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Abstract: This paper analyzes the harmonics and vibration under different fault-tolerant current control when one-phase winding of dual three-phase permanent magnet synchronous motor (PMSM) is an open circuit fault. Firstly, the dual three-phase load condition subdomain model of breadloaf surface-mounted permanent magnet (PM) PMSM is established. Secondly, the working conditions of an open circuit fault of one-phase winding are set from the perspectives of no fault-tolerant (NFT) control, maximum torque (MT) control, minimum copper loss (MCL) control and single three-phase mode (STPM) control. The torque harmonics and electromagnetic force harmonics of the four working conditions are analyzed. Then, considering the multi-physical field of electromagnetic and structural coupling, the electromagnetic force in the motor air gap acts on the stator core, and the vibration under four different working conditions is calculated and analyzed. Through the comparison between the analytical method and finite element method, the accuracy of the established model is verified. The calculation method is effective and accurate, it can quickly predict the motor performance in case of one-phase open circuit fault of dual three-phase PMSM.

Keywords: subdomain method; open circuit fault; harmonic analysis; electromagnetic vibration; dual three-phase motor

1. Introduction

Polyphase permanent magnet synchronous motors (PPMSMs) have attracted more and more attention in ship propulsion, electric vehicles, wind power generation and other applications. Compared with three-phase motors, PPMSMs have the advantages of low voltage, high power, low vibration noise and multi degree of freedom control [1,2], the most prominent is that with the increase of phase numbers, its control degree of freedom increases. Therefore, more fault-tolerant control methods can be realized in case of motor winding fault [3].

The prediction method of motor vibration has always been the focus of many researchers. The finite element(FE) method can accurately simulate and analyze the vibration results of the motor, but the calculation of multi-physical field coupling is very timeconsuming. The sub domain method (SM) divides the motor into multiple structural domains and quickly calculates the air gap magnetic field results of the motor in combination with the boundary conditions. It can achieve the same calculation accuracy as the finite element method [4–7]. Based on the electromagnetic force calculated by analytical method and the modal calculation results, the vibration of the motor can be calculated quickly.

Open circuit fault of winding is very common in AC motors. Solder melting caused by long-term high-temperature operation, action of mechanical force and electromagnetic force, switching fault of inverter and other reasons may lead to open circuit of motor winding [8]. Considering the steady-state operation of the motor, there are many solutions



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Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). to the amplitude and phase of each phase current under this assumption as long as the total magnetomotive force (MMF) before and after the fault is kept unchanged. Reference [9] proposed a fault-tolerant control scheme to minimize copper loss and maximize torque with MT per ampere. In reference [10], in order to obtain the MT of the given maximum peak current, a remedial method for the current amplitude and phase of the remaining phase windings is proposed. In reference [11], a fault-tolerant hybrid current control method is proposed by combining single-phase three-phase mode control with maximum torque control. Reference [12] established the mathematical model of one-phase open circuit dual three-phase PMSM according to four different neutral connection modes under fault state, and derived the MCL vector control strategy under four conditions. Finally, different models were compared. Reference [13] proposed a single index fault detection method, which, together with natural fault-tolerant control, enhances the robustness with minimum complexity. In reference [14], a method based on variable ratio after vector space decomposition is used to study the diagnosis of incomplete phase fault, which can quickly detect and locate the fault phase. According to the principle of constant stator magnetic potential and aiming at the MCL, reference [15] optimized the current of dual three-phase PMSM without one-phase winding.

Although the PPMSMs have high fault-tolerance and many degrees of freedom of fault-tolerant control in case of winding open circuit fault, the motor will inevitably produce torque ripple and more electromagnetic harmonics after a winding fault. Even if the corresponding fault-tolerant control is adopted, the harmonic component will still be increased and electromagnetic vibration will be generated due to the operation of asymmetric phase. Therefore, the prediction and analysis of harmonics and vibration after a polyphase winding fault can better evaluate the operation state and performance after an open circuit fault of motor winding.

In this paper, the magnetic field calculation model of breadloaf PM dual three-phase PMSM under load conditions is established, based on the SM. It is assumed that the open circuit fault of one-phase winding is the unbalanced operation condition of motor fault. Firstly, a variety of fault-tolerant strategy models are established, that is, NFT control, MT control, MCL control and STPM control. Then, under different fault-tolerant conditions, the torque harmonics, electromagnetic force harmonics and vibration of the motor are analyzed by using the FE method and analytical model. The results show the effectiveness and accuracy of the analytical method. It provides a basis for studying the winding open circuit fault of dual three-phase PMSM.

2. Magnetic Field Modeling and Winding Open Circuit Fault Setting

2.1. Subdomain Model of Dual Three-Phase Motor

In this paper, a dual three-phase PMSM with 96 slots and 16 poles is studied, the structural parameters of the motor are shown in Table 1. In order to reduce the vibration of the motor, the motor structure is designed with integer slots and double-stacked windings in the form of spatial arrangement. The PMs are of breadloaf structure and the air gap length is increased to 5 mm. The reason for the long air gap length is that this motor is used in underwater propulsion, therefore, compared with the industrial motor, the designed motor harmonic is lower. If the FE method is used to calculate and analyze the motor, whether it is a single electromagnetic field calculation or multi physical field coupling calculation, the consideration of harmonics and vibration has high requirements for the grid quality in the process of quantitative analysis. This paper uses the mutual verification of analytical method and finite element method to consider the open circuit of one phase of winding.

Symbol	Quantity	Value (Units)
$Q_s/p/m$	Slots/Pole pair/Phase	96/8/6
Ro	Stator outer radials	182.5 (mm)
R_i	Stator inner radials	150 (mm)
L_{st}	Stator core length	20 (mm)
δ	Air gap length	5 (mm)
h_m	Magnet maximum thickness	6 (mm)
Br	Magnet remanence	1.38 (T)
α_p	Pole-arc coefficient	0.89
n _r	Speed mech	600 (rpm)

Table 1. Parameters of the simulated motor.

The structural view of the motor is shown in Figure 1a, and the precise SM of surfacemounted PMSM studied in this paper is established, as shown in Figure 1b and Equation (1), including breadloaf PM sub domain, air gap sub domain, stator slot sub domain and stator slot sub domain [6]. In Equation (1), A_{rj} , A_{air} , A_{soi} and A_{sli} represent the vector magnetic potential in each subdomain, μ_0 is the vacuum permeability, and its value is $4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A}$, M_{rj} and $M_{\theta j}$ represent the magnetization of the permanent magnet, *J* is a matrix, which represents the current density and winding distribution.

$$\begin{cases} \frac{\partial^2 A_{rj}}{\partial r^2} + \frac{1}{r} \frac{\partial A_{rj}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A_{rj}}{\partial \theta^2} = -\frac{\mu_0}{r} \left(M_{\theta j} - \frac{\partial M_{rj}}{\partial \theta} \right) \text{Magnet} \\ \frac{\partial^2 A_{air}}{\partial r^2} + \frac{1}{r} \frac{\partial A_{air}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A_{air}}{\partial \theta^2} = 0 \quad \text{Air-gap} \\ \frac{\partial^2 A_{soi}}{\partial r^2} + \frac{1}{r} \frac{\partial A_{soi}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A_{soi}}{\partial \theta^2} = 0 \quad \text{Slot-opening} \\ \frac{\partial^2 A_{sli}}{\partial r^2} + \frac{1}{r} \frac{\partial A_{sli}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A_{sli}}{\partial \theta^2} = -\mu_0 J \quad \text{Stator-slot} \end{cases}$$
(1)



Figure 1. Dual three-phase motor model: (**a**) Motor structure model; (**b**) SM model; (**c**) Permanent magnet model.

As shown in Figure 1c, for breadloaf PM, the equivalent modeling of multi-section PM is divided by the equal pole arc coefficient method, and the included angle between the magnetic pole of section *j* and the center line of PM θ_j can be expressed as:

$$\theta_j = \frac{\alpha_p \pi}{p} \left(\frac{l - 2j + 1}{2l} \right) \tag{2}$$

where *l* is the number of segments of PM, α_p is the pole arc coefficient of the PM, *p* is the pole-pair number of the motor, the outer radius R_{mj} and inner radius R_j of the *j*-segment PM can be expressed as:

$$R_{mj} = h\cos\theta_j + \sqrt{\left(h\cos\theta_j\right)^2 - \left(h^2 - R_j^2\right)}$$
(3)

$$R_{rj} = R_r \cos[\alpha_p \pi / (2p)] / \cos \theta_j \tag{4}$$

where R_r is the maximum outer diameter of rotor core, R_m is the maximum outer diameter of PM, the center position angle of section *j* PM α_j and polar arc coefficient α_{pj} can be expressed as:

$$\begin{cases} \alpha_j = \frac{\pi}{2p} - \theta_j \\ \alpha_{pj} = \frac{\alpha_p}{l} \end{cases}$$
(5)

Through the variable separation method, the general solution of the equation established in each sub domain is solved. Equation (1) is used to solve the results of each magnetic pole magnetic field. The synthetic magnetic field is equivalent to the superposition of the magnetic field contributions of each segment of PM

$$B_{air,r} = \sum_{j=1}^{l} B_{air,rj} = \sum_{j=1}^{l} \frac{1}{r} \frac{\partial A_{air}(r,\theta)}{\partial \theta}$$

$$B_{air,\theta} = \sum_{j=1}^{l} B_{air,\theta j} = \sum_{j=1}^{l} -\frac{\partial A_{air}(r,\theta)}{\partial r}$$
(6)

For the motor analyzed in this paper, the radial component and tangential component of air gap magnetic flux density calculated by SM method and FE method under load current is shown in Figure 2, it can be seen that the SM result has high calculation accuracy.



Figure 2. Air gap magnetic flux density under load condition: (a) Radial component; (b) Tangential component.

2.2. Open Circuit Fault Condition Setting

When the motor winding works normally in dual three-phase mode (DTPM), the current of each phase is shown in Figure 3a, the current phase difference between the two sets of windings is 30°. The armature magnetic field distribution conforms to the topology principle of dual three-phase motor in Figure 3b.

The motor with two neutral points analyzed is an integer slot multi-phase winding structure with double-layer short distance distribution. It is assumed that the winding is sinusoidal distribution without considering the spatial harmonics of the winding distribution coefficient. As shown in Figure 4, if the F-phase winding suddenly has an open circuit fault, after a short transient process, the current of the F-phase winding immediately becomes 0 A.



Figure 3. Armature characteristics before the fault: (**a**) Phase current waveform; (**b**) FFT2 results of radial component of armature flux density.



Figure 4. Star diagram of winding open circuit fault.

The MMF of the winding before the fault can be expressed as

$$MMF = N_{A}i_{A} + N_{B}i_{B} + N_{C}i_{C} + N_{D}i_{D} + N_{E}i_{E} + N_{F}i_{F}$$

$$= \frac{N_{t}}{2} \Big[i_{A}\cos(\varphi) + i_{B}\cos(\varphi - \frac{2\pi}{3}) + i_{C}\cos(\varphi - \frac{4\pi}{3}) \Big] + \frac{N_{t}}{2} \Big[i_{D}\cos(\varphi - \frac{\pi}{6}) + i_{E}\cos(\varphi - \frac{5\pi}{6}) + i_{F}\cos(\varphi - \frac{9\pi}{6}) \Big]$$

$$= \frac{3}{2} N_{t} I_{m}\cos(\omega t - \varphi)$$
(7)

Winding *MMF* of phase F open circuit will be the sum of the remaining five phases:

$$MMF' = N_A i_A + N_B i_B + N_C i_C + N_D i_D + N_E i_E = \frac{N_t}{2} \left[i_A \cos(\varphi) + i_B \cos(\varphi - \frac{2\pi}{3}) + i_C \cos(\varphi - \frac{4\pi}{3}) + i_D \cos(\varphi - \frac{\pi}{6}) + i_E \cos(\varphi - \frac{5\pi}{6}) \right]$$
(8)

where N_t is the number of winding turns, I_m is the current amplitude and ω is the electrical angular frequency, φ is the winding space angle.

According to Kirchhoff's law, the remaining healthy phase winding current can be expressed as:

$$i_A + i_B + i_C = 0$$

 $i_D + i_E = 0$
(9)

In case of open circuit fault of one-phase winding, the motor will produce a large torque ripple. In order to pursue a stable output torque, the remaining synthetic magnetomotive force should remain the same as before the failure, that is, MMF = MMF', the solution of the reconstructed linear equations is not unique, so it can be used for motor optimization after phase loss [10,11].

Based on the SM, current excitation is applied to the stator slot winding. For the spatial arrangement of double three-phase windings of the unit motor (12 slots 2 poles), as shown in Equation (12), the row vector of the winding matrix M_c represents the slot position, the column vector represents the phase position of the winding, 1 and -1 represent the inflow and outflow of current, and k_f and S_c represent the filling range and area of the winding in the slot

$$J(t) = \frac{k_f}{S_c} i(t) M_c \tag{10}$$

$$i(t) = [i_A \ i_B \ i_C \ i_D \ i_E \ i_F] \tag{11}$$

In this paper, the changes of current amplitude and phase of other healthy phases after missing one phase fault are considered from the perspectives of NFT control, MT control, MCL control and STPM control.

2.2.1. No Fault Tolerant Control

When the F-phase winding has an open-circuit fault, no fault-tolerant strategy is adopted. The motor armature winding can be equivalent to a combination of three-phase motor winding and single-phase motor winding, and the current amplitude of D-phase and E-phase becomes $\sqrt{3}/2$ the original, the phase leads and lags the original 30°, respectively, at this time the phase difference is 180°, the fault tolerant current can be expressed as Equation (13) and Figure 5a.

$$\begin{cases}
i_A = I_m \sin(\omega t) \\
i_B = I_m \sin(\omega t - 2\pi/3) \\
i_C = I_m \sin(\omega t - 4\pi/3) \\
i_D = \sqrt{3}/2 \cdot I_m \sin(\omega t) \\
i_F = \sqrt{3}/2 \cdot I_m \sin(\omega t - \pi)
\end{cases}$$
(13)



Figure 5. Armature characteristics of NFT control: (**a**) Phase current waveform; (**b**) FFT2 results of radial component of armature flux density.

2.2.2. Maximum Torque Control

According to references [10,11], one-phase winding of a dual three-phase PMSM is open circuit, and the remedial measures for the current amplitude and phase of the remaining phases, if the pursuit of MT is the goal, according to Lagrange, the daily multi-

$$T_{\max} = \max\left(i_A^2, i_B^2, i_C^2, i_D^2, i_E^2\right)$$
(14)

$$i_{A} = 0$$

$$i_{B} = \sqrt{3}I_{m}\sin(\omega t)$$

$$i_{C} = \sqrt{3}I_{m}\sin(\omega t - \pi)$$

$$i_{D} = \sqrt{3}I_{m}\sin(\omega t - 3\pi/2)$$

$$i_{F} = \sqrt{3}I_{m}\sin(\omega t - \pi/2)$$
(15)



Figure 6. Armature characteristics of MT control: (**a**) Phase current waveform; (**b**) FFT2 results of radial component of armature flux density.

2.2.3. Minimum Copper Loss Control

According to references [11,14], if the goal is to obtain the MCL, use Equations (7)–(9) to solve Equation (16) by the Lagrangian multiplier method, and amplitude and phase of the fault-tolerant current can be expressed as Equation (17) and Figure 7a.

$$Loss_{Cu} = R_s \left(i_A^2 + i_B^2 + i_C^2 + i_D^2 + i_E^2 \right)$$
(16)

$$\begin{cases} i_{A} = I_{m} \sin(\omega t - \pi/2) \\ i_{B} = -0.5I_{m} \sin(\omega t - \pi/2) + \sqrt{3}I_{m} \sin(\omega t) \\ i_{C} = -0.5I_{m} \sin(\omega t - \pi/2) - \sqrt{3}I_{m} \sin(\omega t) \\ i_{D} = \sqrt{3}/2 \cdot I_{m} \sin(\omega t - \pi/2) \\ i_{E} = -\sqrt{3}/2 \cdot I_{m} \sin(\omega t - \pi/2) \end{cases}$$
(17)



Figure 7. Armature characteristics of MCL control: (a) Phase current waveform; (b) FFT2 results of radial component of armature flux density.

2.2.4. Single Three-Phase Mode Control

(a)

For the dual three-phase PMSM with neutral isolation, whether one or more phases of a set of windings fail, the whole set of windings can be cut off, and the motor is controlled by a single three-phase. At this time, the winding armature current is shown in Figure 8a.



Figure 8. Armature characteristics of STPM control: (**a**) Phase current waveform; (**b**) FFT2 results of radial component of armature flux density.

(b)

As shown in Figure 9, the armature flux density under various working conditions is decomposed by a two-dimensional Fourier transform. This paper is based on the study of sinusoidal current, and the spatial harmonic distribution is shown in Table 2. The harmonic distribution of the armature flux density before winding fault is 12v + 1. The direction of component -1 which is opposite to the fundamental component if motor adopts NFT control, so the rotating magnetic field will produce large harmonic components twice the electrical frequency, the armature magnetic field harmonic with order of $6v \pm 1$ will produce load torque fluctuation with frequency of $|6v \pm 1 - 1|f$, MCL control is the same. If the motor adopts MT control, when v is odd, the motor generates |6v - 2|f torque harmonics, and when v is even, the motor generates |6v|f torque harmonics, STPM control contains only |6v|f torque harmonics. It can be seen that the descending order of the harmonic content of the motor under the four control modes is STPM, MT, MCL, NFT. In the above analysis, v is integral number, f is the electrical frequency of motor operation that it can be calculated by $n_rp/60$.

Table 2. Harmonic distribution of armature magnetic field under different working conditions.

	DTPM	NFT	MT	MCL	STPM
Fundamental component	1	-1 and 1	1	1	1
Harmonic component	12v + 1	$6v \pm 1$	6v - 1, v is odd 6v + 1, v is even	$6v \pm 1$	6v + 1



Figure 9. All the FFTs of the armature flux density.

3. Harmonic Analysis

3.1. Torque Harmonic Analysis

The motor torque result is calculated by integrating the product of the radial component and tangential component of magnetic density on rail in the middle of air gap [6], where R_{air} is the radius in the middle of the air gap, L_{st} is the axial length of motor.

$$T_e = \frac{L_{st}R_{air}^2}{\mu_0} \int_0^{2\pi} B_r(R_{air},\theta) B_t(R_{air},\theta) d\theta$$
(19)

According to the analysis in Section 2, the stable output torque is obtained based on the principle of constant MMF. If the motor can still output the rated torque, the current amplitude under STPM control should be twice the original. After setting the appropriate initial position of the rotor, ensure that the motor output torque under the three faulttolerant controls is close to the rated torque. As shown in Figure 10b, the average torque of MT control and MCL control is 26.95 N·m, and the average value of STPM control is 27.08 N·m. In one electric cycle, the period of output-torque waveform calculated under the four control modes is two, four, two and six. In addition, if the motor adopts NFT control method, although the current amplitude and phase of winding D and winding E will change, as shown in Figure 10a, the imbalance of the motor magnetic field will lead to large torque ripple.



Figure 10. Electromagnetic torque: (a) NFT control; (b) MT control, MCL control and STPM control.

The torque harmonic under the four control modes is shown in Figure 11, it can be seen that the 2nd harmonic accounts for the largest as shown in Figure 11a, which is $6.75 \text{ N} \cdot \text{m}$,

and followed by the 12th harmonic. According to the principle of constant MMF, the motor will output stable torque by MT control and MCL control after one-phase open circuit, but a large number of harmonics are generated. In Figure 11b, the torque harmonic distribution of MT control, MCL control and STPM control is 4*kf* Hz, 2*kf* Hz, and 6*kf* Hz, and *k* is natural number.



Figure 11. Torque harmonics: (a) NFT control; (b) MT, MCL and STPM control.

3.2. Harmonic Analysis of Radial Electromagnetic Force

Although the radial component and tangential component of electromagnetic force will cause radial vibration, generally, the radial component is much larger than the tangential component, so the contribution of tangential component and radial and tangential coupling of electromagnetic force can be ignored. Considering the tangential component of flux density, the radial electromagnetic force density in the air gap is calculated by the Maxwell tensor method [16], the results are shown in Figure 12. Asymmetric phase current leads to the temporal and spatial non-uniformity of radial electromagnetic force, which contains a large number of harmonics.

$$F_r = \frac{B_r^2 - B_t^2}{2\mu_0}$$
(20)

For the integer slot motor analyzed in this paper, because the lowest non-zero space order is $GCD(2p,Q_s) = 16$, therefore, the 0th mode radial electromagnetic force is the main source of motor electromagnetic vibration and noise [16]. The electromagnetic force density distribution at spatial 0th mode is shown in Table 3, harmonic components with amplitude less than 0.5 N/m² in the results are not considered. It can be seen from Table 3 that when the motor adopts NFT control, the maximum radial electromagnetic force is 16th harmonic, the second maximum is 96th harmonic, and it contains a wealth of other orders. The electromagnetic force harmonics of MT control, MCL control and STPM control are distributed in 4kpfr, 2kpfr and 6kpfr, where k is the natural number and fr is the mechanical frequency of motor rotation, that can be calculated by $n_r/60$.

Table 3. Electromagnetic force density distribution at spatial 0th mode.

Radial Force Density (N/m ²)	NFT	MT	MCL	STPM
16fr	4954	-	-	-
32 <i>fr</i>	6.13	24.74	12.26	-
48fr	7.6	-	15.6	30.58
64fr	1.7	6.7	3.4	-
96fr	18.28	22.1	22.09	19.54
128fr	0.52	2.142	1.03	-
144 <i>fr</i>	-	-	0.78	1.6



Figure 12. Radial electromagnetic force density after phase F open circuit fault: (**a**) NFT control; (**b**) MT control; (**c**) MCL control; (**d**) STPM control.

4. Modal and Vibration

4.1. Modal Calculation of Stator Core

The modal natural frequency is the natural property of the structure. When excited by the outside, the structure vibrates naturally according to the specific frequency. Therefore, an accurate structural model is very important for the prediction of electromagnetic vibration and noise. Due to the anisotropy of stator core materials, its parameters are shown in Table 4, and its modes are equivalent to stiffness and mass effects [17].

$$f_m = \frac{1}{2\pi} \left(\frac{K_m}{M_m}\right)^{1/2} \tag{21}$$

Table 4. Equivalent material parameters of stator core.

Material Parameters	Elasticity Modulus (GPa)	Shear Modulus (GPa)	Poisson Ratio	Density (kg/m ³)
Stator core	Ex = Ey = 205.8 $Ez = 150$	Gxy = 80 Gyz = Gxz = 73	0.27	7650

The stator modal shapes obtained by finite element calculation are shown in Figure 13. For the integer slot permanent magnet motor, the lowest non-zero order electromagnetic force is 2p, so 0th mode is usually the main contributor to the low-frequency radiated vibration and noise spectrum.



Figure 13. Modal shape: (a) Mode 0; (b) Mode 1; (c) Mode 2; (d) Mode 3; (e) Mode 4; (f) Mode 5.

The unconstrained free state of stator core is calculated in Table 5. The analytical calculation structure is similar to the finite element calculation structure, and the comparison error is within 5%, so as to better ensure the rationality of the analytical calculation results of vibration and noise.

Table 5. Mode shape and mode frequency of stator.

Mode Order	FEA (Hz)	Analytical (Hz)	Relative Error
0	3532.7	3566.2	0.95%
1	5082.6	4933	-2.94%
2	194	186.6	-3.8%
3	539.8	521.7	-3.36%
4	1013.5	984	-2.91%
5	1599.4	1559	-2.52%
6	2277.8	2233.2	-1.96%
7	3026.2	2992.05	-1.13%

4.2. Electromagnetic Vibration

The modal superposition method is used to simulate the harmonic response of electromagnetic and structural multi physical fields of damped stator core, when the stator core is subjected to electromagnetic excitation force, its vibration process can be expressed as Equation (22) [18]:

$$M\frac{d^2A_{dism}}{dt^2} + C\frac{dA_{dism}}{dt} + KA_{dism} = F_m$$
(22)

where M, C, and K represent the mass, the damping and the stiffness matrices, F_m is the spatiotemporal harmonic vector of electromagnetic force. The vibration displacement A_{dism} and acceleration A_{accm} can be expressed as Equations (23) and (24) [18].

$$A_{dism} = \frac{F_m/m_s}{\sqrt{(\omega_m^2 - \omega_r^2)^2 + 4\xi_m^2 \omega_r^2 \omega_m^2}}$$
(23)

$$A_{accm} = \omega_r^2 A_{dism} \tag{24}$$

$$F_m = 2\pi R_{air} L_{st} F_r \tag{25}$$

where m_s is the mass of stator core, ω_r is the electrical angular speed, and ω_m is m order modal angular frequency, P_r is equivalent to the radial electromagnetic force density applied on the stator, ξ_m is the m-mode circumferential modal damping coefficient [17], which can be calculated by empirical Formula (26), f_m is the m order modal frequency.

$$\xi_m = \frac{1}{2\pi} \left(2.76 \times 10^{-5} f_m + 0.062 \right) \tag{26}$$

When the motor runs at the speed of 600 r/min, Figure 14 shows the comparison of motor vibration acceleration under different fault-tolerance strategies and normal operation. If fault-tolerant measures are not adopted as shown in Figure 14a, it can be seen that the motor will produce a large vibration at 16fr. The second frequency position that produces a large vibration is 96fr, it is caused by stator slotting and related to the number of motor slots. The third frequency position is 3520 Hz, since it is close to the natural frequency of mode 0 (3532.7 Hz), although the electromagnetic force is very small, it can still produce a large vibration.



Figure 14. FE calculation results of vibration acceleration on stator core surface: (**a**) Comparison between NFT and DTPM; (**b**) Comparison between MT and DTPM; (**c**) Comparison between MCL and DTPM; (**d**) Comparison between STPM and DTPM.

When sinusoidal fault-tolerant current is applied, the vibration acceleration of stator surface under three working conditions is compared in Figure 14b–d. It can be seen that the maximum difference is in the frequency band within 1000 Hz. Under the condition that the MMF remains unchanged and the output torque performance of the motor is the same, the frequency position of the maximum vibration of the motor is 96*fr*, this is related to stator slotting, and its amplitude can be weakened by means of a slot skewed in the motor design.

In addition to the maximum vibration at 96*fr*, for MT control, vibration will be generated at 32*fr* and 64*fr* frequencies, MCL control will generate vibration at 32*fr*, 48*fr* and 64*fr*, while STPM control only generates vibration at 48*fr*, but the amplitude is larger than that of the first two fault-tolerant methods, the electromagnetic force harmonic distribution of the third-order analysis is verified. Although these frequencies are lower than the vibration generated at 96*fr* due to the motor slotting, the winding fault-tolerant current waveform is the result of sinusoidal. In practice, due to external factors, such as driver harmonics and mechanical reasons, it will be easier to interact with these order harmonics to produce greater vibration and noise.

In the intermediate frequency region, different fault-tolerant methods produce different components of electromagnetic force harmonics, which will cause the vibration of the corresponding harmonic frequency. Figure 15 shows the comparison of analytical and finite element calculation results under various working conditions, the calculation results of the two are similar, and the finite element calculation result is slightly larger than the analytical calculation result.



Figure 15. Comparison of vibration results between analytical method and FE method: (**a**) NFT control; (**b**) MT control; (**c**) MCL control; (**d**) STPM control.

5. Conclusions

In this paper, the magnetic field model of one-phase open circuit fault of low harmonic dual three-phase PMSM is established, based on the SM. It is assumed that the F-phase winding has an open circuit fault, the NFT control, MT control, MCL control and STPM control are considered, respectively. The torque harmonics and electromagnetic force harmonics under different working conditions are analyzed and the electromagnetic vibration is predicted. Analytical calculation results and finite element calculation results are compared to verify the accuracy of the established analytical model. By analyzing the motor harmonics and vibration distribution of different fault-tolerance methods, the following conclusions can be drawn:

- When there is an open-circuit failure of one-phase winding of the motor winding, if there is no fault-tolerance measure, the asymmetric armature current will cause the motor magnetic field to be asymmetric. A negative sequence magnetic field of order -1 will be generated, which will produce large torque ripple and vibration.
- Under the condition of the conservation of magnetomotive force, redistribute the current amplitude and phase so that the output torque of the motor can still be

the same as before the fault. The maximum vibration produced by the motor is still the slot frequency vibration. The motor under MT control and MCL control produces a lot of low-frequency harmonic components. Although the amplitude of the vibration generated by these harmonics is not obvious compared with the slot frequency vibration, this is based on the assumption of windings. The current is sinusoidal, so these harmonics should not be ignored. Compared with MT and MCL control, STPM control will produce larger vibration at 48*fr*, but its harmonic component is single.

According to the integer slot Motor vibration and noise theory, if the motor vibration and noise performance is required, the motor should have fewer types of harmonics, and STPM control is more suitable. If the motor copper loss is required to be as low as possible, then MCL control has an advantage. The reliability of the established analysis method is verified by the structural parameters of 16 pole 96 slot motor. This analytical method is not limited to the analysis of motors with this structure. It is universal and can analyze double three-phase motors with other topologies.

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