

Article

Research on State of Power Estimation of Echelon-Use Battery Based on Adaptive Unscented Kalman Filter

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Abstract: An echelon-use lithium-ion battery (EULB) refers to a powered lithium-ion battery used in electric vehicles when the battery capacity is attenuated to less than 80% and greater than 20%. Aiming at the degradation of the performance of the EULB and the unclear initial value of the state of energy (SOE), estimations of the state of power (SOP) of an EULB are not accurate. An SOP estimation method based on an adaptive dual unscented Kalman filter (ADUKF) is proposed. First, the second-order resistor-capacitance symmetry equivalent model (SRCSEM) of the EULB is established. Second, an unscented transformation (UT) is introduced and the battery parameters estimated by the ADUKF: (a) the SOE is estimated based on an adaptive unscented Kalman filtering (AUKF) algorithm, that uses the observation noise equation γ_k , R_k and the processes noise equation q_k , Q_k , and (b) the ohmic internal resistance (OIR) and actual capacity (AC) are estimated based on the aforementioned algorithm, which uses the observation noise equation $\gamma_{\theta,k}$, $R_{\theta,k}$ and the process noise equation $q_{\theta,k}$, $Q_{\theta,k}$. Third, the working voltage and OIR are predicted using optimal estimation, and the SOP of the EULB is estimated. MATLAB simulation results show that EULB symmetry capacity decays to 80%, 60%, 40%, and 20% of rated capacity, the proposed algorithm is adaptive regardless of whether the initial SOE value is consistent with the actual value, and the estimation error of the EULB's SOP is less than 3.28%, showing high accuracy. The results of this study can provide valuable reference for estimating EULB parameters, and help to understand the usage behavior of retired batteries.

Keywords: echelon-use lithium-ion battery (EULB); adaptive unscented Kalman filter (AUKF); state of power; state of energy; second-order resistor-capacitance symmetry equivalent model (SRCSEM)



Citation: Hou, E.; Xu, Y.; Qiao, X.; Liu, G.; Wang, Z. Research on State of Power Estimation of Echelon-Use Battery Based on Adaptive Unscented Kalman Filter. *Symmetry* **2022**, *14*, 919. <https://doi.org/10.3390/sym14050919>

Academic Editor: José Carlos R. Alcántud

Received: 28 March 2022

Accepted: 27 April 2022

Published: 30 April 2022

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1. Introduction

With an increase in the number of retired lithium-ion batteries, the EULB has attracted much attention. An EULB refers to a lithium iron phosphate powered lithium battery used in electric vehicles (EVs) when the capacity is attenuated to greater than 20% and less than 80%, which is used for power backup and energy storage of communication base stations.

In order to achieve effective utilization of lithium-ion batteries throughout their life cycle, many researches are devoted to the echelon-use of retired EVs batteries in the references. Neubauer et al. [1] started the study of battery secondary utilization in 2010. In references [2,3], Tong and Omar verified the feasibility of re-application of echelon-use batteries in photovoltaic charging stations, and completed more than 1000 charge discharge cycles at 1 C and 80% discharge depth. Jiang et al. [4], who estimated the residual capacity of echelon-use batteries by the regression method verified the feasibility of the method with an estimated error within 3%. In reference [5], Schuster et al. studied the influence of battery capacity and impedance parameter aging on state of health (SOH) estimation. Zhang et al. [6] conducted performance tests on retired EV battery modules, coordinating the test accuracy and test time. Therefore, the retired batteries were shown to still have a

high ability in discharge, indicating that an EULB energy storage system can be applied to achieve the power requirement of an electrical grid.

Although the retired battery still has a high discharge capacity, it is difficult to ensure the safety of EULB due to performance degradation and capacity attenuation. As a result, large-scale promotion and utilization of echelon-use batteries are difficult. As an important parameter of battery safety control and energy recovery, SOP (state of power) has attracted more and more attention. Common methods to estimate SOP include battery experiment and model method. In references [7,8], the United States Advanced Battery Consortium (USABC) adopted a battery experimental method that has the advantage of strong practicality. In references [9,10], the battery experimental method is a test method adopted by Fang and Yu. Owing to disadvantages of testing being complicated and equipment being required, the use of the estimation method has increased based on the equivalent circuit model. The equivalent circuit model was used to estimate SOP firstly in reference [11]. In references [12–15], the first-order equivalent circuit model was used to estimate SOP. In reference [15], the first-order model was adopted to estimate state of charge (SOC), SOH, and SOP based on an unscented Kalman filter, but observation and process errors were not considered. The second-order resistor-capacitance equivalent model (SRCSEM) was used to estimate SOP in [16–24]. In [23], the adaptive estimation algorithm was adopted, and the estimation accuracy of SOP was 4.8%, which was not high. Reference [24] estimated SOC, SOE (state of energy), SOP, and SOH based on SRCSEM, which achieved high accuracy, but lacked adaptive research. The third-order equivalent model was adopted in [25] to carry out SOP estimation. Owing to the high order of the equivalent model, a large amount of calculation is required. Lin [16,26] proposed a new polarization voltage (NPV) model based on current and time, and the overall error of estimated accuracy of SOP was found to be approximately 5%. Liu [27] estimated SOP based on a fractional model, with a maximum error of 1.34% and high accuracy, but an insufficient adaptive ability.

The traditional SOP estimation method either has poor estimation accuracy or inadequate adaptive ability, and neither of them can be satisfied at the same time. Aiming at the degradation of EULB performance, therefore, an important research direction of battery SOP estimation is to improve the accuracy and adaptive ability of the algorithm.

SOE is an important parameter of the battery management system, which is the ratio of the remaining available energy to the maximum available energy. The performance degradation of EULB reduces the maximum available energy, thus reducing the accuracy of SOE estimation. However, few people have studied this topic in SOE estimation methods at present.

Owing to the above problems of EULB performance, in the present work an adaptive dual unscented Kalman filter (ADUKF) algorithm is first applied based on the SRCSEM. That is, based on the ADUKF algorithm used to estimate the SOE, ohmic internal resistance (OIR), and actual capacity (AC), the optimal estimation is used to predict the working voltage and OIR is used to estimate the EULB's SOP. Finally, the estimation method of the EULB's SOP is established. It can not only improve the estimation accuracy of SOP, but also enhance the self-adaptability of SOP estimation. It provides safety guarantees for the promotion and utilization of the EULB.

The objective of this paper is to propose an effective SOP estimation method for EULB and analyze the influence of the degradation of the performance of EULB. There are four original contributions as follows:

- (1) Owing to the degradation of EULB, the SOE initial value is not clear, thus parameter estimation of EULB is discussed.
- (2) Aiming at unclear SOE of EULB, UT is adopted and an adaptive method used, that can be adjusted adaptively to estimate the accuracy based on the observation noise equation γ_k , R_k and the processes noise equation q_k , Q_k .
- (3) To improve the accuracy of SOP estimation, UT and ADUKF are adopted to estimate the parameters of EULB based on the observation noise equation $\gamma_{\theta,k}$, $R_{\theta,k}$ and the

process noise equation $q_{\theta,k}$, $Q_{\theta,k}$ aimed at performance attenuation of the EULB and the inaccuracy of AC and OIR.

- (4) It provides a theoretical basis for the effective utilization of the whole life cycle of a lithium-ion battery.

2. SRCSEM of EULB

Owing to the complex nonlinear system of the lithium-ion battery, in order to simulate the characteristics of the battery more accurately, it is necessary to select a higher-order battery equivalent model. The SRCSEM can not only better reflect the static and dynamic characteristics of EULB, but also can reduce the order of the model, which can reduce the calculation and makes it easy to apply to engineering [28–30].

As shown in Figure 1, the second-order resistor-capacitance symmetry model is used as the equivalent model of an EULB. $\tau_1 = R_1C_1$ is the long time constant, which is used to simulate the process of rapid change of discharge voltage in the dynamic characteristics of EULB; $\tau_2 = R_2C_2$ is the short time constant, which is used to simulate the process of slow and stable discharge voltage in the dynamic characteristics of EULB. In addition, U_{oc} is the open circuit voltage (OCV) of EULB, U_L is the working voltage of EULB, i_L is the charge/discharge current of EULB, and R_0 is the OIR of the EULB.

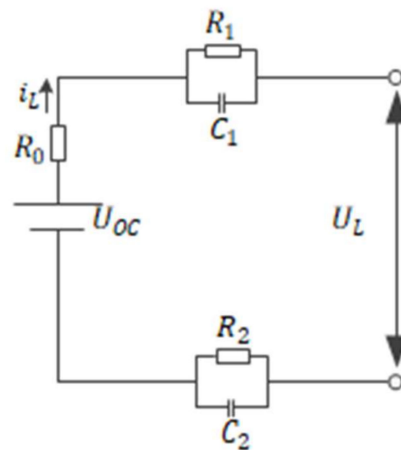


Figure 1. The SRCSEM.

According to Figure 1, the discrete state equation of the second-order resistor-capacitance symmetry equivalent circuit of the EULB is as follows:

$$\begin{bmatrix} U_{k+1}^{R_1C_1} \\ S_{ek+1} \\ U_{k+1}^{R_2C_2} \end{bmatrix} = \begin{bmatrix} \exp\left(\frac{-\Delta t}{\tau_1}\right) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \exp\left(\frac{-\Delta t}{\tau_2}\right) \end{bmatrix} \cdot \begin{bmatrix} U_k^{R_1C_1} \\ S_{ek} \\ U_k^{R_2C_2} \end{bmatrix} + \begin{bmatrix} R_1\left(1 - \exp\left(\frac{-\Delta t}{\tau_1}\right)\right) \\ \frac{-U_k\Delta t}{E} \\ R_2\left(1 - \exp\left(\frac{-\Delta t}{\tau_2}\right)\right) \end{bmatrix} \cdot i_k + \omega_k \quad (1)$$

According to Figure 1, the discrete observation equation of the second-order resistor-capacitance equivalent circuit of the EULB is as follows:

$$U_k = \left[\frac{d(U_{oc}(S_e))}{dS_e} \Big|_{S_e=S_{ek}} - 1 - 1 \right] \cdot \begin{bmatrix} S_{ek} \\ U_k^{R_1C_1} \\ U_k^{R_2C_2} \end{bmatrix} - i_k R_0 + \vartheta_k \quad (2)$$

As

$$A_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \exp\left(\frac{-\Delta t}{\tau_1}\right) & 0 \\ 0 & 0 & \exp\left(\frac{-\Delta t}{\tau_2}\right) \end{bmatrix}, B_k = \begin{bmatrix} \frac{-\Delta t}{E} \\ R_1 \left(1 - \exp\left(\frac{-\Delta t}{\tau_1}\right)\right) \\ R_2 \left(1 - \exp\left(\frac{-\Delta t}{\tau_2}\right)\right) \end{bmatrix}, x_k = \begin{bmatrix} S_{ek} \\ U_k^{R_1 C_1} \\ U_k^{R_2 C_2} \end{bmatrix},$$

$$C_k = \left[\frac{d(U_{oc}(S_e))}{dS_e} \Big|_{S_e=S_{ek}} \quad -1 \quad -1 \right], u_k = i_k.$$

Thus

$$f(x_k, u_k) = A_k x_k + B_k u_k \quad (3)$$

$$g(x_k, u_k) = C_k x_k - R_{0,k} u_k \quad (4)$$

where, S_{ek} , S_{ek+1} are the SOE of the EULB at $k, k+1$ time in discrete state; E is the energy of the EULB; i_k is the charge/discharge current at k time in discrete state; U_k is the working voltage of the EULB at k time in discrete state; Δt is the sampling period; $U_k^{R_1 C_1}$, $U_{k+1}^{R_1 C_1}$ are the estimated voltage values of R_1 at $k, k+1$ time in discrete state; $U_k^{R_2 C_2}$, $U_{k+1}^{R_2 C_2}$ are the estimated voltage values of R_2 at $k, k+1$ time in discrete state; ω_k , ϑ_k are independent system noises, ω_k is a 3-dimensional vector quantity; and $U_{oc}(S_e)$ is the OCV of the EULB corresponding to the SOE value of the EULB at the k time in discrete state.

3. Model Parameter Identification

3.1. OCV

The OCV U_{OC} :

- (1) Charging current is 36 A and cut-off voltage is 3.65 V.
- (2) Discharging current is 36 A and cut-off voltage is 2.5 V. Record the discharge voltage $U_{discharge}$.
- (3) Charging current is 36 A and cut-off voltage is 3.65 V. Record the charge voltage U_{charge} .
- (4) The OCV U_{OC} is as follows:

$$U_{OC} = \frac{U_{discharge} + U_{charge}}{2} \quad (5)$$

3.2. OIR

Based on Ohm's law, the OIR R_0 is as follows:

$$R_0 = \frac{\Delta U}{|i_L|} \quad (6)$$

where i_L is the charge/discharge current of EULB and ΔU is voltage change [31,32].

3.3. Polarization Resistance

The working voltage U_L , charge/discharge current i_L and OCV U_{OC} of the EULB is collected through the charge/discharge test. Model parameter identification based on the least square method is not repeated in this paper because the method is described in detail in references [28,32].

4. SOP Estimation

In this paper, an ADUKF algorithm is applied based on the SRCSEM: the first is to estimate the SOE based on the AUKF algorithm; the second is to estimate the OIR and AC based on the AUKF algorithm [33,34].

4.1. UT

UT is the core technology of unscented Kalman filter (UKF). The idea of UT is to approximate the distribution of a probability density function through a set of carefully selected sample points and the corresponding weights of sample points. A certain number of sigma points are selected from the prior distribution according to a certain strategy, and nonlinear transformation is performed on each sigma point to obtain the corresponding transformed sampling points. In addition, the posterior mean and variance are calculated by weighting these sample points. The sample points obtained by UT are not linearized, nor are higher-order terms lost, thus the accuracy of UT is higher than that of the extended Kalman filter algorithm.

For any nonlinear function $y = g(x)$, around the n -dimensional state variable x with mean value of $\bar{x}$ and variance of P_x , $2n + 1$ sigma points and corresponding weights can be selected through the following UT process.

$$\begin{aligned} \chi_i &= \bar{x} & i &= 0 \\ \chi_i &= \bar{x} + \left(\sqrt{(n + \lambda)P_x} \right)_i & i &= 1, \dots, n \\ \chi_i &= \bar{x} - \left(\sqrt{(n + \lambda)P_x} \right)_{i-n} & i &= n + 1, \dots, 2n \\ \omega_0^m &= \omega_0^c = \frac{\lambda}{n + \lambda} & i &= 0 \\ \omega_i^m &= \omega_i^c = \frac{\lambda}{2n + \lambda} & i &= 1, \dots, 2n \end{aligned} \quad (7)$$

In the above formula, $\left(\sqrt{(n + \lambda)P_x} \right)_i$ is the i column or row of the square root of $(n + \lambda)P_x$. λ is the scale factor used to represent the range of sampling points around equilibrium value points, and ω_i^m is the weight of corresponding sample points. In this paper, $\lambda = 0.1$, $i = 2$.

From χ_i , the sample point with nonlinear transformation can be obtained:

$$y_i = f(\chi_i) \quad (8)$$

Then, statistical characteristics of output sampling points can be obtained by weighted approximation:

$$y = \sum_i^{2n} \omega_i^m y_i \quad (9)$$

$$P_y = \sum_i^{2n} \omega_i^c (y_i - y)(y_i - y)^T \quad (10)$$

4.2. SOE Estimation Based on ADUKF

According to Equations (3) and (4), the variable of the echelon-use battery system is the SOE. In this paper, the ohmic resistance and actual capacity of the echelon-use battery are added into the state variable due to the serious problems of the increase of ohmic resistance and the decline of the actual capacity of the echelon-use battery. The system state variable has three parameters: SOE, ohm internal resistance, and the actual capacity [35,36].

The state and observation equations are as follows:

$$x_{k+1} = f(x_k, u_k, \theta_k) + q_k \quad (11)$$

$$y_{k+1} = g(x_k, u_k, \theta_k) + \gamma_k \quad (12)$$

where, θ_k is the state variable ohmic internal resistance and actual capacity, $\theta_k = [R_{0,k}, C_k]$, to reduce the amount of calculation, whenever $\sum_{k=1}^n (C_k - C_{k-1}) > 0.72$, that is, the reduction per C exceeds 0.72 Ah, the values of E, R_0 in Equations (1) and (2) are updated, $E = C_n$, $R_0 = R_{0,n}$; x_k is the system state variable; u_k is the input of the system and is the echelon-use battery current; and y_k is the system observation variable and is the working voltage of the echelon-use battery.

In this paper, the error covariance matrices of the zero-mean Gaussian white noise $q_k \wedge \gamma_k$ are $Q_k \wedge R_k$.

The algorithm flow is as follows:

- Step 1: Initialize x is as follows:

$$\hat{x}_0 = E(x_0). \quad (13)$$

$$\hat{P}_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \quad (14)$$

- Step 2: Generating sigma points:

$$\chi_i = \left[\bar{x}, \bar{x} + \left(\sqrt{(n + \lambda)P_x} \right)_i, \bar{x} - \left(\sqrt{(n + \lambda)P_x} \right)_{i-n} \right] \quad (15)$$

- Step 3: Time update of the system state x is as follows

$$\chi_{i,k} = f(\chi_{i,k-1}) \quad (16)$$

$$x_k = \sum_{i=0}^{2n} \omega_i^m \chi_{i,k} + q_k \quad (17)$$

$$P_k = \sum_{i=0}^{2n} \omega_i^c [\chi_{i,k} - x_k][\chi_{i,k} - x_k]^T + Q_k \quad (18)$$

$$y_{i,k} = g(\chi_{i,k-1}) \quad (19)$$

$$y_k = \sum_{i=0}^{2n} \omega_i^m y_{i,k} + \gamma_k \quad (20)$$

$$P_{y,k} = \sum_{i=0}^{2n} \omega_i^c [y_{i,k} - y_k][y_{i,k} - y_k]^T + R_k \quad (21)$$

$$P_{xy,k} = \sum_{i=0}^{2n} \omega_i^c [\chi_{i,k} - x_k][y_{i,k} - y_k]^T \quad (22)$$

- Step 4: Status update of the system state x is as follows: The Kalman gain is as follows:

$$K_k = P_{xy,k} P_{y,k}^{-1} \quad (23)$$

The optimal estimation of state variables is as follows:

$$\hat{x}_k = x_k + K_k[y_k - \hat{y}_k] \quad (24)$$

The optimal estimate of the covariance is as follows:

$$\hat{P}_k = P_k - K_k P_{y,k} K_k^T \quad (25)$$

- Step 5: Process noise mean and covariance equation is as follows:

$$q_k = (1 - d_k)q_{k-1} + d_k \left[\hat{x}_k - \sum_{i=0}^{2n} \omega_i^m f(\chi_{i,k-1}, u_k, \theta_k) \right] \quad (26)$$

$$Q_k = (1 - d_k)Q_{k-1} + d_k [K_k(\hat{y}_k - y_k)(\hat{y}_k - y_k)^T K_k^T + P_k - A_{k-1} \hat{P}_{k-1} A_{k-1}^T] \quad (27)$$

- Step 6: Observe the noise mean and covariance equation as follows:

$$\gamma_k = (1 - d_k)\gamma_{k-1} + d_k \left[y_k - \sum_{i=0}^{2n} \omega_i^m g(\chi_{i,k-1}) \right] \quad (28)$$

$$R_k = (1 - d_k)R_{k-1} + d_k [(\hat{y}_k - y_k)(\hat{y}_k - y_k)^T - C_k P_k C_k^T] \quad (29)$$

where $d_k = \frac{1-b}{1-b^k}$, b is the forgetting factor of x , $0 < b < 1$, $k = 1, 2, \dots, n$; x_k and $\hat{x}_k$ are the estimation and optimal estimation of the state variable at k time, respectively; y_k and $\hat{y}_k$ are the estimated value and actual observation value at k time; and P_k and $\hat{P}_k$ are, respectively, the estimation and optimal estimation of the error covariance at k time.

4.3. OIR and AC Estimation Based on AUKF

The state and observation formulas of the system with the newly added state parameters:

$$\theta_{k+1} = \theta_k + q_{\theta,k} \quad (30)$$

$$D_{k+1} = g(x_k, u_k, \theta_k) + \gamma_{\theta,k} \quad (31)$$

where $q_{\theta,k}$ is the noise on the input variable, and it is the zero-mean Gaussian white noise; $\gamma_{\theta,k}$ is the noise on the output variable, and it is the zero-mean Gaussian white noise; the error covariance matrices of $q_{\theta,k}$ and $\gamma_{\theta,k}$ are $Q_{\theta,k}$ and $R_{\theta,k}$; and the state variable θ is estimated based on the AUKF algorithm, and the estimated values of the EULB's OIR and AC are calculated. In order to improve the accuracy, the error between the actual value and the estimated value of the working voltage is optimized [37–41].

In this paper, the error covariance matrices of the zero-mean Gaussian white noise $q_{\theta,k} \wedge \gamma_{\theta,k}$ are $Q_{\theta,k} \wedge R_{\theta,k}$.

The algorithm flow is as follows:

- Step 1: Initialize θ as follows:

$$\hat{\theta}_0 = E(\theta_0) \quad (32)$$

$$P_{\theta,0} = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \quad (33)$$

- Step 2: Generating sigma points:

$$\chi_{\theta,i} = \left[\bar{x}, \bar{x} + \left(\sqrt{(n+\lambda)P_{\theta}} \right)_i, \bar{x} - \left(\sqrt{(n+\lambda)P_{\theta}} \right)_{i-n} \right] \quad (34)$$

- Step 3: Time update of the system state θ is as follows:

$$\chi_{\theta i,k} = f(\chi_{\theta i,k-1}) \quad (35)$$

$$\theta_k = \sum_{i=0}^{2n} \omega_{\theta i}^m \chi_{\theta i,k} + q_{\theta,k} \quad (36)$$

$$P_{\theta,k} = \hat{P}_{\theta,k-1} + Q_{\theta,k} \quad (37)$$

$$y_{\theta i,k} = g(\chi_{\theta i,k-1}) \quad (38)$$

$$y_{\theta,k} = \sum_{i=0}^{2n} \omega_{\theta i}^m y_{\theta i,k} + \gamma_{\theta,k} \quad (39)$$

$$P_{\theta y,k} = \sum_{i=0}^{2n} \omega_{\theta i}^c [y_{\theta i,k} - y_{\theta,k}] [y_{\theta i,k} - y_{\theta,k}]^T + R_{\theta,k} \quad (40)$$

$$P_{\theta xy,k} = \sum_{i=0}^{2n} \omega_{\theta i}^c [\chi_{\theta i,k} - \theta_k] [y_{\theta i,k} - y_{\theta,k}]^T \quad (41)$$

- Step 4: Status update of the system state θ is as follows: The Kalman gain is as follows:

$$K_{\theta,k} = P_{\theta xy,k} P_{\theta y,k}^{-1} \quad (42)$$

The optimal estimation of state variables is as follows:

$$\hat{\theta}_k = \theta_k + K_{\theta,k} [y_k - \hat{y}_k] \quad (43)$$

The optimal estimate of the covariance is as follows:

$$\hat{P}_{\theta,k} = P_{\theta,k} - K_{\theta,k} P_{\theta y,k} K_{\theta,k}^T \quad (44)$$

- Step 5: Process noise mean and covariance equation is as follows:

$$q_{\theta,k} = (1 - d_{\theta,k}) q_{\theta,k-1} + d_{\theta,k} \left[\hat{\theta}_k - \sum_{i=0}^{2n} \omega_{\theta i}^m f(\chi_{\theta i,k-1}, u_k, \theta_k) \right] \quad (45)$$

$$Q_{\theta,k} = (1 - d_{\theta,k})Q_{\theta,k-1} + d_{\theta,k}[K_{\theta,k}(\hat{y}_k - y_k)(\hat{y}_k - y_k)^T K_{\theta,k}^T + P_{\theta,k} - A_{k-1}\hat{P}_{\theta,k-1}A_{k-1}^T] \quad (46)$$

- Step 6: Observe the noise mean and covariance equation as follows:

$$\gamma_{\theta,k} = (1 - d_{\theta,k})\gamma_{\theta,k-1} + d_{\theta,k}\left[y_{\theta,k} - \sum_{i=0}^{2n} \omega_{\theta,i}^m \mathcal{G}(\chi_{\theta,i,k-1})\right] \quad (47)$$

$$R_{\theta,k} = (1 - d_{\theta,k})R_{\theta,k-1} + d_{\theta,k}[(\hat{y}_k - y_k)(\hat{y}_k - y_k)^T - C_k P_{\theta,k} C_k^T] \quad (48)$$

where $d_{\theta,k} = \frac{1-b_{\theta}}{1-b_{\theta}^k}$, b_{θ} is the forgetting factor of θ , $0 < b_{\theta} < 1$, $k = 1, 2, \dots, n$.

4.4. SOP Estimation Based on AUKF

The SOP optimal estimate forecast is updated as follows:

The optimal estimation and prediction working voltage at time $k + 1$:

$$\hat{U}_{k+1} = g(\hat{x}_k, u_k, \hat{\theta}_k) = \left[\frac{d(U_{oc}(S_e))}{dS_e} \Big|_{S_e=S_{ek}} - 1 - 1 \right] \cdot \begin{bmatrix} S_{ek} \\ U_k^{R_1 C_1} \\ U_k^{R_2 C_2} \end{bmatrix} - i_k R_{0,k} \quad (49)$$

where $g(\hat{x}_{k-1}, u_{k-1}, \hat{\theta}_{k-1})$ is the estimated value of the observed variable at k time of sampling.

The optimal estimation and prediction SOP at time $k + 1$:

$$P_{k+1} = \frac{\hat{U}_{k+1}^2}{R_{0,k+1} + R_1 + R_2} \quad (50)$$

The effect of polarization resistance is very small, so it can be ignored. That is:

$$P_{SOP} = \frac{\hat{U}_{k+1}^2}{R_{0,k+1}} \quad (51)$$

4.5. The Flow Chart of SOP Estimation Based on ADUKF

The flow chart of the SOP estimation of an echelon-use battery is shown in Figure 2.

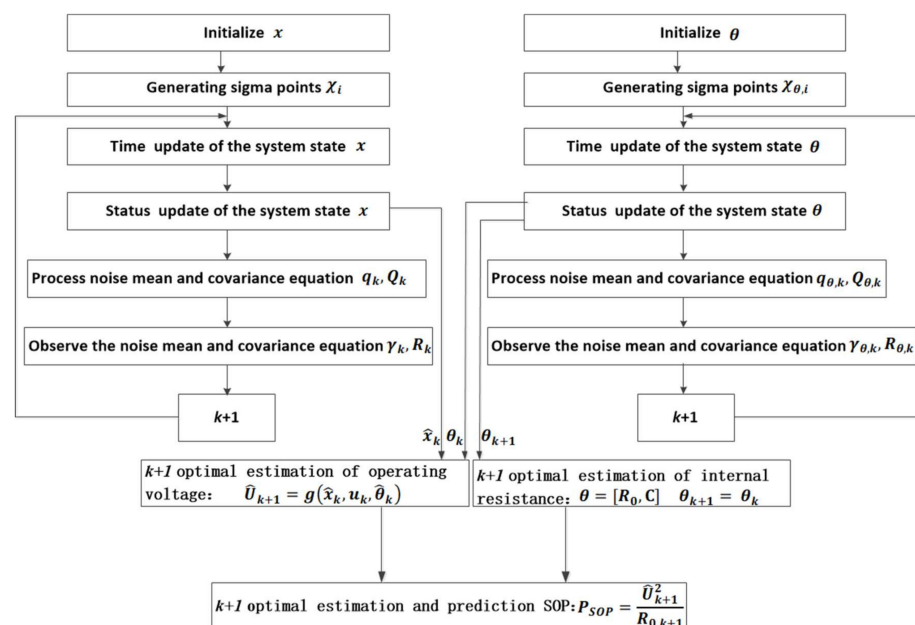


Figure 2. SOP estimation flow chart.

5. Simulation and Discussion

5.1. SOE Estimation Based on ADUKF

According to Figure 3, an EULB (as shown in Table 1) was selected to conduct the charge/discharge experiment using test equipment (BTS20, 5V/4*300A/WD, Hubei Techpow Electric Co., Ltd., Xiangfan, China) at room temperature. In order to simulate the working conditions, the fully charged EULB was discharged several times. Different discharge currents were used each time for simulation verification and analysis based on MATLAB R2021a (MathWorks, Natick, MA, USA).

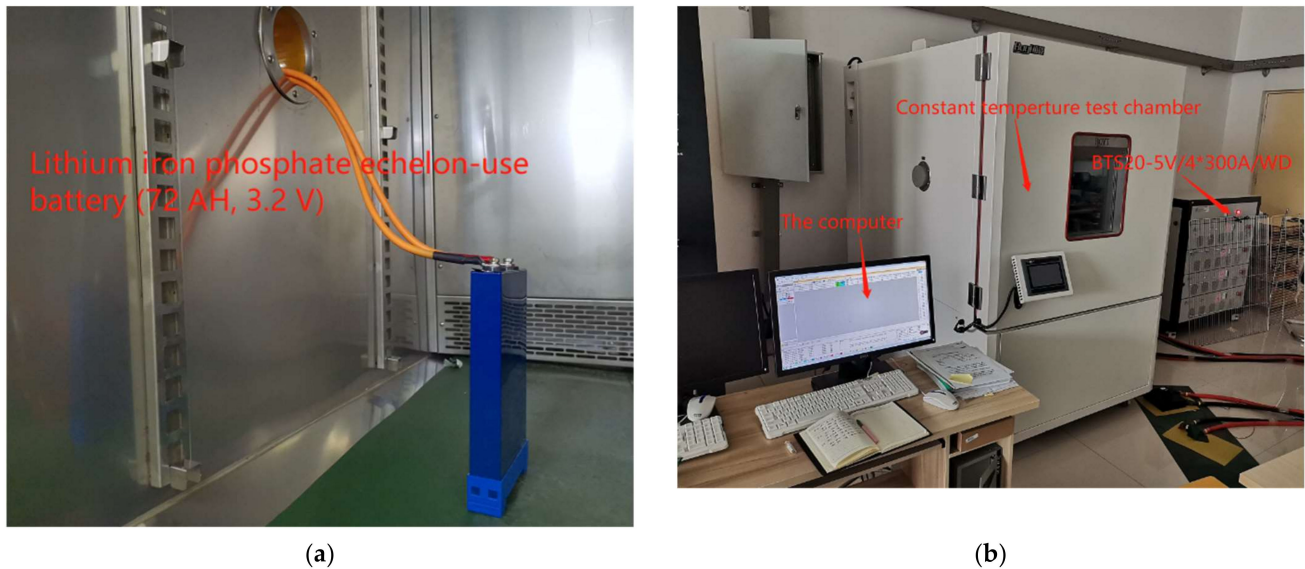


Figure 3. Battery testing system. (a) The lithium iron echelon-use battery. (b) The charge/discharge experiment.

Table 1. Parameters of the EULB.

Items	Parameter	Remarks
capacity	72 Ah	72 A
nominal voltage	3.2 V	
working voltage	2.5–3.65 V	
charging time	3 h	24 A
charging temperature	0–45 °C	
discharging temperature	−20–55 °C	

In order to verify the adaptive characteristics of the ADUKF algorithm, a test experiment was carried out on a fully charged EULB, starting from SOE of 100% and ending at SOE of 25%. In the paper, selected battery symmetry capacity decays to 80%, 60%, 40%, and 20% separately of rated capacity, and the initial SOE values were changed to 70% and 40% separately, and the adaptive and error curves were observed and analyzed.

In the process of simulation verification, the estimated values of SOE, working voltage, and SOP were calculated based on the ADUKF algorithm, and the actual values were acquired by the test equipment (BTS20).

The formula for error is as follows:

According to Equations (1), (2) and (24), $\hat{x}_k = \begin{bmatrix} \hat{S}_{ek} \\ \hat{U}_k^{R_1 C_1} \\ \hat{U}_k^{R_2 C_2} \end{bmatrix}$, $\hat{S}_{ek}$ is the optimal value of SOE.

The SOE error equation is as follows:

$$SOE\ error = \hat{S}_{ek} - S_{actual} \quad (52)$$

where, S_{actual} is the value collected by the device.

According to Equation (29), the optimal estimation of the working voltage is $\hat{U}_{k+1}$ at time $k + 1$.

The working voltage error equation is as follows:

$$Working\ voltage\ error = \hat{U}_{k+1} - U_{actual} \quad (53)$$

where U_{actual} is the value acquired by the test equipment.

According to Equation (31), the optimal estimation SOP is P_{SOP} at time $k + 1$.

The SOP error equation is as follows:

$$SOP\ error = P_{SOP} - P_{actual} \quad (54)$$

where, P_{actual} is the value acquired by the test equipment.

5.2. Capacity Decays to 80% and SOE Starts at 70%

The simulation verification curve when capacity decays to 80% and SOE starts at 70% is shown in Figure 4.

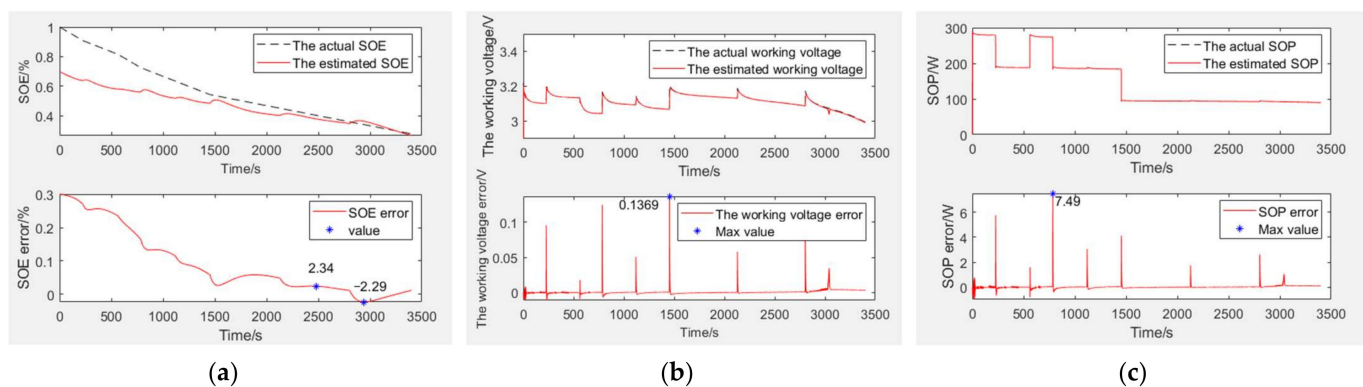


Figure 4. The simulation verification curve when capacity decays to 80% and SOE starts at 70%. Estimation and error curves of (a) SOE, (b) working voltage, and (c) SOP.

SOE estimation curves of EULB are shown in Figure 4a, the bottom graph is the SOE error curve and the top graph is the SOE adaptive estimation curve. As shown in Table 2, the SOE error is -2.29% to 2.34% , and SOE estimation is adaptive through curve comparison.

Table 2. Estimation error of the EULB's parameters.

Estimation Error		SOE	Working Voltage	SOP
capacity decays to 80%	SOE = 70%	-2.29% to 2.34%	4.28%	3.25%
	SOE = 40%	-2.31% to 2.34%	4.28%	3.26%
capacity decays to 60%	SOE = 70%	-2.23% to 2.35%	4.28%	3.26%
	SOE = 40%	-2.25% to 2.35%	4.29%	3.26%
capacity decays to 40%	SOE = 70%	-2.36% to 2.34%	4.32%	3.27%
	SOE = 40%	-2.38% to 2.34%	4.33%	3.28%
capacity decays to 20%	SOE = 70%	-2.37% to 2.34%	4.32%	3.28%
	SOE = 40%	-2.39% to 2.34%	4.33%	3.28%

The working voltage estimation curves of EULB are shown in Figure 4b, the bottom graph is the working voltage error curve and the top graph is the working voltage adaptive estimation curve. As shown in Table 2, the working voltage error is 0.1369 V (within 4.28%), and the working voltage estimation is adaptive through curve comparison.

SOP estimation curves of EULB are shown in Figure 4c, the bottom graph is the SOP error curve and the top graph is the SOP adaptive estimation curve. As shown in Table 2, the SOP error is 7.49 W (within 3.25%), and SOP estimation is adaptive through curve comparison.

5.3. Capacity Decays to 80% and SOE Starts at 40%

The simulation verification curve when capacity decays to 80% and SOE starts at 40% is shown in Figure 5.

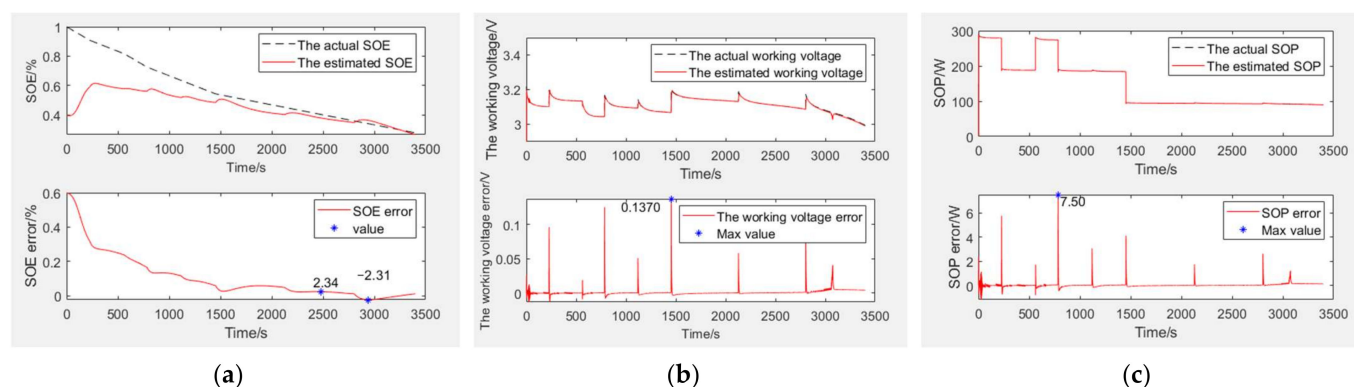


Figure 5. The simulation verification curve when capacity decays to 80% and SOE starts at 40%. Estimation and error curves of (a) SOE, (b) working voltage, and (c) SOP.

SOE estimation curves of the EULB are shown in Figure 5a, the bottom graph is the SOE error curve and the top graph is the SOE adaptive estimation curve. As shown in Table 2, the SOE error is -2.31% to 2.34% , and SOE estimation is adaptive through curve comparison.

The working voltage estimation curves of the EULB are shown in Figure 5b, the bottom graph is the working voltage error curve and the top graph is the working voltage adaptive estimation curve. As shown in Table 2, the working voltage error is 0.1370 V (within 4.28%), and the working voltage estimation is adaptive through curve comparison.

SOP estimation curves of the EULB are shown in Figure 5c, the bottom graph is the SOP error curve and the top graph is the SOP adaptive estimation curve. As shown in Table 2, the SOP error is 7.50 W (within 3.26%), and SOP estimation is adaptive through curve comparison.

5.4. Capacity Decays to 60% and SOE Starts at 70%

The simulation verification curve when capacity decays to 60% and SOE starts at 70% is shown in Figure 6.

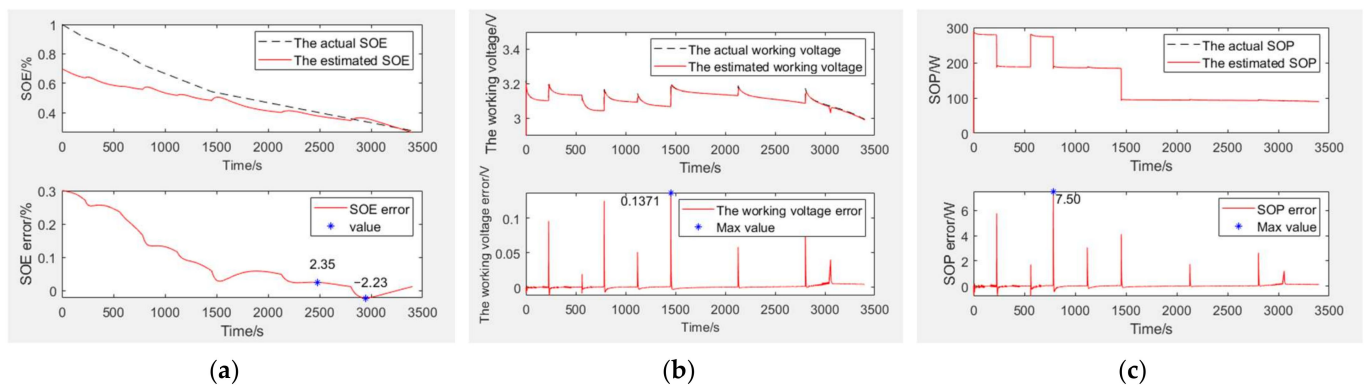


Figure 6. The simulation verification curve when capacity decays to 60% and SOE starts at 70%. Estimation and error curves of (a) SOE, (b) working voltage, and (c) SOP.

SOE estimation curves of the EULB are shown in Figure 6a, the bottom graph is the SOE error curve and the top graph is the SOE adaptive estimation curve. As shown in Table 2, the SOE error is -2.23% to 2.35% , and SOE estimation is adaptive through curve comparison.

The working voltage estimation curves of EULB are shown in Figure 6b, the bottom graph is the working voltage error curve and the top graph is the working voltage adaptive estimation curve. As shown in Table 2, the working voltage error is 0.1371 V (within 4.28%), and the working voltage estimation is adaptive through curve comparison.

SOP estimation curves of the EULB are shown in Figure 6c, the bottom graph is the SOP error curve and the top graph is the SOP adaptive estimation curve. As shown in Table 2, the SOP error is 7.50 W (within 3.26%), and SOP estimation is adaptive through curve comparison.

5.5. Capacity Decays to 60% and SOE Starts at 40%

The simulation verification curve when capacity decays to 60% and SOE starts at 40% in Figure 7.

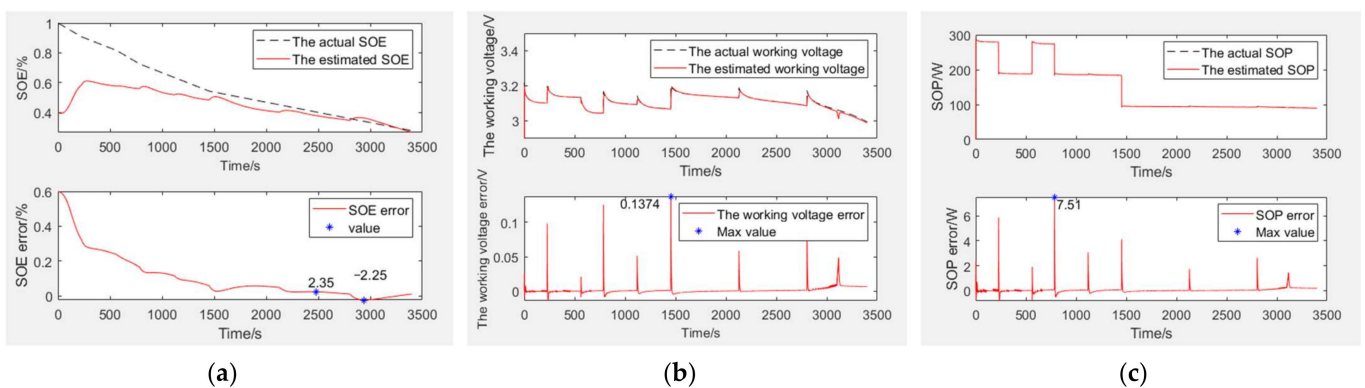


Figure 7. The simulation verification curve when capacity decays to 60% and SOE starts at 40%. Estimation and error curves of (a) SOE, (b) working voltage, and (c) SOP.

SOE estimation curves of the EULB are shown in Figure 7a, the bottom graph is the SOE error curve and the top graph is the SOE adaptive estimation curve. As shown in Table 2, the SOE error is -2.25% to 2.35% , and SOE estimation is adaptive through curve comparison.

The working voltage estimation curves of the EULB are shown in Figure 7b, the bottom graph is the working voltage error curve and the top graph is the working voltage adaptive

estimation curve. As shown in Table 2, the working voltage error is 0.1374 V (within 4.29%), and the working voltage estimation is adaptive through curve comparison.

SOP estimation curves of the EULB are shown in Figure 7c, the bottom graph is the SOP error curve and the top graph is the SOP adaptive estimation curve. As shown in Table 2, the SOP error is 7.51 W (within 3.26%), and SOP estimation is adaptive through curve comparison.

5.6. Capacity Decays to 40% and SOE Starts at 70%

The simulation verification curve when capacity decays to 40% and SOE starts at 70% is shown in Figure 8.

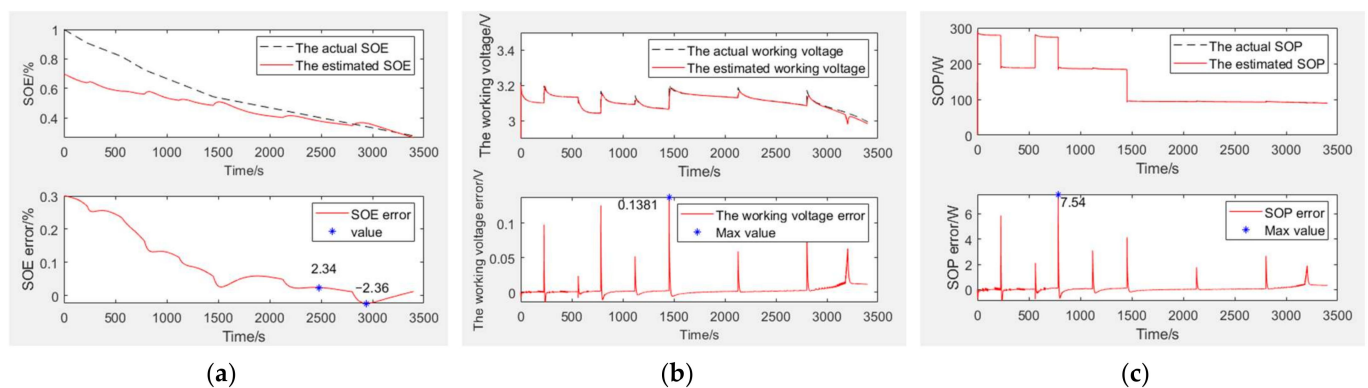


Figure 8. The simulation verification curve when capacity decays to 40% and SOE starts at 70%. Estimation and error curves of (a) SOE, (b) working voltage, and (c) SOP.

SOE estimation curves of the EULB are shown in Figure 8a, the bottom graph is the SOE error curve and the top graph is the SOE adaptive estimation curve. As shown in Table 2, the SOE error is -2.36% to 2.34% , and SOE estimation is adaptive through curve comparison.

The working voltage estimation curves of the EULB are shown in Figure 8b, the bottom graph is the working voltage error curve and the top graph is the working voltage adaptive estimation curve. As shown in Table 2, the working voltage error is 0.1381 V (within 4.32%), and the working voltage estimation is adaptive through curve comparison.

SOP estimation curves of the EULB are shown in Figure 8c, the bottom graph is the SOP error curve and the top graph is the SOP adaptive estimation curve. As shown in Table 2, the SOP error is 7.54 W (within 3.27%), and SOP estimation is adaptive through curve comparison.

5.7. Capacity Decays to 40% and SOE Starts at 40%

The simulation verification curve when capacity decays to 40% and SOE starts at 40% is shown in Figure 9.

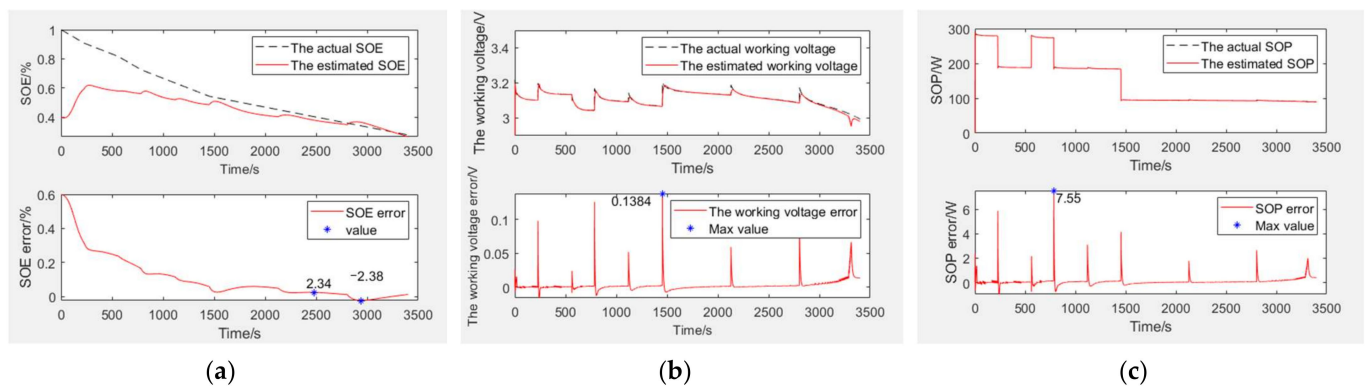


Figure 9. The simulation verification curve when capacity decays to 40% and SOE starts at 40%. Estimation and error curves of (a) SOE, (b) working voltage, and (c) SOP.

SOE estimation curves of the EULB are shown in Figure 9a, the bottom graph is the SOE error curve and the top graph is the SOE adaptive estimation curve. As shown in Table 2, the SOE error is -2.38% to 2.34% , and SOE estimation is adaptive through curve comparison.

The working voltage estimation curves of the EULB are shown in Figure 9b, the bottom graph is the working voltage error curve and the top graph is the working voltage adaptive estimation curve. As shown in Table 2, the working voltage error is 0.1384 V (within 4.33%), and the working voltage estimation is adaptive through curve comparison.

SOP estimation curves of the EULB are shown in Figure 9c, the bottom graph is the SOP error curve and the top graph is the SOP adaptive estimation curve. As shown in Table 2, the SOP error is 7.55 W (within 3.28%), and SOP estimation is adaptive through curve comparison.

5.8. Capacity Decays to 20% and SOE Starts at 70%

The simulation verification curve when capacity decays to 20% and SOE starts at 70% is shown in Figure 10.

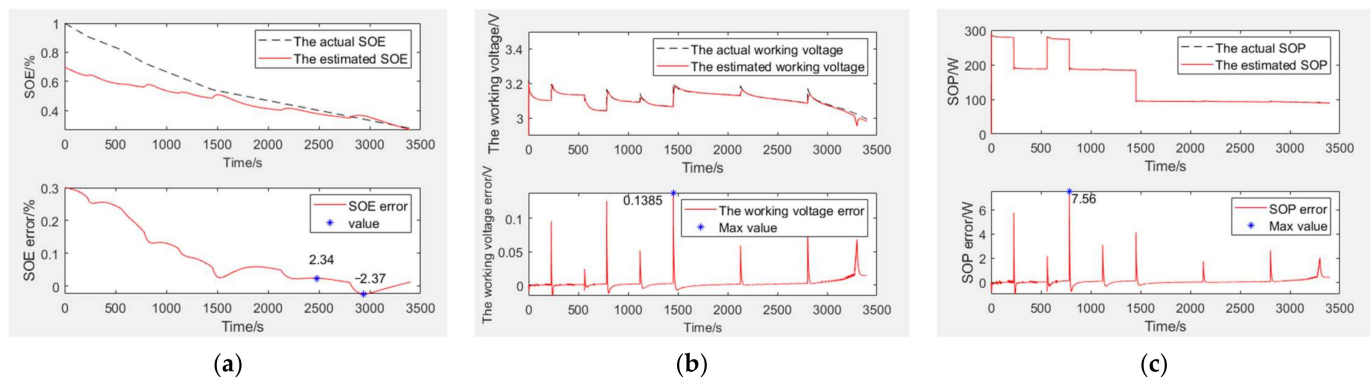


Figure 10. The simulation verification curve when capacity decays to 20% and SOE starts at 70%. Estimation and error curves of (a) SOE, (b) working voltage, and (c) SOP.

SOE estimation curves of the EULB are shown in Figure 10a, the bottom graph is the SOE error curve and the top graph is the SOE adaptive estimation curve. As shown in Table 2, the SOE error is -2.37% to 2.34% , and SOE estimation is adaptive through curve comparison.

The working voltage estimation curves of the EULB are shown in Figure 10b, the bottom graph is the working voltage error curve and the top graph is the working voltage adaptive estimation curve. As shown in Table 2, the working voltage error is 0.1385 V (within 4.33%), and the working voltage estimation is adaptive through curve comparison.

SOP estimation curves of the EULB are shown in Figure 10c, the bottom graph is the SOP error curve and the top graph is the SOP adaptive estimation curve. As shown in Table 2, the SOP error is 7.56 W (within 3.28%), and SOP estimation is adaptive through curve comparison.

5.9. Capacity Decays to 20% and SOE Starts at 40%

The simulation verification curve when capacity decays to 20% and SOE starts at 40% is shown in Figure 11.

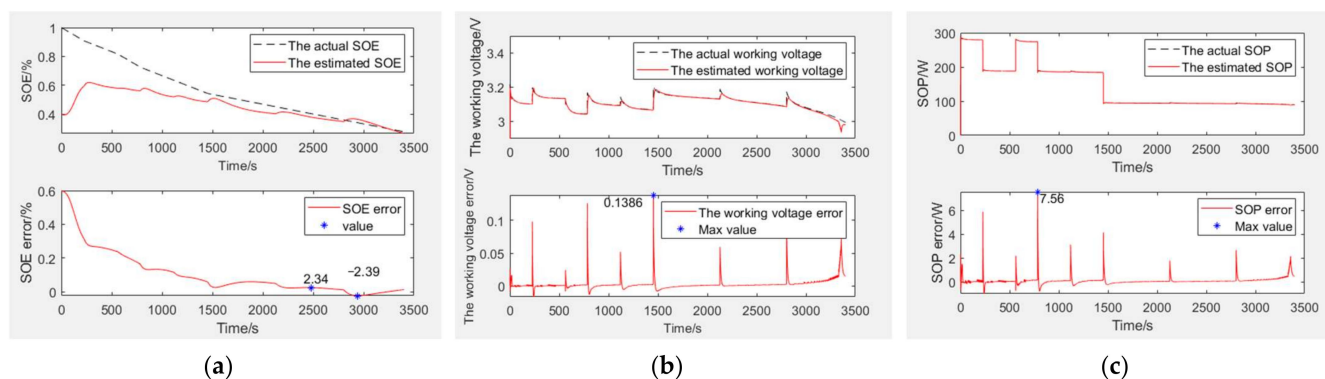


Figure 11. The simulation verification curve when capacity decays to 20% and SOE starts at 40%. Estimation and error curves of (a) SOE, (b) working voltage, and (c) SOP.

SOE estimation curves of the EULB are shown in Figure 11a, the bottom graph is the SOE error curve and the top graph is the SOE adaptive estimation curve. As shown in Table 2, the SOE error is -2.39% to 2.34% , and SOE estimation is adaptive through curve comparison.

The working voltage estimation curves of the EULB are shown in Figure 11b, the bottom graph is the working voltage error curve and the top graph is the working voltage adaptive estimation curve. As shown in Table 2, the working voltage error is 0.1386 V (within 4.33%), and the working voltage estimation is adaptive through curve comparison.

SOP estimation curves of the EULB are shown in Figure 11c, the bottom graph is the SOP error curve and the top graph is SOP adaptive estimation curve. As shown in Table 2, the SOP error is 7.56 W (within 3.28%), and SOP estimation is adaptive through curve comparison.

Simulation results show that the estimation accuracy error of the EULB's SOE is less than 2.38%, that the EULB working voltage is less than 4.33%, and that the EULB's SOP is less than 3.28%. The method is adaptive regardless of whether the initial SOE value is consistent with the actual value.

6. Conclusions

In this paper, the SRCSEM of an EULB is established first, and then the SOP estimation method of such a battery is established based on the ADUKF algorithm. Simulation shows that, at the same degradation level, the greater the initial SOE error, the greater the error of SOP estimation, the more serious the degradation level of the battery, and the greater the error of estimation of SOE, operating voltage, and SOP. Simulation results show that the estimation accuracy error of the EULB's SOP is less than 3.28%. The accuracy is higher than reference [23] and lower than reference [24]; thus, the method has high accuracy. Furthermore, regardless of whether the initial value of the SOE is clear or not, the ADUKF algorithm can be adjusted to an adaptive algorithm.

Therefore, on the basis of ensuring estimation accuracy, the adaptive ability of the SOP estimation method is enhanced.

Author Contributions: Conceptualization, Y.X. and E.H.; methodology, Y.X.; software, E.H.; validation, E.H., Y.X. and X.Q.; formal analysis, G.L.; investigation, G.L.; resources, Y.X.; data curation, E.H.; writing—original draft preparation, E.H.; writing—review and editing, Y.X.; visualization, X.Q.; supervision, Y.X.; project administration, Z.W.; funding acquisition, Y.X. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Science and Technology Major Project of Inner Mongolia Autonomous Region (2020ZD0014).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: This research was conducted within the University of Shandong Jiao Tong, supported by the charging-discharging equipment (BTS20-5V/4*300A/WD). We are thankful to the testing and simulation team members for their support with the experiment.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are most common in this manuscript:

EULB	Echelon-use lithium-ion battery
AUKF	Adaptive unscented Kalman filter
SRCSEM	Second-order resistor-capacitance symmetry equivalent model
SOC	State of charge
SOH	State of health
SOP	State of power
SOE	State of energy
OCV	Open circuit voltage
ADUKF	Adaptive dual unscented Kalman filter
UT	Unscented transformation
EVs	Electric vehicles
UKF	Unscented Kalman filter
USABC	United States Advanced Battery Consortium

References

1. Neubauer, J.; Ahmad, P. The ability of battery second use strategies to impact plug-in electric vehicle prices and serve utility energy storage applications. *J. Power Sources* **2011**, *196*, 10351–10358. [\[CrossRef\]](#)
2. Tong, S.J.; Same, A.; Kootstra, M.A.; Park, J.W. Off-grid photovoltaic vehicle charge using second life lithium batteries: An experimental and numerical investigation. *Appl. Energy* **2013**, *104*, 740–750. [\[CrossRef\]](#)
3. Omar, N.; Daowd, M. Assessment of second life of lithium iron phosphate-based batteries. *Int. Rev. Electr. Eng. (IREE)* **2012**, *7*, 3941–3948.
4. Jiang, Y.; Jiang, J.; Zhang, C.; Zhang, W.; Gao, Y.; Li, N. State of health estimation of second-life LiFePO₄ batteries for energy storage applications. *J. Clean. Prod.* **2018**, *205*, 754–762. [\[CrossRef\]](#)
5. Simon, F.S.; Martin, J.B.; Christian, C.; Markus, G.; Andreas, J. Correlation between capacity and impedance of lithium-ion cells during calendar and cycle life. *J. Power Sources* **2016**, *305*, 191–199. [\[CrossRef\]](#)
6. Zhang, Y.; Li, Y.; Tao, Y.; Ye, J.; Pan, A.; Li, X.; Liao, Q.; Wang, Z. Performance assessment of retired EV battery modules for echelon use. *Energy* **2020**, *993*, 116555. [\[CrossRef\]](#)
7. Hunt, G. *USABC Electric Vehicle Battery Test Procedures Manual. Revision 2*; Technical Reports; Office of Scientific & Technical Information: Washington, DC, USA, 1996.
8. Wik, T.; Fridholm, B.; Kuusisto, H. Implementation and Robustness of an Analytically Based Battery State of Power. *J. Power Sources* **2015**, *287*, 448–457. [\[CrossRef\]](#)
9. Li, F. Test Method for Peak Output Power of Nickel Metal Hydride Power Batteries. Master's Thesis, Central South University, Changsha, China, 2007.
10. Hu, Y. Prediction Status of Peak Power of Battery on HEV. Master's Thesis, Harbin Institute of Technology, Harbin, China, 2012.
11. Plett, G.L. High-performance Battery-pack Power Estimation using a Dynamic Cell Model. *IEEE Trans. Veh. Technol.* **2004**, *53*, 1586–1593. [\[CrossRef\]](#)
12. Wang, S.; Verbrugge, M.; Wang, J.S. Power Prediction from a Battery State Estimator that Incorporates Diffusion Resistance. *J. Power Sources* **2012**, *214*, 399–406. [\[CrossRef\]](#)

13. Wang, S.; Verbrugge, M.; Wang, J.S. Multi-parameter Battery State Estimator Based on the Adaptive and Direct Solution of the Governing Differential Equations. *J. Power Sources* **2011**, *196*, 8735–8741. [\[CrossRef\]](#)
14. Zhou, W.; Zhang, N.; Zhai, H. Enhanced Battery Power Constraint Handling in MPC-Based HEV Energy Management: A Two-Phase Dual-Model Approach. *IEEE Trans. Transp. Electrification* **2021**, *7*, 1236–1248. [\[CrossRef\]](#)
15. Zhang, T.; Guo, N.; Sun, X.; Fan, J.; Yang, N.; Song, J. A Systematic Framework for State of Charge, State of Health and State of Power Co-Estimation of Lithium-Ion Battery in Electric Vehicles. *Sustainability* **2021**, *13*, 5166. [\[CrossRef\]](#)
16. Lin, P.; Wang, Z.; Jin, P.; Hong, J. Novel Polarization Voltage Model: Accurate Voltage and State of Power Prediction. *IEEE Access* **2020**, *8*, 92039–92049. [\[CrossRef\]](#)
17. Lu, J.; Chen, Z.; Yang, Y.; Lv, M. Online Estimation of State of Power for Lithium-Ion Batteries in Electric Vehicles Using Genetic Algorithm. *IEEE Access* **2018**, *6*, 20868–20880. [\[CrossRef\]](#)
18. Nejad, S.; Gladwin, D.T.; Stone, D.A. On-chip implementation of Extended Kalman Filter for adaptive battery states monitoring. In Proceedings of the IECON 2016—42nd Annual Conference of the IEEE Industrial Electronics Society, Florence, Italy, 23–26 October 2016; pp. 5513–5518. [\[CrossRef\]](#)
19. Nejad, S.; Gladwin, D.T. Online Battery State of Power Prediction Using PRBS and Extended Kalman Filter. *IEEE Trans. Ind. Electron.* **2020**, *67*, 3747–3755. [\[CrossRef\]](#)
20. Chen, Z.; Lu, J.; Yang, Y.; Xiong, R. Online estimation of state of power for lithium-ion battery considering the battery aging. In Proceedings of the 2017 Chinese Automation Congress (CAC), Jinan, China, 20–22 October 2017; pp. 3112–3116. [\[CrossRef\]](#)
21. Shrivastava, P.; Soon, T.K.; Idris, M.Y.I.B.; Mekhilef, S. Overview of Model-Based Online State-of-Charge Estimation Using Kalman Filter Family for Lithium-Ion Batteries. *Renew. Sustain. Energy Rev.* **2019**, *113*, 109233. [\[CrossRef\]](#)
22. Cheng, W.; Yi, Z.; Liang, J.; Song, Y.; Liu, D. An SOC and SOP Joint Estimation Method of Lithium-ion Batteries in Unmanned Aerial Vehicles. In Proceedings of the 2020 International Conference on Sensing, Measurement & Data Analytics in the Era of Artificial Intelligence (ICSMD), Xi'an, China, 5–17 October 2020; pp. 247–252. [\[CrossRef\]](#)
23. Hou, E.; Xu, Y.; Qiao, X.; Liu, G.; Wang, Z. State of Power Estimation of Echelon-Use Battery Based on Adaptive Dual Extended Kalman Filter. *Energies* **2021**, *14*, 5579. [\[CrossRef\]](#)
24. Shrivastava, P.; Soon, T.K.; Idris, M.Y.I.B.; Mekhilef, S.; Adnan, S.B.R.S. Model-based state of X estimation of lithium-ion battery for electric vehicle applications. *Int. J. Energy Res.* **2022**, 1–20. [\[CrossRef\]](#)
25. Rahimifard, S.; Ahmed, R.; Habibi, S. Interacting Multiple Model Strategy for Electric Vehicle Batteries State of Charge/Health/Power Estimation. *IEEE Access* **2021**, *9*, 109875–109888. [\[CrossRef\]](#)
26. Lin, P.; Jin, P.; Hong, J.; Wang, Z. Battery voltage and state of power prediction based on an improved novel polarization voltage model. *Energy Rep.* **2020**, *6*, 2299–2308. [\[CrossRef\]](#)
27. Liu, C.; Hu, M.; Jin, G.; Xu, Y.; Zhai, J. State of power estimation of lithium-ion battery based on fractional-order equivalent circuit model. *J. Energy Storage* **2021**, *41*, 102954. [\[CrossRef\]](#)
28. Hou, E.; Qiao, X.; Liu, G. Modeling and Simulation of Power Lithium-ion Battery SOC. *Chin. Comput. Simul.* **2014**, *31*, 193–196.
29. Yi, F.; Lu, D.; Wang, X.; Pan, C.; Tao, Y.; Zhou, J.; Zhao, C. Energy Management Strategy for Hybrid Energy Storage Electric Vehicles Based on Pontryagin's Minimum Principle Considering Battery Degradation. *Sustainability* **2022**, *14*, 1214. [\[CrossRef\]](#)
30. Hou, E.; Qiao, X.; Liu, G.; Li, Y. Influence of temperature on parameters of power lithium-ion battery based on RC equivalent circuit. *Chin. J. Power Sources* **2015**, *39*, 287–289.
31. Cao, L.; Huang, J.; Cao, M.; Yang, K. SOC and internal resistance estimation of lithium battery based on DKF. *J. Nanchang Univ. Eng. Technol.* **2018**, *40*, 179–183.
32. Hou, E.; Qiao, X.; Liu, G. Estimation of power lithium-ion battery SOC based on fuzzy optimal decision. *Chin. J. Power Sources* **2017**, *41*, 920–922.
33. Shi, J.; Li, B.; Liu, M.F. Study on degenerate of lithium-ion battery based on fuzzy inference system. *Chin. J. Power Sources* **2018**, *42*, 1488–1490.
34. Gu, Y.; Zhang, Y. SOC Estimation of Lithium Battery Based on Double Kalman Filter. *Chin. J. Power Sources* **2016**, *40*, 986–989.
35. Liu, X.; He, Y.; Zeng, G.J. Power State Estimation of Lithium Battery Considering Temperature Effect. *Chin. J. Power Technol.* **2016**, *31*, 155–163.
36. She, L.Y. Research on Joint Estimation of SOC and SOP for Vehicle Power Battery. Master's Thesis, Wuhan University of Technology, Wuhan, China, 2018.
37. Peng, S.; Zhu, X.; Xing, Y.; Shi, H.; Cai, X.; Pecht, M. An adaptive state of charge estimation approach for lithium-ion series-connected battery system. *J. Power Sources* **2018**, *392*, 48–59. [\[CrossRef\]](#)
38. Zhang, S.; Guo, X.; Zhang, X. An improved adaptive unscented kalman filtering for state of charge online estimation of lithium-ion battery. *J. Energy Storage* **2020**, *32*, 101980. [\[CrossRef\]](#)
39. Li, Y.; Yuan, M.; Chadli, M.; Wang, Z.; Zhao, D. Unknown input functional observer design for discrete time interval type-2 Takagi-Sugeno fuzzy systems. *IEEE Trans. Fuzzy Syst.* **2022**. [\[CrossRef\]](#)
40. Lin, X.; Tang, Y.; Ren, J.; Wei, Y. State of charge estimation with the adaptive unscented Kalman filter based on an accurate equivalent circuit model. *J. Energy Storage* **2021**, *41*, 102840. [\[CrossRef\]](#)
41. Li, Y.; Liu, S.; Zhao, D.; Shi, X.; Cui, Y. Event-triggered fault estimation for discrete time-varying systems subject to sector-bounded nonlinearity: A Krein space-based approach. *Int. J. Robust Nonlinear Control* **2021**, *31*, 5360–5380. [\[CrossRef\]](#)