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Types of Maintenance Based on Uncertain Data Envelope Analysis

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Abstract: Nowadays, one of the main challenges of marine maintenance is how to select an optimum maintenance strategy for each component of the complex ship machinery system. The uncertainty of the parameters is one of the main difficulties encountered. For example, engineers and experts are always questioning the credibility and integrity of the data collected, and historical maintenance records may also be lost during maintenance. The above data asymmetry will also lead to parameter uncertainty, which directly affects the accuracy of the prediction results. This paper proposes a method for determining maintenance types of ship equipment based on uncertainty theory and the Data Envelopment Analysis (DEA) model. Firstly, this paper constructs an uncertain maintenance optimization model (UMOM) based on the classic Data Envelopment Analysis model. Then, we converted the UMOM into an equivalent deterministic model for easy calculation by the uncertainty theory. Finally, a case study is given to verify this model. The results will conclude that the UMOM can meet the need for a reasonable classification of maintenance types of mechanical equipment in a marine system and provide valuable information for systemic management and storage.

Keywords: preventive maintenance; Data Envelopment Analysis; uncertainty system; marine industry

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1. Introduction

Determining maintenance types of components in a complex machinery system has become an urgent issue to be solved. There are two types of maintenance: preventive and corrective [1]. Corrective maintenance is performed when a component has been 'run-to-fail' state, which implies the consequences and cost of this failure are considered acceptable and affordable, at least compared to its preventive maintenance expense. Therefore, it is always applied to low-cost and non-critical components. Meanwhile, it does not take the opportunity cost of downtime and the loss of productivity.

Preventive maintenance is the maintenance that occurs before components fail to increase the safety, quality, and availability of systems by detection, systematic inspections, and prevention of incipient failure [2]. Methodologies of preventive maintenance have been developed since the 1970s. From the beginning, the UK industry introduced the Terotechnology model to discover the balance between maintenance costs and profits [3]. Integrated Logistic Support (ILS) and Logistic Support Analysis (LSA) are applied for more complicated and restrictive systems industries, which means the lack of flexibility. This shortcoming limits their application in many industries like the maritime industry [4]. Another widely used method is Business Centered Maintenance (BCM), which treats preventive maintenance activities as a part of holism [5]. Compared with the extensive cost of BCM, Asset Management (AM) is more effective and goal-oriented [6]. Both BCM and AM are business-oriented, while Total Productive Maintenance (TPM) is preventive maintenance-oriented. Cooke [7] also pointed out that one of the deficiencies of TPM is lacking autonomous maintenance and multi-tasked groups. The safety of equipment relies heavily on reliability, and the reliability of equipment decreases over time. It must be inspected frequently and repaired regularly to restore its reliability. Generally speaking, the more frequent and intensive the preventive maintenance, the more reliable the equipment. However, for complex systems, expensive preventive maintenance and inspection costs are a big problem. Therefore, compared to traditional maintenance, Reliability Centered Maintenance (RCM) has been widely used in the maritime industry. However, RCM is usually time consuming and resource intensive. Therefore, it is difficult to implement in some complex situations. For the maritime industry, RCM lacks a flexible enough management system to deal with various emergencies at sea.

The processes of both preventive and corrective maintenance are closely related to availability, indicating the probability that the system will be operational at a given time t when it is required for use. Therefore, the relevant work section should also emphasize the availability aspects spread over all types of technical documentation. Di Mauro, M. [8] proposed the HASFC framework designed to support MANO infrastructure deployments of SFCs with optimal availability/cost trade-offs through a dedicated REST interface. Zhao, X. and Yang, JH. [9] developed an optimal Condition Based Maintenance (CBM) strategy for a single machine system during a two-phase failure, a process that includes a normal phase and a delayed time phase. Schaefer, DR. et al. [10] proposed replicated execution of workflows to ensure availability in the event of a failure. All of these availability-related tasks are an important part of explaining preventive maintenance.

Preventive maintenance does present more advantages, but another thing to consider is the balance of utility and expense of preventive maintenance. In other words, providing optimal system reliability and safety performance at the lowest cost is exactly the aim of an optimal maintenance policy [11]. Determining the maintenance type of components is critical and urgently needed.

The key is to achieve a balance of multiple considerations such as cost, availability, reliability, and safety. For example, simple components can be maintained by the crew with technical data and faulty equipment provided by the site. Therefore, preventive maintenance seems to not be a good idea for those low-cost and non-critical components. However, more complex instruments need professional technicians organized in the naval repair yard and their downtime cost is extremely high as well. Apparently, these core system components are more suitable candidates for preventive maintenance. One of the classic Multiple Criteria Decision-Making (MCDM) problems is the selection of maintenance strategies. Azadivar and Shu [12] used a method to decide on a proper maintenance strategy for each class of systems that are in a just-in-time environment. Wang et al. [13] proposed a fuzzy modification of the Analytic Hierarchy Process (AHP) method to evaluate different maintenance strategies for different parts, which translates the uncertain factors into fuzzy numbers by an evaluation tool. In the maritime industry, Lazakis et al. [14] enhanced a fuzzy multiple attributive group decision-making technique by Analytical Hierarchy Process to gain a better maintenance weighting. Emovon et al. [15] developed a hybrid MCDM method including Delphi-AHP and Delphi-AHP-PROMETHEE which can provide suitable maintenance strategies for ship machinery systems. Moreover, a larger group of experts and additional alternatives should be included for further enhancement. Jardine, A.K.S. [16] discussed maintenance resource planning for utility wood poles at an electric distribution company and presented the associated preventive maintenance model. Replacing poles above a threshold age during periodic inspections reduces the number of future failures and thus reduces the unplanned demand on maintenance resources. To determine the threshold age of poles, the failure times of poles are assumed to follow the Weibull distribution and their parameters are estimated by the maximum likelihood method. To demonstrate the need for preventive replacement, the delayed renewal process theorem was then used to calculate the expected number of failures in any given interval in the future. Finally, Jardine, A.K.S. proposed a mathematical programming model to determine the threshold age to ensure that the expected number of failures in any given interval in the future is finite.

In real life, samples are often difficult to obtain or even vacant, and there are information asymmetries in the only data that exist. When historical data is missing, for example, experts will make subjective judgments based on experience and historical data. Often there is an asymmetry of subjectivity and objectivity in such empirical data. Uncertainty and certainty are symmetric, while uncertainty can be considered absolute. Thus, uncertainty theory is introduced together with the Data Envelopment Analysis (DEA) model to construct an uncertain maintenance optimization model. DEA is a linear programming model expressed as the ratio of output to input. It attempts to maximize the efficiency of a service unit by comparing the efficiency of a particular unit with the performance of a group of similar units providing the same service. It is a method of operations research and the study of economic production boundaries and is generally used to measure the production efficiency of some decision-making departments. Charnes et al. [17] and Banker et al. [18] used DEA to evaluate the relative efficiency of decision-making units (DMUs) with multiple inputs and outputs. To handle imprecise inputs and outputs in DEA, Imprecise Data Envelopment Analysis (IDEA) method was proposed by Cooper et al. [19]. Wen and Li further improved the classic DEA models to a fuzzy framework based on credibility measures [20]. In 2007, Liu [21] proposed the uncertainty theory to deal with belief degree. We can regard the belief degree as the uncertain variable's uncertainty distribution, then apply Liu's [22] theory. In this paper, an uncertain maintenance optimization model (UMOM) is based on the DEA model. It is a systematic analysis method that specifically takes into account the uncertainty of the influencing factors to determine the optimal level of maintenance activities for each system, equipment, etc. Compared with previous studies, the main innovation and contribution of this paper is the application of the uncertain DEA model to the evaluation of maintenance categories. Uncertain DEA models can better evaluate some factors that cannot be specifically quantified, which is what we call uncertainty.

This paper is well organized. Section 2 briefly introduces some basic concepts and properties of uncertainty theory. In Section 3 we establish the evaluation system and classify the uncertain parameters, constant parameters, and qualitative parameters. In Section 4, the UMOM is developed and further reduced to a simpler equivalent deterministic UMOM model based on uncertainty theory for easier calculation. Moreover, a simplified model is given when all uncertain variables obey a zigzag distribution. A numerical example of the approach is given in Section 5. Finally, Section 6 gives the overall conclusion.

2. Preliminaries

In order to deal with the likelihood that something will happen, there are two axiomatic mathematical systems including probability theory and uncertainty theory. Uncertainty theory is a branch of mathematics founded by Liu in 2007 [21] concerned with the analysis of belief degree. Some basic axioms, definitions, and theorems will be represented in this section. Let Γ denote an empty set, and L a σ-algebra over Γ. Each element $\Lambda \in L$ is an event. Each event Λ will obtain a number $M\{\Lambda\} \in [0,1]$ to indicate the belief degree. The uncertain measure M must satisfy the following axioms suggested by Liu [22,23]:

Axiom 1. *(Normality Axiom) $M\{\Gamma\} = 1$ for the universal set Γ.*

Axiom 2. *(Duality Axiom) $M\{\Lambda\} + M\{\Lambda^c\} = 1$ for any event Λ.*

Axiom 3. *(Subadditivity Axiom) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots$, we have*

$$M\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty} M\{\Lambda_i\} \tag{1}$$

Axiom 4. *(Product Axiom) Let (Γ_k, L_k, M_k) be uncertainty spaces for $k = 1, 2, \ldots$ The product uncertain measure M is an uncertain measure satisfying*

$$M\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \min_{k\ge 0} M_k\{\Lambda_k\} \tag{2}$$

where Λ_k are arbitrarily chosen events from L_k for $k = 1, 2, \ldots$ respectively.

Definition 2.1. *(Liu [21]) Let Γ be a nonempty set, and $\mathcal{L}$ a σ-algebra over Γ, and M an uncertain measure. Then the triplet (Γ, L, M) is called an uncertainty space.*

Definition 2.2. (*Liu [21]) The uncertainty distribution Φ of an uncertain variable ξ is defined by*

$$\Phi(x) = M\{\xi \leq x\} \tag{3}$$

for any real number x.

Example 2.1. *An uncertain variable ξ is called zigzag if it has a zigzag uncertainty distribution*

$$\Phi(x) = \begin{cases} 0, & if\ x \leq a \\ \dfrac{x-a}{2(b-a)}, & if\ a < x \leq b \\ \dfrac{x+c-2b}{2(c-b)}, & if\ b < x \leq c \\ 1, & if\ c < x \end{cases} \tag{4}$$

denoted by L (a, b) where a, b are real numbers with a < b.

Definition 2.3. (*Liu [21]) Let ξ be an uncertain variable with regular uncertainty distribution $\Phi(x)$. Then the inverse function $\Phi^{-1}(\alpha)(\alpha \in (0,1))$ is called the inverse uncertainty distribution of ξ.*

Example 2.2. *The inverse uncertainty distribution of zigzag uncertain variable $\mathcal{Z}(a, b, c)$ is*

$$\Phi^{-1}(\alpha) = \begin{cases} (1-2\alpha)a + 2\alpha b, & if\ \alpha < 0.5 \\ (2-2\alpha)b + (2\alpha - 1)c, & if\ \alpha \geq 0.5 \end{cases}$$

Theorem 2.1. *(Liu [21]) Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distribution $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If the function $f(\xi_1, \xi_2, \ldots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \ldots, \xi_m$ and strictly decreasing with respect to $\xi_{1+m}, \xi_{2+m}, \ldots, \xi_n$, then the uncertain variable $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ has an inverse uncertainty distribution*

$$\Phi^{-1}(\alpha) = f\Big(\Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)\Big). \tag{5}$$

3. The Establishment of Evaluation System

Maintenance work has an indispensable role in ensuring the safety of the ship an3d completing the task. Excessive maintenance increases maintenance costs and wastes unnecessary time and effort, while lack of timely maintenance causes financial losses and can even lead to mission failure and serious safety incidents, resulting in casualties. Therefore, our goal is to establish a reasonable evaluation system to determine the maintenance type of components.

In this paper, maintenance work is divided into preventive maintenance and corrective maintenance. Preventive maintenance refers to routine, scheduled protective maintenance performed before equipment failure occurs. It is used to avoid, as far as possible, a series of major effects that a failure may bring. The main operations include regular maintenance, operator monitoring, use inspection, functional testing, regular disassembly and repair, regular scrapping, and comprehensive work. Corrective maintenance, on the other hand, refers to repairs that can be performed after equipment failure.

Generally speaking, the more important and difficult it is to repair the components in the ship's system, the more we tend to do preventive maintenance to avoid the impact on the mission accomplishment and the safety of the ship. We know that there is always uncertainty about the type of failure and the cause of failure that may occur in a component. The uncertainty of the failure leads to the uncertainty of the maintenance. We extract a number of parameters from the influencing factors as evaluation factors to determine the type of maintenance. The factors are divided into four areas: expense, time, consequence, and operation. See Table 1 for details.

Table 1. The influencing factors and parameters.

Influencing Factors	Parameters	Direct Parameters
Expense	Maintenance cost Acquisition cost Storage cost	Cost
Time	Maintenance time Time between faults Maintenance cycle	Maintenance time Maintenance cycle Time between faults
Consequence	Direct consequence Indirect consequence	Mission completion Economic loss Security
Operation	Maintenance environment Maintenance level Replacing ability Maintenance feasibility Confidentiality level	Maintenance level Spare number Standard part or not Confidentiality Maintenance experience

The above parameters such as Economic loss, Security, and Maintenance level are qualitative metrics, which usually cannot be observed as specific quantitative changes. Here we use some scalars to artificially represent their different levels of performance. These qualitative indicators are specified in Table 2.

Table 2. Evaluation criterion of qualitative parameters.

	1	3	5	7	9
Mission completion	Little task consequence	Minor task consequence	General task consequence	Serious task consequence	Particularly severe task consequence
Economic loss	Few economic loss	Minor economic loss	General economic loss	Serious economic loss	Particularly severe economic loss
Security	Little security consequence	Minor security consequence	General security consequence	Serious security consequence	Particularly severe security consequence
Maintenance level	Crew-level maintenance	Between crew R. and relay R.	Relay-level maintenance	Between relay R. and depot R.	Depot-level maintenance
Standard part or not	Standard part	Non-standard part			
Maintenance experience	Rich maintenance experience	Much maintenance experience	Some maintenance experience	Little maintenance experience	No maintenance experience
Confidentiality	Low confidentiality	Between low C. and medium C.	Medium confidentiality	Between medium C. and high C.	High confidentiality

We further divide parameters into quantitative and qualitative parameters. Usually, if sufficient historical data are available, we can use probability distributions to reflect the characteristics of the dynamic parameters. When there is not enough sample, they will be evaluated by experts and considered as uncertain variables. Then, we use uncertain variables for modeling. Their details are shown in Table 3.

Table 3. Qualitative parameters and quantitative parameters.

Sign	Qualitative Parameters	Sign	Quantitative Parameters
E	Maintenance experience	C	Cost
MI	Mission completion	M	Maintenance cycle
L	Economic loss	N	Spare number
S	Security	T	Maintenance time
R	Maintenance level	B	Time between faults
P	Standard part or not		
F	Confidentiality		

4. Discussion

Authors should discuss the results and how they can be interpreted from the perspective of previous studies and of the working hypotheses. The findings and their implications should be discussed in the broadest context possible. Future research directions may also be highlighted.

DEA divides the indicators in an indexing system into inputs and outputs. Due to the error nature of the measurements, researchers have proposed to model DEA with different theories, such as probability theory [24], chance constrained planning [25,26], fuzzy theory [27], and uncertainty theory [28]. In this paper, we want to develop an optimization model that includes uncertain variables for the classification of maintenance types. In addition, some limitations in practical situations are ignored in the study. Therefore, some assumptions are made before modeling.

(1) The equipment is divided into standard and non-standard parts.
(2) All uncertain parameters have their own uncertainty distribution functions.
(3) All items are repairable. The equipment that does preventive maintenance is more important than the parts that do restorative maintenance.
(4) All inputs and outputs are independent of each other.

In addition, assuming that there are n DMUs, and the relevant symbols and notations are introduced as follows:

DMU_k: the kth DMU, k = 1, 2, . . . , n
DMU_0: target DMU;
$\widetilde{T}_k$: the uncertain maintenance time of DMU_k;
$\widetilde{B}_k$: the uncertain time between faults of DMU_k;
$\widetilde{E}_k$: the uncertain maintenance experience of DMU_k;
$\widetilde{MI}_k$: the uncertain mission completion level of DMU_k;
$\widetilde{L}_k$: the uncertain economic loss level of DMU_k;
$\widetilde{S}_k$: the uncertain security level of DMU_k;
$\widetilde{R}_k$: the uncertain maintenance level of DMU_k;
$\widetilde{P}_k$: the uncertain standard part or not of DMU_k;
$\widetilde{F}_k$: the uncertain confidentiality level of DMU_k;
$\widetilde{C}_k$: the uncertain cost of DMU_k;
$\widetilde{M}_k$: the uncertain maintenance cycle of DMU_k;
$\widetilde{N}_k$: the uncertain spare number of DMU_k;
$\Phi_{\widetilde{T}_k}$: the uncertainty distribution of $\widetilde{T}_k$;
$\Phi_{\widetilde{B}_k}$: the uncertainty distribution of $\widetilde{B}_k$;
$\Phi_{\widetilde{E}_k}$: the uncertainty distribution of $\widetilde{E}_k$;
$\Phi_{\widetilde{MI}_k}$: the uncertainty distribution of $\widetilde{MI}_k$;
$\Phi_{\widetilde{L}_k}$: the uncertainty distribution of $\widetilde{L}_k$;

$\Phi_{\widetilde{S}_k}$: the uncertainty distribution of $\widetilde{S}_k$;
$\Phi_{\widetilde{R}_k}$: the uncertainty distribution of $\widetilde{R}_k$;
$\Phi_{\widetilde{P}_k}$: the uncertainty distribution of $\widetilde{P}_k$;
$\Phi_{\widetilde{F}_k}$: the uncertainty distribution of $\widetilde{F}_k$;
$\Phi_{\widetilde{C}_k}$: the uncertainty distribution of $\widetilde{C}_k$;
$\Phi_{\widetilde{M}_k}$: the uncertainty distribution of $\widetilde{M}_k$;
$\Phi_{\widetilde{N}_k}$: the uncertainty distribution of $\widetilde{N}_k$;
M: the uncertain measure;
α: the belief degree of between 0 and 1;
λ_k: the weight of DMU_k;
s_i^-: the slack of each i-th input;
s_j^+: the slack of each j-th output.

DEA models have multiple inputs and outputs. Through a comprehensive analysis of the input and output data of the DMU, we obtain the relative indicators of the efficiency of each DMU, and then rank all the DMU efficiency indicators to determine the relatively effective DMU. It is known that a DMU with smaller input and larger output will be more efficient than other DMUs. In this paper, different kinds of devices are considered as DMUs for evaluation. If we need to pay more attention to making it smaller, the index should be classified as an input. Conversely, if the index is larger, the demand is more important, and this index is used as an output. For example, the number of spare parts is considered an input, since maintenance must pay more attention to parts with fewer spare parts. Maintenance costs are considered an output. If the equipment is more expensive to repair, it means that preventive maintenance is more important for this equipment. Similarly, other indices can be classified according to this principle. Then the input vector and output vector are:

$$X_k = \{\widetilde{B}_k,\ \widetilde{N}_k, \widetilde{M}_k,\ \widetilde{E}_k\},\ k = 1, 2,\ \ldots,\ n.$$

$$Y_k = \{\widetilde{MI}_k, \widetilde{T}_k,\ \widetilde{S}_k, \widetilde{P}_k, \widetilde{L}_k, \widetilde{F}_k, \widetilde{C}_k, \widetilde{R}_k\},\ k = 1, 2,\ \ldots,\ n.$$

Then the UMOM is:

$$\begin{cases} \max \theta = \sum\limits_{i=1}^{4} s_i^- + \sum\limits_{j=1}^{8} s_j^+ \\ subjectto \\ M\left\{ \sum\limits_{k=1}^{n} X_{ki}\lambda_k \leq X_{0i} - s_i^- \right\} \geq \alpha, i = 1, 2, 3, 4 \\ M\left\{ \sum\limits_{k=1}^{n} Y_{kj}\lambda_k \leq Y_{0j} + s_j^+ \right\} \geq \alpha, j = 1, 2, \ldots, 8 \\ \sum\limits_{k=1}^{n} \lambda_k = 1 \\ \lambda_k \geq 0, k = 1, 2, \ldots, n \\ s_i^- \geq 0, i = 1, 2, 3, 4 \\ s_i^+ \geq 0, j = 1, 2, \ldots, 8 \end{cases} \tag{6}$$

where $X_{k1} = \widetilde{B}_k$, $X_{k2} = \widetilde{N}_k$, $X_{k3} = \widetilde{M}_k$, $X_{k4} = \widetilde{E}_k$, and $Y_{k1} = \widetilde{R}_k$, $Y_{k2} = \widetilde{S}_k$, $Y_{k3} = \widetilde{T}_k$, $Y_{k4} = \widetilde{P}_k$, $Y_{k5} = \widetilde{L}_k$, $Y_{k6} = \widetilde{MI}_k$, $Y_{k7} = \widetilde{F}_k$, $Y_{k8} = \widetilde{C}_k$.

Criterion. *The larger the value of* θ*, the more the component needs preventive maintenance.*

θ is the sum of the slack of all input and output variables of the target DMU. The smaller the input, the larger the output, and the larger value of θ, which in this paper indicates that the component should be subjected to preventive maintenance. The above uncertain maintenance optimization model is a nonlinear programming model, so it has an infinite number of optimal solutions. Therefore, when the input and output are uncertain variables, we can reduce the uncertainty model to an equivalent deterministic model according to Liu's uncertainty theory mentioned in Section 2. Then, by solving (6) to evaluate all the DMUs, we can sort the evaluation results and then get the maintenance types of the components.

Theorem 4.1. *Assume $\widetilde{T}_1, \widetilde{T}_2, \ldots, \widetilde{T}_n$ are independent uncertain variables defined on uncertain space (Γ, L, M), in which $\widetilde{T}_0$ is any variable belongs to $\widetilde{T}_1, \widetilde{T}_2, \ldots, \widetilde{T}_n$. The uncertainty distributions of $\widetilde{T}_1, \widetilde{T}_2, \ldots, \widetilde{T}_n$ are Φ_1, Φ_2, ... , Φ_n. Then*

$$M\left\{\sum_{k=1}^{n} \widetilde{T}_k\lambda_k \le \widetilde{T}_0 - s_1^-\right\} \ge \alpha \tag{7}$$

holds if and only if

$$\sum_{k=1,k\neq 0}^{n} \lambda_k\Phi_k^{-1}(\alpha) + \lambda_0\Phi_0^{-1}(1-\alpha) \le \Phi_0^{-1}(1-\alpha) - s_1^-. \tag{8}$$

Proof. We consider the equation

$$M\left\{\sum_{k=1}^{n} \widetilde{T}_k\lambda_k \le \widetilde{T}_0 - s_1^-\right\} \ge \alpha \tag{9}$$

Equation (9) can be rewrite as

$$M\left\{\sum_{k=1}^{n} \widetilde{T}_k\lambda_k - (1-\lambda_k)\widetilde{T}_0 \le -s_1^-\right\} \ge \alpha \tag{10}$$

Because $-(1-\lambda_k)\widetilde{T}_0$ is an uncertain variable which is decreasing with respect to $\widetilde{T}_0$, its inverse uncertainty distribution is

$$\gamma_{T_0}^{-1}(\alpha) = -(1-\lambda_0)\Phi_0^{-1}(1-\alpha). \quad 0<\alpha<1 \tag{11}$$

For each $1 \le k \le n$ and $k \neq 0$, $\widetilde{T}_k\lambda_k$ is an uncertain variable which is increasing with respect to $\widetilde{T}_k$, its inverse uncertainty distribution is

$$\gamma_{T_k}^{-1}(\alpha) = \lambda_k\Phi_k^{-1}(\alpha). \quad 0<\alpha<1 \tag{12}$$

It follows from the operational law that the inverse uncertainty distribution of the sum $\sum_{k=1,k\neq 0}^{n} \widetilde{T}_k\lambda_k - (1-\lambda_k)\widetilde{T}_0$ is

$$\gamma^{-1}(\alpha) = \sum_{k=1}^{n} \gamma_{T_k}^{-1}(\alpha) = \sum_{k=1,k\neq 0}^{n} \lambda_k\Phi_k^{-1}(\alpha) - (1-\lambda_0)\Phi_0^{-1}(1-\alpha), 0<\alpha<1 \tag{13}$$

Similarly, we may derive the result for the uncertain variables, getting the other equivalent constraints. The UMOM (6) could be converted to the following model:

$$\begin{cases} \max\theta = \sum\limits_{i=1}^{4} s_i^- + \sum\limits_{j=1}^{8} s_j^+ \\ subjectto \\ \sum\limits_{k=1,k\neq 0}^{n} \lambda_k\Phi_{X_{ki}}^{-1}(\alpha) + \lambda_0\Phi_{X_{0i}}^{-1}(1-\alpha) \le \Phi_{X_{0i}}^{-1}(1-\alpha) - s_i^-, i = 1,2,3,4 \\ \sum\limits_{k=1,k\neq 0}^{n} \lambda_k\Psi_{Y_{kj}}^{-1}(1-\alpha) + \lambda_0\Psi_{Y_{0j}}^{-1}(\alpha) \ge \Psi_{Y_{0j}}^{-1}(\alpha) + s_j^+, j = 1,2,\ldots,8 \\ \sum\limits_{k=1}^{n} \lambda_k = 1 \\ \lambda_k \ge 0, k = 1,2,\ldots,n \\ s_i^- \ge 0, i = 1,2,3,4 \\ s_j^+ \ge 0, j = 1,2,\ldots,8 \end{cases} \tag{14}$$

Particularly, when all the independent uncertain variables follow the zigzag distributions $\widetilde{T}_k \sim Z(a_{T_k}, b_{T_k}, c_{T_k})$, respectively, then it can be simplified as:

$$\begin{cases} \max\theta = \sum\limits_{i=1}^{4} s_i^- + \sum\limits_{j=1}^{8} s_j^+ \\ subjectto \\ \sum\limits_{k=1,k\neq 0}^{n} \lambda_k\left[(2-2\alpha)b_{X_{ki}} + (2\alpha-1)c_{X_{ki}}\right] + \lambda_0\left[(1-2\alpha)a_{X_{0i}} + 2\alpha b_{X_{0i}}\right] \\ \leq \left[(1-2\alpha)a_{X_{0i}} + 2\alpha b_{X_{0i}}\right] - s_i^-, i = 1,2,3,4 \\ \sum\limits_{k=1,k\neq 0}^{n} \lambda_k\left[(1-2\alpha)a_{Y_{kj}} + 2\alpha b_{Y_{kj}}\right] + \lambda_0\left[(2-2\alpha)b_{Y_{0j}} + (2\alpha-1)c_{Y_{0j}}\right] \\ \geq \left[(2-2\alpha)b_{Y_{0j}} + (2\alpha-1)c_{Y_{0j]}}\right] + s_j^+, j = 1,2,3,4,5,6,7,8 \\ \sum\limits_{k=1}^{n} \lambda_k = 1 \\ \lambda_k \geq 0, k = 1,2,\ldots,n \\ s_i^- \geq 0, i = 1,2,3,4 \\ s_j^+ \geq 0, j = 1,2,3,4,5,6,7,8. \end{cases} \tag{15}$$

□

5. Case Study

To illustrate the practicality of this model, the case study considers the diesel engine system of bulk carriers. The ship's diesel generator is responsible for the entire power supply of the ship. Diesel engine generators have been increasingly used on ships due to their unique advantages. In addition to being a conventional unit for providing ship's power, they are also widely used for electric propulsion. The use of full or semi-electric propulsion has become a trend for ships of several thousand tons or more. Power failure of a ship due to diesel generator failure can cause serious damage to the ship's main or auxiliary engines and the marine environment. Therefore, the maintenance of each component of diesel generators must be carried out properly. Determining the type of maintenance they are to undergo is a crucial step in the maintenance work.

This paper selects 18 parts of the diesel engine system as analysis objects and assumes that they have been in the working state of performing tasks during the time period we measured. In this paper, parameters are all uncertainty parameters. As assumed in Section 4, each uncertain variable has its uncertainty distribution function. Their distribution is shown in the table below.

In this case, we set the confidence level = 0.6. According to the rank criterion, the relative DMUs can be ranked with. The uncertain variables involved in Table 4 are extracted from the diesel engine system of bulk carriers, with 18 representative components involved in the maintenance work. The analyzed data obey the zigzag distribution. The specific data are as follows:

Table 4. The distribution of uncertain variables.

	Idle Gear	Frame	Bedplate	Crankshaft	Air Filter	Air Valve	Turbocharger	Valve Guide	Water Pump
$\widetilde{F}_k$	Z(1,3,3)	Z(1,3,5)	Z(1,3,5)	Z(1,3,5)	Z(3,3,5)	Z(1,3,5)	Z(1,3,3)	Z(1,3,5)	Z(1,3,5)
$\widetilde{E}_k$	Z(1,3,5)	Z(1,1,1)	Z(1,3,5)	Z(1,3,5)	Z(3,3,5)	Z(1,3,5)	Z(1,5,5)	Z(5,5,7)	Z(1,3,5)
$\widetilde{M}_k$(h)	Z(20,20.5,20.6)	Z(18,18.5,18.6)	Z(12,14,20)	Z(9,18.9,19)	Z(12,13.5,16)	Z(5,22,22.2)	Z(13.5,15.6,16)	Z(21.3,22.6,26.2)	Z(18.8,19.6,20.3)
$\widetilde{C}_k$($)	Z(5487,5800,6512)	Z(2000,2103,2566)	Z(5645,6023,6570)	Z(100,7766.7767)	Z(7899,8220,8660)	Z(230,7900.5,8000)	Z(7290,8006,8038)	Z(6589,6856,6956)	Z(6215,6546,6687)
$\widetilde{R}_k$	Z(1,3,3)	Z(3,5,7)	Z(3,5,7)	Z(3,5,7)	Z(1,3,5)	Z(1,3,5)	Z(3,3,5)	Z(1,1,3)	Z(1,3,3)
$\widetilde{S}_k$	Z(3,5,7)	Z(1,5,5)	Z(3,5,7)	Z(1,3,3)	Z(3,5,7)	Z(3,3,5)	Z(1,3,5)	Z(3,5,7)	Z(1,3,3)
$\widetilde{T}_k$(h)	Z(5,5.2,6)	Z(10,12,15)	Z(2,3,4)	Z(4,4.01,4.02)	Z(11.7,12.3,15)	Z(2.9,3,4)	Z(7.3,7.5,7.7)	Z(2.8,3.0,3.3)	Z(2.6,2.7,2.9)
$\widetilde{N}_k$	Z(24,26,27)	Z(100,210,260)	Z(8.8,19.2,21.1)	Z(10,13,16)	Z(10,19.2,21)	Z(50,51,52)	Z(33,35,35.5)	Z(21,23,29)	Z(47,48.5,49)
$\widetilde{P}_k$	Z(1,3,5)	Z(1,3,5)	Z(1,3,5)	Z(1,3,5)	Z(1,3,5)	Z(1,3,5)	Z(3,3,5)	Z(3,3,5)	Z(1,1,3)
$\widetilde{L}_k$	Z(1,3,5)	Z(1,3,5)	Z(1,3,5)	Z(3,3,5)	Z(1,3,5)	Z(1,3,3)	Z(1,3,5)	Z(3,5,5)	Z(3,3,5)
$\widetilde{MI}_k$	Z(1,3,5)	Z(1,3,5)	Z(3,5,7)	Z(3,3,5)	Z(1,3,5)	Z(1,3,3)	Z(1,3,5)	Z(3,5,7)	Z(1,3,5)
$\widetilde{B}_k$(h)	Z(6122,6150,6201)	Z(25645,26001,27010)	Z(15000,26000,56200)	Z(5000,5120,5300)	Z(23100,27550,34000)	Z(18880,20120,23000)	Z(8687,9856,10422)	Z(3567,3756,3945)	Z(12300,12431,12543)
	Muffler	**Fan**	**Injection pump**	**Cylinder**	**Oil filter**	**Fuel supply pump**	**Cinder cleaning hole**	**Strainer screen**	**Main bearing**
$\widetilde{F}_k$	Z(3,5,7)	Z(3,3,5)	Z(5,7,9)	Z(3,5,7)	Z(3,3,7)	Z(3,5,7)	Z(5,5,7)	Z(1,3,5)	Z(3,5,7)
$\widetilde{E}_k$	Z(3,3,5)	Z(1,3,5)	Z(3,5,5)	Z(5,7,9)	Z(1,3,3)	Z(1,5,5)	Z(1,3,5)	Z(3,5,5)	Z(1,3,5)
$\widetilde{M}_k$(h)	Z(23.5,25.2,36)	Z(50,52.2,53)	Z(34,35.2,36.7)	Z(56,58,59.5)	Z(45,47,48)	Z(20.1,24.3,30.5)	Z(32.3,33.8,35.6)	Z(5.6,6.3,7.8)	Z(22,26,33)
$\widetilde{C}_k$($)	Z(8565,8664,8765)	Z(3546,3654,3782)	Z(6578,6854,6987)	Z(5894,5987,6023)	Z(2564,2654,2756)	Z(3265,3467,3587)	Z(7235,7356,7456)	Z(5623,5746,6013)	Z(4143,4256,4354)
$\widetilde{R}_k$	Z(3,3,5)	Z(1,3,5)	Z(1,3,3)	Z(1,3,3)	Z(1,3,5)	Z(1,1,3)	Z(1,3,5)	Z(1,3,5)	Z(1,1,3)
$\widetilde{S}_k$	Z(3,5,7)	Z(3,5,7)	Z(1,3,3)	Z(1,3,5)	Z(1,3,5)	Z(1,3,5)	Z(1,3,5)	Z(1,1,3)	Z(1,1,3)
$\widetilde{T}_k$(h)	Z(8.9,9.2,10.3)	Z(2.3,3.2,4.2)	Z(3,5,6)	Z(4.5,6.7,7.6)	Z(2.3,3.2,3.6)	Z(4.6,4.8,5.2)	Z(9.5,10.3,11.5)	Z(9.6,9.8,10.6)	Z(3.6,5.3,5.6)
$\widetilde{N}_k$	Z(27,28.6,29)	Z(11.6,12.3,13.5)	Z(10.5,10.8,11)	Z(17.5,17.9,18.5)	Z(32,32.5,34)	Z(22.3,25.2,27.8)	Z(13.5,16.5,17.5)	Z(32.1,34.2,36)	Z(12.2,13.2,13.5)
$\widetilde{P}_k$	Z(1,3,3)	Z(1,3,3)	Z(1,1,3)	Z(3,3,5)	Z(3,3,5)	Z(1,3,5)	Z(1,3,5)	Z(1,1,3)	Z(1,3,5)
$\widetilde{L}_k$	Z(3,5,5)	Z(1,1,3)	Z(3,5,5)	Z(1,3,5)	Z(3,5,7)	Z(1,3,3)	Z(1,3,5)	Z(3,5,7)	Z(1,3,3)
$\widetilde{MI}_k$	Z(3,5,5)	Z(3,5,5)	Z(1,3,5)	Z(3,3,5)	Z(1,1,3)	Z(1,3,5)	Z(1,3,5)	Z(1,3,5)	Z(1,3,5)
$\widetilde{B}_k$(h)	Z(865,876,986)	Z(560,578,596)	Z(2635,2657,2753)	Z(564,586,689)	Z(1254,1354,1368)	Z(13354,13654,13785)	Z(4130,4211,4362)	Z(2134,2315,2356)	Z(1023,1102,1230)

Based on the previous analysis and the characteristics of the uncertain DEA model, we know that this is a typical multi-input linear programming problem, and MATLAB is a good choice to deal with such problems. The basic principle is to keep the input or output of DMU constant, determine the relatively effective production frontier with the help of our linear programming and statistical data, and judge the relative effectiveness by comparing the degree of deviation of the decision unit from the frontier. In this case, we do linear programming once for each spare part, i.e., 18 times linear programming is done in total. By linear programming we derive the value of θ for each spare part, according to the criteria, and a larger A means that the more important this spare part is, the more preventive maintenance is needed.

Given that the criterion is 2 in the case study, DMUs whose value of θ is greater than 2 need preventive maintenance. Therefore, Table 5 shows that Idle gear, Crankshaft, Injection pump, Cylinder, and Main bearing need preventive maintenance, while others do not. This result is also somewhat in line with people's general intuition. Take the cylinder as an example. It is the most basic component of a diesel engine, and it is also the key for a diesel engine to generate power and output power to the outside. During the operation of a diesel engine, the cylinder's condition has a decisive influence on the performance of the diesel engine. This also explains why the θ of the cylinder is so high—because preventive maintenance is very necessary to keep it in proper working condition. The failure of the air filter, on the other hand, does not significantly affect the power generation tasks of the diesel engine or cause serious damage to any structure. Considering the not-so-low cost of preventive maintenance, it is best to perform corrective maintenance on the air filter, that is, just replace it when it is too dirty.

Table 5. Results of evaluating the DMUs with $\alpha = 0.6$.

	Idle Gear	Frame	Bedplate	Crankshaft	Air Filter	Air Valve	Turbocharger	Valve Guide	Water Pump
θ	2.3798×10^4	0	0	2.8254×10^4	0	1.2820×10^4	2.0542×10^4	0	1.8239×10^4
	Muffler	**Fan**	**Injection pump**	**Cylinder**	**Oil filter**	**Fuel supply pump**	**Cinder cleaning hole**	**Strainer screen**	**Main bearing**
θ	0	0	7.0459×10^4	2.9548×10^4	0	1.3938×10^4	0	0	2.7275×10^4

6. Conclusions

The bulk of commodities is transported by ocean-going vessels, while ships involved in accidents that can be catastrophic results in production loss and damage to both the environment and personnel, which are irreversible. Therefore, the choice of ship maintenance strategy has become one of the most widely discussed topics in the marine industry today. To address the asymmetry of data and uncertainty of variables, this paper established an uncertain maintenance optimization model (UMOM) based on the classical DEA model and the uncertainty theory of Liu [21]. To simplify the calculation, UMOM is transformed into an equivalent deterministic model under zigzag distribution. The first application of the uncertain data envelope approach to the evaluation of maintenance types in the ship industry is the main innovation and contribution of this paper. In the case study, the diesel generator system of bulk carriers is used to confirm the previous uncertainty model. As previously stated, the method aims to give the criteria of maintenance type for different spares. There is still a lot of room for improvement. The sensitivity and stability of the model await further explanation. Moreover, the existing model cannot tackle conditions where both aleatory uncertainty and epistemic uncertainty exist. Despite the limitations, this study provides a viable solution for the issue of determining the type of maintenance. Due to the numerous types and sizes of the global trade ships, the studies may focus on applications for different ship types and different operational tasks in the future.

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