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Revisiting the Okubo–Marshak Argument

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Abstract: Modular localization and the theory of string-localized fields have revolutionized several key aspects of quantum field theory. They reinforce the contention that *local* symmetry emerges directly from quantum theory, but global gauge invariance remains in general an unwarranted assumption to be examined case by case. Armed with those modern tools, we reconsider here the classical Okubo–Marshak argument on the non-existence of a "strong CP problem" in quantum chromodynamics.

Keywords: string independence; local symmetry; strong CP problem

To the memory of Bob Marshak and Daniel Testard

check for updates

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1. Introduction: String-Localized Fields

In this paper, the case by Okubo and Marshak [1] against the existence of the "strong CP problem" in quantum chromodynamics, which was based on the covariant approach to Yang–Mills theory by Kugo and Ojima [2], is reassessed from a different theoretical standpoint. For this purpose, we bring to bear the theory of string-localized quantum fields (SLFs).

SLFs are not a recent invention. The paper by Dirac [3] should be regarded as a precedent. Charged SLFs, recognizably similar to the modern version, were a brainchild of Mandelstam [4]. In a different vein, they were treated by Buchholz and Fredenhagen [5]. In the 1980s, they were further developed by Steinmann [6–8]. They resurfaced over 15 years ago in important papers by one of us with Schroer and Yngvason [9,10]. Interest in SLFs had meanwhile been sustained and renewed by their obvious connection to modular structures [11], with roots in deep mathematical results of Hilbert algebra theory [12] and a geometrical interpretation through the Bisognano–Wichmann relations [13,14]. Among the harbingers of their revival we count papers [15–17], bearing upon the proper concept of locality in modern quantum field theory.

The gist of [9] was to show that within the SLF dispensation, Wigner particles of mass zero and unbounded helicity possess associated quantum fields; those had long before been excluded from the standard framework by Yngvason himself [18]. The detailed treatment in [10], apart from reviewing this fact, directly and comprehensively relates SLF to point-localized fields of the ordinary sort.

At the price of an extra variable, SL fields offer important advantages. Two of these are: (a) string-localized fields slip past the theorem in Section 5.9 of [19]—that it is impossible to construct on Hilbert space tensor fields of rank r for massless particles with helicity r, such as photons and gluons; (b) their short-distance behaviour, both for massive and massless particles, is the *same for all bosons* as for scalars and for all fermions as for spin-$\frac{1}{2}$ particles. (*Free* scalar and spin-$\frac{1}{2}$ particles are non-stringy).

The primary upshot is that large no-go territories for QFT are now being trespassed. The separation of helicities in the massless limit of higher spin fields is clarified in [20]. The van Dam-Veltman–Zakharov discontinuity [21,22] at the $m \downarrow 0$ limit of massive gravitons is resolved [23]. Unimpeachable stress–energy-momentum (SEM) tensors for massless fields of *any* helicity are constructed [20,23], allowing for gravitational interaction, and flouting the Weinberg–Witten theorems in particular. A good candidate SEM for unbounded helicity particles now also exists in SL field theory [24]. The Velo–Zwanziger instability [25] is exorcised [26]. The prohibition for covariant spinorial field types (A, B) to represent massless Wigner particles of helicity r, unless $r = B - A$, argued in Section 5.9 of [19], has been made void. SL field theory is also helping to deal with profound, age-old problems of QED [27].

In summary, SL fields sit comfortably midway between "ordinary" and algebraic QFT. Furthermore, the renormalization of QFT models is to take place without calling upon ghost fields, BRS invariance and the like; since, for SL fields, one need not surrender the positivity of the energy nor of the state spaces for the physical particles.

Regarding the Standard Model, rigorous perturbative field theory [28] together with the *principle of string independence* of the $\mathbb{S}$-matrix allowed us to show some time ago that the chirality of the electroweak interactions does not have to be put by hand, as in the GWS model; rather, it follows ineluctably from the massive character of their interaction carriers—a conclusion irrespective of "mechanisms" conjuring the mass [29]. The proof built on outstanding work by Aste, Dütsch and Scharf at the turn of the century on a *quantum* formulation for gauge invariance [30,31] (see [32] as well). Following Marshak, henceforth we refer to electroweak theory as quantum flavourdynamics (QFD).

SLFs have their own drawbacks, to be sure. Practical calculations of loop graphs with SL fields in internal lines are rather challenging, and a proof of normalizability at all orders is still pending. The applications of SLF so far mostly deal with issues of principle that are badly dealt with within conventional QFT. Of which there are plenty. This paper addresses one of them.

Massless Bosonic SLF: General Theory

The term "string" (not to be confused with the strings of string theory) in the present context precisely denotes a ray starting at a point x in Minkowski space $\mathbb{M}_4$ that extends to infinity in a spacelike or lightlike direction. This is the natural limit of the spacelike cones in the intrinsic localization procedure of [17]. The set of such strings can be parametrized by the one-sheet hyperboloid $H_3 \subsetneq \mathbb{M}_4$ ("de Sitter space") with neck radius equal to 1 (the use of the limiting case of lightlike strings, parametrized by the celestial spheres $S^2 \cup S^2 \subsetneq \mathbb{M}_4$, simplifies some formulae; they are a little troublesome from the functional-analytic viewpoint, however).

By way of example, let us look first at the "Abelian" case of helicity $r = 1$. Let $d\mu(p) = (2\pi)^{-3/2}\, d^3\boldsymbol{p}/2|\boldsymbol{p}|$. The quantum electromagnetic field strength, built on Wigner's helicity ± 1 unirreps of the Poincaré group, is written as follows:

$$F_{\mu\nu}(x) = \sum_{\sigma=\pm} \int d\mu(p)\, \big[e^{i(px)} u^{\sigma}_{\mu\nu}(p) a^{\dagger}(p,\sigma) + e^{-i(px)} \bar{u}^{\sigma}_{\mu\nu}(p) a(p,\sigma)\big], \tag{1}$$

where $(px) \equiv g_{\mu\nu} p^{\mu} x^{\nu}$ with mostly negative metric $(g_{\mu\nu})$; the intertwiners are of the form $u^{\sigma}_{\mu\nu}(p) = ie^{\sigma}_{\nu}(p) p_{\mu} - ie^{\sigma}_{\mu}(p) p_{\nu}$, for $e^{\pm}$ a polarization zweibein, with $\bar{u}$ denoting the complex conjugate of u.

The corresponding string-localized vector field for photons is given by

$$A_{\mu}(x,e) = \sum_{\sigma=\pm} \int d\mu(p)\, \big[e^{i(px)} u^{\sigma}_{\mu}(p,e) a^{\dagger}(p,\sigma) + e^{-i(px)} \bar{u}^{\sigma}_{\mu}(p,e) a(p,\sigma)\big], \tag{2}$$

$$\text{its intertwiners } u^{\sigma}_{\mu}(p,e) \text{ being of the form } \quad \frac{(pe)e^{\pm}_{\mu}(p) - (e^{\pm}(p)e)p_{\mu}}{(pe)}. \tag{3}$$

The denominator (pe) in $u^\sigma_\mu(p,e)$ is shorthand for the distribution $\big((pe)+i0\big)^{-1}$ to be smeared in the string variable e. The following key relations—integral and differential, respectively—are effortlessly derived:

$$A_\mu(x,e) = \int_0^\infty dt\, F(x+te)_{\mu\nu}\, e^\nu; \quad \partial_\mu A_\nu(x,e) - \partial_\nu A_\mu(x,e) = F_{\mu\nu}(x); \tag{4}$$

so that $A_\mu(x,e)$ is a *bona fide* potential for $F_{\mu\nu}(x)$. For the first identity,

$$i\big(p_\mu e^\pm_\lambda(p) - p_\lambda e^\pm_\mu(p)\big)e^\lambda \lim_{\varepsilon\downarrow 0}\int_0^\infty dt\, e^{it((pe)+i\varepsilon)} = e^\pm_\mu(p) - p^\mu \frac{(e^\pm(p)\,e)}{(pe)}. \tag{5}$$

It should be clear that the vector potential $A_\mu(x,e)$ fulfills the equations $e^\mu A_\mu(x,e) = \partial^\mu A_\mu(x,e) = 0$. These "transversality" properties are necessary for the free field $A_\mu(x,e)$ acting on the physical Hilbert space: see Section 5 of [10]. Their role is to reduce the number of degrees of freedom, as required for on-shell photons, in much the same way that the six components of the electromagnetic field reduced by the Maxwell equations propagate two degrees of freedom. (Only the second could perhaps be taken as a Lorenz "gauge condition").

The reader should fully appreciate the deep differences between $A_\mu(x,e)$ and the $A_\mu(x)$ potentials of standard QFT. In particular, there are no "pure gauge" configurations in QED or QCD (understood in this paper in the narrow sense of gluodynamics, the theory of pure Yang–Mills fields), when working in SL field theory. The field $A_\mu(x,e)$ lives on the same Fock space as $F_{\mu\nu}$. Thus, the second equation in (4) is an operator relation, which is not the case for the similar one in the usual framework. All this makes easier the physical interpretation of the present case.

In contrast with the standard formalism, $A_\mu(x,e)$ is perfectly covariant: for Λ a Lorentz transformation, c a translation, and U the second quantization of the mentioned unirrep pair of the Poincaré group:

$$U(c,\Lambda)A_\mu(x,e)U^\dagger(c,\Lambda) = (\Lambda^{-1})_\mu^{\ \lambda} A_\lambda(\Lambda x + c, \Lambda e). \tag{6}$$

It is important to realize that the string-differential of the photon field is a gradient:

$$\begin{aligned} d_e A_\mu(x,e) &:= \sum_\rho de^\rho \frac{\partial A_\mu(x,e)}{\partial e^\rho} = -\sum_{\sigma=\pm}\int d\mu(p)\,\bigg[e^{i(px)}\bigg(\frac{p_\mu e^\sigma_\rho}{(pe)} - \frac{p_\rho p_\mu (e^\sigma e)}{(pe)^2}\bigg) a^\dagger(p,\sigma) \\ &\qquad + e^{-i(px)}\bigg(\frac{p_\mu e^\sigma_\rho}{(pe)} - \frac{p_\rho p_\mu (e^\sigma e)}{(pe)^2}\bigg)^{-} a(p,\sigma)\bigg]\, de^\rho \\ &= i\partial_\mu \sum_{\sigma=\pm}\int d\mu(p)\,\bigg[e^{i(px)}\bigg(\frac{e^\sigma_\rho}{(pe)} - \frac{p_\rho (e^\sigma e)}{(pe)^2}\bigg) a^\dagger(p,\sigma) \\ &\qquad - e^{-i(px)}\bigg(\frac{e^\sigma_\rho}{(pe)} - \frac{p_\rho (e^\sigma e)}{(pe)^2}\bigg)^{-} a(p,\sigma)\bigg]\, de^\rho =: \partial_\mu u(x,e). \end{aligned} \tag{7}$$

Naturally, u satisfies the wave equation and $(d_e)^2 A_\mu = \partial_\mu d_e u(x,e) = 0$.

Last, but not least, locality

$$[A_\mu(x,e), A_\nu(x',e')] = 0 \tag{8}$$

holds whenever the strings $x + \mathbb{R}^+\tilde{e}$ and $x' + \mathbb{R}^+ e'$ are spacelike separated for all $\tilde{e}$ in some open neighbourhood of e: "causally disjoint". A proof is found in Appendix C of [29].

2. Dealing with String Independence

Perturbation theory for the SLF is attacked here in the spirit of "renormalization without regularization" of rigorous $\mathbb{S}$-matrix theory [28]. The method engages the construction of a Bogoliubov-type functional scattering operator $\mathbb{S}[g;h]$ dependent on a multiplet g

of external fields and a test function $h \in \mathcal{D}(H_3)$ with integral 1 that averages over the string directions. $\mathbb{S}[g;h]$ is submitted to the customary conditions of causality, unitarity and covariance. One looks for it as a formal power series in g,

$$\mathbb{S}[g;h] := 1 + \sum_{n=1}^{\infty} \frac{i^n}{n!} \prod_{k=1}^{n} \prod_{l=1}^{m} \int d^4x_k \int d\sigma(e_{k,l})\, g(x_k) h(e_{k,l}) S_n(x_1, \boldsymbol{e}_1; \dots; x_n, \boldsymbol{e}_n), \tag{9}$$

where σ is the measure on H_3. Only the first-order vertex coupling $S_1 = S_1(x, \boldsymbol{e})$, a Wick polynomial in the free fields, is postulated, already under severe restrictions. It depends on an array $\boldsymbol{e} = (e_1, \dots, e_m)$ of string coordinates, with m the maximum number of SLFs appearing in a sub-monomial of S_1. For $n \geq 2$, the S_n are time-ordered products that need to be constructed. Two sets of strings cannot be ordered, after chopping them into segments if necessary [33], if and only if they touch each other (see the Appendix A for details). The resulting exceptional set

$$\mathbb{D}_2 := \{\, (x, \boldsymbol{e}; x', \boldsymbol{e}') : (x + \mathbb{R}^+ e_k) \cap (x' + \mathbb{R}^+ e'_l) \neq \emptyset \text{ for some } k, l \,\} \tag{10}$$

is a set of measure zero that includes the diagonal $x = x'$. A similar statement holds for $n > 2$. Only that part of S_n contributes to $\mathbb{S}[g;h]$ that is symmetric under permuting the string coordinates, which are smeared with the same test function h (this symmetry is heavily exploited in the following derivations). The extension of the S_n-products across $\mathbb{D}_n$ is the renormalization problem in a nutshell [34].

The natural and essential hypothesis of interacting SLF theory is simple enough: physical observables and quantities closely related to them, particularly the $\mathbb{S}$-matrix, cannot depend on the string coordinates. This is the intrinsically quantum **string independence** principle: colloquially, the strings "ought not to be seen". In this paper, it will replace the "gauge principle" with advantage.

For the physics of the model described by S_1 to be string-independent, one must require that a vector field $Q^\mu(x, \boldsymbol{e})$ exist such that, after appropriate symmetrization in the string variables

$$d_{e_1} S_1^{\text{sym}} = (\partial Q) \equiv \partial_\mu Q^\mu; \tag{11}$$

thus, on applying integration by parts in the "adiabatic limit", as g goes to a set of constants, the contribution from the divergence vanishes. As the covariant $\mathbb{S}[g;h]$ approaches the invariant physical scattering matrix $\mathbb{S}$, $U(a, \Lambda) \mathbb{S}\, U^\dagger(a, \Lambda) = \mathbb{S}$, all dependence on the strings disappears.

On the face of it, existence of the adiabatic limit is the property that S_k be integrable distributions. Due to severe infrared problems, the latter does not hold in QCD, which involves us here. However, a recent breakthrough [35] rigorously establishes the existence of a suitable weaker adiabatic limit (WAL) in QCD, and so the above reasoning can proceed. (The cited work is concerned with point-local fields. It can be generalized to the string-local setting in low orders. A proof of WAL at all orders in the SLF context is still awaiting.)

2.1. The Aste–Scharf Argument Recast in SLF Theory

Proposition 1. *Suppose that we are given n massless fields $A_{\mu a}$, $(a = 1, \dots, n)$. For their mutual cubic coupling modulo divergences, string independence at the first order enables the Wick product combination:*

$$S_1(x, e_1, e_2) = \frac{g}{2} f_{abc}\, A_{\mu a}(x, e_1) A_{\nu b}(x, e_2) F_c^{\mu\nu}(x), \tag{12}$$

where f_{abc} are completely skew-symmetric *coefficients (subindices that appear twice are summed over).*

Before proceeding, we note that this vertex promulgates a *renormalizable* theory by power counting. Note also that S_1 is intrinsically symmetric in the string coordinates, and that

$$d_{e_1} S_1 = \frac{g}{2} \partial_\mu \big[f_{abc}\, u_a(x, e_1) A_{\nu b}(x, e_2) F_c^{\mu\nu}(x) \big] =: \partial_\mu Q^\mu(x, e_1, e_2). \tag{13}$$

Proof of Proposition 1. We abbreviate $A^i \equiv A(x, e_i)$ for the SLF and make the Ansatz

$$S_1'(x, e_1, e_2, e_3) = g f^1_{abc} A^1_{\mu a} A^2_{\nu b}\, \partial^\mu A^{3\nu}_c \tag{14}$$

for the cubic coupling of the fields, with the coefficients f^1_{abc} being unknown a priori. We shall show that string independence forces this to be of the form S_1 as in (12), where thos e numerical coefficients f_{abc} are completely skew-symmetric. We first split $f^1_{abc} =: d_{abc} + f^2_{abc}$ into a symmetric ($d_{abc} = d_{acb}$) and a skew-symmetric part ($f^2_{abc} = -f^2_{acb}$) under exchange of the second and third indices. After symmetrizing (14) in $e_2 \leftrightarrow e_3$, the contribution of d_{abc} yields a divergence:

$$d_{abc} A^1_{\mu a} (A^2_{\nu b} \partial^\mu A^{3\nu}_c + A^3_{\nu b} \partial^\mu A^{2\nu}_c) = \partial^\mu (d_{abc}\, A^1_{\mu a} A^2_{\nu b} A^{3\nu}_c). \tag{15}$$

We can therefore replace $S_1'(x, e_1, e_2, e_3)$ by

$$S_1''(x, e_1, e_2, e_3) := g f^2_{abc} A^1_{\mu a} A^2_{\nu b} \partial^\mu A^{3\nu}_c. \tag{16}$$

Its symmetrized version satisfies

$$d_{e_1} \textstyle\sum_{\pi \in S_3} S_1''(x, e_{\pi(1)}, e_{\pi(2)}, e_{\pi(3)}) = 2g f^2_{abc} (\partial_\nu A^2_{\mu a}\, \partial^\mu A^{3\nu}_b + \partial_\nu A^2_{\mu b}\, \partial^\mu A^{3\nu}_a) u^1_c + \text{div}. \tag{17}$$

We next split the coefficients $f^2_{abc} = f^+_{abc} + f^-_{abc}$ into symmetric and skew-symmetric parts under exchange of the first two indices, $f^\pm_{abc} = \pm f^\pm_{bac}$. The skew-symmetric part f^-_{abc} enters into (17), whereas the symmetric part f^+_{abc} contributes

$$d_{e_1} \sum_{\pi \in S_3} S_1''(x, e_{\pi(1)}, e_{\pi(2)}, e_{\pi(3)}) = 4g f^+_{abc} \partial_\nu A^2_{\mu a}\, \partial^\mu A^{3\nu}_b u^1_c + \text{div}. \tag{18}$$

By our basic postulate, this must be a divergence. Since the operators $\partial_\nu A^2_{\mu a}\, \partial^\mu A^{3\nu}_b u^1_c$ are linearly independent, that can happen if and only if the symmetric part f^+_{abc} is identically zero. This leads to complete skew-symmetry of the $f^2_{abc} \equiv f^-_{abc} =: f_{abc}$; that is, the string independence principle constrains S_1'' in (16) to the form

$$S_1''' = g f_{abc} A^1_{\mu a} A^2_{\nu b} \partial^\mu A^{3\nu}_c = \frac{g}{2} f_{abc}\, A^1_{\mu a} A^2_{\nu b} F^{\mu\nu}_c =: S_1, \tag{19}$$

so that the dependence on e_3 is trivial and Formula (12) with the stated skew-symmetry condition is established. □

It is worth pointing out that the above reasoning for the form of S_1 becomes simpler in our SLF context than in the quantum gauge invariance approach; see Section 3.1 of [30], the inference there being in terms of the customary fields and their ungainly retinue of unphysical fields.

2.2. Dealing with String Independence at Second Order: Preliminaries

Perturbative string independence should hold at every order in the couplings, surviving renormalization. Now, the f_{abc} do not yet a Lie algebra make; for that, one needs to prove a *Jacobi identity*. This will be obtained from string independence in constructing the functional $\mathbb{S}$-matrix at second order in the couplings.

Outside the exceptional set $\mathbb{D}_2$ from (10), time-ordered products of string-localized fields reduce to ordinary products where the order of terms is determined by the geometric time-ordering of the string segments. There, string variation and derivatives commute with time ordering, and we have

$$\begin{aligned} d_{e_1} S_2(x, e_1, e_2; x', e_1', e_2') &= d_{e_1} \mathrm{T}(S_1(x, e_1, e_2) S_1(x', e_1', e_2')) \\ &= \partial_\mu \mathrm{T}(Q^\mu(x, e_1, e_2) S_1(x', e_1', e_2')). \end{aligned} \tag{20}$$

Above, T denotes a generic time-ordering recipe, that is, an extension of the time ordering across $\mathbb{D}_2$. It does *not* automatically follow that (20) holds over the *whole* $(x, e_1, e_2; x', e'_1, e'_2)$ set. The challenge is to manufacture a time-ordered product S_2 so that this property holds everywhere after appropriate symmetrization in the string variables. The construction of S_2 by solving the *obstructions* to such an identity will *impose* the couplings of "non-Abelian gauge theory". The vector quantity Q^μ plays a central role in our development.

Candidate extensions across $\mathbb{D}_2$ are restricted by the requirement that the Wick expansion hold everywhere. We are concerned only with the tree graph for gluon–gluon scattering. Its corresponding amplitude is of the general form

$$\mathrm{T}(UV')_{\mathrm{tree}} = \sum_{\varphi,\chi'} \frac{\partial U}{\partial \varphi} \langle\langle \mathrm{T}\, \varphi\, \chi' \rangle\rangle \frac{\partial V'}{\partial \chi'}, \tag{21}$$

where $\langle\langle - \rangle\rangle$ denotes a vacuum expectation value, the sum in the brackets goes over all the free fields entering the monomials $U(x)$, $V(x')$, and we employ formal derivation within the Wick polynomials.

For time-ordered products of the fields entering S_2 in our model, we naturally consider, in the first place,

$$\langle\langle \mathrm{T}_0\, \varphi(x,e)\, \chi(x',e') \rangle\rangle := \frac{i}{(2\pi)^4} \int d^4p\, \frac{e^{-i(p(x-x'))}}{p^2 + i0}\, M^{\varphi\chi}(p,e,e'), \tag{22}$$

where the $M^{\varphi\chi}(p,e,e')$ are given by

$$M^{\varphi\chi}_{*\bullet} := \sum_\sigma \overline{u^{\sigma;\varphi}_*(p,e)}\, u^{\sigma;\chi}_\bullet(p,e'), \tag{23}$$

for the appropriate spacetime indices $*, \bullet$, in terms of the respective intertwiners. We need

$$M^{FA}_{\mu\nu,\lambda}(p,e') = i\bigg(p_\mu g_{\nu\lambda} - p_\nu g_{\mu\lambda} + p_\lambda \frac{p_\nu e'_\mu - p_\mu e'_\nu}{(pe') + i0} \bigg), \tag{24}$$

$$M^{FF}_{\mu\nu,\kappa\lambda}(p) = p_\nu p_\kappa\, g_{\mu\lambda} + p_\mu p_\lambda\, g_{\nu\kappa} - p_\nu p_\lambda\, g_{\mu\kappa} - p_\mu p_\kappa\, g_{\nu\lambda}, \tag{25}$$

to be found for instance in our [29]. Note that in all generality,

$$d_e \langle\langle \mathrm{T}_0\, \varphi(x,e)\, \chi(x',e') \rangle\rangle = \langle\langle \mathrm{T}_0\, d_e\varphi(x,e)\, \chi(x',e') \rangle\rangle, \tag{26}$$

and similarly for $d_{e'}$.

The problem of resolving the obstructions to Equation (20) reduces to an extension problem for numerical distributions by carefully constructing the contractions (21). At present we have, with completely skew-symmetric coefficients f_{abc},

$$S_1 = \frac{g}{2} f_{abc}\, A^1_{\mu a} A^2_{\nu b} F^{\mu\nu}_c \quad \text{and} \quad Q^\mu(x,e_1,e_2) = \frac{g}{2} f_{abc}\, u^1_a A^2_{\nu b} F^{\mu\nu}_c. \tag{27}$$

Clearly S_1 is symmetric under exchange of the string variables. We want the expression $S_2(x, e_1, e_2; x', e'_1, e'_2)$ to be symmetric under the exchange $(x, e_1, e_2) \leftrightarrow (x', e'_1, e'_2)$. Therefore, resolving all obstructions to Equation (20) is equivalent to inspecting the obstruction

$$\begin{aligned} \mathcal{O}^1 &:= d_{e_1}\big(\mathrm{T}_0(S_1(x,e_1,e_2) S_1(x',e'_1,e'_2))\big)_{\mathrm{tree}} - \partial_\mu\, \mathrm{T}_0\big(Q^\mu(x,e_1,e_2) S_1(x',e'_1,e'_2)\big)_{\mathrm{tree}} \\ &= \mathrm{T}_0\big(\partial_\mu Q^\mu(x,e_1,e_2) S_1(x',e'_1,e'_2)\big)_{\mathrm{tree}} - \partial_\mu\, \mathrm{T}_0\big(Q^\mu(x,e_1,e_2) S_1(x',e'_1,e'_2)\big)_{\mathrm{tree}}, \end{aligned} \tag{28}$$

to investigate how it can be made to vanish after appropriate symmetrization. To this end, with the delta-function δ_e along the string e defined as

$$\delta_e(x - x') := \int_0^\infty dt\, \delta(x + te - x'), \tag{29}$$

one obtains, for similarly coloured gluons,

$$\partial_\mu \langle\!\langle \mathrm{T}_0(F^{\mu\nu}(x)A^\lambda(x',e)) \rangle\!\rangle = i\big[g^{\nu\lambda}\delta(x-x') - e^\nu\partial^\lambda\delta_{-e}(x-x')\big]; \tag{30}$$

$$\partial_\mu \langle\!\langle \mathrm{T}_0(F^{\mu\nu}(x)F^{\kappa\lambda}(x')) \rangle\!\rangle = i(g^{\nu\kappa}\partial^\lambda - g^{\nu\lambda}\partial^\kappa)\,\delta(x-x'). \tag{31}$$

These formulae are easily derived by use of Equations (22), (24) and (25), respectively.

2.3. The Jacobi Identity Emerges

We compute, with an eye on (21) and (28),

$$\begin{aligned}\mathcal{O}^1 &= -\frac{g}{2} f_{abc} \sum_{\chi'} u_a^1 A_{\nu b}^2 \partial_\mu \langle\!\langle \mathrm{T}_0\, F_c^{\mu\nu} \chi' \rangle\!\rangle \frac{\partial S_1(x',e_1',e_2')}{\partial \chi'} \\ &= -\frac{g^2}{4} f_{abc} f_{dfc} u_a^1 A_{\nu b}^2 \big[\partial_\mu \langle\!\langle \mathrm{T}_0\, F^{\mu\nu} F'^{\kappa\lambda} \rangle\!\rangle A'^{1'}_{\kappa d} A'^{2'}_{\lambda f} \\ &\qquad + \partial_\mu \langle\!\langle \mathrm{T}_0\, F^{\mu\nu} A'^{1'}_{\kappa} \rangle\!\rangle A'^{2'}_{\lambda d} F'^{\kappa\lambda}_f - \partial_\mu \langle\!\langle \mathrm{T}_0\, F^{\mu\nu} A'^{2'}_{\lambda} \rangle\!\rangle A'^{1'}_{\kappa d} F'^{\kappa\lambda}_f\big],\end{aligned} \tag{32}$$

where $A'^{1'}_{\kappa d} \equiv A_{\kappa d}(x',e_1')$, $A'^{2'}_{\lambda f} \equiv A_{\lambda f}(x',e_2')$, and similarly. Employing (30) and (31), this obstruction equals

$$\begin{aligned}&-i\frac{g^2}{4} f_{abc} f_{dfc} u_a^1 A_{\nu b}^2 \big[A^{1'}_{\kappa d} A^{2'}_{\lambda f} (g^{\nu\kappa}\partial^\lambda - g^{\nu\lambda}\partial^\kappa)\,\delta(x-x') \\ &\qquad + A'^{2'}_{\lambda d} F'^{\kappa\lambda}_f \big(g^\nu_\kappa\,\delta(x-x') - e_1^\nu \partial_\kappa \delta_{-e_1'}(x-x')\big) \\ &\qquad - A'^{1'}_{\kappa d} F'^{\kappa\lambda}_f \big(g^\nu_\lambda\,\delta(x-x') - e_2^\nu \partial_\lambda \delta_{-e_2'}(x-x')\big)\big].\end{aligned} \tag{33}$$

Exploiting $\partial^\kappa \delta_{-e_i'}(x-x') = -\partial^\kappa_{x'} \delta_{-e_i'}(x-x')$ as well as the skew-symmetry of f_{dfc} and Maxwell's equations for $F'^{\kappa\lambda}_f$, we see that terms of the type

$$A'^{2'}_{\lambda d} F'^{\kappa\lambda}_f\, \partial_\kappa\, \delta_{-e_1'}(x-x') = -\partial'_\kappa\big[A'^{2'}_{\lambda d} F'^{\kappa\lambda}_f\, \delta_{-e_1'}(x-x')\big] \tag{34}$$

form a divergence of an expression supported at the exceptional set $\mathbb{D}_2$. Integrating by parts in the first line (33), the obstruction reads, up to a divergence supported at $\mathbb{D}_2$,

$$\begin{aligned}&-i\frac{g^2}{4} f_{abc} f_{dfc} \big[-\partial^\lambda u_a^1 A_{\nu b}^2 A_d^{1'\nu} A^{2'}_{\lambda f} - u_a^1 \partial^\lambda A_{\nu b}^2 A_d^{1'\nu} A^{2'}_{\lambda f} + \partial^\kappa u_a^1 A_{\nu b}^2 A^{1'}_{\kappa d} A_f^{2'\nu} \\ &\qquad + u_a^1 \partial^\kappa A_{\nu b}^2 A^{1'}_{\kappa d} A_f^{2'\nu} + u_a^1 A_{\nu b}^2 A^{2'}_{\lambda d} F_f^{\nu\lambda} - u_a^1 A_{\nu b}^2 A^{1'}_{\kappa d} F_f^{\kappa\nu}\big]\,\delta(x-x') \\ &= -i\frac{g^2}{4} f_{abc} f_{dfc} u_a^1 \big[F_b^{\mu\nu} A^{1'}_{\mu d} A^{2'}_{\nu f} + A^2_{\mu b} A^{2'}_{\nu d} F_f^{\mu\nu} + A^2_{\mu b} A^{1'}_{\nu d} F_f^{\mu\nu}\big]\,\delta(x-x') \end{aligned} \tag{35}$$

$$-i\frac{g^2}{4} f_{abc} f_{dfc} d_{e_1} \big[A_a^{1\mu} A_{\nu b}^2 A^{1'}_{\mu d} A_f^{2'\nu} - A_a^{1\mu} A_{\nu b}^2 A_d^{1'\nu} A^{2'}_{\mu f}\big]\,\delta(x-x'). \tag{36}$$

We have used that $\partial^\mu u_a^1 = d_{e_1} A_a^{1\mu}$. On symmetrizing in the variables e_2, e_1', e_2', the line (35) becomes proportional to

$$-i\,g^2 u_a^1 F_b^{\mu\nu} \big[A^{1'}_{\mu d} A^{2'}_{\nu f} + A^2_{\mu d} A^{2'}_{\nu f} + A^2_{\mu d} A^{1'}_{\nu f}\big] \big[f_{abc} f_{cdf} + f_{afc} f_{cbd} + f_{adc} f_{cfb}\big]\,\delta(x-x'). \tag{37}$$

It follows that the Jacobi identity

$$f_{abc} f_{cdf} + f_{afc} f_{cbd} + f_{adc} f_{cfb} = 0 \tag{38}$$

is a *necessary* condition for the obstruction to string independence to vanish.

2.4. The Quartic Term

We still have to deal with the term of the type $\partial uAAA \sim d_{e_1} AAAA$ in (36). Using the skew-symmetry of the f_{abc}, symmetrization of (36) in the variables e'_1, e'_2 yields the identical symmetrization of

$$d_{e_1}\left[-i\frac{g^2}{2} f_{abc}f_{dfc}A_a^\mu(x,e_1)A_b^\nu(x,e_2)A_{\mu d}(x,e'_1)A_{\nu f}(x,e'_2)\right]\delta(x-x'). \tag{39}$$

Thus, to keep string independence, a summand

$$i\,\frac{g^2}{2}\,f_{abc}f_{dfc}A_a^\mu(x,e_1)A_b^\nu(x,e_2)A_{\mu d}(x,e'_1)A_{\nu f}(x,e'_2)\,\delta(x-x'), \tag{40}$$

whose string-derivative d_{e_1} cancels the expression (39), must be added to S_2. This is the four-gluon coupling, usually attributed to the "covariant derivative" present in the "kinetic" QCD Lagrangian. Now, since in the present dispensation a g^2AAAA term is also renormalizable to begin with, we could have introduced it from the outset. Then, a discussion parallel to the above leads again to Equation (40), with precisely the same second-order coefficient in the coupling constant.

In summary, string independence of the $\mathbb{S}$-matrix at second order holds *if and only if* the Jacobi identity with completely skew-symmetric f_{abc} for the cubic coupling (12) holds *and* the above quartic term (40) is present at that order in the $\mathbb{S}$-matrix.

3. Discussion

The outcome of the previous arguments, together with the lessons on QFD in [29], is that Lie algebra structures of the compact type of necessity govern interactions in QFT. Compactness is related to complete skew-symmetry: a finite-dimensional Lie algebra structure with generators X_a defined by $[X_a, X_b] = \sum_c f_{abc}X_c$ does require $f_{abc} = -f_{bac}$ and the Jacobi identity, but not $f_{abc} = -f_{acb}$ in general. The extra requirement imposed by string independence leads to a negative definite Killing form and thus semisimple compactness (see for instance Section 3.6 of [36]).

The authors currently know of *five* different arguments within perturbative QFT for the reductive Lie algebra structure of the interactions: the already classical one by Cornwall, Levin and Tiktopoulos from unitarity bounds at high energy [37]; the Aste–Scharf analysis referred to in Section 2.1; the argument in this paper; and two more found in the book [38]: in its Chapter 27, one is made by elaborating on the spinor-helicity formalism for gluon scattering calculations, and another—most charming of them all—in its Chapter 9 by a variant of Weinberg's "soft limit" reasoning, long ago applied to link helicity 1 particles with charge conservation and helicity 2 particles with universality of gravity. The irrelevance of the gauge condition in the old direct construction of scattering amplitudes by Zwanziger [39] also comes to mind.

This is a good place to take stock of the lessons from [29]. Our argument there was also motivated, in a somewhat contrarian way, by Marshak's thoughts on the "neutrino paradigm" in his posthumous book *Conceptual Foundations of Modern Particle Physics* [40], Chapters 1 and 6 (see also [41]). The sole inputs of our treatment of flavourdynamics in it are the *physical* particle types, masses and charges: "spontaneous symmetry breaking", which comes in succour of the conventional gauge models, is not required, since the SLF models are renormalizable to begin with, irrespectively of mass. String independence at first and second order rules the bosonic couplings, just as in gluodynamics; the non-vanishing masses bring only minor complications. Couplings between bosons and fermions a priori are just asked to respect electric charge conservation and Lorentz invariance. Chirality of the couplings is the outcome of general string independence. The proof requires the presence of the scalar particle and is done with Dirac fermions: from the standpoint of SLFs, it does not make sense to say that lepton or quark currents are "chiral" in QFD: their couplings are. (In the current parlance, fermions in the Standard Model are schizophrenic:

non-chiral in QCD, chiral in QFD. Certainly, from the SLF treatment, one can reengineer the GWS model and its warts, hypercharges and all, in reverse. This was done in Appendix D of [29]).

The cited book by Marshak is still a very good guide for the discussion that follows, witness perhaps to the lack of progress and "deeply disturbing features" [42] affecting the present theoretical apparatus, in contrast with tremendous progress in the experimental realm during recent years.

3.1. Story of Two Principles

The "gauge principle", a *top-down, classical-geometrical* principle which has ruled particle theory for over 60 years, is foreign to QFT. Looking back, a defect—the unavailability in conventional QFT of a Hilbert space framework for massless particles—was elevated into a doctrine. By now, *bottom-up, inherently quantum* principles for the construction of interactions deserve their place in the sun.

If only for sound epistemological reasons, one should ascertain whither the bottom-up approach leads, in present-day particle physics. A Lie algebra and a local Lie group amount to the same thing. Now, the mathematical beauty and power of spin-offs such as extended field configurations in classical Yang–Mills geometry is not to be denied—it is enough to recall [43]. In turn, it is perfectly legitimate to look for *global* Lie group features in modern, bottom-up QFT. However, we contend that those need to be constructed as *quantum* field theory entities and—as anyone who has had to grapple with, e.g., the rigorous definition of Wilson loops for interacting fields in SLF formulations well knows—this is more intricate than presuming classical geometry entities to possess non-perturbative quantum counterparts.

Consider, for instance, Dirac's ad-hoc non-integrable phase and magnetic monopole (and their progeny of 't Hooft–Polyakov monopoles). The basic idea was seductive, and led directly and elegantly to electric charge discreteness. Nevertheless, humbler, essentially perturbative arguments on cancellation of anomalies—up to and including the mixed gravitational-gauge anomaly [44] recalled by Marshak (see Chapter 7 of [40] and [45], among others [46])—are known to provide an explanation for that discreteness (Section 30.4 of [38]). Whereas magnetic monopoles of any kind have stubbornly refused to show up.

This is perhaps the place to comment on the Aharonov–Bohm and Aharonov–Casher effects being held as "proofs" of the "fundamental" character of the standard electromagnetic gauge potential, since the calculation via the electromagnetic field depends on a region where the test particle is not allowed. This is merely a misunderstanding: these effects can be computed by means of the SL A-field, which contains the same information as the F-field. The deep reasons lie in the mind-boggling entanglement properties of QFT, as compared to ordinary quantum mechanics; concretely, in the failure of *Haag duality* for all quantum massless fields with helicity $r \geq 1$. This was shown over 40 years ago [47]. Consult [48] in this respect as well.

3.2. Reassessing the Okubo–Marshak Argument

Marshak was very open to topological and differential-geometric constructions in QFT, and actually Chapter 10 of his book [40], particularly Section 10.3 and Subsection 10.3.c, is still a very good place to learn about instantons and "vacuum tunneling" into topologically inequivalent vacua, apparently leading to the degenerate θ-vacuum—the "strong CP problem"—since the neutron's electric dipole moment is vanishingly small and, according to some, to the "axion".

The steps to the claim of existence of this problem are well known. In the Euclidean setting, first, the finiteness of the classical action functional of course requires $\lim_{|x|\uparrow\infty} F_{\mu\nu}(x) = 0$, for which it is enough to demand that $\lim_{|x|\uparrow\infty} A_\mu(x) \to U^\dagger(x)\,\partial_\mu U(x)$. It is then said that A is "pure gauge". By using an A_0 gauge, a winding number n is related directly to the Euclidean action; in fact, one is dealing with the homotopy group of the three-dimensional sphere, which is the group of integer numbers. The next step is to define the vacuum states in Minkowski space in such a way that instantons become "tunneling events" between two different Minkowski vacua $|m\rangle$, $|m'\rangle$ with respective winding numbers m, m' satisfying $m' - m = n$. Then, it is argued that the "true vacuum" is of the form $|\theta\rangle = \sum_m e^{-im\theta} |m\rangle$ with $\langle\theta' \mid \theta\rangle = 2\pi\delta(\theta - \theta')$, and the value of θ is anyone's guess. This is aptly described by Marshak in Subsection 10.3.c of [40].

Nevertheless, he came to regard the axion hypothesis as a bridge too far. In [2], it was rigorously proved that the BRS charge "kills" the *physical* vacuum, which if cyclic must be unique. However, that charge and the antiunitary operator for CPT invariance commute, and this obviously demands a zero (or π) value for the θ-parameter of the instanton makeup [1]. One could contend that the θ-vacua are non-normalizable (a sign of trouble in itself) and that the physical vacuum is a superposition of them. However, by means of a simple procedure, Okubo and Marshak showed how in that case CPT invariance still guarantees the experimentally measurable value of θ to be zero.

The existence of the original "visible" axion had been already disallowed by experiment and observation [49] by the time of the writing of [40], leading to a succession of "invisible" axions, that must be "super-light", but still face experimental limitations. Marshak concludes: "It does seem that the odds of finding the 'invisible' axion are rapidly diminishing and that the incentive to carry on the ingenious searches for the 'invisible' axions is fueled more by astrophysical and cosmological interest than by any hope of salvaging the Peccei–Quinn-type solution of the 'strong CP problem' in QCD." This rings even truer 29 years later.

Making a contention to the same purpose in SLF theory is straightforward. The string-localized vector potentials live on Hilbert space and act cyclically on the vacuum. In fact, every local subalgebra of operators enjoys this property (this is part of the Reeh–Schlieder property [50]). Therefore, θ-vacua are not allowed.

In plainer language: $F^{\mu\nu}(x) \downarrow 0$ at spatial infinity implies $A^{\mu}(x, e) \downarrow 0$. Thus, only the $m = 0$ vacuum (so to speak) occurs: there are no instantons, and no θ-vacua, and the so-called strong CP problem is solved. The moral of this part of the story is that quantum field theory should stand on its own two feet, rather than on classical crutches.

3.3. Coda

The "strong CP problem" and the axion ideas have undergone mutations from the time of their inception to the present day. The relations of the present-day "invisible" axion with the $U_A(1)$ anomaly remain murky; for a recent review of the latter, consult [51]. The contemporary main selling point is still the "axiverse"; in other words, the search for the axion is nowadays essentially model-free. Absence of evidence is not evidence of absence, to be sure, and so the hunting for ALPs is bound to go on [52].

A weak point of most analyses on the present subject is that confinement is not taken into account. However, it does appear to be inimical to the violation of CP invariance [53].

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Appendix A. Time Ordering Outside the String Diagonal

For a vertex coupling S_1 of the form (12), we show $S_2 := \mathrm{T}\big(S_1(x, e_1, e_2)S_1(x', e_1', e_2')\big)$ is fixed outside the string diagonal $\mathbb{D}_2$ from (10) by the causal factorization property

$$\mathrm{T}(S_1 S_1') = S_1 S_1' \quad \text{or} \quad S_1' S_1 \,, \tag{A1}$$

depending on the time ordering of the respective localization regions.

In [33], the corresponding statement was shown for the case of *first-degree* string-localized fields by "chopping" the strings, and it was indicated why the construction runs into difficulties for higher-degree Wick polynomials. In the present case, we are fortunate that the relevant Wick products are just ordinary products (see (A4)), and we are back to the first-degree case. We present the argument in detail.

Let $(x, e_1, e_2; x', e_1', e_2') \in \mathbb{D}_2$ with $e_1 \neq e_2$, $e_1' \neq e_2'$. The localization region of $S_1(x, e_1, e_2)$ consists of the two strings $x + \mathbb{R}^+ e_k$, each of which is chopped into a small compact "head" $\mathcal{R}_k$ containing the vertex x and an infinite "tail" $\mathcal{S}_k$:

$$x + \mathbb{R}^+ e_k = \mathcal{R}_k \cup \mathcal{S}_k, \quad \text{where} \quad \mathcal{R}_k := x + [0, s_k]\, e_k, \quad \mathcal{S}_k := x + [s_k, \infty)\, e_k \,. \tag{A2}$$

Accordingly, the string-localized potential decomposes as $A_\mu(x, e_1) = A_\mu^{\mathrm{H}}(x, e_1) + A_\mu^{\mathrm{T}}(x, e_1)$, where A_μ^{H} is localized in the compact "head" $\mathcal{R}_1$ and A_μ^{T} in the "tail" $\mathcal{S}_1$. The same is true for $A_\mu(x, e_2)$. Then, the vertex coupling S_1 is a linear combination of four terms:

$$S_1 = S_1^{\mathrm{HH}} + S_1^{\mathrm{HT}} + S_1^{\mathrm{TH}} + S_1^{\mathrm{TT}}, \qquad S_1^{\mathrm{KL}} := \frac{g}{2} f_{abc} \,{:}A_{\mu a}^{\mathrm{K}}(e_1) A_{\nu b}^{\mathrm{L}}(e_2) F_c^{\mu\nu}{:}\,. \tag{A3}$$

The terms S_1^{TH} and S_1^{TT} can be written as

$$S_1^{\mathrm{TH}} = \frac{g}{2} f_{abc} A_{\mu a}^{\mathrm{T}}(e_1) \,{:}A_{\nu b}^{\mathrm{H}}(e_2) F_c^{\mu\nu}{:}\,, \qquad S_1^{\mathrm{TT}} = \frac{g}{2} f_{abc} A_{\mu a}^{\mathrm{T}}(e_1)\, A_{\nu b}^{\mathrm{T}}(e_2) F_c^{\mu\nu} \tag{A4}$$

and similarly for S_1^{HT}. We have taken $A_{\mu a}^{\mathrm{T}}(e_1)$ and $A_{\nu b}^{\mathrm{T}}(e_2)$ out of the Wick product; this can be done because all contractions between $A_{\mu a}$, $A_{\nu b}$ and $F_c^{\mu\nu}$ are zero since the indices a, b, c are distinct by the skew-symmetry of f_{abc}.

The localization regions of these terms are as follows. The operator product in S_1^{TT} is the ordinary product of three linear fields: one localized in the string $\mathcal{S}_1$, another in $\mathcal{S}_2$ and a third at x. S_1^{TH} is the product of a linear field localized in the string $\mathcal{S}_1$ and a Wick product localized in the compact interval $\mathcal{R}_2$ that can be made arbitrarily small around x. Similarly for S_1^{HT}. The term S_1^{HH} is a Wick product localized in $\mathcal{R}_1 \cup \mathcal{R}_2$ which also can be made arbitrarily small around x.

We decompose $S_1(x', e_1', e_2')$ in like manner. Then, the second order S_2 is a sum of terms of the form $\mathrm{T}\big[S_1^{\mathrm{KL}}(e_1, e_2)\, S_1^{\mathrm{K'L'}}(e_1', e_2')\big]$, where the operator in brackets is the product of string-localized linear fields and (almost) point-localized Wick products. By our hypothesis that $e_1 \neq e_2$ and $e_1' \neq e_2'$, their localization regions are mutually disjoint. By Proposition 2.2 of [33], such regions can be chronologically ordered, eventually after chopping them into segments, and the corresponding operators can be time-ordered according to (A1). As in the proof of in Proposition 3.2 of [33], one sees that the result is Wick's expansion, where the time-ordering appears only within two-point functions (in contrast to the case considered in [33], here there are products of time-ordered two-point functions, but these are well-defined since they have disjoint arguments). Again by the skew-symmetry of

f_{abc}, only contractions between fields localized on e- and e'-strings occur. Therefore, the restriction $e_1 \neq e_2$ and $e'_1 \neq e'_2$ can be removed. The proof is complete.

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