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Editorial

Special Issue: Symmetry in Nonequilibrium Statistical Mechanics and Dynamical Systems

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Abstract: The recent developments in dynamical systems theory and non-equilibrium statistical mechanics have allowed the birth of new challenges and research perspectives. In particular, different frameworks have been proposed for the modeling of complex emerging phenomena occurring in nature and society. This editorial article introduces the topic and the contributions of this Special Issue. This Special Issue focuses, on the one hand, on the development of new methods, frameworks and models coming from dynamical system theory and the equilibrium/non-equilibrium statistical mechanics and, on the other hand, opens problems related to the existing frameworks. The Special Issue also includes applications to physical, biological and engineering systems.

Keywords: kinetic theory; fractional equations; numerical methods; natural transform; Hamiltonian dynamics; Onsager reciprocal relations; Lipschitz stability in time; oscillation properties; neutral differential equations; Lyapunov functions

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Introduction

Dynamical systems are still an attractive research domain, and recently, the number of applications have increased as a result of the amount of attention being payed to the modeling of complex systems [1], in particular of a living fashion. A dynamical system is a model that describes the time evolution of a system. The main ingredient to derive a dynamical system is the selection of variables, called state variables, which allows the complete time description of the system under consideration. The state space, which is the set of all possible values of the state variables, can be discrete or continuous. If the state space is continuous and finite-dimensional, then it is called the phase space. The state of the system obeys differential/difference equations and the investigation of analytical or numerical solutions is a fundamental step in their solution. In particular, the asymptotic analysis of the solution of a dynamical system is a fruitful research domain.
Dynamical system theory is a well-established research domain [2,3], and today, many perspectives are open in particular on applications in biology [4], engineering [5], and economics [6].

From the application point of view, the dynamical system model has allowed the birth of different frameworks coming from different tools proposed by different scholars involved in modeling approaches, e.g., generalized kinetic theory and fractional differential equations.

In the modeling of complex systems, an important step is also the possibility of linking dynamics occurring at different scales. Usually, the main representation scales are microscopic, mesoscopic and macroscopic. Models based on dynamical system theory have been proposed at different scales. However, the ultimate framework requires the definition of multiscale approaches, which allow linking different models at the different scales, that is, to link the phenomena occurring at the macroscopic scale with the particle interactions occurring at the microscopic and mesoscopic scale.

The main aim of statistical mechanics is the macroscopic description of a phenomenon by analyzing the microscopic behaviour [7]. Statistical methods and probability theory are employed in this description. Differently from classical mechanics, which considers the behaviour of a single state, in statistical mechanics the notion of an ensemble is introduced, which consists of a large collection of independent copies of the system in various states. The statistical ensemble is a probability distribution over all possible states of the system.

Equilibrium statistical mechanics, also called statistical thermodynamics, aims at deriving the thermodynamics of a material by analyzing the properties and the interactions among the constituent particles. Statistical equilibrium does not mean that the particles have stopped moving (mechanical equilibrium), only that the ensemble is not evolving.

In the modeling of complex phenomena, there is the role of the interactions among the different components composing the complex system. A preliminary phenomenological analysis is thus followed by the modeling of the interaction terms that are derived by the means of the definition of the interaction kernels and parameters on which some symmetry assumptions are made.

This Special Issue focuses, on the one hand, on the development of new methods, frameworks and models coming from the dynamical system theory and the equilibrium/non-equilibrium statistical mechanics. On the other hand, it focuses on the open problems related to the existing frameworks.

The paper [8] is devoted to an important development of the thermostatted kinetic theory for active particles [9]. Specifically, and to the best of our knowledge, the paper [8] proposes for the first time the introduction of a vectorial activity variable in the microscopic state of the particles composing the complex system. In particular, the vectorial variable can attain discrete or continuous values, and thus the frameworks are based on nonlinear, partial-integro differential equations or ordinary differential equations. After the introduction of the new frameworks, ref. [8] focuses on the global existence and uniqueness of the solution of the related Cauchy problem. The content of this paper [8] is in between nonequilibrum statistical mechanics and dynamical systems.

The paper [10] deals with an important issue in statistical mechanics and specifically with the time reversal invariance of particles systems. As it is known, for systems under the action of a magnetic field, the relation between time reversal invariance and Onsager reciprocal relations has been thoroughly investigated [11]. The main aim of this paper is to identify the most general time reversal operation compatible with a classical Hamiltonian system. Moreover, the paper analyzes the minimal coupling with a generic magnetic field, formulating sufficient conditions for the magnetic field and for the force potential that make the Onsager reciprocal relations hold.

The paper [12] is concerned with an important application: autoimmune diseases [13]. Specifically, the paper, by employing a statistical mechanics framework coming from the generalized kinetic theory, proposes a Boltzmann-type model, which is based on the interaction between the immune system cells and a virus. A brief qualitative analysis on the existence and uniqueness of the solution of the model is presented; a quantitative analysis based on the numerical simulations of the solution is also performed. The numerical simulations consist of a sensitivity analysis on some of the parameters and show different scenarios of autoimmune diseases.

The paper [14] presents a mathematical analysis of a class of neutral differential equations. Specifically, a second-order, nonlinear, neutral differential equation with sublinear neutral terms is considered. After a preliminary analysis, the paper establishes sufficient conditions for the oscillation of the solution. Finally an illustrative example is provided to validate the main results.

The paper [15] is devoted to the mathematical analysis of a fractional differential equation. Specifically, the paper deals with a system of nonlinear Riemann–Liouville fractional differential equations with non-instantaneous impulses. In particular, after preliminary results, the paper introduces the Lipschitz stability in time and establishes sufficient conditions. Illustrative examples complete the paper.

The focus of the paper [16] is the analytical and numerical solution of a class of fractional differential equations based on the Caputo derivative. The integral transform method, called the Fractional Decomposition method, is implemented. From the application point of view, the method is employed to solve two nonlinear, fractional, ordinary differential equations and to obtain the solution of a model with time-fractional diffusion.

Bearing all of the above in mind, this Special Issue has contributed to this research field by generalizing existing results in dynamical systems (with classical or fractional derivatives), introducing new techniques and models, and validating issues in non-equilibrium statistical mechanics. The reader will find the new results, methods, and models contained with this issue to be of great interest.

The authors that have contributed to this Special Issue and the guest editor hope that the readers will benefit from the contents of this Special Issue in the development and pursuit of their research.

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