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Editorial

Symmetry in Chaotic Systems and Circuits

Christos Volos

Laboratory of Nonlinear Systems, Circuits & Complexity (LaNSCom), Department of Physics, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece; volos@physics.auth.gr

Chaos theory is currently one of the most fascinating fields in modern science, revolutionizing our understanding of organization and patterns in nature. On the other hand, symmetry is a traditional and highly developed area of mathematics, a field that seems to lie at the opposite end of the spectrum to nature. However, in the last few years, scientists have found connections between these two areas, and these connections can have profound consequences on our understanding of the complex behavior in many physical, chemical, biological, and mechanical systems.

Therefore, symmetry can play an important role in the field of nonlinear systems, especially in the field of designing nonlinear circuits that produce chaos. In more detail, from designing chaotic systems and circuits with symmetric nonlinear terms to the study of system's equilibria with symmetry in the case of self-excited attractors, or symmetric line of equilibria in the case of hidden attractors, the feature of symmetry can play a significant role in these systems.

The overall purpose of this Special Issue is to present the latest scientific advances in nonlinear chaotic systems and circuits that introduce various kinds of symmetries. Applications of chaotic systems and circuits with symmetries, or a deliberate lack of symmetry, are also presented in this Special Issue. The volume contains 14 published papers, with authors from countries all over the world (Algeria, Bulgaria, China, Greece, Iran, Iraq, Malaysia, Mexico, Russia, Taiwan, Thailand, Turkey, and Vietnam). This reflects the high impact of the proposed topic and the sophisticated organization of this Special Issue.

In the first paper of this Special Issue, entitled "A Simple Chaotic Flow with Hyperbolic Sinusoidal Function and Its Application to Voice Encryption", S. Mobayen et al. studied a new chaotic system with hyperbolic sinusoidal function. The proposed chaotic system provides a new category of chaotic flows, which offers a better perception of chaotic attractors. In more detail, in the proposed chaotic flow, according to the changes in system parameters, a self-excited attractor and two forms of hidden attractors occur. The dynamic behavior of the proposed chaotic flow is studied through eigenvalues, bifurcation diagrams, phase portraits, and a spectrum of Lyapunov exponents. Moreover, the existence of double-scroll attractors in the real word is considered via the Orcard-PSpice software through the electronic execution of new chaotic flows, and illustrative results between the numerical simulations and Orcard-PSpice outcomes are obtained. Furthermore, a random number generator (RNG) design based on the proposed system is presented, and a novel voice encryption algorithm is proposed [1].

In the next paper, entitled "A New Hyperchaotic Map for a Secure Communication Scheme with an Experimental Realization", Nadia M. G. Al-Saidi et al. present a new 2D chaotic map, namely, the 2D Infinite-Collapse-Sine Model (2D-ICSM). By using various metrics, including Lyapunov exponents and bifurcation diagrams complex dynamics, the robust hyperchaotic behavior of the 2D-ICSM is demonstrated. Furthermore, the cross-correlation coefficient, phase space diagram, and sample entropy algorithm prove that the 2D-ICSM has a high sensitivity to initial values and parameters, extreme complexity performance, and a much larger hyperchaotic range than existing maps. Finally, in order to empirically verify the efficiency and simplicity of the 2D-ICSM in practical applications, a symmetric secure communication system using the 2D-ICSM is presented [2].

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In the paper, entitled "Three-Saddle-Foci Chaotic Behavior of a Modified Jerk Circuit with Chua's Diode", Pattrawut Chansangiam investigates the chaotic behavior of a modified jerk circuit with Chua's diode. The Chua's diode considered in this study is a nonlinear resistor with a symmetric piecewise linear voltage–current characteristic. To describe the system, we apply fundamental laws of electrical circuit theory in order to formulate a mathematical model in terms of a third-order (jerk) nonlinear differential equation, or equivalently, a system of three first-order differential equations. The system's analysis shows that it has three collinear equilibrium points. Furthermore, a numerical simulation illustrates that the system's oscillations are dense, have no time period, are highly sensitive to initial conditions, and have a chaotic hidden attractor [3].

In the next paper, entitled "The Effect of a Non-Local Fractional Operator in an Asymmetrical Glucose-Insulin Regulatory System: Analysis, Synchronization and Electronic Implementation", Jesus M. Munoz-Pacheco et al. analyze the dynamics of a glucose–insulin regulatory system by applying a non-local fractional operator in order to represent the memory of the underlying system whose state variables define the population densities of insulin, glucose, and β-cells, respectively. The authors mainly focused on four parameters that are associated with different disorders (type 1 and type 2 diabetes mellitus, hypoglycemia, and hyperinsulinemia) to determine their observation ranges in relation to their fractional order. Similar to many preceding works in biosystems, the resulting analysis showed chaotic behaviors related to the fractional-order and system parameters. Subsequently, an active control scheme for forcing the chaotic regime (an illness) to follow a periodic oscillatory state (i.e., a disorder-free equilibrium) is proposed. Finally, the electronic realization of a fractional glucose–insulin regulatory model to prove the conceptual findings is also presented [4].

The paper, entitled "A Nonlinear Five-Term System: Symmetry, Chaos, and Prediction", by Vo Phu Thoai et al. presents a simple symmetrical system including only five nonlinear terms. By using various tools from nonlinear theory, such as phase portraits, bifurcation diagrams, Lyapunov exponents, and entropy, this system's rich dynamical behavior is discovered. Interestingly, multi-stability is also observed when changing system's initial conditions. Chaotic behavior of such a system is predicted by applying a machine learning approach based on a neural network [5].

In the next paper, "A Two-Parameter Modified Logistic Map and Its Application to Random Bit Generation", L. Moysis et al. propose a modified logistic map based on the system previously proposed by Han in 2019. The constructed map exhibits interesting chaos-related phenomena, such as antimonotonicity, crisis, and coexisting attractors. In addition, the Lyapunov exponent of the map can achieve higher values, so the behavior of the proposed map is generally more complex than the original. The map is then successfully applied to the problem of random bit generation using techniques such as the comparison between maps, XOR and bit reversal. The proposed algorithm passes all the NIST tests, shows good correlation characteristics, and has a high key space [6].

The paper "Symmetry Evolution in Chaotic System", by C. Li et al. presents a comprehensive exploration of symmetry and conditional symmetry from the evolution of symmetry. In this study, unlike other chaotic systems of conditional symmetry, it is derived from the symmetric diffusionless Lorenz system. The transformation from symmetry and asymmetry to conditional symmetry was examined by constant planting and dimension growth, proving that the offset boosting of some necessary variables is the key factor for reestablishing polarity balance in a dynamical system [7].

In the next paper, "A Novel Method for Performance Improvement of Chaos-Based Substitution Boxes", Fırat Artuğer and Fatih Özkaynak examine chaotic behavior in the field of information security. In this context, a novel method is proposed in order to improve the performance of chaos-based substitution box structures. Substitution box structures play a special role in block cipher algorithms since they are the only nonlinear components in substitution permutation network architectures. However, the substitution box structures used in modern block encryption algorithms contain various vulnerabilities to side-channel attacks. Recent studies have shown that chaos-based designs can offer a variety of opportunities to prevent side-channel attacks. The problem of chaos-based designs is that substitution box performance criteria are worse than designs based on mathematical transformation. Therefore, in this study, a post processing algorithm is proposed to improve the performance of chaos-based designs. The analysis results show that the proposed method can improve the performance criteria. The importance of these results is that chaos-based designs may offer opportunities for other practical applications in addition to the prevention of side-channel attacks [8].

The paper, "A Symmetric Controllable Hyperchaotic Hidden Attractor", by Xin Zhang et al. discovered, by introducing a simple feedback, a hyperchaotic hidden attractor in the newly proposed Lorenz-like chaotic system. Some variables of the equilibria-free system can be controlled in amplitude and offset by an independent knob. Furthermore, a circuit experiment based on Multisim, which was previously presented, is proven to be consistent with the theoretic analysis and numerical simulation [9].

In the next paper, "Two New Asymmetric Boolean Chaos Oscillators with No Dependence on Incommensurate Time-Delays and Their Circuit Implementation", Jesus M. Munoz-Pacheco et al. propose two new chaotic oscillators based on autonomous Boolean networks (ABN), preserving asymmetrical logic functions. This means that the ABNs require a combination of XOR–XNOR logic functions. The two ABNs do not have fixed points; therefore, they can evolve into Boolean chaos. Using the Lyapunov exponent's method, the chaotic behavior of the proposed oscillators is proved to be insensitive to incommensurate time-delay paths. As a result, they can be implemented using distinct electronic circuits. More specifically, logic-gates-, GAL-, and FPGA-based implementations verify theoretical findings. An integrated circuit using a CMOS 180 nm fabrication technology is also presented to obtain a compact chaos oscillator with a relatively high frequency. The dynamical behaviors of those implementations are analyzed using time series, time-lag embedded attractors, frequency spectra, Poincaré maps, and Lyapunov exponents [10].

G. Zhang et al., in the paper entitled "Image Encryption Algorithm Based on Tent Delay-Sine Cascade with Logistic Map", present a new chaotic map combined with delay and cascade called a tent delay-sine cascade with logistic map (TDSCL). Compared with the original one-dimensional simple map, the proposed map has increased initial value sensitivity, internal randomness and a larger chaotic parameter interval. The chaotic sequence generated by TDSCL has a pseudo-randomness and is suitable for image encryption. Based on this chaotic map, an image encryption algorithm with a symmetric structure, which can achieve confusion and diffusion at the same time, is proposed. The simulation results show that after encryption using the proposed algorithm, the entropy of the cipher is extremely close to the ideal value of eight, and the correlation coefficients between the pixels are lower than 0.01, thus the algorithm can resist statistical attacks. Moreover, the number of pixel change rate (NPCR) and the unified average changing intensity (UACI) of the proposed algorithm are very close to the ideal value, which indicates that it can efficiently resist to a chosen plain text attack [11].

In the next paper, "Symmetric Key Encryption Based on Rotation-Translation Equation", Borislav Stoyanov and Gyurhan Nedzhibov propose an improved encryption algorithm based on numerical methods and a rotation–translation equation. A new encryption–decryption algorithm is developed using the concept of symmetric key instead of public key. The goal of this study is to improve an existing encryption algorithm using a faster convergent iterative method, providing secure convergence of the corresponding numerical scheme, and improved security by a using rotation–translation formula [12].

Xinhe Zhu and Wei-Shih Du in the paper entitled "New Chaotic Systems with Two Closed Curve Equilibrium Passing the Same Point: Chaotic Behavior, Bifurcations, and Synchronization," propose a chaotic system with infinite equilibrium points laying on two closed curves passing the same point. The proposed system belongs to a class of systems with hidden attractors. The dynamical properties of the new system were investigated by means of phase portraits, equilibrium points, Poincaré sections, bifurcation diagram, Kaplan–Yorke dimension, and maximal Lyapunov exponents. The anti-synchronization of systems is obtained using the active control. This study broadens the current knowledge of systems with infinite equilibria [13].

In the last paper, entitled "Dynamic Symmetry in Dozy-Chaos Mechanics", Vladimir V. Egorov presents all kinds of dynamic symmetries in dozy–chaos (quantum–classical) mechanics by taking into account the chaotic dynamics of the joint electron–nuclear motion in the transient state of molecular "quantum" transitions. The reason for the emergence of chaotic dynamics is associated with a certain new property of electrons, consisting of the provocation of chaos (dozy chaos) in a transient state, which appears as a result of the binding of atoms by electrons into molecules and condensed matter, providing the possibility of reorganizing a very heavy nuclear subsystem as a result of transitions of light electrons. Various dynamic symmetries appearing in theory are associated with the emergence of dynamic organization in electronic–vibrational transitions, in particular with the emergence of an electron–nuclear–reorganization resonance (the so-called Egorov resonance) and its antisymmetric (chaotic) "twin" with direct and reverse transitions, as well as with different values of the electron–phonon interaction in the initial and final states of the system. All these dynamic symmetries are investigated using the simplest example of quantum–classical mechanics, namely, the example of quantum–classical mechanics of elementary electron–charge transfers in condensed media [14].

The Guest Editor hopes you will enjoy reading this Special Issue focused on cutting-edge research in the field of symmetry in chaotic systems and circuits. We expect the collected studies will motivate researchers to continue groundbreaking work in this emerging area.

Finally, the Guest Editor would like to thank the authors of all the papers submitted to this Special Issue, as well as the anonymous reviewers, some of whom helped with multiple review assignments. Additionally, the Guest Editor would like to thank the journal's editorial board for being very encouraging and accommodative regarding this Special Issue.

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