{\frac{-\psi}{u}}\mathbb{W}(\psi)d\psi, \ \ \psi>0, \ \ u\in(-\psi_1,\psi_2), \tag{2}$$

having an inverse YT as:

$$Y^{-1}\{M(u)\} = \mathbb{W}(\psi). \tag{3}$$

Definition 3. *The nth derivative YT is stated as [47,48]:*

$$Y\{\mathbb{W}^n(\psi)\} = \frac{M(u)}{u^n} - \sum_{k=0}^{n-1}\frac{\mathbb{W}^k(0)}{u^{n-k-1}}, \ \ \forall \ \ n=1,2,3,\cdots \tag{4}$$

Definition 4. *The YT of derivative having fractional-order is stated as [47,48]:*

$$Y\{\mathbb{W}^{\varrho}(\psi)\} = \frac{M(u)}{u^{\varrho}} - \sum_{k=0}^{n-1}\frac{\mathbb{W}^k(0)}{u^{\varrho-(k+1)}}, \ \ n-1<\varrho\leq n. \tag{5}$$

3. Construction of HPTM

To demonstrate the basic process of HPTM, we take a general nonlinear fractional partial differential equation as follows:

$$D_{\psi}^{\varrho}\mathbb{W}(\xi,\psi) = \mathcal{P}_1[\xi]\mathbb{W}(\xi,\psi) + \mathcal{Q}_1[\xi]\mathbb{W}(\xi,\psi), \ \ 0<\varrho\leq 1, \tag{6}$$

concerning the initial values

$$\mathbb{W}(\xi,0) = \chi(\xi).$$

Here, $D_{\psi}^{\varrho} = \frac{\partial^{\varrho}}{\partial\psi^{\varrho}}$ demonstrate the Caputo operator, and $\mathcal{P}_1[\xi]$, $\mathcal{Q}_1[\xi]$ are linear and nonlinear terms.

Using the YT, we have:

$$Y[D_{\psi}^{\varrho}\mathbb{W}(\xi,\psi)] = Y[\mathcal{P}_1[\xi]\mathbb{W}(\xi,\psi) + \mathcal{Q}_1[\xi]\mathbb{W}(\xi,\psi)], \tag{7}$$

$$\frac{1}{u^{\varrho}}\{M(u) - u\mathbb{W}(0)\} = Y[\mathcal{P}_1[\xi]\mathbb{W}(\xi,\psi) + \mathcal{Q}_1[\xi]\mathbb{W}(\xi,\psi)]. \tag{8}$$

Then, we have:

$$M(u) = u\mathbb{W}(0) + u^{\varrho}Y[\mathcal{P}_1[\xi]\mathbb{W}(\xi,\psi) + \mathcal{Q}_1[\xi]\mathbb{W}(\xi,\psi)]. \tag{9}$$

Let us use the inverse YT on both sides:

$$\mathbb{W}(\xi,\psi) = \mathbb{W}(0) + Y^{-1}[u^{\varrho}Y[\mathcal{P}_1[\xi]\mathbb{W}(\xi,\psi) + \mathcal{Q}_1[\xi]\mathbb{W}(\xi,\psi)]]. \tag{10}$$

On using the HPM:

$$\mathbb{W}(\xi,\psi) = \sum_{k=0}^{\infty} \epsilon^k \mathbb{W}_k(\xi,\psi). \tag{11}$$

having the parameter $\epsilon \in [0,1]$.

The nonlinear components are ultimately addressed as:

$$\mathcal{Q}_1[\xi]\mathbb{W}(\xi,\psi) = \sum_{k=0}^{\infty} \epsilon^k H_n(\mathbb{W}). \tag{12}$$

In addition, He's polynomials $H_k(\mathbb{W})$ are stated as:

$$H_n(\mathbb{W}_0,\mathbb{W}_1,\ldots,\mathbb{W}_n) = \frac{1}{\Gamma(n+1)} D_\epsilon^k \left[\mathcal{Q}_1\left(\sum_{k=0}^{\infty} \epsilon^i \mathbb{W}_i\right)\right]_{\epsilon=0}, \tag{13}$$

with $D_\epsilon^k = \frac{\partial^k}{\partial \epsilon^k}$.

On substituting (14) and (15) in (12), we obtain:

$$\sum_{k=0}^{\infty} \epsilon^k \mathbb{W}_k(\xi,\psi) = \mathbb{W}(0) + \epsilon \times \left(Y^{-1}\left[u^{\varrho}Y\{\mathcal{P}_1 \sum_{k=0}^{\infty} \epsilon^k \mathbb{W}_k(\xi,\psi) + \sum_{k=0}^{\infty} \epsilon^k H_k(\mathbb{W})\}\right]\right). \tag{14}$$

By computing ϵ coefficients, we have:

$$\begin{aligned}
&\epsilon^0 : \mathbb{W}_0(\xi,\psi) = \mathbb{W}(0),\\
&\epsilon^1 : \mathbb{W}_1(\xi,\psi) = Y^{-1}[u^{\varrho}Y(\mathcal{P}_1[\xi]\mathbb{W}_0(\xi,\psi) + H_0(\mathbb{W}))],\\
&\epsilon^2 : \mathbb{W}_2(\xi,\psi) = Y^{-1}[u^{\varrho}Y(\mathcal{P}_1[\xi]\mathbb{W}_1(\xi,\psi) + H_1(\mathbb{W}))],\\
&.\\
&.\\
&.\\
&\epsilon^k : \mathbb{W}_k(\xi,\psi) = Y^{-1}[u^{\varrho}Y(\mathcal{P}_1[\xi]\mathbb{W}_{k-1}(\xi,\psi) + H_{k-1}(\mathbb{W}))],\\
&k > 0, k \in N.
\end{aligned} \tag{15}$$

Likewise, the series is capable of estimating the analytical solution as:

$$\mathbb{W}(\xi,\psi) = \lim_{M\to\infty} \sum_{k=1}^{M} \mathbb{W}_k(\xi,\psi). \tag{16}$$

4. Construction of YTDM

To demonstrate the basic process of YTDM, we take a general nonlinear fractional partial differential equation as follows:

$$D_\psi^{\varrho}\mathbb{W}(\xi,\psi) = \mathcal{P}_1(\xi,\psi) + \mathcal{Q}_1(\xi,\psi), 0 < \varrho \le 1, \tag{17}$$

concerning initial values

$$\mathbb{W}(\xi,0) = \chi(\xi).$$

Here. $D_{\psi}^{\varrho} = \frac{\partial^{\varrho}}{\partial \psi^{\varrho}}$ demonstrate the Caputo operator, and $\mathcal{P}_1$ and $\mathcal{Q}_1$ are linear and non-linear terms.

Using the YT, we have:

$$\begin{aligned} &Y[D_{\psi}^{\varrho}\mathbb{W}(\xi,\psi)] = Y[\mathcal{P}_1(\xi,\psi) + \mathcal{Q}_1(\xi,\psi)], \\ &\frac{1}{u^{\varrho}}\{M(u) - u\mathbb{W}(0)\} = Y[\mathcal{P}_1(\xi,\psi) + \mathcal{Q}_1(\xi,\psi)]. \end{aligned} \tag{18}$$

Then, we have:

$$M(u) = u\mathbb{W}(0) + u^{\varrho}Y[\mathcal{P}_1(\xi,\psi) + \mathcal{Q}_1(\xi,\psi)]. \tag{19}$$

Let us use the inverse YT on both sides:

$$\mathbb{W}(\xi,\psi) = \mathbb{W}(0) + Y^{-1}[u^{\varrho}Y[\mathcal{P}_1(\xi,\psi) + \mathcal{Q}_1(\xi,\psi)]. \tag{20}$$

On using the YTDM:

$$\mathbb{W}(\xi,\psi) = \sum_{m=0}^{\infty} \mathbb{W}_m(\xi,\psi). \tag{21}$$

The nonlinear components are ultimately addressed as:

$$\mathcal{Q}_1(\xi,\psi) = \sum_{m=0}^{\infty} \mathcal{A}_m. \tag{22}$$

with

$$\mathcal{A}_m = \frac{1}{m!}\left[\frac{\partial^m}{\partial \ell^m}\left\{\mathcal{Q}_1\left(\sum_{k=0}^{\infty}\ell^k\xi_k, \sum_{k=0}^{\infty}\ell^k\psi_k\right)\right\}\right]_{\ell=0}. \tag{23}$$

On substituting (24) and (26) into (23), we obtain:

$$\sum_{m=0}^{\infty}\mathbb{W}_m(\xi,\psi) = \mathbb{W}(0) + Y^{-1}u^{\varrho}\left[Y\left\{\mathcal{P}_1(\sum_{m=0}^{\infty}\xi_m, \sum_{m=0}^{\infty}\psi_m) + \sum_{m=0}^{\infty}\mathcal{A}_m\right\}\right]. \tag{24}$$

Thus, we obtain

$$\begin{aligned} \mathbb{W}_0(\xi,\psi) &= \mathbb{W}(0), \\ \mathbb{W}_1(\xi,\psi) &= Y^{-1}[u^{\varrho}Y\{\mathcal{P}_1(\xi_0,\psi_0) + \mathcal{A}_0\}], \end{aligned} \tag{25}$$

Hence, in general for $m \geq 1$, we have:

$$\mathbb{W}_{m+1}(\xi,\psi) = Y^{-1}[u^{\varrho}Y\{\mathcal{P}_1(\xi_m,\psi_m) + \mathcal{A}_m\}].$$

5. Numerical Examples

Example 1. *Let us assume the nonlinear FDE as:*

$$D_{\psi}^{\varrho}\mathbb{W}(\psi) + \mathbb{W}^2(\psi) = 2\mathbb{W}(\psi) + 1, \quad 0 < \varrho \leq 1, \tag{26}$$

concerning initial value

$$\mathbb{W}(0) = 0.$$

Using Definition (4) and the YT, we have:

$$Y\Big(D_{\psi}^{\varrho}\mathbb{W}(\psi)\Big) = Y\Big(2\mathbb{W}(\psi) - \mathbb{W}^2(\psi) + 1\Big), \tag{27}$$

Then, we have:

$$\frac{1}{u^{\varrho}}\{M(u) - u\mathbb{W}(0)\} = Y\Big(2\mathbb{W}(\psi) - \mathbb{W}^2(\psi) + 1\Big), \tag{28}$$

$$M(u) = u\mathbb{W}(0) + u^{\varrho} Y\Big(2\mathbb{W}(\psi) - \mathbb{W}^2(\psi) + 1\Big). \tag{29}$$

Let us use the inverse YT on both sides:

$$\begin{aligned} \mathbb{W}(\psi) &= \mathbb{W}(0) + Y^{-1}\Big[u^{\varrho}\Big\{Y\Big(2\mathbb{W}(\psi) - \mathbb{W}^2(\psi) + 1\Big)\Big\}\Big], \\ \mathbb{W}(\psi) &= Y^{-1}[u^{\varrho}\{Y(1)\}] + Y^{-1}\Big[u^{\varrho}\Big\{Y\Big(2\mathbb{W}(\psi) - \mathbb{W}^2(\psi)\Big)\Big\}\Big]. \end{aligned} \tag{30}$$

On using the HPM:

$$\sum_{k=0}^{\infty} \epsilon^k \mathbb{W}_k(\psi) = Y^{-1}[u^{\varrho}\{Y(1)\}] + \epsilon\left(Y^{-1}\left[u^{\varrho} Y\left[2\left(\sum_{k=0}^{\infty} \epsilon^k \mathbb{W}_k(\psi)\right) - \left(\sum_{k=0}^{\infty} \epsilon^k H_k(\mathbb{W})\right)\right]\right]\right). \tag{31}$$

The nonlinear components in term of He's polynomial are ultimately addressed as:

$$\sum_{k=0}^{\infty} \epsilon^k H_k(\mathbb{W}) = \mathbb{W}^2(\psi) \tag{32}$$

Additionally, the first few He's polynomials are stated as:

$$\begin{aligned} H_0(\mathbb{W}) &= \mathbb{W}_0^2, \\ H_1(\mathbb{W}) &= 2\mathbb{W}_0\mathbb{W}_1 \\ H_2(\mathbb{W}) &= 2\mathbb{W}_0\mathbb{W}_2 + (\mathbb{W}_1)^2 \end{aligned}$$

By computing ϵ coefficients, we have:

$$\begin{aligned} &\epsilon^0 : \mathbb{W}_0(\psi) = \frac{\psi^{\varrho}}{\Gamma(\psi+1)}, \\ &\epsilon^1 : \mathbb{W}_1(\psi) = Y^{-1}\left(u^{\varrho} Y\left[2(\mathbb{W}_0) - H_0(\mathbb{W})\right]\right) = \frac{2\psi^{2\varrho}}{\Gamma(2\psi+1)} - \frac{\Gamma(2\psi+1)\psi^{3\varrho}}{\Gamma(3\psi+1)(\Gamma(\psi+1))^2}, \\ &\epsilon^2 : \mathbb{W}_2(\psi) = Y^{-1}\left(u^{\varrho} Y\left[2(\mathbb{W}_1) - H_1(\mathbb{W})\right]\right) = \frac{4\psi^{3\varrho}}{\Gamma(3\psi+1)} - \left[\frac{2\Gamma(2\varrho+1)}{(\Gamma(\psi+1))^2} + \frac{4\Gamma(3\varrho+1)}{\Gamma(\psi+1)\Gamma(2\psi+1)}\right] \\ &\frac{\psi^{4\varrho}}{\Gamma(4\psi+1)} - \frac{2\Gamma(2\psi+1)\Gamma(4\psi+1)\psi^{5\varrho}}{(\Gamma(\psi+1))^3\Gamma(3\psi+1)\Gamma(5\psi+1)}, \\ &\vdots \end{aligned}$$

Likewise, the series is capable of estimating the analytical solution as:

$$\begin{aligned} \mathbb{W}(\psi) &= \mathbb{W}_0(\psi) + \mathbb{W}_1(\psi) + \mathbb{W}_2(\psi) + \cdots \\ \mathbb{W}(\psi) &= \frac{\psi^{\varrho}}{\Gamma(\psi+1)} + \frac{2\psi^{2\varrho}}{\Gamma(2\psi+1)} - \frac{\Gamma(2\psi+1)\psi^{3\varrho}}{\Gamma(3\psi+1)(\Gamma(\psi+1))^2} + \frac{4\psi^{3\varrho}}{\Gamma(3\psi+1)} - \left[\frac{2\Gamma(2\varrho+1)}{(\Gamma(\psi+1))^2} + \frac{4\Gamma(3\varrho+1)}{\Gamma(\psi+1)\Gamma(2\psi+1)}\right] \\ &\frac{\psi^{4\varrho}}{\Gamma(4\psi+1)} - \frac{2\Gamma(2\psi+1)\Gamma(4\psi+1)\psi^{5\varrho}}{(\Gamma(\psi+1))^3\Gamma(3\psi+1)\Gamma(5\psi+1)} + \cdots \end{aligned}$$

If we choose $\varrho = 1$, we have:

$$\mathbb{W}(\psi) = 1 + \sqrt{2}\tanh\left(\sqrt{2}\psi + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right). \tag{33}$$

Solution by means of YTDM
Using Definition (4) and the YT, we have:

$$Y\left\{\frac{\partial^{\varrho}\mathbb{W}}{\partial\psi^{\varrho}}\right\} = Y\Big(2\mathbb{W}(\psi) - \mathbb{W}^2(\psi) + 1\Big), \tag{34}$$

Then, we have:

$$\frac{1}{u^{\varrho}}\{M(u) - u\mathbb{W}(0)\} = Y\Big(2\mathbb{W}(\psi) - \mathbb{W}^2(\psi) + 1\Big), \tag{35}$$

$$M(u) = u\mathbb{W}(0) + u^{\varrho}Y\Big(2\mathbb{W}(\psi) - \mathbb{W}^2(\psi) + 1\Big). \tag{36}$$

Let us use the inverse YT on both sides:

$$\begin{aligned} \mathbb{W}(\psi) &= \mathbb{W}(0) + Y^{-1}\Big[u^{\varrho}\Big\{Y\Big(2\mathbb{W}(\psi) - \mathbb{W}^2(\psi) + 1\Big)\Big\}\Big], \\ \mathbb{W}(\psi) &= Y^{-1}[u^{\varrho}\{Y(1)\}] + Y^{-1}\Big[u^{\varrho}\Big\{Y\Big(2\mathbb{W}(\psi) - \mathbb{W}^2(\psi)\Big)\Big\}\Big]. \end{aligned} \tag{37}$$

The series form solution is determined as:

$$\mathbb{W}(\psi) = \sum_{m=0}^{\infty}\mathbb{W}_m(\psi). \tag{38}$$

The nonlinear components are ultimately addressed as $\mathbb{W}^2(\psi) = \sum_{m=0}^{\infty}\mathcal{A}_m$. *Thus, we obtain:*

$$\begin{aligned} \sum_{m=0}^{\infty}\mathbb{W}_m(\psi) &= Y^{-1}[u^{\varrho}\{Y(1)\}] + Y^{-1}\left[u^{\varrho}Y\left[2\mathbb{W}(\psi) + \sum_{m=0}^{\infty}\mathcal{A}_m\right]\right], \\ \sum_{m=0}^{\infty}\mathbb{W}_m(\psi) &= Y^{-1}[u^{\varrho}\{Y(1)\}] + Y^{-1}\left[u^{\varrho}Y\left[2\mathbb{W}(\psi) + \sum_{m=0}^{\infty}\mathcal{A}_m\right]\right]. \end{aligned} \tag{39}$$

In addition, the first few nonlinear terms are stated as:

$$\begin{aligned} \mathcal{A}_0 &= \mathbb{W}_0^2, \\ \mathcal{A}_1 &= 2\mathbb{W}_0\mathbb{W}_1, \\ \mathcal{A}_2 &= 2\mathbb{W}_0\mathbb{W}_2 + (\mathbb{W}_1)^2. \end{aligned}$$

By computing both sides, we have:

$$\mathbb{W}_0(\psi) = \frac{\psi^{\varrho}}{\Gamma(\psi+1)},$$

On $m = 0$

$$\mathbb{W}_1(\psi) = \frac{2\psi^{2\varrho}}{\Gamma(2\psi+1)} - \frac{\Gamma(2\psi+1)\psi^{3\varrho}}{\Gamma(3\psi+1)(\Gamma(\psi+1))^2},$$

On $m = 1$

$$\mathbb{W}_2(\psi) = \frac{4\psi^{3\varrho}}{\Gamma(3\psi+1)} - \left[\frac{2\Gamma(2\varrho+1)}{(\Gamma(\psi+1))^2} + \frac{4\Gamma(3\varrho+1)}{\Gamma(\psi+1)\Gamma(2\psi+1)}\right]\frac{\psi^{4\varrho}}{\Gamma(4\psi+1)} - \frac{2\Gamma(2\psi+1)\Gamma(4\psi+1)\psi^{5\varrho}}{(\Gamma(\psi+1))^3\Gamma(3\psi+1)\Gamma(5\psi+1)},$$

Hence, the other components for $(m \geq 2)$ *are easy to obtain:*

$$\mathbb{W}(\psi) = \sum_{m=0}^{\infty}\mathbb{W}_m(\psi) = \mathbb{W}_0(\psi) + \mathbb{W}_1(\psi) + \mathbb{W}_2(\psi) + \mathbb{W}_3(\psi) + \cdots$$

$$\mathbb{W}(\psi) = \frac{\psi^{\varrho}}{\Gamma(\psi+1)} + \frac{2\psi^{2\varrho}}{\Gamma(2\psi+1)} - \frac{\Gamma(2\psi+1)\psi^{3\varrho}}{\Gamma(3\psi+1)(\Gamma(\psi+1))^2} + \frac{4\psi^{3\varrho}}{\Gamma(3\psi+1)} - \left[\frac{2\Gamma(2\varrho+1)}{(\Gamma(\psi+1))^2} + \frac{4\Gamma(3\varrho+1)}{\Gamma(\psi+1)\Gamma(2\psi+1)}\right]$$
$$\frac{\psi^{4\varrho}}{\Gamma(4\psi+1)} - \frac{2\Gamma(2\psi+1)\Gamma(4\psi+1)\psi^{5\varrho}}{(\Gamma(\psi+1))^3\Gamma(3\psi+1)\Gamma(5\psi+1)} + \cdots$$

If we choose $\varrho = 1$, we have:

$$\mathbb{W}(\psi) = 1 + \sqrt{2}\tanh\left(\sqrt{2}\psi + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right) \tag{40}$$

In Figure 1, the solution of the proposed methods of various values of Example 1. In Table 1, numerical comparison of the exact and suggested techniques solution of ϱ for Example 1.

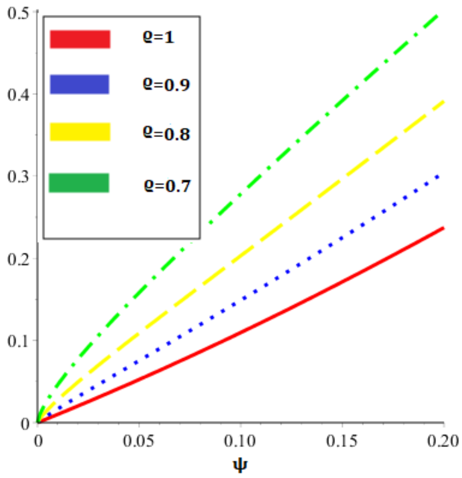

Figure 1. The solutions for various values of ϱ of Example 1.

Table 1. Numerical comparison of the exact and suggested solutions at various values of ϱ for Example 1.

ψ	$\varrho = 0.97$	$\varrho = 0.98$	$\varrho = 0.99$	$\varrho = 1\ (approx)$	$\varrho = 1\ (exact)$
0.01	0.01176337	0.01118070	0.01062659	0.01010032	0.01010032
0.02	0.02330104	0.02229010	0.021322844	0.02040261	0.02040261
0.03	0.03490317	0.03351089	0.03217427	0.03090871	0.03090871
0.04	0.04662356	0.04487682	0.04319605	0.04162043	0.04162043
0.05	0.05848489	0.05640176	0.05439387	0.05253943	0.05253943
0.06	0.07049871	0.06809248	0.06576994	0.06366731	0.06366731
0.07	0.08267136	0.07995237	0.07732490	0.07500552	0.07500552
0.08	0.09500633	0.09198297	0.08905846	0.08655544	0.08655544
0.09	0.10750540	0.10418476	0.10096983	0.09831830	0.09831830
0.10	0.12016928	0.11655748	0.11305785	0.11029519	0.11029519

Example 2. *Let us assume the diffusion FDE as:*

$$D^{\varrho}_{\psi}\mathbb{W}(\xi,\psi) = \mathbb{W}_{\xi\xi}(\xi,\psi) + \mathbb{W}(\xi,\psi), \quad 0 < \varrho \leq 1, \tag{41}$$

concerning initial value

$$\mathbb{W}(\xi,0) = \cos(\pi\xi).$$

Using Definition (4) and the YT, we have:

$$Y\Big(D_{\psi}^{\varrho}\mathbb{W}(\xi,\psi)\Big) = Y\big(\mathbb{W}_{\xi\xi}(\xi,\psi) + \mathbb{W}(\xi,\psi)\big), \tag{42}$$

Then, we have:

$$\frac{1}{u^{\varrho}}\{M(u) - u\mathbb{W}(0)\} = Y\big(\mathbb{W}_{\xi\xi}(\xi,\psi) + \mathbb{W}(\xi,\psi)\big), \tag{43}$$

$$M(u) = u\mathbb{W}(0) + u^{\varrho}Y\big(\mathbb{W}_{\xi\xi}(\xi,\psi) + \mathbb{W}(\xi,\psi)\big). \tag{44}$$

Let us use the inverse YT on both sides:

$$\begin{aligned}\mathbb{W}(\xi,\psi) &= \mathbb{W}(0) + Y^{-1}\big[u^{\varrho}\big\{Y\big(\mathbb{W}_{\xi\xi}(\xi,\psi) + \mathbb{W}(\xi,\psi)\big)\big\}\big],\\ \mathbb{W}(\xi,\psi) &= \cos(\pi\xi) + Y^{-1}\big[u^{\varrho}\big\{Y\big(\mathbb{W}_{\xi\xi}(\xi,\psi) + \mathbb{W}(\xi,\psi)\big)\big\}\big].\end{aligned} \tag{45}$$

On using the HPM:

$$\sum_{k=0}^{\infty}\epsilon^k\mathbb{W}_k(\xi,\psi) = \cos(\pi\xi) + \epsilon\left(Y^{-1}\left[u^{\varrho}Y\left[\left(\sum_{k=0}^{\infty}\epsilon^k\mathbb{W}_k(\xi,\psi)\right)_{\xi\xi} + \left(\sum_{k=0}^{\infty}\epsilon^k\mathbb{W}_k(\xi,\psi)\right)\right]\right]\right). \tag{46}$$

By computing ϵ coefficients, we have:

$$\begin{aligned}&\epsilon^0 : \mathbb{W}_0(\xi,\psi) = \cos(\pi\xi),\\ &\epsilon^1 : \mathbb{W}_1(\xi,\psi) = Y^{-1}\left(u^{\varrho}Y\left[(\mathbb{W}_0)_{\xi\xi} + \mathbb{W}_0\right]\right) = (1-\pi^2)\cos(\pi\xi)\frac{\psi^{\varrho}}{\Gamma(\varrho+1)},\\ &\epsilon^2 : \mathbb{W}_2(\xi,\psi) = Y^{-1}\left(u^{\varrho}Y\left[(\mathbb{W}_1)_{\xi\xi} + \mathbb{W}_1\right]\right) = (1-\pi^2)^2\cos(\pi\xi)\frac{\psi^{2\varrho}}{\Gamma(2\varrho+1)}\\ &\epsilon^3 : \mathbb{W}_3(\xi,\psi) = Y^{-1}\left(u^{\varrho}Y\left[(\mathbb{W}_2)_{\xi\xi} + \mathbb{W}_2\right]\right) = (1-\pi^2)^3\cos(\pi\xi)\frac{\psi^{3\varrho}}{\Gamma(3\varrho+1)}\\ &\vdots\end{aligned}$$

Likewise, the series is capable of estimating the analytical solution as:

$$\begin{aligned}\mathbb{W}(\xi,\psi) &= \mathbb{W}_0(\xi,\psi) + \mathbb{W}_1(\xi,\psi) + \mathbb{W}_2(\xi,\psi) + \mathbb{W}_3(\xi,\psi) + \cdots\\ \mathbb{W}(\xi,\psi) &= \cos(\pi\xi) + (1-\pi^2)\cos(\pi\xi)\frac{\psi^{\varrho}}{\Gamma(\varrho+1)} + (1-\pi^2)^2\cos(\pi\xi)\frac{\psi^{2\varrho}}{\Gamma(2\varrho+1)} + (1-\pi^2)^3\cos(\pi\xi)\frac{\psi^{3\varrho}}{\Gamma(3\varrho+1)} + \cdots\end{aligned}$$

If we choose $\varrho = 1$, we have:

$$\mathbb{W}(\psi) = \cos(\pi\xi)e^{(1-\pi^2)\psi} \tag{47}$$

Solution by means of YTDM

Using Definition (4) and the YT, we have:

$$Y\left\{\frac{\partial^{\varrho}\mathbb{W}}{\partial\psi^{\varrho}}\right\} = Y\big(\mathbb{W}_{\xi\xi}(\xi,\psi) + \mathbb{W}(\xi,\psi)\big), \tag{48}$$

Then, we have:

$$\frac{1}{u^{\varrho}}\{M(u) - u\mathbb{W}(0)\} = Y\big(\mathbb{W}_{\xi\xi}(\xi,\psi) + \mathbb{W}(\xi,\psi)\big), \tag{49}$$

$$M(u) = u\mathbb{W}(0) + u^{\varrho}Y\big(\mathbb{W}_{\xi\xi}(\xi,\psi) + \mathbb{W}(\xi,\psi)\big). \tag{50}$$

Let us use the inverse YT on both sides:

$$\begin{aligned}\mathbb{W}(\xi,\psi) &= \mathbb{W}(0) + \Upsilon^{-1}\big[u^{\varrho}\big\{\Upsilon\big(\mathbb{W}_{\xi\xi}(\xi,\psi) + \mathbb{W}(\xi,\psi)\big)\big\}\big],\\ \mathbb{W}(\xi,\psi) &= \cos(\pi\xi) + \Upsilon^{-1}\big[u^{\varrho}\big\{\Upsilon\big(\mathbb{W}_{\xi\xi}(\xi,\psi) + \mathbb{W}(\xi,\psi)\big)\big\}\big].\end{aligned} \tag{51}$$

The series form solution is determined as:

$$\begin{aligned}\mathbb{W}(\xi,\psi) &= \sum_{m=0}^{\infty}\mathbb{W}_m(\xi,\psi),\\ \sum_{m=0}^{\infty}\mathbb{W}_m(\xi,\psi) &= \cos(\pi\xi) + \Upsilon^{-1}\big[u^{\varrho}\big\{\Upsilon\big(\mathbb{W}_{\xi\xi}(\xi,\psi) + \mathbb{W}(\xi,\psi)\big)\big\}\big].\end{aligned} \tag{52}$$

By computing both sides, we have:

$$\mathbb{W}_0(\xi,\psi) = \cos(\pi\xi),$$

On $m = 0$

$$\mathbb{W}_1(\xi,\psi) = (1-\pi^2)\cos(\pi\xi)\frac{\psi^{2\varrho}}{\Gamma(2\varrho+1)},$$

On $m = 1$

$$\mathbb{W}_2(\xi,\psi) = (1-\pi^2)^2\cos(\pi\xi)\frac{\psi^{2\varrho}}{\Gamma(2\varrho+1)},$$

On $m = 2$

$$\mathbb{W}_3(\xi,\psi) = (1-\pi^2)^3\cos(\pi\xi)\frac{\psi^{2\varrho}}{\Gamma(2\varrho+1)},$$

Hence, the other components for $(m \geq 3)$ *are easy to obtain:*

$$\mathbb{W}(\xi,\psi) = \sum_{m=0}^{\infty}\mathbb{W}_m(\xi,\psi) = \mathbb{W}_0(\xi,\psi) + \mathbb{W}_1(\xi,\psi) + \mathbb{W}_2(\xi,\psi) + \mathbb{W}_3(\xi,\psi) + \cdots$$

$$\begin{aligned}\mathbb{W}(\xi,\psi) &= \mathbb{W}_0(\xi,\psi) + \mathbb{W}_1(\xi,\psi) + \mathbb{W}_2(\xi,\psi) + \mathbb{W}_3(\xi,\psi) + \cdots\\ \mathbb{W}(\xi,\psi) &= \cos(\pi\xi) + (1-\pi^2)\cos(\pi\xi)\frac{\psi^{\varrho}}{\Gamma(\varrho+1)} + (1-\pi^2)^2\cos(\pi\xi)\frac{\psi^{2\varrho}}{\Gamma(2\varrho+1)} + (1-\pi^2)^3\cos(\pi\xi)\frac{\psi^{3\varrho}}{\Gamma(3\varrho+1)} + \cdots\end{aligned}$$

If we choose $\varrho = 1$*, we have:*

$$\mathbb{W}(\psi) = \cos(\pi\xi)e^{(1-\pi^2)\psi} \tag{53}$$

In Figure 2, Plots illustrating the precise and suggested approaches to the solution of Example 2*. In Figure 3, plots of the suggested approaches for Example* 2 *at* $\varrho = 0.8, 0.6$*. In Figure 4, plots of the suggested approaches results at various* ϱ *orders of Example* 2*. In Table 2, numerical comparison of the exact and suggested solutions at various values of* ϱ *for Example* 2*.*

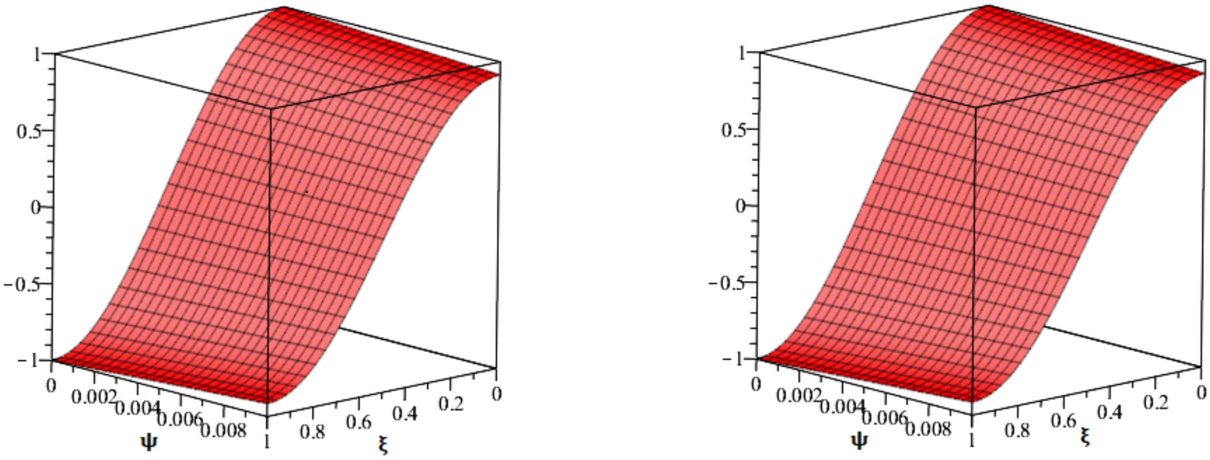

Figure 2. Plots illustrating the precise and suggested approaches to the solution of Example 2.

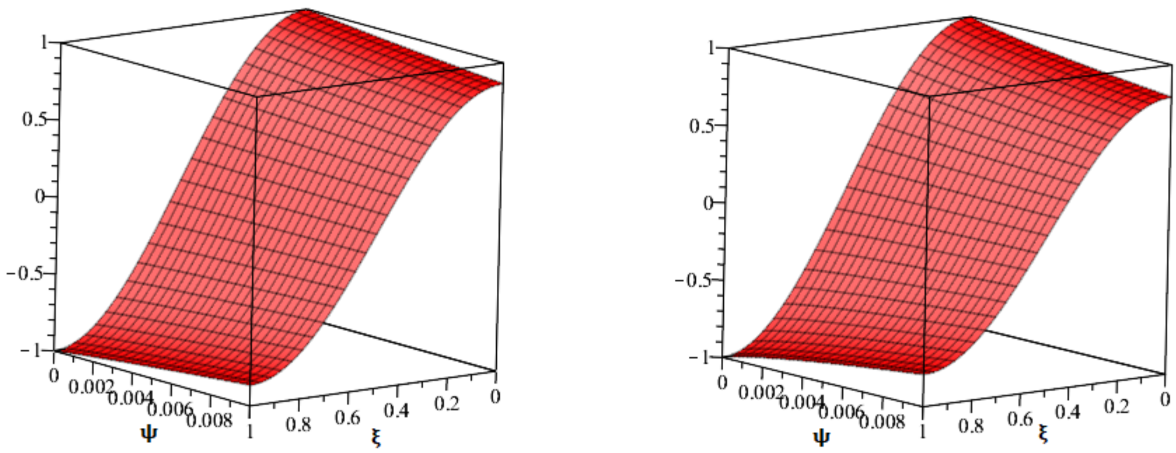

Figure 3. Plots of the suggested approaches for Example 2 at $\varrho = 0.8, 0.6$.

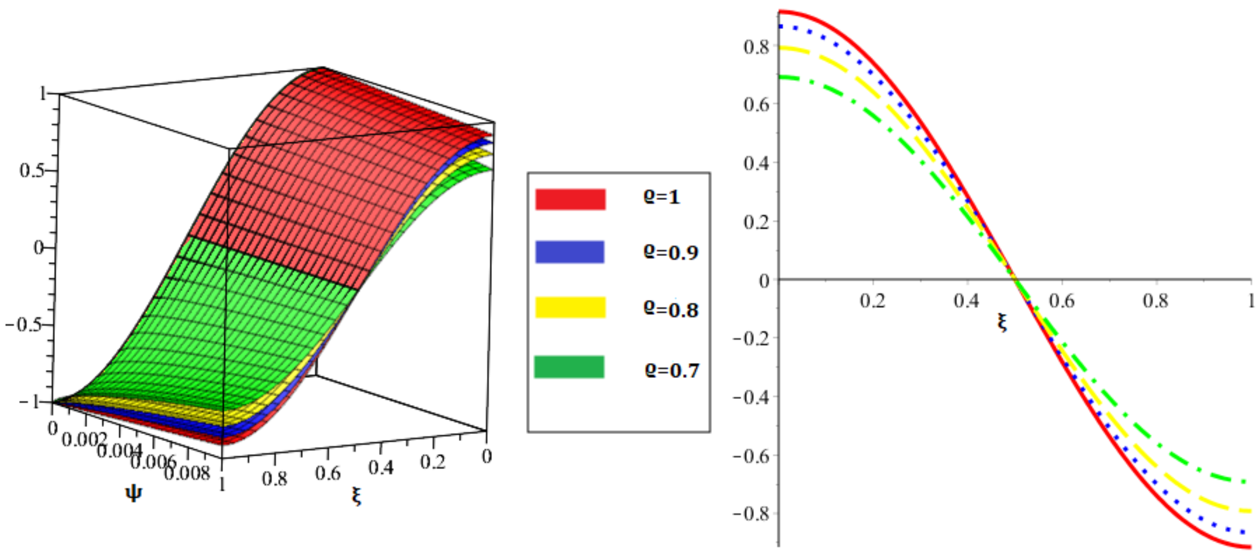

Figure 4. Plots of the suggested approaches' results at various ϱ orders of Example 2.

Table 2. Numerical comparison of the exact and suggested solutions at various values of ϱ for Example 2.

ψ	ξ	$\varrho = 0.7$	$\varrho = 0.8$	$\varrho = 0.9$	$\varrho = 1(approx)$	$\varrho = 1(exact)$
0.01	0.2	0.800752	0.801332	0.801820	0.801873	0.801873
	0.4	0.305860	0.306081	0.306268	0.306288	0.306288
	0.6	−0.305860	−0.306081	−0.306268	−0.306288	−0.306288
	0.8	−0.800752	−0.801332	−0.801820	−0.801873	−0.801873
	1	−0.989784	−0.990501	−0.991104	−0.991169	−0.991169
0.02	0.2	0.792798	0.793827	0.794698	0.794792	0.794792
	0.4	0.302822	0.303214	0.303547	0.303583	0.303583
	0.6	−0.302822	−0.303214	−0.303547	−0.303583	−0.303583
	0.8	−0.792798	−0.793827	−0.794698	−0.794792	−0.794792
	1	−0.979952	−0.981224	−0.982301	−0.982417	−0.982417
0.03	0.2	0.785006	0.786431	0.787643	0.787773	0.787773
	0.4	0.299845	0.300389	0.300852	0.300902	0.300902
	0.6	−0.299845	−0.300389	−0.300852	−0.300902	−0.300902
	0.8	−0.785006	−0.786431	−0.787643	−0.787773	−0.787773
	1	−0.970321	−0.972082	−0.973580	−0.973742	−0.973742
0.04	0.2	0.777344	0.779129	0.780652	0.780817	0.780817
	0.4	0.296919	0.297600	0.298182	0.298245	0.298245
	0.6	−0.296919	−0.297600	−0.298182	−0.298245	−0.298245
	0.8	−0.777344	−0.779129	−0.780652	−0.780817	−0.780817
	1	−0.960850	−0.963056	−0.964940	−0.965143	−0.965143
0.05	0.2	0.769797	0.771914	0.773726	0.773922	0.773922
	0.4	0.294036	0.294844	0.295537	0.295612	0.295612
	0.6	−0.294036	−0.294844	−0.295537	−0.295612	−0.295612
	0.8	−0.769797	−0.771914	−0.773726	−0.773922	−0.773922
	1	−0.951521	−0.954138	−0.956378	−0.956620	−0.956620

6. Conclusions

In this study, novel methodologies known as the homotopy perturbation transform method (HPTM) and Yang transform decomposition method (YTDM) were introduced for various types of nonlinear PDEs as well as fractional PDEs and were studied in detail, including figures and tabulated numerical data. The simplest technique to solve FPDEs is by combining the Caputo fractional derivative with the Yang transform, the HPTM, and the YTDM to appropriately handle both time and space derivatives. This also makes analyzing the fractional component much easier. Many linear and nonlinear FDEs have been implemented using the HPTM and YTDM, and employing the new mechanism has not presented any problems for us. To solve applications of the HPTM and YTDM, testing realistic series that converge quickly and applying it to additional fractional differential equation applications are some of our future goals. The suggested methods can be used to resolve any linear or nonlinear physical problem that occurs in applied sciences and engineering.

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