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Interval Valued T-Spherical Fuzzy Soft Average Aggregation Operators and Their Applications in Multiple-Criteria Decision Making

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Abstract: This paper deals with uncertainty, asymmetric information, and risk modelling in a complex power system. The uncertainty is managed by using probability and decision theory methods. Multiple-criteria decision making (MCDM) is a very effective and well-known tool to investigate fuzzy information more effectively. However, the selection of houses cannot be done by utilizing symmetry information, because enterprises do not have complete information, so asymmetric information should be used when selecting enterprises. In this paper, the notion of soft set $\left(S_{ft}S\right)$ and interval-valued T-spherical fuzzy set (*IVT-SFS*) are combined to produce a new and more effective notion called interval-valued T-spherical fuzzy soft set $\left(IVT-SFS_{ft}S\right)$. It is a more general concept and provides more space and options to decision makers (DMs) for making their decision in the field of fuzzy set theory. Moreover, some average aggregation operators like interval-valued T-spherical fuzzy soft weighted average $\left(IVT-SFS_{ft}WA\right)$ operator, interval-valued T-spherical fuzzy soft ordered weighted average $\left(IVT-SFS_{ft}OWA\right)$ operator, and interval-valued T-spherical fuzzy soft hybrid average $\left(IVT-SFS_{ft}HA\right)$ operators are explored. Furthermore, the properties of these operators are discussed in detail. An algorithm is developed and an application example is proposed to show the validity of the present work. This manuscript shows how to make a decision when there is asymmetric information about an enterprise. Further, in comparative analysis, the established work is compared with another existing method to show the advantages of the present work.

Keywords: interval-valued T-spherical fuzzy soft set; average aggregation operators; multiple-criteria decision making

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Citation: Mahmood, T.; Ahmmad, J.; Ali, Z.; Pamucar, D.; Marinkovic, D. Interval Valued T-Spherical Fuzzy Soft Average Aggregation Operators and Their Applications in Multiple-Criteria Decision Making. *Symmetry* **2021**, *13*, 829. https://doi.org/10.3390/sym13050829

Academic Editor: Jian-Qiang Wang

Received: 6 April 2021
Accepted: 4 May 2021
Published: 9 May 2021

1. Introduction

Multi-criteria decision making (MCDM) is a process that can give the ranking results for the finite alternatives according to the attribute values of different alternatives, and it is an important aspect of decision sciences. In recent years, the development of enterprises and social decision making in all aspects is related to the issue of MCDM, so it is widely applied in all kinds of fields. In the real decision-making process, an important problem is how to express the attribute value more efficiently and accurately. In the real world, because of the complexity of decision-making problems and the fuzziness of decision-making environments, it is not enough to express attribute values of alternatives by exact values. For this, the concept of fuzzy set (FS) was proposed by Zadeh [1], and many extensions have been established by researchers and many new notions were developed over time. Since FS only deals with membership grade (MG) "α" with the condition that $0 \leq \alpha \leq 1$, which is the limited idea, so the idea of FS was further generalized into an interval-valued fuzzy set [2] (IVFS). In many practical examples, we have to deal not only with MG but also consider the non-membership grade (NMG) "γ". Since in FS the NMG is not under consideration, which is a drawback of FS, the concept of intuitionistic fuzzy set (IFS) was established by Atanassov [3] having the characteristics that $0 \leq \alpha + \gamma \leq 1$. In addition, some prioritized IF aggregation operators are discussed in [4]. Moreover, IF interaction aggregation operators and IF hybrid arithmetic and geometric aggregation operators are established in [5,6]. To provide more space to DMs, Atanassov [7] generalized IFS into IVIFS, and some IVIF aggregation operators are given in [8]. Aggregation operators are a valuable tool to deal with the fuzzy information because it converts the whole data into a single value which is helpful in the decision-making process. When DMs provide "0.6" as MG and "0.5" as NMG, then IFS fails to deal with such types of information. To overcome this issue, the idea of IFS was further extended into Pythagorean fuzzy set $\left(P_y FS\right)$ [9] having the condition that $0 \leq \alpha^2 + \gamma^2 \leq 1$. It is a stronger apparatus and it can tackle fuzzy information more effectively. Based on Einstein's t-norm and t-co norm, some generalized fuzzy geometric aggregation operators are given by Garg et al. [10]. This idea is further extended into $IVP_y FS$ and some aggregation operators are provided in [11]. $P_y FS$s also limited notion because when DMs provide 0.7 as MG and "0.9" as NMG, then $P_y FS$ cannot tackle this type of data. To overcome this complexity, this notion is further generalized into q-rung orthopair fuzzy set (q-ROFS) established by Yager [12] having the necessary condition that $0 \leq \alpha^q + \gamma^q \leq 1$. Some q-ROF point weighted aggregation operators are explored in [13]. Some IVq-ROF Archimedean Muirhead Mean operators are discussed in [14]. Molodtsov [15] established the idea of a soft set $\left(S_{ft}S\right)$ which is a parameterization structure to deal with uncertainty in data. Maji et al. [16] explored some new operations and proposed application of $S_{ft}S$. Ali et al. [17] explored the application of $S_{ft}S$ in decision-making problems. Since the idea of $S_{ft}S$ has been established, some new notions are established like a fuzzy soft set $\left(FS_{ft}S\right)$ established by Maji et al. [18], which is the combination of FS and $S_{ft}S$. Some considerable extensions have been developed keeping in view the idea of $FS_{ft}S$ and then IVFS and $S_{ft}S$ are combined by Yang et al. [19] to introduce the new idea called $IVFS_{ft}S$. Since $FS_{ft}S$ is a limited structure, so notions of IF soft set $\left(IFS_{ft}S\right)$ [20] have been developed. Moreover, generalized and group-based generalized intuitionistic fuzzy soft sets with their applications in decision making have been explored in [21,22]. In addition, due to the drawback of $IFS_{ft}S$, the further idea of $IFS_{ft}S$ has been extended into a Pythagorean fuzzy soft set $\left(P_y FS_{ft}S\right)$ [23]. Further q-rung orthopair fuzzy soft set $\left(q - ROFS_{ft}S\right)$ proposed by Hussain et al. [24] developed the notion of $P_y FS_{ft}S$ and also explored some $q - ROFS_{ft}WA$, $q - ROFS_{ft}OWA$ and $q - ROFS_{ft}HA$ operators.

From the mentioned literature, it is clear that all the fuzzy information deals with only MG and NMG. Sometimes, DMs consider the obstinacy grade AG "β" along with MG "α" and NMG "γ" in their information, and there are many practical examples which can be provided in this regard, so due to this reason, the idea of picture fuzzy set (PFS) [25] has been developed, which also considers the AG, which is more general information and provides more space to deal with vagueness in data with condition that $0 \leq \alpha + \beta + \gamma \leq 1$. Similarly, as the idea of IFS is generalized into $P_y FS$, the notion of PFS set is extended into the spherical fuzzy set (SFS) by Mahmood et al. [26] with condition that $0 \leq \alpha^2 + \beta^2 + \gamma^2 \leq 1$. Moreover, Ashraf et al. [27] established the spherical fuzzy Dombi aggregation and proposed their application in group decision-making problems. SFS is a limited idea because if DMs provide "0.9" as an MG, 0.8 as an NMG, and 0.7 as an AG, then both PFS and SFS fail to deal with such types of information, so to overcome this complexity, the notion of T-spherical fuzzy set (T-SFS) has been established by Ullah et al. [28] with condition that $0 \leq \alpha^q + \beta^q + \gamma^q \leq 1$ and exploring some similarities measures based on T-SFNs. Some

T-SF power Muirhead mean operators based on novel operational law have been developed in [29]. Further, Quek et al. [30] established the generalized T-spherical fuzzy weighted aggregation operators on neutrosophic sets. Correlation coefficients for T-SFS and their application in clustering and multi-attribute decision making have been established by Ullah et al. [31] and a note on geometric aggregation operators in the T-SF environment is given in [32]. Furthermore, Ullah et al. [33] proposed T-SF Hamacher aggregation operators. Some T-SF Einstein hybrid aggregation operators and their application in multi-attribute decision-making problems have been proposed by Munir et al. [34]. Based on improved interactive aggregation operators, an algorithm for T-SF multi-attribute decision making has been established by Garg et al. [35]. The idea of T-SFS has been extended to interval-valued T-spherical fuzzy set (IVT-SFS) established by Ullah et al. [36] and they have explored the evaluation of investment policy based on multi-attribute decision making using IVT-SF aggregation operators. Keeping in view the idea of $FS_{ft}S$, $IFS_{ft}S$, $P_yS_{ft}S$ and $q-ROFS_{ft}S$, the notion of PF soft set $\left(PFS_{ft}S\right)$ has been proposed by Yang et al. [37], which generalizes all the above literature due to parameterization structure. The idea of a multi-valued picture fuzzy soft set was proposed by Jan et al. [38]. The study of aggregation operators and their application in decision making can be seen in [39,40]. Perveen et al. [41] extended the idea of $PFS_{ft}S$ into the spherical fuzzy soft set $\left(SFS_{ft}S\right)$, which is the combination of $S_{ft}S$ and SFS. Since T-SFS is more general than SFS, so the concept of $SFS_{ft}S$ is further extended into a T-spherical fuzzy soft set $\left(T-SFS_{ft}S\right)$ proposed by Guleria et al. [42]. Moreover, some new operations on interval-valued picture fuzzy soft set $\left(IVPFS_{ft}\right)$ are discussed in [43] and interval-valued spherical fuzzy weighted arithmetic means (IVSFWAM) and interval-valued spherical fuzzy weighted geometric mean (IVSFWGM) operators are established in [44].

The notion of interval-valued T-spherical fuzzy sets and soft sets is very closely related to the notion of symmetry. Based on symmetry, we can talk about the mixture of both theories. We can extend the notion of interval-valued T-spherical fuzzy to interval-valued T-spherical fuzzy soft sets, especially when determining the aggregate interval-valued T-spherical fuzzy soft number estimated by several experts and in a situation where there is imperfect knowledge (when one party has different information to another).

MCDM is a very effective and well-known tool to investigate fuzzy information more effectively. Thus, from the mentioned literature, it is clear that the interval-valued structures are more general and gain more attention in decision-making problems. To the best of our knowledge, there is no work on combining the notion of IVT-SFS and $S_{ft}S$. Hence, in this paper, the notion of $S_{ft}S$ and IVT-SFS are combined to produce a new notion called the $IVT-SFS_{ft}S$. It is a more general concept and provides more space to DMs for making their decision in the field of fuzzy set theory. Moreover, some new average aggregation operators like $IVT-SFS_{ft}WA$ operator and $IVT-SFS_{ft}OWA$ operators are explored. $IVT-SFS_{ft}WA$ can only find the $IVT-SFS_{ft}$ values and $IVT-SFS_{ft}OWA$ weight the ordered position. Hence, due to this drawback, the $IVT-SFS_{ft}HA$ operators are explored, as they can account for both aspects. Furthermore, the properties of these operators are discussed in detail. An algorithm is developed, and an application example is proposed to show the validity of the proposed work. In a comparative analysis, the present work is compared with another existing method to show the advantages the present work offers.

The manuscript is structured as follows: Section 2 deals with basic notions of PFS, SFS, T-SFS, $S_{ft}S$, $PFS_{ft}S$, $SFS_{ft}S$ and $T-SFS_{ft}S$. Moreover, their operations are discussed. Section 3 deals with the basic notion of $IVT-SFS_{ft}S$ and some fundamental operations on this notion are discussed in detail. In Section 4, we have established some new operators called $IVT-SFS_{ft}WA$, $IVT-SFS_{ft}OWA$ and $IVT-SFS_{ft}HA$ operator. In Section 5, we have established an algorithm and an illustrative example is given to show the validity of the present work. In addition, we have provided a comparative analysis of the present work to demonstrate its advantages compared to the approaches from the literature. Finally, Section 6 provides concluding remarks.

2. Preliminaries

This section deals with the basic notion of SFS, T-SFS, $S_{ft}S$, $SFS_{ft}S$ and $T - SFS_{ft}S$. Moreover, their basic properties are discussed which will help us in further sections.

Definition 1 [26]. *An SFS for a non-empty set X is given by*

$$P = \{< x, \alpha(x), \beta(x), \gamma(x) > | x \in X\}$$

where $\boldsymbol{\alpha(x)} : \boldsymbol{X \to [0, 1]}$ *is the MG,* $\boldsymbol{\beta(x) : X \to [0, 1]}$ *is the AG and* $\boldsymbol{\gamma(x) : X \to [0, 1]}$ *is the NMG with condition that* $\boldsymbol{0 \leq \alpha(x)^2 + \beta(x)^2 + \gamma(x)^2 \leq 1}$.

Definition 2 [26]. *A T-SFS for a non-empty set X is given by*

$$P = \{< x, \alpha(x), \beta(x), \gamma(x) > | x \in X\}$$

where $\alpha(x) : X \to [0, 1]$ *is the MG,* $\beta(x) : X \to [0, 1]$ *is the AG and* $\gamma(x) : X \to [0, 1]$ *is NMG with the condition that* $0 \leq (\alpha(x))^q + (\beta(x))^q + (\gamma(x))^q \leq 1$.

Definition 3 [15]. *Let be a fixed set and E be a set of parameters and* $H \subseteq E$, *then the pair* (F, H) *is said to be* $S_{ft}S$ *over the universal set , where F is the map given by* $F : H \to P()$, *where* $P()$ *is the power set of .*

Definition 4 [18]. *Let be a fixed set and E be a set of parameters and* $H \subseteq E$, *then the pair* (F, H) *is said to be* $FS_{ft}S$ *over the universal set , where F is the map given by* $F : H \to FS^{()}$, *where* $FS^{()}$ *is the family of all FS over given as*

$$\boldsymbol{F(s_j) = \{x_i, \alpha_j(x_i) \mid x \in \mho\}}$$

Definition 5 [41]. Let *be a fixed set and E be a set of parameters and* $H \subseteq E$, *then the pair* (F, H) *is said to be* $SFS_{ft}S$ *over the universal set , where F is the map given by* $F : H \to SFS^{()}$, *where* $SFS^{()}$ *is the family of all SFS over given as*

$$F(s_j) = \{x_i, \alpha_j(x_i), \beta_j(x_i), \gamma_j(x_i) | x \in \}$$

with condition that $0 \leq (\alpha_j(x_i))^2 + (\beta_j(x_i))^2 + (\gamma_j(x_i))^2 \leq 1$.

Definition 6 [42]. *Let be a fixed set and E be a set of parameters and* $H \subseteq E$, *then the pair* (F, H) *is said to be* $T - SFS_{ft}S$ *over the universal set , where F is the map given by* $F : H \to T - SFS^{()}$, *where* $T - SFS^{()}$ *is the family of all SFS over given as*

$$F(s_j) = \{x_i, \alpha_j(x_i), \beta_j(x_i), \gamma_j(x_i) | x \in \}$$

with condition that $0 \leq (\alpha_j(x_i))^q + (\beta_j(x_i))^q + (\gamma_j(x_i))^q \leq 1$.

Definition 7 [36]. *An IVT-SFS for a non-empty set X is given by*

$$P = \{< x, \alpha(x), \beta(x), \gamma(x) > | x \in X\}$$

where $\alpha(x) : X \to [0, 1]$ *such that* $\alpha(x) = [\alpha^L(x), \alpha^U(x)]$ *is the MG,* $\beta(x) : X \to [0, 1]$ *such that* $\beta(x) = [\beta^L(x), \beta^U(x)]$ *is the AG and* $\gamma(x) : X \to [0, 1]$ *such that* $\gamma(x) = [\gamma^L(x), \gamma^U(x)]$ *is NMG with the condition that* $0 \leq ((\alpha^U(x)))^q + (\beta^U(x))^q + (\gamma^U(x))^q \leq 1$.

Definition 8 [36]. *Let* $F_1 = ([\alpha^L{}_1, \alpha^U{}_1], [\beta^L{}_1, \beta^U{}_1], [\gamma^L{}_1, \gamma^U{}_1])$, $F_2 = ([\alpha^L{}_2, \alpha^U{}_2], [\beta^L{}_2, \beta^U{}_2], [\gamma^L{}_2, \gamma^U{}_2])$ *and* $F = ([\alpha^L, \alpha^U], [\beta^L, \beta^U], [\gamma^L, \gamma^U])$ *be three IVT-SFN and*

$K > 0$. Let '$\vee$' denote the maximum and '$\wedge$' denote the minimum. Then basic operation on IVT-SFN is defined by

1. $F_1 \subseteq F_2$ *Iff* $\alpha^L{}_1 \le \alpha^L{}_2$, $\alpha^U{}_1 \le \alpha^U{}_2$, $\beta^L{}_1 \le \beta^L{}_2$, $\beta^U{}_1 \le \beta^U{}_2$ *and* $\gamma^L{}_1 \ge \gamma^L{}_2$, $\gamma^U{}_1 \ge \gamma^U{}_2$.
2. $F_1 = F_2$ *Iff* $F_1 \subseteq F_2$ *and* $F_2 \subseteq F_1$.
3. $F_1 \cup F_2 = \left\{ \begin{array}{c} \left[\vee\left(\alpha^L{}_1,\ \alpha^L{}_2\right),\ \vee\left(\alpha^U{}_1,\ \alpha^U{}_2\right)\right],\ \left[\wedge\left(\beta^L{}_1,\ \beta^L{}_2\right),\ \wedge\left(\beta^U{}_1,\ \beta^U{}_2\right)\right], \\ \left[\wedge\left(\gamma^L{}_1,\ \gamma^L{}_2\right),\ \wedge\left(\gamma^U{}_1,\ \gamma^U{}_2\right)\right] \end{array} \right\}.$
4. $F_1 \cap F_2 = \left\{ \begin{array}{c} \left[\wedge\left(\alpha^L{}_1,\ \alpha^L{}_2\right),\ \wedge\left(\alpha^U{}_1,\ \alpha^U{}_2\right)\right],\ \left[\wedge\left(\beta^L{}_1,\ \beta^L{}_2\right),\ \wedge\left(\beta^U{}_1,\ \beta^U{}_2\right)\right], \\ \left[\vee\left(\gamma^L{}_1,\ \gamma^L{}_2\right),\ \vee\left(\gamma^U{}_1,\ \gamma^U{}_2\right)\right] \end{array} \right\}.$
5. $F^c = \left(\left[\gamma^L,\ \gamma^U\right],\ \left[\beta^L,\ \beta^U\right],\ \left[\alpha^L,\ \alpha^U\right]\right).$
6. $F_1 \oplus F_2 = \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{\left(\alpha^L{}_1\right)^q +\ \left(\alpha^L{}_2\right)^q - \left(\alpha^L{}_1\right)^q\left(\alpha^L{}_2\right)^q}, \\ \sqrt[q]{\left(\alpha^U{}_1\right)^q +\ \left(\alpha^U{}_2\right)^q - \left(\alpha^U{}_1\right)^q\left(\alpha^U{}_2\right)^q} \end{array} \right],\ \left[\beta^L{}_1\beta^L{}_2,\ \beta^U{}_1\beta^U{}_2\right], \\ \left[\gamma^L{}_1\gamma^L{}_2,\ \gamma^U{}_1\gamma^U{}_2\right] \end{array} \right).$
7. $F_1 \otimes F_2 = \left(\begin{array}{c} \left[\alpha^L{}_{11}\alpha^L{}_{12},\ \alpha^U{}_{11}\alpha^U{}_{12}\right],\ \left[\beta^L{}_{11}\beta^L{}_{12},\ \beta^U{}_{11}\beta^U{}_{12}\right], \\ \left[\begin{array}{c} \sqrt[q]{\left(\gamma^L{}_1\right)^q +\ \left(\gamma^L{}_2\right)^q - \left(\gamma^L{}_1\right)^q\left(\gamma^L{}_2\right)^q}, \\ \sqrt[q]{\left(\gamma^U{}_1\right)^q +\ \left(\gamma^U{}_2\right)^q - \left(\gamma^U{}_1\right)^q\left(\gamma^U{}_2\right)^q} \end{array} \right] \end{array} \right).$
8. $F^K = \left(\left[\left(\alpha^L\right)^K,\ \left(\alpha^U\right)^K\right],\ \left[\left(\beta^L\right)^K,\ \left(\beta^U\right)^K\right], \left[\sqrt[q]{1 -\ \left(\gamma^{Lq}\right)^k}, \sqrt[q]{1 -\ \left(\gamma^{Uq}\right)^k}\right]\right).$
9. $KF = \left(\left[\sqrt[q]{1 - \left(\alpha^{Lq}\right)^k}, \sqrt[q]{1 - \left(\alpha^{Uq}\right)^k}\right],\ \left[\left(\beta^L\right)^K,\ \left(\beta^U\right)^K\right], \left[\left(\gamma^L\right)^K,\ \left(\gamma^U\right)^K\right]\right).$

3. Interval-Valued T-Spherical Fuzzy Soft Set $\left(\boldsymbol{IVT - SFS_{ft}S}\right)$

This section deals with the fundamental notion of $IVT - SFS_{ft}S$. Furthermore, some basic operations are defined according to this new notion. Moreover, we define score function (SF) and accuracy function (AF) based on $IVT - SFS_{ft}$ numbers.

Definition 9. *Consider a soft set (, E) and $H \subseteq E$. A pair $(F,\ H)$ is said to be an Interval-valued T-spherical fuzzy soft set $\left(IVT - SFS_{ft}S\right)$ over the universal set , where F is the map given by $F : H \to IVT - SFS$, which is defined to be*

$$F_{s_j}(x_i) = \left\{<\ x_i,\ \left[\alpha^L{}_j(x_i), \alpha^U{}_j(x_i)\right], \left[\beta^L{}_j(x_i), \beta^U{}_j(x_i)\right],\ \left[\gamma^L{}_j(x_i), \gamma^U{}_j(x_i)\right]\ > \middle| x_i \in \right\}$$

where $IVT - SFS$ represent the collection of all interval-valued T-spherical fuzzy sets over . Here $\left[\alpha^L{}_j(x_i), \alpha^U{}_j(x_i)\right]$, $\left[\beta^L{}_j(x_i), \beta^U{}_j(x_i)\right]$ and $\left[\gamma^L{}_j(x_i), \gamma^U{}_j(x_i)\right]$, represent the membership grade, obstinacy grade, and non-membership grade of an object $x_i \in$ to a set F_{s_j}, respectively, with the condition that $0 \le \left(\alpha^U{}_j(x_i)\right)^q +\ \left(\beta^U{}_j(x_i)\right)^q + \left(\gamma^U{}_j(x_i)\right)^q \le 1$. For the sake of simplicity $F_{s_j}(x_i) = \left\{<\ x_i,\ \left[\alpha^L{}_j(x_i), \alpha^U{}_j(x_i)\right], \left[\beta^L{}_j(x_i), \beta^U{}_j(x_i)\right],\ \left[\gamma^L{}_j(x_i), \gamma^U{}_j(x_i)\right] >\right\}$ is denoted by $F_{s_{ij}} = \left(\left[\alpha^L{}_j(x_i), \alpha^U{}_j(x_i)\right],\ \left[\beta^L{}_j(x_i), \beta^U{}_j(x_i)\right],\ \left[\gamma^L{}_j(x_i), \gamma^U{}_j(x_i)\right]\right)$, which is called interval-valued T-Spherical fuzzy soft number $\left(IVT - SFS_{ft}N\right)$. Moreover, refusal degree is defined by

$$\delta_{F_{s_{ij}}} = \left[\begin{array}{c} \sqrt[q]{1 - \left(\alpha^L{}_j(x_i)\right)^q +\ \left(\beta^L{}_j(x_i)\right)^q + \left(\gamma^L{}_j(x_i)\right)^q}, \\ \sqrt[q]{1 - \left(\alpha^U{}_j(x_i)\right)^q +\ \left(\beta^U{}_j(x_i)\right)^q + \left(\gamma^U{}_j(x_i)\right)^q} \end{array} \right].$$

Definition 10. *Let $F_{s_{11}} = \left(\left[\alpha^L{}_{11},\ \alpha^U{}_{11}\right],\ \left[\beta^L{}_{11},\ \beta^U{}_{11}\right],\ \left[\gamma^L{}_{11},\ \gamma^U{}_{11}\right]\right)$, $F_{s_{12}} = \left(\left[\alpha^L{}_{12},\ \alpha^U{}_{12}\right],\ \left[\beta^L{}_{12},\ \beta^U{}_{12}\right],\ \left[\gamma^L{}_{12},\ \gamma^U{}_{12}\right]\right)$ and $F = \left(\left[\alpha^L,\ \alpha^U\right],\ \left[\beta^L,\ \beta^U\right],\ \left[\gamma^L,\ \gamma^U\right]\right)$ be three $IVT - SFS_{ft}Ns$ and $K > 0$. Then basic operation on $IVT - SFS_{ft}Ns$ are defined by*

1. $F_{s_{11}} \subseteq F_{s_{12}}$ *Iff* $\alpha^L{}_{11} \le \alpha^L{}_{12}$, $\alpha^U{}_{11} \le \alpha^U{}_{12}$, $\beta^L{}_{11} \le \beta^L{}_{12}$, $\beta^U{}_{11} \le \beta^U{}_{12}$ *and* $\gamma^L{}_{11} \ge \gamma^L{}_{12}$, $\gamma^U{}_{11} \ge \gamma^U{}_{12}$.
2. $F_{s_{11}} = F_{s_{12}}$ *Iff* $F_{s_{11}} \subseteq F_{s_{12}}$ *and* $F_{s_{12}} \subseteq F_{s_{11}}$.
3. $F_{s_{11}} \cup F_{s_{12}} = \left\{ \begin{array}{c} \left[\vee\left(\alpha^L{}_{11},\ \alpha^L{}_{12}\right),\ \vee\left(\alpha^U{}_{11},\ \alpha^U{}_{12}\right)\right],\ \left[\wedge\left(\beta^L{}_{11},\ \beta^L{}_{12}\right),\ \wedge\left(\beta^U{}_{11},\ \beta^U{}_{12}\right)\right], \\ \left[\wedge\left(\gamma^L{}_{11},\ \gamma^L{}_{12}\right),\ \wedge\left(\gamma^U{}_{11},\ \gamma^U{}_{12}\right)\right] \end{array} \right\}.$

4. $F_{s_{11}} \cap F_{s_{12}} = \left\{ \begin{array}{c} [\wedge(\alpha^L{}_{11}, \alpha^L{}_{12}), \wedge(\alpha^U{}_{11}, \alpha^U{}_{12})], [\wedge(\beta^L{}_{11}, \beta^L{}_{12}), \wedge(\beta^U{}_{11}, \beta^U{}_{12})], \\ [\vee(\gamma^L{}_{11}, \gamma^L{}_{12}), \vee(\gamma^U{}_{11}, \gamma^U{}_{12})] \end{array} \right\}$.
5. $F^c = \left([\gamma^L, \gamma^U], [\beta^L, \beta^U], [\alpha^L, \alpha^U]\right)$.
6. $F_{s_{11}} \oplus F_{s_{12}} = \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{(\alpha^L{}_{11})^q + (\alpha^L{}_{12})^q - (\alpha^L{}_{11})^q(\alpha^L{}_{12})^q}, \\ \sqrt[q]{(\alpha^U{}_{11})^q + (\alpha^U{}_{12})^q - (\alpha^U{}_{11})^q(\alpha^U{}_{12})^q} \end{array} \right], [\beta^L{}_{11}\beta^L{}_{12}, \beta^U{}_{11}\beta^U{}_{12}], \\ [\gamma^L{}_{11}\gamma^L{}_{12}, \gamma^U{}_{11}\gamma^U{}_{12}] \end{array} \right)$.
7. $F_{s_{11}} \otimes F_{s_{12}} = \left(\begin{array}{c} [\alpha^L{}_{11}\alpha^L{}_{12}, \alpha^U{}_{11}\alpha^U{}_{12}], [\beta^L{}_{11}\beta^L{}_{12}, \beta^U{}_{11}\beta^U{}_{12}], \\ \left[\begin{array}{c} \sqrt[q]{(\gamma^L{}_{11})^q + (\gamma^L{}_{12})^q - (\gamma^L{}_{11})^q(\gamma^L{}_{12})^q}, \\ \sqrt[q]{(\gamma^U{}_{11})^q + (\gamma^U{}_{12})^q - (\gamma^U{}_{11})^q(\gamma^U{}_{12})^q} \end{array} \right] \end{array} \right)$.
8. $F^K = \left(\left[(\alpha^L)^K, (\alpha^U)^K \right], \left[(\beta^L)^K, (\beta^U)^K \right], \left[\sqrt[q]{1 - (\gamma^{Lq})^k}, \sqrt[q]{1 - (\gamma^{Uq})^k} \right] \right)$.
9. $KF = \left(\left[\sqrt[q]{1 - (\alpha^{Lq})^k}, \sqrt[q]{1 - (\alpha^{Uq})^k} \right], \left[(\beta^L)^K, (\beta^U)^K \right], \left[(\gamma^L)^K, (\gamma^U)^K \right] \right)$.

Example 1. *Suppose a coach of a German team wants to select the best football player from a set of alternatives given as* $= \{x_1, x_2, x_3, x_4, x_5\}$. *Suppose* $E = \{s_1 = fitness,\ s_2 = experience,\ s_3 = performance\ record,\ s_4 = consistency\}$ *be the corresponding set of parameters. Using the above given information, the decision maker assesses the alternatives according to their parameter values and gives information in the form of* $IVT - SFS_{ft}Ns$ *given in Table 1.*

Table 1. Tabular representation of $IVT - SFS_{ft}S(F, H)$ for $q \geq 3$.

	s_1	s_2	s_3	s_4
x_1	([0.3, 0.5], [0.3, 0.8], [0.2, 0.6])	([0.4, 0.5], [0.3, 0.51], [0.3, 0.52])	([0.6, 0.7], [0.3, 0.5], [0.3, 0.6])	([0.5, 0.6], [0.5, 0.5], [0.5, 0.7])
x_2	([0.3, 0.5], [0.4, 0.5], [0.5, 0.5])	([0.2, 0.6], [0.4, 0.5], [0.2, 0.4])	([0.5, 0.8], [0.4, 0.6], [0.4, 0.5])	([0.1, 0.3], [0.1, 0.4], [0.2, 0.9])
x_3	([0.4, 0.6], [0.3, 0.6], [0.2, 0.6])	([0.5, 0.6], [0.3, 0.3], [0.5, 0.7])	([0.5, 0.5], [0.3, 0.4], [0.6, 0.8])	([0.1, 0.6], [0.2, 0.7], [0.3, 0.4])
x_4	([0.2, 0.7], [0.2, 0.8], [0.2, 0.5])	([0.2, 0.6], [0.3, 0.8], [0.4, 0.5])	([0.3, 0.5], [0.4, 0.5], [0.6, 0.8])	([0.1, 0.3], [0.2, 0.4], [0.1, 0.5])
x_5	([0.6, 0.6], [0.6, 0.7], [0.6, 0.7])	([0.3, 0.5], [0.4, 0.5], [0.5, 0.5])	([0.2, 0.6], [0.3, 0.4], [0.4, 0.5])	([0.2, 0.6], [0.3, 0.7], [0.3, 0.51])

Definition 11. *For* $IVT - SFS_{ft}SF_{s_{11}} = \left([\alpha^L{}_{11}, \alpha^U{}_{11}], [\beta^L{}_{11}, \beta^U{}_{11}], [\gamma^L{}_{11}, \gamma^U{}_{11}]\right)$, *the score function (SF) is defined by*

$$SC(F_{s_{11}}) = \frac{(\alpha^L{}_{11})^q\left(1 - (\beta^L{}_{11})^q - (\gamma^L{}_{11})^q\right) + (\alpha^U{}_{11})^q\left(1 - (\beta^U{}_{11})^q - (\gamma^U{}_{11})^q\right)}{3}$$

Note that $SC(F_{s_{11}}) \in [-1, 1]$.

Definition 12. *Let* $F_{s_{11}} = \left([\alpha^L{}_{11}, \alpha^U{}_{11}], [\beta^L{}_{11}, \beta^U{}_{11}], [\gamma^L{}_{11}, \gamma^U{}_{11}]\right)$, $F_{s_{12}} = \left([\alpha^L{}_{12}, \alpha^U{}_{12}], [\beta^L{}_{12}, \beta^U{}_{12}], [\gamma^L{}_{12}, \gamma^U{}_{12}]\right)$ *be two* $IVT - SFS_{ft}Ns$, *then*

1. *If* $SC(F_{s_{11}}) > SC(F_{s_{12}})$, *then* $F_{s_{11}} \geq F_{s_{12}}$.
2. *If* $SC(F_{s_{11}}) < SC(F_{s_{12}})$, *then* $F_{s_{11}} \leq F_{s_{12}}$.
3. $SC(F_{s_{11}}) = SC(F_{s_{12}})$, *then*
   (1) *If* $\delta_{F_{s_{11}}} > \delta_{F_{s_{12}}}$, *then* $F_{s_{11}} > F_{s_{12}}$.
   (2) *If* $\delta_{F_{s_{11}}} = \delta_{F_{s_{12}}}$, *then* $F_{s_{11}} = F_{s_{12}}$.

Theorem 1. *Let* $F_{s_{11}} = \left([\alpha^L{}_{11}, \alpha^U{}_{11}], [\beta^L{}_{11}, \beta^U{}_{11}], [\gamma^L{}_{11}, \gamma^U{}_{11}]\right)$,

$F_{s_{12}} = \left(\left[\alpha^{L}{}_{12},\ \alpha^{U}{}_{12}\right],\ \left[\beta^{L}{}_{12},\ \beta^{U}{}_{12}\right],\ \left[\gamma^{L}{}_{12},\ \gamma^{U}{}_{12}\right]\right)$ *be two* $IVT-SFS_{ft}Ns$ *and* $K>0$. Then the following properties hold.

1. $F_{s_{11}} \oplus F_{s_{12}} = F_{s_{21}} \oplus F_{s_{11}}$.
2. $F_{s_{11}} \otimes F_{s_{12}} = F_{s_{21}} \otimes F_{s_{11}}$.
3. $K(F_{s_{11}} \oplus F_{s_{12}}) = (KF_{s_{11}} \oplus KF_{s_{12}})$.
4. $(K_1+K_2)(F_{s_{11}}) = K_1(F_{s_{11}}) + K_2(F_{s_{11}})$.
5. $(F_{s_{11}})^{K_1+K_2} = (F_{s_{11}})^{K_1} \otimes (F_{s_{11}})^{K_2}$.
6. $(F_{s_{11}})^{K} \otimes (F_{s_{11}})^{K} = (F_{s_{11}} \otimes F_{s_{11}})^{K}$.

Proof. Proofs are straightforward. □

4. Interval-Valued T-Spherical Fuzzy Soft Average $\left(IVT-SFS_{ft}A\right)$ Aggregation Operator

In this section, the detailed study of $IVT-SFS_{ft}WA$, $IVT-SFS_{ft}OWA$ and $IVT-SFS_{ft}HA$ operators is discussed and further, we will discuss the properties of these operators.

4.1. Interval-Valued T-Spherical Fuzzy Soft Weighted Average $\left(IVT-SFS_{ft}WA\right)$ Aggregation Operators

Here, we discuss the detailed structure of $IVT-SFS_{ft}WA$ operators and their properties are discussed in detail.

Definition 13. *Let* $F_{s_{ij}} = \left(\left[\alpha^{L}{}_{ij},\ \alpha^{U}{}_{ij}\right],\ \left[\beta^{L}{}_{ij},\ \beta^{U}{}_{ij}\right],\ \left[\gamma^{L}{}_{ij},\ \gamma^{U}{}_{ij}\right]\right)$ *for* $i=1,\ 2,\ \ldots,\ n$ *and* $j=1,\ 2,\ \ldots,\ m$, *be the family of* $IVT-SFS_{ft}Ns$, $\varpi=\{\varpi_1,\ \varpi_2,\ \ldots,\ \varpi_n\}$ *denote the weight vector (WV) of* e_i *experts and* $p=\{p_1,\ p_2,\ \ldots,\ p_m\}$ *denote the WV of parameters* s_j *with condition* $\varpi_i,\ p_j \in [0,\ 1]$ *with* $\sum_{i=1}^{n}\varpi_i = 1$ *and* $\sum_{i=1}^{n} p_i = 1$, *then* $IVT-SFS_{ft}WA$ *operator is the function defined as* $IVT-SFS_{ft}WA : \mathbb{Q}^n \to \mathbb{Q}$, *where* ($\mathbb{Q}$ *is the family of all* $IVT-SFS_{ft}Ns$)

$$IVT-SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) = \oplus_{j=1}^{m} p_j\left(\oplus_{i=1}^{n}\varpi_i F_{s_{ij}}\right).$$

Theorem 2. *Let* $F_{s_{ij}} = \left(\left[\alpha^{L}{}_{ij},\ \alpha^{U}{}_{ij}\right],\ \left[\beta^{L}{}_{ij},\ \beta^{U}{}_{ij}\right],\ \left[\gamma^{L}{}_{ij},\ \gamma^{U}{}_{ij}\right]\right)$ *for* $i=1,\ 2,\ \ldots,\ n$ *and* $j=1,\ 2,\ \ldots,\ m$, *be the family of* $IVT-SFS_{ft}Ns$. *Then the aggregated result for* $IVT-SFS_{ft}WA$ operator is given as

$$\begin{aligned} &IVT-SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) = \oplus_{j=1}^{m} p_j\left(\oplus_{i=1}^{n}\varpi_i F_{s_{ij}}\right) \\ &= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{L}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}}, \\ \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{U}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}} \end{array}\right], \\ \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^{L}{}_{ij}\right)^{\varpi_i}\right)^{p_i},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^{U}{}_{ij}\right)^{\varpi_i}\right)^{p_i}\right], \\ \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^{L}{}_{ij}\right)^{\varpi_i}\right)^{p_i},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^{U}{}_{ij}\right)^{\varpi_i}\right)^{p_i},\right] \end{array}\right) \end{aligned} \tag{1}$$

where $\varpi=\{\varpi_1,\ \varpi_2,\ \ldots,\ \varpi_n\}$ *denote the WV of* e_i *experts and* $p=\{p_1,\ p_2,\ \ldots,\ p_m\}$ *denote the WV of parameters* s_j *with condition* $\varpi_i,\ p_j \in [0,\ 1]$ *with* $\sum_{i=1}^{n}\varpi_i = 1$ *and* $\sum_{i=1}^{n} p_i = 1$.

Proof. We will use the mathematical induction method to prove this result.

We know by the operational laws that

$$F_{s_{11}} \oplus F_{s_{12}} = \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{(\alpha^L{}_{11})^q + (\alpha^L{}_{12})^q - (\alpha^L{}_{11})^q (\alpha^L{}_{12})^q}, \\ \sqrt[q]{(\alpha^U{}_{11})^q + (\alpha^U{}_{12})^q - (\alpha^U{}_{11})^q (\alpha^U{}_{12})^q} \end{array} \right], \\ \left[\beta^L{}_{11}\beta^L{}_{12},\ \beta^U{}_{11}\beta^U{}_{12}\right],\ \left[\gamma^L{}_{11}\gamma^L{}_{12},\ \gamma^U{}_{11}\gamma^U{}_{12}\right] \end{array} \right)$$

And

$$KF_s = \left(\left[\sqrt[q]{1 - \left(1 - (\alpha^{Lq})^k\right)}, \sqrt[q]{1 - \left(1 - (\alpha^{Uq})^k\right)} \right],\ \left[\left(\beta^L\right)^K,\ \left(\beta^U\right)^K \right], \left[\left(\gamma^L\right)^K,\ \left(\gamma^U\right)^K \right] \right) \text{ for } k \geq 1.$$

First of all, we will show that Equation (1) is true for $n = 2$ and $m = 2$, so we have

$$\begin{aligned}
IVT - SFS_{ft}S(F_{s_{11}},\ F_{s_{12}}) &= \oplus_{j=1}^{2} p_j \left(\oplus_{i=1}^{2} \varpi_i F_{s_{ij}} \right) = p_1 \left(\oplus_{i=1}^{2} \varpi_i F_{s_{i1}} \right) \oplus p_2 \left(\oplus_{i=1}^{2} \varpi_i F_{s_{i2}} \right) \\
&= p_1 \left(\varpi_1 F_{s_{11}} \oplus \varpi_2 F_{s_{21}} \right) \oplus p_2 \left(\varpi_1 F_{s_{12}} \oplus \varpi_2 F_{s_{22}} \right)
\end{aligned}$$

$$= p_1 \left\{ \left(\begin{array}{c} \left[\sqrt[q]{1 - (1 - \alpha^L{}_{11}{}^q)^{\varpi_1}}, \sqrt[q]{1 - (1 - \alpha^U{}_{11}{}^q)^{\varpi_1}}, \right], \\ \left[\beta^L{}_{11}{}^{\varpi_1},\ \beta^U{}_{11}{}^{\varpi_1}\right],\ \left[\gamma^L{}_{11}{}^{\varpi_1},\ \gamma^U{}_{11}{}^{\varpi_1}\right] \end{array} \right) \right\} \oplus \left(\begin{array}{c} \left[\sqrt[q]{1 - (1 - \alpha^L{}_{21}{}^q)^{\varpi_2}}, \sqrt[q]{1 - (1 - \alpha^U{}_{21}{}^q)^{\varpi_2}}, \right], \\ \left[\beta^L{}_{21}{}^{\varpi_1},\ \beta^U{}_{21}{}^{\varpi_2}\right],\ \left[\gamma^L{}_{21}{}^{\varpi_1},\ \gamma^U{}_{21}{}^{\varpi_2}\right] \end{array} \right)$$

$$\oplus p_2 \left\{ \begin{array}{c} \left(\begin{array}{c} \left[\sqrt[q]{1 - (1 - \alpha^L{}_{12}{}^q)^{\varpi_1}}, \sqrt[q]{1 - (1 - \alpha^U{}_{12}{}^q)^{\varpi_1}}, \right], \\ \left[\beta^L{}_{12}{}^{\varpi_1},\ \beta^U{}_{12}{}^{\varpi_1}\right],\ \left[\gamma^L{}_{12}{}^{\varpi_1},\ \gamma^U{}_{12}{}^{\varpi_1}\right] \end{array} \right) \\ \oplus \left(\begin{array}{c} \left[\sqrt[q]{1 - (1 - \alpha^L{}_{22}{}^q)^{\varpi_2}}, \sqrt[q]{1 - (1 - \alpha^U{}_{22}{}^q)^{\varpi_2}}, \right], \\ \left[\beta^L{}_{22}{}^{\varpi_1},\ \beta^U{}_{22}{}^{\varpi_2}\right],\ \left[\gamma^L{}_{22}{}^{\varpi_1},\ \gamma^U{}_{22}{}^{\varpi_2}\right] \end{array} \right) \end{array} \right\}$$

$$= p_1 \left(\begin{array}{c} \left[\sqrt[q]{1 - \prod_{i=1}^{2} (1 - \alpha^L{}_{i1}{}^q)^{\varpi_i}},\ \sqrt[q]{1 - \prod_{i=1}^{2} (1 - \alpha^U{}_{i1}{}^q)^{\varpi_i}} \right], \\ \left[\prod_{i=1}^{2} \beta^L{}_{i1}{}^{\varpi_i},\ \prod_{i=1}^{2} \beta^U{}_{i1}{}^{\varpi_i} \right],\ \left[\prod_{i=1}^{2} \gamma^L{}_{i1}{}^{\varpi_i},\ \prod_{i=1}^{2} \gamma^U{}_{i1}{}^{\varpi_i} \right] \end{array} \right)$$

$$\oplus p_2 \left(\begin{array}{c} \left[\sqrt[q]{1 - \prod_{i=1}^{2} (1 - \alpha^L{}_{i2}{}^q)^{\varpi_i}},\ \sqrt[q]{1 - \prod_{i=1}^{2} (1 - \alpha^U{}_{i2}{}^q)^{\varpi_i}} \right], \\ \left[\prod_{i=1}^{2} \beta^L{}_{i2}{}^{\varpi_i},\ \prod_{i=1}^{2} \beta^U{}_{i2}{}^{\varpi_i} \right],\ \left[\prod_{i=1}^{2} \gamma^L{}_{i2}{}^{\varpi_i},\ \prod_{i=1}^{2} \gamma^U{}_{i2}{}^{\varpi_i} \right] \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1 - \left(\prod_{i=1}^{2} (1 - \alpha^L{}_{i1}{}^q)^{\varpi_i} \right)^{p_1}}, \\ \sqrt[q]{1 - \left(\prod_{i=1}^{2} (1 - \alpha^U{}_{i1}{}^q)^{\varpi_i} \right)^{p_1}}, \end{array} \right], \left[\begin{array}{c} \left(\prod_{i=1}^{2} \beta^L{}_{i1}{}^{\varpi_i} \right)^{p_1}, \\ \left(\prod_{i=1}^{2} \beta^U{}_{i1}{}^{\varpi_i} \right)^{p_1} \end{array} \right], \\ \left[\left(\prod_{i=1}^{2} \gamma^L{}_{i1}{}^{\varpi_i} \right)^{p_1},\ \left(\prod_{i=1}^{2} \gamma^U{}_{i1}{}^{\varpi_i} \right)^{p_1} \right] \end{array} \right)$$

$$\oplus \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1 - \left(\prod_{i=1}^{2} (1 - \alpha^L{}_{i2}{}^q)^{\varpi_i} \right)^{p_2}}, \\ \sqrt[q]{1 - \left(\prod_{i=1}^{2} (1 - \alpha^U{}_{i2}{}^q)^{\varpi_i} \right)^{p_2}}, \end{array} \right], \left[\begin{array}{c} \left(\prod_{i=1}^{2} \beta^L{}_{i2}{}^{\varpi_i} \right)^{p_2}, \\ \left(\prod_{i=1}^{2} \beta^U{}_{i2}{}^{\varpi_i} \right)^{p_2} \end{array} \right], \\ \left[\left(\prod_{i=1}^{2} \gamma^L{}_{i2}{}^{\varpi_i} \right)^{p_2},\ \left(\prod_{i=1}^{2} \gamma^U{}_{i2}{}^{\varpi_i} \right)^{p_2} \right] \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1 - \prod_{j=1}^{2} \left(\prod_{i=1}^{2} (1 - \alpha^L{}_{ij}{}^q)^{\varpi_i} \right)^{p_j}}, \\ \sqrt[q]{1 - \prod_{j=1}^{2} \left(\prod_{i=1}^{2} (1 - \alpha^U{}_{ij}{}^q)^{\varpi_i} \right)^{p_j}} \end{array} \right], \\ \left[\prod_{j=1}^{2} \left(\prod_{i=1}^{2} \beta^L{}_{ij}{}^{\varpi_i} \right)^{p_j},\ \prod_{j=1}^{2} \left(\prod_{i=1}^{2} \beta^U{}_{ij}{}^{\varpi_i} \right)^{p_j} \right],\ \prod_{j=1}^{2} \left(\prod_{i=1}^{2} \gamma_{ij}{}^{\varpi_i} \right)^{p_j} \end{array} \right)$$

Hence the result is true for $n = 2$ and $m = 2$.

Next, suppose that Equation (1) is true for $n = z_1$ and $m = z_2$

$$IVT - SFS_{ft}WA\left(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{z_1 z_2}}\right) = \oplus_{j=1}^{z_2} p_j\left(\oplus_{i=1}^{z_1}\varpi_i F_{s_{ij}}\right)$$

$$= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1-\prod\limits_{j=1}^{z_2}\left(\prod\limits_{i=1}^{z_1}\left(1-\left(\alpha^L{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}}, \\ \sqrt[q]{1-\prod\limits_{j=1}^{z_2}\left(\prod\limits_{i=1}^{z_1}\left(1-\left(\alpha^U{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}} \end{array}\right], \\ \left[\prod\limits_{j=1}^{z_2}\left(\prod\limits_{i=1}^{z_1}\left(\beta^L{}_{ij}\right)^{\varpi_i}\right)^{p_j},\ \prod\limits_{j=1}^{z_2}\left(\prod\limits_{i=1}^{z_1}\left(\beta^U{}_{ij}\right)^{\varpi_i}\right)^{p_j}\right], \\ \left[\prod\limits_{j=1}^{m}\left(\prod\limits_{i=1}^{n}\left(\gamma^L{}_{ij}\right)^{\varpi_i}\right)^{p_j},\ \prod\limits_{j=1}^{m}\left(\prod\limits_{i=1}^{n}\left(\gamma^U{}_{ij}\right)^{\varpi_i}\right)^{p_j}\right] \end{array}\right)$$

Further, suppose that Equation (1) is true for $n = z_1 + 1$ and $m = z_2 + 1$

$$IVT - SFS_{ft}S\left(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{(z_{1+1})(z_{2+1})}}\right) = \left\{\oplus_{j=1}^{z_2} p_j\left(\oplus_{i=1}^{z_1}\varpi_i F_{s_{ij}}\right)\right\} \oplus p_{z_1+1}\left(\varpi_{z_2+1}F_{s_{(z_{1+1})(z_{2+1})}}\right)$$

$$= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1-\prod\limits_{j=1}^{z_2}\left(\prod\limits_{i=1}^{z_1}\left(1-\left(\alpha^L{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}}, \\ \sqrt[q]{1-\prod\limits_{j=1}^{z_2}\left(\prod\limits_{i=1}^{z_1}\left(1-\left(\alpha^U{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}} \end{array}\right], \\ \left[\begin{array}{c} \prod\limits_{j=1}^{z_2}\left(\prod\limits_{i=1}^{z_1}\left(\beta^L{}_{ij}\right)^{\varpi_i}\right)^{p_j}, \\ \prod\limits_{j=1}^{z_2}\left(\prod\limits_{i=1}^{z_1}\left(\beta^U{}_{ij}\right)^{\varpi_i}\right)^{p_j} \end{array}\right], \\ \left[\begin{array}{c} \prod\limits_{j=1}^{z_2}\left(\prod\limits_{i=1}^{z_1}\left(\gamma^L{}_{ij}\right)^{\varpi_i}\right)^{p_j}, \\ \prod\limits_{j=1}^{z_2}\left(\prod\limits_{i=1}^{z_1}\left(\gamma^U{}_{ij}\right)^{\varpi_i}\right)^{p_j} \end{array}\right] \end{array}\right) \oplus p_{z_1+1}\left(\varpi_{z_2+1}F_{s_{(z_{1+1})(z_{2+1})}}\right)$$

$$= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1-\prod_{j=1}^{z_2+1}\left(\prod_{i=1}^{z_1+1}\left(1-\left(\alpha^L{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}}, \\ \sqrt[q]{1-\prod_{j=1}^{z_2+1}\left(\prod_{i=1}^{z_1+1}\left(1-\left(\alpha^U{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}} \end{array}\right], \left[\begin{array}{c} \prod_{j=1}^{z_2+1}\left(\prod_{i=1}^{z_1+1}\left(\beta^L{}_{ij}\right)^{\varpi_i}\right)^{p_j}, \\ \prod_{j=1}^{z_2+1}\left(\prod_{i=1}^{z_1+1}\left(\beta^U{}_{ij}\right)^{\varpi_i}\right)^{p_j} \end{array}\right], \\ \left[\prod_{j=1}^{z_2+1}\left(\prod_{i=1}^{z_1+1}\left(\gamma^L{}_{ij}\right)^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{z_2+1}\left(\prod_{i=1}^{z_1+1}\left(\gamma^U{}_{ij}\right)^{\varpi_i}\right)^{p_j}\right] \end{array}\right)$$

It is clear from the above expression that $IVT - SFS_{ft}WA$ is again an $IVT - SFS_{ft}N$. Hence, given Equation (1) is true for $n = z_1 + 1$ and $m = z_2 + 1$.

Hence it is true for all $m,\ n \geq 1$. □

Remark 1.

1. *Using $q = 1$, then established $IVT - SFS_{ft}WA$ operator will reduce to $IVPFS_{ft}WA$ operator.*
2. *Using $q = 2$, then established $IVT - SFS_{ft}WA$ operator will reduce to $IVSFS_{ft}WA$ operator.*
3. *If we neglect the obstinacy grade that is $\beta_{ij} = 0$, and using $q = 2$, the proposed $IVT - SFS_{ft}WA$ operator will reduce to $IVP_yFS_{ft}WA$ operator.*
4. *If we neglect the obstinacy grade that is $\beta_{ij} = 0$, and using $q = 1$, the proposed $IVT - SFS_{ft}WA$ operator will reduce to an interval-valued intuitionistic fuzzy soft weighted average $\left(IVIFS_{ft}WA\right)$ operator.*
5. *Moreover, if we put only one parameter that is s_1 (mean m = 1), then $IVT - SFS_{ft}WA$ operator reduces to an interval-valued T-spherical fuzzy weighted average (IVT-SFWA) operator.*

Hence it is clear that $IVPFS_{ft}WA,\ IVSFS_{ft}WA,\ IVP_yFS_{ft}WA,\ IVIFS_{ft}WA$ and IVT-SFWA operators are the special cases of $IVT - SFS_{ft}WA$ operator. The present work is more general.

Example 2. *A person desires to buy a car from a set of five car brands as alternatives* $= \{x_1 = BMW,\ x_2 = Suzuki,\ x_3 = Tata,\ x_4 = Hyundai\ Motors,\ x_5 = Mercedes\ \}$*. Let* $E = \{s_1 = Comfortability,\ s_2 = Good\ shape,\ x_3 = Reasonable\ price,\ s_4 = Automatic\ system\}$*. Let the set* $\varpi = \{0.25,\ 0.15,\ 0.14,\ 0.3,\ 0,16\}$ *denote the weight vector of "*e_i*" experts and* $p = \{0.27,\ 0.19,\ 0.29,\ 0.25\}$ *denote the weight vector of "*s_j*" parameters. The experts provide their information in the form of* $IVT - SFS_{ft}Ns$ *as given in Table 2.*

By using Equation (1), we have

$$IVT - SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{54}})$$

$$= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{L}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}}, \\ \sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{U}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}} \end{array} \right], \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^{L}{}_{ij}\right)^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^{U}{}_{ij}\right)^{\varpi_i}\right)^{p_j} \right], \\ \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^{L}{}_{ij}\right)^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^{U}{}_{ij}\right)^{\varpi_i}\right)^{p_j}, \right] \end{array} \right)$$

$$= \left[\begin{array}{c} \left(\left[\begin{array}{c} \sqrt[3]{\begin{array}{c} 1 - \left\{\left(1-0.2^3\right)^{0.25}\left(1-0.5^3\right)^{0.15}\left(1-0.3^3\right)^{0.14}\left(1-0.5^3\right)^{0.3}\left(1-0.6^3\right)^{0.16}\right\}^{0.27} \\ \left\{\left(1-0.1^3\right)^{0.25}\left(1-0.1^3\right)^{0.15}\left(1-0.2^3\right)^{0.14}\left(1-0.1^3\right)^{0.3}\left(1-0.5^3\right)^{0.16}\right\}^{0.19} \\ \left\{\left(1-0.6^3\right)^{0.25}\left(1-0.2^3\right)^{0.15}\left(1-0.3^3\right)^{0.14}\left(1-0.4^3\right)^{0.3}\left(1-0.3^3\right)^{0.16}\right\}^{0.29} \\ \left\{\left(1-0.3^3\right)^{0.25}\left(1-0.2^3\right)^{0.15}\left(1-0.4^3\right)^{0.14}\left(1-0.2^3\right)^{0.3}\left(1-0.5^3\right)^{0.16}\right\}^{0.25} \end{array}}, \\ \sqrt[3]{\begin{array}{c} 1 - \left\{\left(1-0.6^3\right)^{0.25}\left(1-0.5^3\right)^{0.15}\left(1-0.5^3\right)^{0.14}\left(1-0.8^3\right)^{0.3}\left(1-0.7^3\right)^{0.16}\right\}^{0.27} \\ \left\{\left(1-0.3^3\right)^{0.25}\left(1-0.6^3\right)^{0.15}\left(1-0.6^3\right)^{0.14}\left(1-0.3^3\right)^{0.3}\left(1-0.53^3\right)^{0.16}\right\}^{0.19} \\ \left\{\left(1-0.6^3\right)^{0.25}\left(1-0.3^3\right)^{0.15}\left(1-0.5^3\right)^{0.14}\left(1-0.6^3\right)^{0.3}\left(1-0.7^3\right)^{0.16}\right\}^{0.29} \\ \left\{\left(1-0.5^3\right)^{0.25}\left(1-0.6^3\right)^{0.15}\left(1-0.5^3\right)^{0.14}\left(1-0.6^3\right)^{0.3}\left(1-0.6^3\right)^{0.16}\right\}^{0.25} \end{array}} \end{array} \right] \right), \\ \left(\left[\begin{array}{c} \left(\begin{array}{c} \left\{\left(1-0.3^3\right)^{0.25}\left(1-0.3^3\right)^{0.15}\left(1-0.4^3\right)^{0.14}\left(1-0.4^3\right)^{0.3}\left(1-0.3^3\right)^{0.16}\right\}^{0.27} \\ \left\{\left(1-0.2^3\right)^{0.25}\left(1-0.2^3\right)^{0.15}\left(1-0.3^3\right)^{0.14}\left(1-0.1^3\right)^{0.3}\left(1-0.5^3\right)^{0.16}\right\}^{0.19} \\ \left\{\left(1-0.6^3\right)^{0.25}\left(1-0.2^3\right)^{0.15}\left(1-0.4^3\right)^{0.14}\left(1-0.3^3\right)^{0.3}\left(1-0.3^3\right)^{0.16}\right\}^{0.29} \\ \left\{\left(1-0.4^3\right)^{0.25}\left(1-0.3^3\right)^{0.15}\left(1-0.3^3\right)^{0.14}\left(1-0.4^3\right)^{0.3}\left(1-0.3^3\right)^{0.16}\right\}^{0.25} \end{array} \right) \\ \left(\begin{array}{c} \left\{\left(1-0.8^3\right)^{0.25}\left(1-0.4^3\right)^{0.15}\left(1-0.5^3\right)^{0.14}\left(1-0.6^3\right)^{0.3}\left(1-0.5^3\right)^{0.16}\right\}^{0.27} \\ \left\{\left(1-0.4^3\right)^{0.25}\left(1-0.7^3\right)^{0.15}\left(1-0.7^3\right)^{0.14}\left(1-0.3^3\right)^{0.3}\left(1-0.61^3\right)^{0.16}\right\}^{0.19} \\ \left\{\left(1-0.7^3\right)^{0.25}\left(1-0.6^3\right)^{0.15}\left(1-0.5^3\right)^{0.14}\left(1-0.6^3\right)^{0.3}\left(1-0.71^3\right)^{0.16}\right\}^{0.29} \\ \left\{\left(1-0.5^3\right)^{0.25}\left(1-0.56^3\right)^{0.15}\left(1-0.7^3\right)^{0.14}\left(1-0.5^3\right)^{0.3}\left(1-0.7^3\right)^{0.16}\right\}^{0.25} \end{array} \right) \end{array} \right] \right), \\ \left(\left[\begin{array}{c} \left(\begin{array}{c} \left\{\left(1-0.4^3\right)^{0.25}\left(1-0.6^3\right)^{0.15}\left(1-0.6^3\right)^{0.14}\left(1-0.4^3\right)^{0.3}\left(1-0.3^3\right)^{0.16}\right\}^{0.27} \\ \left\{\left(1-0.1^3\right)^{0.25}\left(1-0.3^3\right)^{0.15}\left(1-0.3^3\right)^{0.14}\left(1-0.2^3\right)^{0.3}\left(1-0.5^3\right)^{0.16}\right\}^{0.19} \\ \left\{\left(1-0.3^3\right)^{0.25}\left(1-0.2^3\right)^{0.15}\left(1-0.5^3\right)^{0.14}\left(1-0.2^3\right)^{0.3}\left(1-0.2^3\right)^{0.16}\right\}^{0.29} \\ \left\{\left(1-0.5^3\right)^{0.25}\left(1-0.4^3\right)^{0.15}\left(1-0.3^3\right)^{0.14}\left(1-0.2^3\right)^{0.3}\left(1-0.5^3\right)^{0.16}\right\}^{0.25} \end{array} \right) \\ \left(\begin{array}{c} \left\{\left(1-0.5^3\right)^{0.25}\left(1-0.7^3\right)^{0.15}\left(1-0.8^3\right)^{0.14}\left(1-0.51^3\right)^{0.3}\left(1-0.6^3\right)^{0.16}\right\}^{0.27} \\ \left\{\left(1-0.5^3\right)^{0.25}\left(1-0.4^3\right)^{0.15}\left(1-0.42^3\right)^{0.14}\left(1-0.9^3\right)^{0.3}\left(1-0.72^3\right)^{0.16}\right\}^{0.19} \\ \left\{\left(1-0.5^3\right)^{0.25}\left(1-0.5^3\right)^{0.15}\left(1-0.5^3\right)^{0.14}\left(1-0.6^3\right)^{0.3}\left(1-0.55^3\right)^{0.16}\right\}^{0.29} \\ \left\{\left(1-0.5^3\right)^{0.25}\left(1-0.45^3\right)^{0.15}\left(1-0.6^3\right)^{0.14}\left(1-0.7^3\right)^{0.3}\left(1-0.7^3\right)^{0.16}\right\}^{0.25} \end{array} \right) \end{array} \right] \right) \end{array} \right]$$

$$= ([0.431381,\ 0.622008],\ [0.276481,\ 0.533375],\ [0.270999,\ 0.544335])$$

Table 2. Tabular representation of $IVT - SFS_{ft}S(F,\ H)$ for $q \geq 3$.

	s_1	s_2	s_3	s_4
x_1	([0.2, 0.6], [0.3, 0.8], [0.4, 0.5])	([0.1, 0.3], [0.2, 0.4], [0.1, 0.5])	([0.6, 0.6], [0.6, 0.7], [0.3, 0.5])	([0.3, 0.5], [0.4, 0.5], [0.5, 0.5])
x_2	([0.5, 0.5], [0.3, 0.4], [0.6, 0.7])	([0.1, 0.6], [0.2, 0.7], [0.3, 0.4])	([0.2, 0.3], [0.2, 0.6], [0.2, 0.5])	([0.2, 0.6], [0.3, 0.56], [0.4, 0.45])
x_3	([0.3, 0.5], [0.4, 0.5], [0.6, 0.8])	([0.2, 0.6], [0.3, 0.7], [0.3, 0.42])	([0.3, 0.5], [0.4, 0.5], [0.5, 0.5])	([0.4, 0.5], [0.3, 0.7], [0.3, 0.6])
x_4	([0.5, 0.8], [0.4, 0.6], [0.4, 0.51])	([0.1, 0.3], [0.1, 0.3], [0.2, 0.9])	([0.4, 0.6], [0.3, 0.6], [0.2, 0.6])	([0.2, 0.6], [0.4, 0.5], [0.2, 0.7])
x_5	([0.6, 0.7], [0.3, 0.5], [0.3, 0.6])	([0.5, 0.53], [0.5, 0.61], [0.5, 0.72])	([0.3, 0.7], [0.3, 0.71], [0.2, 0.55])	([0.5, 0.6], [0.3, 0.7], [0.5, 0.7])

Theorem 3. *Let* $F_{s_{ij}} = \left(\left[\alpha^L{}_{ij},\ \alpha^U{}_{ij}\right],\ \left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right],\ \left[\gamma^L{}_{ij},\ \gamma^U{}_{ij}\right]\right)$ *for* $i = 1,\ 2,\ \ldots,\ n$ *and* $j = 1,\ 2,\ \ldots,\ m$, *be the family of* $IVT - SFS_{ft}Ns$, $\varpi = (\varpi_1,\ \varpi_2,\ \ldots,\ \varpi_n)^T$ *denote the weight vector of* e_i *experts and* $p = (p_1,\ p_2,\ \ldots,\ p_m)^T$ *denote the weight vector of parameters* s_j *with condition* $\varpi_i,\ p_j \in [0,\ 1]$ *with* $\sum_{i=1}^{n} \varpi_i = 1$ *and* $\sum_{i=1}^{n} p_i = 1$. *Then* $IVT - SFS_{ft}WA$ *operator holds the foowing properties:*

1. *(Idempotency). Let* $F_{s_{ij}} = \left(\left[\alpha^L{}_{ij},\ \alpha^U{}_{ij}\right],\ \left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right],\ \left[\gamma^L{}_{ij},\ \gamma^U{}_{ij}\right]\right) = F_s$ *for all* $i = 1,\ 2,\ \ldots,\ n$ *and* $j = 1,\ 2,\ \ldots,\ m$, *where* $F_s = \left(\left[\alpha^L,\ \alpha^U\right], \left[\beta^L,\ \beta^U\right],\ \left[\gamma^L,\ \gamma^U\right]\right)$, *then* $IVT - SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) = F_s$.
2. *(Boundedness). If* $F^-_{s_{ij}} = \begin{pmatrix} \left\{\left[min_j min_i\left(\alpha^L{}_{ij}\right),\ min_j min_i\left(\alpha^U{}_{ij}\right)\right]\right\},\ \left\{\left[max_j max_i\left(\beta^L{}_{ij}\right),\ max_j max_i\left(\beta^U{}_{ij}\right)\right]\right\}, \\ \left\{\left[max_j max_i\left(\gamma^L{}_{ij}\right),\ max_j max_i\left(\gamma^U{}_{ij}\right)\right]\right\} \end{pmatrix}$ *And* $F^+_{s_{ij}} = \begin{pmatrix} \left\{\left[max_j max_i\left(\alpha^L{}_{ij}\right), max_j max_i\left(\alpha^U{}_{ij}\right)\right]\right\}, \left\{\left[min_j min_i\left(\beta^L{}_{ij}\right), min_j min_i\left(\beta^U{}_{ij}\right)\right]\right\}, \\ \left\{\left[min_j min_i\left(\gamma^L{}_{ij}\right), min_j min_i\left(\gamma^U{}_{ij}\right)\right]\right\} \end{pmatrix}$, *then* $F^-_{s_{ij}} \leq IVT - SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) \leq F^+_{s_{ij}}$.
3. ***(Monotonicity).*** *Let* $F'_{s_{ij}} = \left(\left[\alpha'^L{}_{ij},\ \alpha'^U{}_{ij}\right],\ \left[\beta'^L{}_{ij},\ \beta'^U{}_{ij}\right],\ \left[\gamma'^L{}_{ij},\ \gamma'^U{}_{ij}\right]\right)$ *be any other collection of* $IVT - SFS_{ft}Ns$ *for all* $i = 1,\ 2,\ \ldots,\ n$ *and* $j = 1,\ 2,\ \ldots,\ m$ *such that* $\alpha^L{}_{ij} \leq \alpha'^L{}_{ij}$, $\alpha^U{}_{ij} \leq \alpha'^U{}_{ij}$, $\beta^L{}_{ij} \geq \beta'^L{}_{ij}$, $\beta^U{}_{ij} \geq \beta'^U{}_{ij}$ *and* $\gamma^L{}_{ij} \geq \gamma'^L{}_{ij}$, $\gamma^U{}_{ij} \geq \gamma'^U{}_{ij}$, then
$$IVT - SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) \leq IVT - SFS_{ft}WA\left(F'_{s_{11}},\ F'_{s_{12}},\ \ldots,\ F'_{s_{nm}}\right).$$
4. ***(Shift Invariance).*** *If* $F_s = \left(\left[\alpha^L,\ \alpha^U\right], \left[\beta^L,\ \beta^U\right],\ \left[\gamma^L,\ \gamma^U\right]\right)$ *is another* $IVT - SFS_{ft}N$, then

$$IVT - SFS_{ft}WA(F_{s_{11}} \oplus F_s,\ F_{s_{12}} \oplus F_s,\ \ldots,\ F_{s_{nm}} \oplus F_s) = IVT - SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) \oplus F_s.$$

5. ***(Homogeneity).*** *For any real number*$K > 0$

$$IVT - SFS_{ft}WA(KF_{s_{11}},\ KF_{s_{12}},\ \ldots,\ KF_{s_{nm}}) = K\Big(IVT - SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}})\Big)$$

Proof.

1. (Idempotency). Let $F_{s_{ij}} = \left(\left[\alpha^L{}_{ij},\ \alpha^U{}_{ij}\right],\ \left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right],\ \left[\gamma^L{}_{ij},\ \gamma^U{}_{ij}\right]\right) = F_s$ for all $i = 1,\ 2,\ \ldots,\ n$ and $j = 1,\ 2,\ \ldots,\ m$, where $F_s = \left(\left[\alpha^L,\ \alpha^U\right], \left[\beta^L,\ \beta^U\right],\ \left[\gamma^L,\ \gamma^U\right]\right)$, then from Theorem 1, we have

$$IVT - SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) = \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - (\alpha^{L}{}_{ij})^{q}\right)^{\varpi_i}\right)^{p_j}}, \\ \sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - (\alpha^{U}{}_{ij})^{q}\right)^{\varpi_i}\right)^{p_j}} \end{array} \right], \\ \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}(\beta^{L}{}_{ij})^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}(\beta^{U}{}_{ij})^{\varpi_i}\right)^{p_j}, \right], \\ \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}(\gamma^{L}{}_{ij})^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}(\gamma^{U}{}_{ij})^{\varpi_i}\right)^{p_j}, \right] \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - (\alpha^{L})^{q}\right)^{\varpi_i}\right)^{p_j}}, \\ \sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - (\alpha^{U})^{q}\right)^{\varpi_i}\right)^{p_j}} \end{array} \right],\ \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}(\beta^{L})^{\varpi_i}\right)^{p_i},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}(\beta^{U})^{\varpi_i}\right)^{p_i}, \right] \\ \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}(\gamma^{L})^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}(\gamma^{U})^{\varpi_i}\right)^{p_j}, \right] \end{array} \right)$$

$$= \left(\left[\sqrt[q]{1 - \left(1 - (\alpha^{L})^{q}\right)},\ \sqrt[q]{1 - \left(1 - (\alpha^{U})^{q}\right)} \right],\ \left[\beta^{L},\ \beta^{U}\right],\ \left[\gamma^{L},\ \gamma^{U}\right] \right) = \left(\left[\alpha^{L},\ \alpha^{U}\right], \left[\beta^{L},\ \beta^{U}\right],\ \left[\gamma^{L},\ \gamma^{U}\right]\right) = F_s.$$

Hence $IVT - SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) = F_s$.

2. (Boundedness). As $F^{-}_{s_{ij}} = \left(\begin{array}{c} \left\{\left[min_j min_i\left(\alpha^{L}{}_{ij}\right),\ min_j min_i\left(\alpha^{U}{}_{ij}\right)\right]\right\},\ \left\{\left[max_j max_i\left(\beta^{L}{}_{ij}\right),\ max_j max_i\left(\beta^{U}{}_{ij}\right)\right]\right\}, \\ \left\{\left[max_j max_i\left(\gamma^{L}{}_{ij}\right),\ max_j max_i\left(\gamma^{U}{}_{ij}\right)\right]\right\} \end{array} \right)$ and $F^{+}_{s_{ij}} = \left(\begin{array}{c} \left\{\left[max_j max_i\left(\alpha^{L}{}_{ij}\right),\ max_j max_i\left(\alpha^{U}{}_{ij}\right)\right]\right\},\ \left\{\left[min_j min_i\left(\beta^{L}{}_{ij}\right),\ min_j min_i\left(\beta^{U}{}_{ij}\right)\right]\right\}, \\ \left\{\left[min_j min_i\left(\gamma^{L}{}_{ij}\right),\ min_j min_i\left(\gamma^{U}{}_{ij}\right)\right]\right\} \end{array} \right)$, then we have to prove that $F^{-}_{s_{ij}} \leq IVT - SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) \leq F^{+}_{s_{ij}}$. Now for each $i = 1,\ 2,\ \ldots,\ n$ and $j = 1,\ 2,\ \ldots,\ m$

$$
\begin{aligned}
&\left\{\left[min_j min_i\left(\alpha^{L}{}_{ij}\right),\ min_j min_i\left(\alpha^{U}{}_{ij}\right)\right]\right\} \leq \left\{\left[\alpha^{L}{}_{ij},\ \alpha^{U}{}_{ij}\right]\right\} \\
&\leq \left\{\left[max_j max_i\left(\alpha^{L}{}_{ij}\right),\ max_j max_i\left(\alpha^{U}{}_{ij}\right)\right]\right\} \\
&\Leftrightarrow \left\{\left[1-max_j max_i\left(\alpha^{L}{}_{ij}{}^{q}\right),\ 1-max_j max_i\left(\alpha^{U}{}_{ij}{}^{q}\right)\right]\right\} \\
&\leq \left[1-\alpha^{L}{}_{ij}{}^{q},\ 1-\alpha^{U}{}_{ij}{}^{q}\right] \leq \left\{\left[min_j min_i\left(\alpha^{L}{}_{ij}{}^{q}\right),\ min_j min_i\left(\alpha^{U}{}_{ij}{}^{q}\right)\right]\right\} \\
&\Leftrightarrow \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-max_j max_i\left(\alpha^{L}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-max_j max_i\left(\alpha^{U}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}\right] \\
&\leq \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{L}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{U}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}\right] \\
&\leq \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-min_j min_i\left(\alpha^{L}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-min_j min_i\left(\alpha^{U}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}\right] \\
&\Leftrightarrow \left[\left(\left(1-max_j max_i\left(\alpha^{L}{}_{ij}\right)^{q}\right)^{\sum_{i=1}^{n}\varpi_i}\right)^{\sum_{j=1}^{m}p_j},\ \left(\left(1-max_j max_i\left(\alpha^{U}{}_{ij}\right)^{q}\right)^{\sum_{i=1}^{n}\varpi_i}\right)^{\sum_{j=1}^{m}p_j}\right] \\
&\leq \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{L}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{U}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}\right] \\
&\leq \left[\begin{array}{c}\left(\left(1-min_j min_i\left(\alpha^{L}{}_{ij}\right)^{q}\right)^{\sum_{i=1}^{n}\varpi_i}\right)^{\sum_{j=1}^{m}p_j}, \\ \left(\left(1-min_j min_i\left(\alpha^{U}{}_{ij}\right)^{q}\right)^{\sum_{i=1}^{n}\varpi_i}\right)^{\sum_{j=1}^{m}p_j}\end{array}\right] \\
&\Leftrightarrow \left[1-max_j max_i\left(\alpha^{L}{}_{ij}\right)^{q},\ 1-max_j max_i\left(\alpha^{U}{}_{ij}\right)^{q}\right] \\
&\leq \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{L}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{U}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}\right] \\
&\leq \left[1-min_j min_i\left(\alpha^{L}{}_{ij}\right)^{q},\ 1-min_j min_i\left(\alpha^{U}{}_{ij}\right)^{q}\right] \\
&\Leftrightarrow \left[1-\left(1-min_j min_i\left(\alpha^{L}{}_{ij}\right)^{q}\right),\ 1-\left(1-min_j min_i\left(\alpha^{U}{}_{ij}\right)^{q}\right)\right] \\
&\leq \left[1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{L}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j},\ 1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{U}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}\right] \\
&\leq \left[1-\left(1-max_j max_i\left(\alpha^{L}{}_{ij}\right)^{q}\right),\ 1-\left(1-max_j max_i\left(\alpha^{U}{}_{ij}\right)^{q}\right)\right]
\end{aligned}
$$

Hence

$$
\begin{aligned}
&\left[min_j min_i\left(\alpha^{L}{}_{ij}\right),\ min_j min_i\left(\alpha^{U}{}_{ij}\right)\right] \\
&\leq \left[\sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{L}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}},\ \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{U}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}}\right] \\
&\leq \left[max_j max_i\left(\alpha^{L}{}_{ij}\right),\ max_j max_i\left(\alpha^{U}{}_{ij}\right)\right]
\end{aligned}
\tag{2}
$$

Now for each $i = 1,\ 2,\ \ldots,\ n$ and $j = 1,\ 2,\ \ldots,\ m$, we have

$$\begin{gathered}
\left[min_j min_i\left(\beta^L{}_{ij}\right),\ min_j min_i\left(\beta^U{}_{ij}\right)\right] \le \left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right] \\
\le \left[max_j max_i\left(\beta^L{}_{ij}\right),\ max_j max_i\left(\beta^U{}_{ij}\right)\right] \\
\Leftrightarrow \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(min_j min_i\left(\beta^L{}_{ij}\right)\right)^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(min_j min_i\left(\beta^U{}_{ij}\right)\right)^{\varpi_i}\right)^{p_j}\right] \\
\le \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^L{}_{ij}\right)^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^U{}_{ij}\right)^{\varpi_i}\right)^{p_j}\right] \\
\le \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(max_j max_i\left(\beta^L{}_{ij}\right)\right)^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(max_j max_i\left(\beta^U{}_{ij}\right)\right)^{\varpi_i}\right)^{p_j}\right] \\
\Leftrightarrow \left[\left(\left(min_j min_i\left(\beta^L{}_{ij}\right)\right)^{\sum_{i=1}^{n}\varpi_i}\right)^{\sum_{j=1}^{m}p_j},\ \left(\left(min_j min_i\left(\beta^U{}_{ij}\right)\right)^{\sum_{i=1}^{n}\varpi_i}\right)^{\sum_{j=1}^{m}p_j}\right] \\
\le \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^L{}_{ij}\right)^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^U{}_{ij}\right)^{\varpi_i}\right)^{p_j}\right] \le \left[\begin{array}{c}\left(\left(max_j max_i\left(\beta^L{}_{ij}\right)\right)^{\sum_{i=1}^{n}\varpi_i}\right)^{\sum_{j=1}^{m}p_j}, \\ \left(\left(max_j max_i\left(\beta^U{}_{ij}\right)\right)^{\sum_{i=1}^{n}\varpi_i}\right)^{\sum_{j=1}^{m}p_j}\end{array}\right] \\
\Rightarrow \left[\begin{array}{c}min_j min_i\left(\beta^L{}_{ij}\right), \\ min_j min_i\left(\beta^U{}_{ij}\right)\end{array}\right] \le \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^L{}_{ij}\right)^{\varpi_i}\right)^{p_j},\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^U{}_{ij}\right)^{\varpi_i}\right)^{p_j}\right] \\
\le \left[max_j max_i\left(\beta^L{}_{ij}\right),\ max_j max_i\left(\beta^U{}_{ij}\right)\right]
\end{gathered} \tag{3}$$

Moreover, for each $i = 1,\ 2,\ \ldots,\ n$ and $j = 1,\ 2,\ \ldots,\ m$, we have

$$\begin{gathered}
\left[min_j min_i\left(\gamma^L{}_{ij}\right),\ min_j min_i\left(\gamma^U{}_{ij}\right)\right] \le \left[max_j max_i\left(\gamma^L{}_{ij}\right),\ max_j max_i\left(\gamma^U{}_{ij}\right)\right] \\
\Leftrightarrow \left[\begin{array}{c}\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(min_j min_i\left(\gamma^L{}_{ij}\right)\right)^{\varpi_i}\right)^{p_j}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(min_j min_i\left(\gamma^U{}_{ij}\right)\right)^{\varpi_i}\right)^{p_j}\end{array}\right] \le \left[\begin{array}{c}\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^L{}_{ij}\right)^{\varpi_i}\right)^{p_j}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^U{}_{ij}\right)^{\varpi_i}\right)^{p_j}\end{array}\right] \\
\le \left[\begin{array}{c}\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(max_j max_i\left(\gamma^L{}_{ij}\right)\right)^{\varpi_i}\right)^{p_j}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(max_j max_i\left(\gamma^U{}_{ij}\right)\right)^{\varpi_i}\right)^{p_j}\end{array}\right] \Leftrightarrow \left[\begin{array}{c}\left(\left(min_j min_i\left(\gamma^L{}_{ij}\right)\right)^{\sum_{i=1}^{n}\varpi_i}\right)^{\sum_{j=1}^{m}p_j}, \\ \left(\left(min_j min_i\left(\gamma^U{}_{ij}\right)\right)^{\sum_{i=1}^{n}\varpi_i}\right)^{\sum_{j=1}^{m}p_j}\end{array}\right] \\
\le \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^L{}_{ij}\right)^{\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^U{}_{ij}\right)^{\varpi_i}\right)^{p_j}\right] \le \left[\begin{array}{c}\left(\left(max_j max_i\left(\gamma^L{}_{ij}\right)\right)^{\sum_{i=1}^{n}\varpi_i}\right)^{\sum_{j=1}^{m}p_j}, \\ \left(\left(max_j max_i\left(\gamma^U{}_{ij}\right)\right)^{\sum_{i=1}^{n}\varpi_i}\right)^{\sum_{j=1}^{m}p_j}\end{array}\right] \\
\Rightarrow \left[\begin{array}{c}min_j min_i\left(\gamma^L{}_{ij}\right), \\ min_j min_i\left(\gamma^U{}_{ij}\right)\end{array}\right] \le \left[\begin{array}{c}\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^L{}_{ij}\right)^{\varpi_i}\right)^{p_j}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^U{}_{ij}\right)^{\varpi_i}\right)^{p_j}\end{array}\right] \le \left[\begin{array}{c}max_j max_i\left(\gamma^L{}_{ij}\right), \\ max_j max_i\left(\gamma^U{}_{ij}\right)\end{array}\right]
\end{gathered} \tag{4}$$

Therefore from Equations (2)–(4), it is clear that

$$\left[\begin{array}{c}min_j min_i\left(\alpha^L{}_{ij}\right), \\ min_j min_i\left(\alpha^U{}_{ij}\right)\end{array}\right] \le \left[\begin{array}{c}\sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^L{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}}, \\ \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^U{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}}\end{array}\right] \le \left[\begin{array}{c}max_j max_i\left(\alpha^L{}_{ij}\right), \\ max_j max_i\left(\alpha^U{}_{ij}\right)\end{array}\right]$$

$$\left[\begin{array}{c}min_j min_i\left(\beta^L{}_{ij}\right), \\ min_j min_i\left(\beta^U{}_{ij}\right)\end{array}\right] \le \left[\begin{array}{c}\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^L{}_{ij}\right)^{\varpi_i}\right)^{p_j}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^U{}_{ij}\right)^{\varpi_i}\right)^{p_j}\end{array}\right] \le \left[\begin{array}{c}max_j max_i\left(\beta^L{}_{ij}\right), \\ max_j max_i\left(\beta^U{}_{ij}\right)\end{array}\right]$$

And

$$\begin{bmatrix} min_j min_i\left(\gamma^L{}_{ij}\right), \\ min_j min_i\left(\gamma^U{}_{ij}\right) \end{bmatrix} \leq \begin{bmatrix} \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^L{}_{ij}\right)^{\varpi_i}\right)^{p_j}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^U{}_{ij}\right)^{\varpi_i}\right)^{p_j} \end{bmatrix} \leq \begin{bmatrix} max_j max_i\left(\gamma^L{}_{ij}\right), \\ max_j max_i\left(\gamma^U{}_{ij}\right) \end{bmatrix}$$

Let $\xi = IVT - SFS_{ft}WA(F_{s_{11}}, F_{s_{12}}, \ldots, F_{s_{nm}}) = \left(\left[\alpha^L{}_{\xi}, \alpha^U{}_{\xi}\right], \left[\beta^L{}_{\xi}, \beta^U{}_{\xi}\right], \left[\gamma^L{}_{\xi}, \gamma^U{}_{\xi}\right]\right)$, then according to the definition of score function given in Definition 11, we obtain

$$SC(\xi) = \frac{\left(\alpha^L{}_{\xi}\right)^q\left(1-\left(\beta^L{}_{\xi}\right)^q-\left(\gamma^L{}_{\xi}\right)^q\right)+\left(\alpha^U{}_{\xi}\right)^q\left(1-\left(\beta^U{}_{\xi}\right)^q-\left(\gamma^U{}_{\xi}\right)^q\right)}{3}$$
$$\leq \left(\frac{\begin{array}{c} max_j max_i\left(\alpha^L{}_{ij}\right)^q\left(1-min_j min_i\left(\beta^L{}_{ij}\right)^q-min_j min_i\left(\gamma^L{}_{ij}\right)^q\right) \\ +max_j max_i\left(\alpha^U{}_{ij}\right)^q\left(1-min_j min_i\left(\beta^U{}_{ij}\right)^q-min_j min_i\left(\gamma^U{}_{ij}\right)^q\right)\end{array}}{3}\right)$$
$$= SC\left(F^+_{s_{ij}}\right) \Rightarrow Sc(\xi) \leq SC\left(F^+_{s_{ij}}\right)$$

and

$$Sc(\xi) = \frac{\left(\alpha^L{}_{\xi}\right)^q\left(1-\left(\beta^L{}_{\xi}\right)^q-\left(\gamma^L{}_{\xi}\right)^q\right)+\left(\alpha^U{}_{\xi}\right)^q\left(1-\left(\beta^U{}_{\xi}\right)^q-\left(\gamma^U{}_{\xi}\right)^q\right)}{3}$$
$$\geq \left(\frac{\begin{array}{c} min_j min_i\left(\alpha^L{}_{ij}\right)^q\left(1-max_j max_i\left(\beta^L{}_{ij}\right)^q-max_j max_i\left(\gamma^L{}_{ij}\right)^q\right) \\ +min_j min_i\left(\alpha^U{}_{ij}\right)^q\left(1-max_j max_i\left(\beta^U{}_{ij}\right)^q-max_j max_i\left(\gamma^U{}_{ij}\right)^q\right)\end{array}}{3}\right)$$
$$= SC\left(F^-_{s_{ij}}\right) \Rightarrow Sc(\xi) \geq SC\left(F^-_{s_{ij}}\right).$$

According to this condition, we have the following cases **Case i.** If $Sc(\xi) < SC\left(F^+_{s_{ij}}\right)$ and $Sc(\xi) > SC\left(F^-_{s_{ij}}\right)$, then by Definition 12, we have

$$\left(F^-_{s_{ij}}\right) < IVT - SFS_{ft}WA(F_{s_{11}}, F_{s_{12}}, \ldots, F_{s_{nm}}) < \left(F^+_{s_{ij}}\right).$$

Case ii. If $Sc(\xi) = SC\left(F^+_{s_{ij}}\right)$, that is

$$\frac{\left(\alpha^L{}_{\xi}\right)^q\left(1-\left(\beta^L{}_{\xi}\right)^q-\left(\gamma^L{}_{\xi}\right)^q\right)+\left(\alpha^U{}_{\xi}\right)^q\left(1-\left(\beta^U{}_{\xi}\right)^q-\left(\gamma^U{}_{\xi}\right)^q\right)}{3}$$
$$= \left(\frac{\begin{array}{c} max_j max_i\left(\alpha^L{}_{ij}\right)^q\left(1-min_j min_i\left(\beta^L{}_{ij}\right)^q-min_j min_i\left(\gamma^L{}_{ij}\right)^q\right) \\ +max_j max_i\left(\alpha^U{}_{ij}\right)^q\left(1-min_j min_i\left(\beta^U{}_{ij}\right)^q-min_j min_i\left(\gamma^U{}_{ij}\right)^q\right)\end{array}}{3}\right)$$

Then by using the above inequalities, we get

$$\left[\alpha^L{}_{\xi}, \alpha^U{}_{\xi}\right] = \left[max_j max_i\left(\alpha^L{}_{ij}\right), max_j max_i\left(\alpha^U{}_{ij}\right)\right]$$

and $\left[\beta^L{}_{\xi},\ \beta^L{}_{\xi}\right] = \left[min_j min_i\left(\beta^L{}_{ij}\right),\ min_j min_i\left(\beta^U{}_{ij}\right)\right]$ and $\left[\gamma^L{}_{\xi},\ \gamma^U{}_{\xi}\right] = \left[min_j min_i\left(\gamma^L{}_{ij}\right),\ min_j min_i\left(\gamma^U{}_{ij}\right)\right]$. Thus $\left(\delta_{\xi}\right)^q = \delta_{F^+_{s_{ij}}}{}^q$ implies that $IVT - SFS_{ft}WA$ $\left(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}\right) = \left(F^+_{s_{ij}}\right)$. **Case iii.** If $Sc(\xi) = Sc\left(F^-_{s_{ij}}\right)$, then

$$Sc(\xi) = \frac{\left(\alpha^L{}_{\xi}\right)^q\left(1-\left(\beta^L{}_{\xi}\right)^q-\left(\gamma^L{}_{\xi}\right)^q\right)+\left(\alpha^U{}_{\xi}\right)^q\left(1-\left(\beta^U{}_{\xi}\right)^q-\left(\gamma^U{}_{\xi}\right)^q\right)}{3}$$
$$= \left(\frac{\begin{array}{c} min_j min_i\left(\alpha^L{}_{ij}\right)^q\left(1 - max_j max_i\left(\beta^L{}_{ij}\right)^q - max_j max_i\left(\gamma^L{}_{ij}\right)^q\right) \\ +min_j min_i\left(\alpha^U{}_{ij}\right)^q\left(1 - max_j max_i\left(\beta^U{}_{ij}\right)^q - max_j max_i\left(\gamma^U{}_{ij}\right)^q\right)\end{array}}{3}\right)$$

Then by using the above inequalities, we obtain

$$\left[\alpha^L{}_{\xi},\ \alpha^U{}_{\xi}\right] = \left[min_j min_i\left(\alpha^L{}_{ij}\right),\ min_j min_i\left(\alpha^U{}_{ij}\right)\right]$$

and $\left[\beta^L{}_{\xi},\ \beta^L{}_{\xi}\right] = \left[max_j max_i\left(\beta^L{}_{ij}\right),\ max_j max_i\left(\beta^U{}_{ij}\right)\right]$ and $\left[\gamma^L{}_{\xi},\ \gamma^U{}_{\xi}\right] = \left[max_j max_i\left(\gamma^L{}_{ij}\right),\ max_j max_i\left(\gamma^U{}_{ij}\right)\right]$. Thus $\left(\delta_{\xi}\right)^q = \delta_{F^-_{s_{ij}}}{}^q$ implies that

$$IVT - SFS_{ft}WA\left(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}\right) = \left(F^-_{s_{ij}}\right).$$

Hence it is proved that

$$\left(F^-_{s_{ij}}\right) \leq IVT - SFS_{\mathrm{ft}}WA\left(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}\right) \leq \left(F^+_{s_{ij}}\right)$$

3. (Monotonicity). $\alpha^L{}_{ij} \leq \alpha'^L{}_{ij}, \alpha^U{}_{ij} \leq \alpha'^U{}_{ij},\ \beta^L{}_{ij} \geq \beta'^L{}_{ij},\ \beta^U{}_{ij} \geq \beta'^U{}_{ij}$ and $\gamma^L{}_{ij} \geq \gamma'^L{}_{ij},\ \gamma^U{}_{ij} \geq \gamma'^U{}_{ij}$, then

$$\begin{array}{c}
\left[\alpha^L{}_{ij},\ \alpha^U{}_{ij}\right] \leq \left[\alpha'^L{}_{ij},\ \alpha'^U{}_{ij}\right] \Rightarrow \left[1-\alpha'^L{}_{ij},\ 1-\alpha'^U{}_{ij}\right] \leq \left[1-\alpha^L{}_{ij},\ 1-\alpha^U{}_{ij}\right] \\
\Rightarrow \left[1-\alpha'^L{}_{ij}{}^q,\ 1-\alpha'^U{}_{ij}{}^q\right] \leq \left[1-\alpha^L{}_{ij}{}^q,\ 1-\alpha^U{}_{ij}{}^q\right] \\
\Rightarrow \left[\begin{array}{c} \prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left(\alpha'^L{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}, \\ \prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left(\alpha'^U{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j} \end{array}\right] \leq \left[\begin{array}{c} \prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left(\alpha^L{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}, \\ \prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left(\alpha^U{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j} \end{array}\right] \\
\Rightarrow \left[\begin{array}{c} 1-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left(\alpha^L{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}, \\ 1-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left(\alpha^U{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j} \end{array}\right] \leq \left[\begin{array}{c} 1-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left(\alpha'^L{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}, \\ 1-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left(\alpha'^U{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j} \end{array}\right] \\
\Rightarrow \left[\begin{array}{c} \sqrt[q]{1-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left(\alpha^L{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}}, \\ \sqrt[q]{1-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left(\alpha^U{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}} \end{array}\right] \leq \left[\begin{array}{c} \sqrt[q]{1-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left(\alpha'^L{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}}, \\ \sqrt[q]{1-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left(\alpha'^U{}_{ij}\right)^q\right)^{\varpi_i}\right)^{p_j}} \end{array}\right]
\end{array} \quad (5)$$

and

$$\begin{array}{c}
\left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right] \geq \left[\beta'^L{}_{ij},\ \beta'^U{}_{ij}\right] \\
\Rightarrow \left[\prod_{i=1}^n\left(\beta^L{}_{ij}\right)^{\varpi_i},\ \prod_{i=1}^n\left(\beta^U{}_{ij}\right)^{\varpi_i}\right] \geq \left[\prod_{i=1}^n\left(\beta'^L{}_{ij}\right)^{\varpi_i},\ \prod_{i=1}^n\left(\beta'^U{}_{ij}\right)^{\varpi_i}\right] \\
\Rightarrow \left[\begin{array}{c} \prod_{j=1}^m\left(\prod_{i=1}^n\left(\beta^L{}_{ij}\right)^{\varpi_i}\right)^{p_j}, \\ \prod_{j=1}^m\left(\prod_{i=1}^n\left(\beta^U{}_{ij}\right)^{\varpi_i}\right)^{p_j} \end{array}\right] \geq \left[\begin{array}{c} \prod_{j=1}^m\left(\prod_{i=1}^n\left(\beta'^L{}_{ij}\right)^{\varpi_i}\right)^{p_j}, \\ \prod_{j=1}^m\left(\prod_{i=1}^n\left(\beta'^U{}_{ij}\right)^{\varpi_i}\right)^{p_j} \end{array}\right]
\end{array} \quad (6)$$

Moreover,

$$\begin{gathered}[\gamma^{L}{}_{ij},\ \gamma^{U}{}_{ij}] \geq [\gamma'^{L}{}_{ij},\ \gamma'^{U}{}_{ij}] \\ \Rightarrow \left[\prod_{i=1}^{n}\left(\gamma^{L}{}_{ij}\right)^{\varpi_i},\ \prod_{i=1}^{n}\left(\gamma^{U}{}_{ij}\right)^{\varpi_i}\right] \geq \left[\prod_{i=1}^{n}\left(\gamma'^{L}{}_{ij}\right)^{\varpi_i},\ \prod_{i=1}^{n}\left(\gamma'^{U}{}_{ij}\right)^{\varpi_i}\right] \\ \Rightarrow \begin{bmatrix}\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^{L}{}_{ij}\right)^{\varpi_i}\right)^{p_j}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^{U}{}_{ij}\right)^{\varpi_i}\right)^{p_j}\end{bmatrix} \geq \begin{bmatrix}\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma'^{L}{}_{ij}\right)^{\varpi_i}\right)^{p_j}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma'^{U}{}_{ij}\right)^{\varpi_i}\right)^{p_j}\end{bmatrix}\end{gathered} \tag{7}$$

Let $\xi_F = IVT - SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) = ([\alpha^{L}{}_{\xi_F},\ \alpha^{U}{}_{\xi_F}],\ [\beta^{L}{}_{\xi_F},\ \beta^{U}{}_{\xi_F}],\ [\gamma^{L}{}_{\xi_F},\ \gamma^{U}{}_{\xi_F}])$ and $\xi_{F'} = IVT - SFS_{ft}WA(F'_{s_{11}},\ F'_{s_{12}},\ \ldots,\ F'_{s_{nm}}) = \left(\left[\alpha'^{L}{}_{\xi_{F'}},\ \alpha'^{U}{}_{\xi_{F'}}\right],\ \left[\beta'^{L}{}_{\xi_{F'}},\ \beta'^{U}{}_{\xi_{F'}}\right],\ \left[\gamma'^{L}{}_{\xi_{F'}},\ \gamma'^{U}{}_{\xi_{F'}}\right]\right)$, then from Equations (5)–(7), we obtain

$$\left[\alpha^{L}{}_{\xi_F},\ \alpha^{U}{}_{\xi_F}\right] \leq \left[\alpha'^{L}{}_{\xi_{F'}},\ \alpha'^{U}{}_{\xi_{F'}}\right],\ \left[\beta^{L}{}_{\xi_F},\ \beta^{U}{}_{\xi_F}\right] \geq \left[\beta'^{L}{}_{\xi_{F'}},\ \beta'^{U}{}_{\xi_{F'}}\right] \text{ and } \left[\gamma^{L}{}_{\xi_F},\ \gamma^{U}{}_{\xi_F}\right] \geq \left[\gamma'^{L}{}_{\xi_{F'}},\ \gamma'^{U}{}_{\xi_{F'}}\right]$$

Now, by using Definition 11 of the score function, we obtain $Sc(\xi_F) \leq Sc(\xi_{F'})$. Here, we have the following cases **Case i.** If $Sc(\xi_F) < Sc(\xi_{F'})$, then by using the comparison result of two $IVT - SFS_{ft}Ns$, we have

$$IVT - SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) < IVT - SFS_{ft}WA\left(F'_{s_{11}},\ F'_{s_{12}},\ \ldots,\ F'_{s_{nm}}\right).$$

Case ii. If $Sc(\xi_F) = Sc(\xi_{F'})$, then

$$\begin{aligned}Sc(\xi_F) &= \frac{\left(\alpha^{L}{}_{\xi_F}\right)^q\left(1-\left(\beta^{L}{}_{\xi_F}\right)^q-\left(\gamma^{L}{}_{\xi_F}\right)^q\right)+\left(\alpha^{U}{}_{\xi_F}\right)^q\left(1-\left(\beta^{U}{}_{\xi_F}\right)^q-\left(\gamma^{U}{}_{\xi_F}\right)^q\right)}{3} \\ &= \frac{\left(\alpha^{L}{}_{\xi_{F'}}\right)^q\left(1-\left(\beta^{L}{}_{\xi_{F'}}\right)^q-\left(\gamma^{L}{}_{\xi_{F'}}\right)^q\right)+\left(\alpha^{U}{}_{\xi_{F'}}\right)^q\left(1-\left(\beta^{U}{}_{\xi_{F'}}\right)^q-\left(\gamma^{U}{}_{\xi_{F'}}\right)^q\right)}{3}\end{aligned}$$

Hence by using the above inequality, we obtain $[\alpha^{L}{}_{\xi_F},\ \alpha^{U}{}_{\xi_F}] = \left[\alpha^{L}{}_{\xi_{F'}},\ \alpha^{U}{}_{\xi_{F'}}\right]$, $[\beta^{L}{}_{\xi_F},\ \beta^{U}{}_{\xi_F}] = \left[\beta^{L}{}_{\xi_{F'}},\ \beta^{U}{}_{\xi_{F'}}\right]$. and $[\gamma^{L}{}_{\xi_F},\ \gamma^{U}{}_{\xi_F}] = \left[\gamma^{L}{}_{\xi_{F'}},\ \gamma^{U}{}_{\xi_{F'}}\right]$. So we obtain $\delta_{\xi_F}{}^q = \delta_{\xi_{F'}}{}^q \Rightarrow ([\alpha^{L}{}_{\xi_F},\ \alpha^{U}{}_{\xi_F}],\ [\beta^{L}{}_{\xi_F},\ \beta^{U}{}_{\xi_F}],\ [\gamma^{L}{}_{\xi_F},\ \gamma^{U}{}_{\xi_F}]) = \left(\left[\alpha'^{L}{}_{\xi_{F'}},\ \alpha'^{U}{}_{\xi_{F'}}\right],\ \left[\beta'^{L}{}_{\xi_{F'}},\ \beta'^{U}{}_{\xi_{F'}}\right],\ \left[\gamma'^{L}{}_{\xi_{F'}},\ \gamma'^{U}{}_{\xi_{F'}}\right]\right)$. Hence it is proved that

$$IVT - SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) \leq IVT - SFS_{ft}WA\left(F'_{s_{11}},\ F'_{s_{12}},\ \ldots,\ F'_{s_{nm}}\right).$$

4. **(Shift Invariance).** Let $F_s = ([\alpha^L,\ \alpha^U],\ [\beta^L,\ \beta^U],\ [\gamma^L,\ \gamma^L])$ and $F_{s_{ij}} = \left(\begin{array}{c}\begin{bmatrix}\alpha^{L}{}_{ij}, \\ \alpha^{U}{}_{ij}\end{bmatrix}, \\ \begin{bmatrix}\beta^{L}{}_{ij}, \\ \beta^{U}{}_{ij}\end{bmatrix}, \begin{bmatrix}\gamma^{L}{}_{ij}, \\ \gamma^{U}{}_{ij}\end{bmatrix}\end{array}\right)$ be family of $IVT - SFS_{ft}Ns$, then

$$F_{s_{ij}} \oplus F_s = \left(\left[\sqrt[q]{1-\left(1-\alpha^{L}{}_{ij}{}^{q}\right)\left(1-\alpha^{Lq}\right)},\ \sqrt[q]{1-\left(1-\alpha^{U}{}_{ij}{}^{q}\right)\left(1-\alpha^{Uq}\right)}\right],\ \left[\beta^{L}{}_{ij}\beta^{L},\ \beta^{U}{}_{ij}\beta^{U}\right],\ \left[\gamma^{L}{}_{ij}\gamma^{L},\ \gamma^{U}{}_{ij}\gamma^{U}\right]\right)$$

Therefore,

$$IVT - SFS_{ft}WA(F_{s_{11}} \oplus F_s,\ F_{s_{12}} \oplus F_s,\ \ldots,\ F_{s_{nm}} \oplus F_s) = \oplus_{j=1}^{m} p_j \left(\oplus_{i=1}^{n} \varpi_i \left(F_{s_{ij}} \oplus F_s \right) \right)$$

$$= \begin{pmatrix} \begin{bmatrix} \sqrt[q]{1 - \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} \left(1 - ({\alpha^L}_{ij})^q\right)^{\varpi_i} \left(1 - (\alpha^L)^q\right)^{\varpi_i} \right)^{p_j}}, \\ \sqrt[q]{1 - \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} \left(1 - ({\alpha^U}_{ij})^q\right)^{\varpi_i} \left(1 - (\alpha^U)^q\right)^{\varpi_i} \right)^{p_j}} \end{bmatrix}, \\ \begin{bmatrix} \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\beta^L}_{ij})^{\varpi_i} \left(\beta^{L\varpi_i}\right) \right)^{p_j}, \\ \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\beta^U}_{ij})^{\varpi_i} \left(\beta^{U\varpi_i}\right) \right)^{p_j} \end{bmatrix} \\ \begin{bmatrix} \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\gamma^L}_{ij})^{\varpi_i} \left(\gamma^{L\varpi_i}\right) \right)^{p_j},\ \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\gamma^U}_{ij})^{\varpi_i} \left(\gamma^{U\varpi_i}\right) \right)^{p_j} \end{bmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} \begin{bmatrix} \sqrt[q]{1 - \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} \left(1 - ({\alpha^L}_{ij})^q\right)^{\varpi_i} \left(1 - (\alpha^L)^q\right)^{\varpi_i} \right)^{p_j}}, \\ \sqrt[q]{1 - \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} \left(1 - ({\alpha^U}_{ij})^q\right)^{\varpi_i} \left(1 - (\alpha^U)^q\right)^{\varpi_i} \right)^{p_j}} \end{bmatrix}, \\ \begin{bmatrix} \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\beta^L}_{ij})^{\varpi_i} \left(\beta^{L\varpi_i}\right) \right)^{p_i}, \\ \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\beta^U}_{ij})^{\varpi_i} \left(\beta^{U\varpi_i}\right) \right)^{p_i} \end{bmatrix}, \\ \begin{bmatrix} \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\gamma^L}_{ij})^{\varpi_i} \left(\gamma^{L\varpi_i}\right) \right)^{p_i},\ \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\gamma^U}_{ij})^{\varpi_i} \left(\gamma^{U\varpi_i}\right) \right)^{p_i} \end{bmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} \begin{bmatrix} \sqrt[q]{1 - \left(1 - (\alpha^L)^q\right) \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} \left(1 - ({\alpha^L}_{ij})^q\right)^{\varpi_i} \right)^{p_j}}, \\ \sqrt[q]{1 - \left(1 - (\alpha^U)^q\right) \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} \left(1 - ({\alpha^U}_{ij})^q\right)^{\varpi_i} \right)^{p_j}} \end{bmatrix}, \\ \begin{bmatrix} \beta^L \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\beta^L}_{ij})^{\varpi_i} \right)^{p_j}, \\ \beta^U \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\beta^U}_{ij})^{\varpi_i} \right)^{p_j} \end{bmatrix}, \\ \begin{bmatrix} \gamma^L \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\gamma^L}_{ij})^{\varpi_i} \right)^{p_j},\ \gamma^U \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\gamma^U}_{ij})^{\varpi_i} \right)^{p_j} \end{bmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} \begin{bmatrix} \sqrt[q]{1 - \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} \left(1 - ({\alpha^L}_{ij})^q\right)^{\varpi_i} \right)^{p_j}}, \\ \sqrt[q]{1 - \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} \left(1 - ({\alpha^U}_{ij})^q\right)^{\varpi_i} \right)^{p_j}} \end{bmatrix}, \\ \begin{bmatrix} \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\beta^L}_{ij})^{\varpi_i} \right)^{p_j}, \\ \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\beta^U}_{ij})^{\varpi_i} \right)^{p_j} \end{bmatrix}, \\ \begin{bmatrix} \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\gamma^L}_{ij})^{\varpi_i} \right)^{p_j},\ \prod\limits_{j=1}^{m} \left(\prod\limits_{i=1}^{n} ({\gamma^U}_{ij})^{\varpi_i} \right)^{p_j} \end{bmatrix} \end{pmatrix} \oplus \left([\alpha^L,\ \alpha^U], [\beta^L,\ \beta^U],\ [\gamma^L,\ \gamma^U] \right)$$

$$= IVT - SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) \oplus F_s$$

Hence the required result is proved.

5. **(Homogeneity).** Let $K \geq 0$ be any real number and $F_{s_{ij}} = \left([{\alpha^L}_{ij}, {\alpha^U}_{ij}], [{\beta^L}_{ij}, {\beta^U}_{ij}],\ [{\gamma^L}_{ij}, {\gamma^U}_{ij}] \right)$ be family of $IVT - SFS_{ft}Ns$, then

$$KF_{s_{ij}} = \left(\left[\sqrt[q]{\left(1-\left(1-\alpha^{L}{}_{ij}{}^{q}\right)^{K}\right)},\ \sqrt[q]{\left(1-\left(1-\alpha^{U}{}_{ij}{}^{q}\right)^{K}\right)} \right],\ \left[\beta^{L}{}_{ij}{}^{K},\ \beta^{U}{}_{ij}{}^{K}\right],\ \left[\gamma^{L}{}_{ij}{}^{K},\ \gamma^{U}{}_{ij}{}^{K}\right] \right)$$

Now

$$\begin{aligned}
&IVT-SFS_{ft}WA(KF_{s_{11}},\ KF_{s_{12}},\ \ldots,\ KF_{s_{nm}}) \\
&= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{L}{}_{ij}\right)^{q}\right)^{K\varpi_i}\right)^{p_j}}, \\ \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{U}{}_{ij}\right)^{q}\right)^{K\varpi_i}\right)^{p_j}} \end{array} \right], \\ \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^{L}{}_{ij}\right)^{K\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^{U}{}_{ij}\right)^{K\varpi_i}\right)^{p_j} \right], \\ \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^{L}{}_{ij}\right)^{K\varpi_i}\right)^{p_j},\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^{U}{}_{ij}\right)^{K\varpi_i}\right)^{p_j} \right] \end{array} \right) \\
&= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1-\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{L}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}\right)^{K}}, \\ \sqrt[q]{1-\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left(\alpha^{U}{}_{ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}\right)^{K}} \end{array} \right], \\ \left[\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^{L}{}_{ij}\right)^{\varpi_i}\right)^{p_j}\right)^{K},\ \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\beta^{U}{}_{ij}\right)^{\varpi_i}\right)^{p_j}\right)^{K} \right], \\ \left[\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^{L}{}_{ij}\right)^{\varpi_i}\right)^{p_j}\right)^{K},\ \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\gamma^{U}{}_{ij}\right)^{\varpi_i}\right)^{p_j}\right)^{K} \right] \end{array} \right) \\
&= K\left(IVT-SFS_{ft}WA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}})\right)
\end{aligned}$$

Hence the result is proved. □

4.2. Interval-Valued T-Spherical Fuzzy Soft Ordered Weighted Average $\left(IVT-SFS_{ft}OWA\right)$ Operator

From the above discussion, it is clear that $IVT-SFS_{ft}WA$ operator only weighted the value of $IVT-SFS_{ft}Ns$. However, on the other hand, the $IVT-SFS_{ft}OWA$ operator weights the ordered position by scoring the $IVT-SFS_{ft}$ values. Here, we will discuss the $IVT-SFS_{ft}OWA$ operator and also its properties.

Definition 14. *Let* $F_{s_{ij}} = \left(\left[\alpha^{L}{}_{ij},\ \alpha^{U}{}_{ij}\right],\ \left[\beta^{L}{}_{ij},\ \beta^{U}{}_{ij}\right],\ \left[\gamma^{L}{}_{ij},\ \gamma^{U}{}_{ij}\right]\right)$ *for* $i = 1,\ 2,\ \ldots,\ n$ *and* $j = 1,\ 2,\ \ldots,\ m$*, be the family of* $IVT-SFS_{ft}Ns$*,* $\varpi = \{\varpi_1,\ \varpi_2,\ \ldots,\ \varpi_n\}$ *denote the weight vector of* e_i *experts and* $p = \{p_1,\ p_2,\ \ldots,\ p_m\}$ *denote the weight vector of parameters* s_j *with condition* $\varpi_i,\ p_j \in [0,\ 1]$ *and* $\sum_{i=1}^{n}\varpi_i = 1$, $\sum_{i=1}^{n} p_i = 1$*. Then* $IVT-SFS_{ft}OWA$ *operator is the mapping defined by* $IVT-SFS_{ft}OWA : \mathbb{Q}^n \to \mathbb{Q}$*, where (*$\mathbb{Q}$ *is the family of all* $IVT-SFS_{ft}Ns$*)*

$$IVT-SFS_{ft}OWA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) \oplus_{j=1}^{m} p_j = \oplus_{j=1}^{m} p_j \oplus_{i=1}^{n} \varpi_i F_{\partial s_{ij}}.$$

Theorem 4. *Let* $F_{s_{ij}} = \left(\left[\alpha^{L}{}_{ij},\ \alpha^{U}{}_{ij}\right],\ \left[\beta^{L}{}_{ij},\ \beta^{U}{}_{ij}\right],\ \left[\gamma^{L}{}_{ij},\ \gamma^{U}{}_{ij}\right]\right)$ *for* $i = 1,\ 2,\ \ldots,\ n$ *and* $j = 1,\ 2,\ \ldots,\ m$*, be the family of* $IVT-SFS_{ft}Ns$*. Then* $IVT-SFS_{ft}OWA$ operator is given as

$$IVT - SFS_{ft}OWA(F_{s_{11}}, F_{s_{12}}, \ldots, F_{s_{nm}}) = \oplus_{j=1}^{m} p_j \left(\oplus_{i=1}^{n} \varpi_i F_{\partial s_{ij}} \right)$$
$$= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1 - \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \left(\alpha^L{}_{\partial ij} \right)^q \right)^{\varpi_i} \right)^{p_j}}, \\ \sqrt[q]{1 - \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \left(\alpha^U{}_{\partial ij} \right)^q \right)^{\varpi_i} \right)^{p_j}} \end{array} \right], \left[\begin{array}{c} \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(\beta^L{}_{\partial ij} \right)^{\varpi_i} \right)^{p_j}, \\ \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(\beta^U{}_{\partial ij} \right)^{\varpi_i} \right)^{p_j} \end{array} \right], \\ \left[\prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(\gamma^L{}_{\partial ij} \right)^{\varpi_i} \right)^{p_j}, \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(\gamma^U{}_{\partial ij} \right)^{\varpi_i} \right)^{p_j} \right] \end{array} \right) \quad (8)$$

where $F_{\partial s_{ij}} = \left(\left[\alpha^L{}_{\partial ij}, \alpha^U{}_{\partial ij} \right], \left[\beta^L{}_{\partial ij}, \beta^U{}_{\partial ij} \right], \left[\gamma^L{}_{\partial ij}, \gamma^U{}_{\partial ij} \right] \right)$ *denote the permutation of ith and jth largest object of the collection of* $i \times j$ $IVT - SFS_{ft}NsF_{s_{ij}} = \left(\left[\alpha^L{}_{ij}, \alpha^U{}_{ij} \right], \left[\beta^L{}_{ij}, \beta^U{}_{ij} \right], \left[\gamma^L{}_{ij}, \gamma^U{}_{ij} \right] \right)$.

Proof. The proof is similar to Theorem 2. □

Remark 2.

1. *Using* $q = 1$, *then established* $IVT - SFS_{ft}OWA$ *operator will reduce to* $IVPFS_{ft}OWA$ *operator.*
2. *Using* $q = 2$, *then established* $IVT - SFS_{ft}OWA$ *operator will reduce to* $IVSFS_{ft}OWA$ *operator.*
3. *If we neglect the obstinacy grade that is* $\beta_{ij} = 0$, *and using* $q = 2$, *the proposed* $IVT - SFS_{ft}OWA$ *operator will reduce to* $IVP_yFS_{ft}OWA$ *operator.*
4. *If we neglect the obstinacy grade that is* $\beta_{ij} = 0$, *and using* $q = 1$, *the proposed* $IVT - SFS_{ft}OWA$ *operator will reduce to an interval-valued intuitionistic fuzzy soft ordered weighted average* $\left(IVIFS_{ft}OWA \right)$ *operator.*
5. *Moreover, if we put only one parameter that is* s_1 *(mean m = 1), then* $IVT - SFS_{ft}OWA$ *operator reduces to the interval-valued T-spherical fuzzy ordered weighted average IVT-SFOWA operator.*

Hence it is clear that $IVPFS_{ft}OWA$, $IVSFS_{ft}OWA$, $IVP_yFS_{ft}OWA$, $IVIFS_{ft}OWA$ and IVT-SFOWA operators are the special cases of $IVT - SFS_{ft}OWA$ operator. The present work is more general.

Example 3. *Consider the collection of* $IVT - SFS_{ft}NsF_{s_{ij}} = \left(\left[\alpha^L{}_{ij}, \alpha^U{}_{ij} \right], \left[\beta^L{}_{ij}, \beta^U{}_{ij} \right], \left[\gamma^L{}_{ij}, \gamma^U{}_{ij} \right] \right)$ *as given in Table 2 of example 2, then tabular depiction of* $F_{\partial s_{ij}} = \left(\left[\alpha^L{}_{\partial ij}, \alpha^U{}_{\partial ij} \right], \left[\beta^L{}_{\partial ij}, \beta^U{}_{\partial ij} \right], \left[\gamma^L{}_{\partial ij}, \gamma^U{}_{\partial ij} \right] \right)$ *is given in Table 3.*

Table 3. $IVT - SFS_{ft}S$, $F_{\partial s_{ij}} = \left(\left[\alpha^L{}_{\partial ij}, \alpha^U{}_{\partial ij} \right], \left[\beta^L{}_{\partial ij}, \beta^U{}_{\partial ij} \right], \left[\gamma^L{}_{\partial ij}, \gamma^U{}_{\partial ij} \right] \right)$ for $q \geq 3$.

	s_1	s_2	s_3	s_4
x_1	([0.6, 0.7], [0.4, 0.6], [0.6, 0.8])	([0.5, 0.53], [0.5, 0.61], [0.5, 0.72])	([0.6, 0.6], [0.6, 0.7], [0.5, 0.5])	([0.5, 0.6], [0.4, 0.5], [0.5, 0.7])
x_2	([0.5, 0.8], [0.4, 0.5], [0.6, 0.7])	([0.2, 0.6], [0.3, 0.7], [0.3, 0.42])	([0.4, 0.6], [0.4, 0.5], [0.3, 0.5])	([0.4, 0.5], [0.4, 0.5], [0.5, 0.5])
x_3	([0.5, 0.5], [0.3, 0.8], [0.4, 0.51])	([0.1, 0.6], [0.2, 0.7], [0.3, 0.40])	([0.3, 0.7], [0.3, 0.71], [0.2, 0.6])	([0.3, 0.5], [0.3, 0.7], [0.4, 0.45])
x_4	([0.3, 0.5], [0.3, 0.5], [0.4, 0.50])	([0.1, 0.3], [0.2, 0.4], [0.2, 0.9])	([0.3, 0.5], [0.3, 0.6], [0.2, 0.55])	([0.2, 0.6], [0.3, 0.7], [0.3, 0.6])
x_5	([0.2, 0.6], [0.3, 0.4], [0.3, 0.6])	([0.1, 0.3], [0.1, 0.3], [0.1, 0.5])	([0.2, 0.3], [0.2, 0.6], [0.2, 0.5])	([0.2, 0.6], [0.3, 0.56], [0.2, 0.7]).

Now, by using Equation (8) of Theorem 4, we have

$$IVT-SFS_{ft}OWA\left(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}\right)$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}\sqrt[q]{1-\prod\limits_{j=1}^{m}\left(\prod\limits_{i=1}^{n}\left(1-\left({\alpha^{L}}_{\partial ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}},\\ \sqrt[q]{1-\prod\limits_{j=1}^{m}\left(\prod\limits_{i=1}^{n}\left(1-\left({\alpha^{U}}_{\partial ij}\right)^{q}\right)^{\varpi_i}\right)^{p_j}}\end{array}\right],\left[\begin{array}{c}\prod\limits_{j=1}^{m}\left(\prod\limits_{i=1}^{n}\left({\beta^{L}}_{\partial ij}\right)^{\varpi_i}\right)^{p_j},\\ \prod\limits_{j=1}^{m}\left(\prod\limits_{i=1}^{n}\left({\beta^{U}}_{\partial ij}\right)^{\varpi_i}\right)^{p_j}\end{array}\right],\\ \left[\prod\limits_{j=1}^{m}\left(\prod\limits_{i=1}^{n}\left({\gamma^{L}}_{\partial ij}\right)^{\varpi_i}\right)^{p_j},\ \prod\limits_{j=1}^{m}\left(\prod\limits_{i=1}^{n}\left({\gamma^{U}}_{\partial ij}\right)^{\varpi_i}\right)^{p_j}\right]\end{array}\right)$$

$$=\left[\begin{array}{c}\left(\left[\begin{array}{c}\sqrt[3]{\begin{array}{c}1-\left\{\left(1-0.6^3\right)^{0.25}\left(1-0.5^3\right)^{0.15}\left(1-0.5^3\right)^{0.14}\left(1-0.5^3\right)^{0.3}\left(1-0.2^3\right)^{0.16}\right\}^{0.27}\\ \left\{\left(1-0.5^3\right)^{0.25}\left(1-0.2^3\right)^{0.15}\left(1-0.1^3\right)^{0.14}\left(1-0.1^3\right)^{0.3}\left(1-0.1^3\right)^{0.16}\right\}^{0.19}\\ \left\{\left(1-0.6^3\right)^{0.25}\left(1-0.4^3\right)^{0.15}\left(1-0.3^3\right)^{0.14}\left(1-0.3^3\right)^{0.3}\left(1-0.2^3\right)^{0.16}\right\}^{0.29}\\ \left\{\left(1-0.5^3\right)^{0.25}\left(1-0.4^3\right)^{0.15}\left(1-0.3^3\right)^{0.14}\left(1-0.2^3\right)^{0.3}\left(1-0.2^3\right)^{0.16}\right\}^{0.25}\end{array}},\\ \sqrt[3]{\begin{array}{c}1-\left\{\left(1-0.8^3\right)^{0.25}\left(1-0.7^3\right)^{0.15}\left(1-0.6^3\right)^{0.14}\left(1-0.5^3\right)^{0.3}\left(1-0.5^3\right)^{0.16}\right\}^{0.27}\\ \left\{\left(1-0.6^3\right)^{0.25}\left(1-0.6^3\right)^{0.15}\left(1-0.53^3\right)^{0.14}\left(1-0.3^3\right)^{0.3}\left(1-0.3^3\right)^{0.16}\right\}^{0.19}\\ \left\{\left(1-0.7^3\right)^{0.25}\left(1-0.6^3\right)^{0.15}\left(1-0.6^3\right)^{0.14}\left(1-0.5^3\right)^{0.3}\left(1-0.3^3\right)^{0.16}\right\}^{0.29}\\ \left\{\left(1-0.6^3\right)^{0.25}\left(1-0.6^3\right)^{0.15}\left(1-0.6^3\right)^{0.14}\left(1-0.5^3\right)^{0.3}\left(1-0.5^3\right)^{0.16}\right\}^{0.25}\end{array}}\end{array}\right]\right),\\ \left(\left[\begin{array}{c}\left(\begin{array}{c}\left\{\left(1-0.4^3\right)^{0.25}\left(1-0.4^3\right)^{0.15}\left(1-0.3^3\right)^{0.14}\left(1-0.3^3\right)^{0.3}\left(1-0.3^3\right)^{0.16}\right\}^{0.27}\\ \left\{\left(1-0.5^3\right)^{0.25}\left(1-0.3^3\right)^{0.15}\left(1-0.2^3\right)^{0.14}\left(1-0.2^3\right)^{0.3}\left(1-0.1^3\right)^{0.16}\right\}^{0.19}\\ \left\{\left(1-0.6^3\right)^{0.25}\left(1-0.4^3\right)^{0.15}\left(1-0.3^3\right)^{0.14}\left(1-0.3^3\right)^{0.3}\left(1-0.2^3\right)^{0.16}\right\}^{0.29}\\ \left\{\left(1-0.4^3\right)^{0.25}\left(1-0.4^3\right)^{0.15}\left(1-0.3^3\right)^{0.14}\left(1-0.3^3\right)^{0.3}\left(1-0.3^3\right)^{0.16}\right\}^{0.25}\end{array}\right),\\ \left(\begin{array}{c}\left\{\left(1-0.8^3\right)^{0.25}\left(1-0.6^3\right)^{0.15}\left(1-0.5^3\right)^{0.14}\left(1-0.5^3\right)^{0.3}\left(1-0.4^3\right)^{0.16}\right\}^{0.27}\\ \left\{\left(1-0.7^3\right)^{0.25}\left(1-0.7^3\right)^{0.15}\left(1-0.61^3\right)^{0.14}\left(1-0.4^3\right)^{0.3}\left(1-0.3^3\right)^{0.16}\right\}^{0.19}\\ \left\{\left(1-0.71^3\right)^{0.25}\left(1-0.7^3\right)^{0.15}\left(1-0.6^3\right)^{0.14}\left(1-0.6^3\right)^{0.3}\left(1-0.5^3\right)^{0.16}\right\}^{0.29}\\ \left\{\left(1-0.7^3\right)^{0.25}\left(1-0.7^3\right)^{0.15}\left(1-0.56^3\right)^{0.14}\left(1-0.5^3\right)^{0.3}\left(1-0.5^3\right)^{0.16}\right\}^{0.25}\end{array}\right)\end{array}\right]\right),\\ \left(\left[\begin{array}{c}\left(\begin{array}{c}\left\{\left(1-0.6^3\right)^{0.25}\left(1-0.6^3\right)^{0.15}\left(1-0.4^3\right)^{0.14}\left(1-0.4^3\right)^{0.3}\left(1-0.3^3\right)^{0.16}\right\}^{0.27}\\ \left\{\left(1-0.5^3\right)^{0.25}\left(1-0.3^3\right)^{0.15}\left(1-0.3^3\right)^{0.14}\left(1-0.2^3\right)^{0.3}\left(1-0.1^3\right)^{0.16}\right\}^{0.19}\\ \left\{\left(1-0.5^3\right)^{0.25}\left(1-0.3^3\right)^{0.15}\left(1-0.2^3\right)^{0.14}\left(1-0.2^3\right)^{0.3}\left(1-0.2^3\right)^{0.16}\right\}^{0.29}\\ \left\{\left(1-0.5^3\right)^{0.25}\left(1-0.5^3\right)^{0.15}\left(1-0.4^3\right)^{0.14}\left(1-0.3^3\right)^{0.3}\left(1-0.2^3\right)^{0.16}\right\}^{0.25}\end{array}\right),\\ \left(\begin{array}{c}\left\{\left(1-0.8^3\right)^{0.25}\left(1-0.7^3\right)^{0.15}\left(1-0.6^3\right)^{0.14}\left(1-0.51^3\right)^{0.3}\left(1-0.5^3\right)^{0.16}\right\}^{0.27}\\ \left\{\left(1-0.9^3\right)^{0.25}\left(1-0.72^3\right)^{0.15}\left(1-0.5^3\right)^{0.14}\left(1-0.42^3\right)^{0.3}\left(1-0.4^3\right)^{0.16}\right\}^{0.19}\\ \left\{\left(1-0.6^3\right)^{0.25}\left(1-0.55^3\right)^{0.15}\left(1-0.5^3\right)^{0.14}\left(1-0.5^3\right)^{0.3}\left(1-0.5^3\right)^{0.16}\right\}^{0.29}\\ \left\{\left(1-0.7^3\right)^{0.25}\left(1-0.7^3\right)^{0.15}\left(1-0.6^3\right)^{0.14}\left(1-0.5^3\right)^{0.3}\left(1-0.45^3\right)^{0.16}\right\}^{0.25}\end{array}\right)\end{array}\right]\right)\end{array}\right]$$

$$=\left([0.414648,\ 0.59571],\ [0.269404,\ 0.52013],\ [0.27383,\ 0.522914]\right)$$

Theorem 5. *Consider the family of* $IVT-SFS_{ft}N, F_{s_{ij}}=\left(\left[{\alpha^{L}}_{ij},\ {\alpha^{U}}_{ij}\right],\ \left[{\beta^{L}}_{ij},\ {\beta^{U}}_{ij}\right],\ \left[{\gamma^{L}}_{ij},\ {\gamma^{U}}_{ij}\right]\right)$ *for* $i=1,\ 2,\ \ldots,\ n$ *and* $j=1,\ 2,\ \ldots,\ m$. *Let* $\varpi=\{\varpi_1,\ \varpi_2,\ \ldots,\ \varpi_n\}$ *denote the weight vector of* e_i *experts and* $p=\{p_1,\ p_2,\ \ldots,\ p_m\}$ *denote the weight vector of parameters* s_j *with condition*

$\varpi_i,\ p_j \in [0,\ 1]$ *and* $\sum_{i=1}^{n} \varpi_i = 1,\ \sum_{i=1}^{n} p_i = 1$. *Then* $IVT - SFS_{ft}OWA$ operator has the following properties.

1. *(Idempotency). Let* $F_{s_{ij}} = \left(\left[\alpha^L{}_{ij},\ \alpha^U{}_{ij}\right],\ \left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right],\ \left[\gamma^L{}_{ij},\ \gamma^U{}_{ij}\right]\right) = F_{\partial s}$ *for all* $i = 1,\ 2,\ \ldots,\ n$ *and* $j = 1,\ 2,\ \ldots,\ m$, *where* $F_{\partial s} = \left(\left[\alpha^L,\ \alpha^U\right], \left[\beta^L,\ \beta^U\right],\ \left[\gamma^L,\ \gamma^U\right]\right)$, *then* $IVT - SFS_{ft}OWA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) = F_{\partial s}$.
2. *(Boundedness). If*

$$F^-_{\partial s_{ij}} = \begin{pmatrix} \left\{\left[min_j min_i\left(\alpha^L{}_{\partial ij}\right),\ min_j min_i\left(\alpha^U{}_{\partial ij}\right)\right]\right\}, \\ \left\{\left[max_j max_i\left(\beta^L{}_{\partial ij}\right),\ max_j max_i\left(\beta^U{}_{\partial ij}\right)\right]\right\}, \\ \left\{\left[max_j max_i\left(\gamma^L{}_{\partial ij}\right),\ max_j max_i\left(\gamma^U{}_{\partial ij}\right)\right]\right\} \end{pmatrix} \text{ and}$$
$$F^+_{\partial s_{ij}} = \begin{pmatrix} \left\{\left[max_j max_i\left(\alpha^L{}_{\partial ij}\right),\ max_j max_i\left(\alpha^U{}_{\partial ij}\right)\right]\right\}, \\ \left\{\left[min_j min_i\left(\beta^L{}_{\partial ij}\right),\ min_j min_i\left(\beta^U{}_{\partial ij}\right)\right]\right\}, \\ \left\{\left[min_j min_i\left(\gamma^L{}_{\partial ij}\right),\ min_j min_i\left(\gamma^U{}_{\partial ij}\right)\right]\right\} \end{pmatrix}, \text{ then}$$
$$F^-_{\partial s_{ij}} \le IVT - SFS_{ft}OWA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) \le F^+_{\partial s_{ij}}.$$

3. *(Monotonicity). Let* $F'_{s_{ij}} = \left(\left[\alpha'^L{}_{ij},\ \alpha'^U{}_{ij}\right],\ \left[\beta'^L{}_{ij},\ \beta'^U{}_{ij}\right],\ \left[\gamma'^L{}_{ij},\ \gamma'^U{}_{ij}\right]\right)$ *be any other collection of* $IVT - SFS_{ft}Ns$ *for all* $i = 1,\ 2,\ \ldots,\ n$ *and* $j = 1,\ 2,\ \ldots,\ m$ *such that* $\alpha^L{}_{ij} \le \alpha'^L{}_{ij}$, $\alpha^U{}_{ij} \le \alpha'^U{}_{ij}$, $\beta^L{}_{ij} \ge \beta'^L{}_{ij}$, $\beta^U{}_{ij} \ge \beta'^U{}_{ij}$ *and* $\gamma^L{}_{ij} \ge \gamma'^L{}_{ij}$, $\gamma^U{}_{ij} \ge \gamma'^U{}_{ij}$, then

$$IVT - SFS_{ft}OWA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) \le IVT - SFS_{ft}OWA\left(F'_{s_{11}},\ F'_{s_{12}},\ \ldots,\ F'_{s_{nm}}\right).$$

4. *(Shift Invariance). If* $F_s = \left(\left[\alpha^L,\ \alpha^U\right], \left[\beta^L,\ \beta^U\right],\ \left[\gamma^L,\ \gamma^U\right]\right)$ *is another family of* $IVT - SFS_{ft}Ns$, then

$$IVT - SFS_{ft}OWA(F_{s_{11}} \oplus F_s,\ F_{s_{12}} \oplus F_s,\ \ldots,\ F_{s_{nm}} \oplus F_s) = IVT - SFS_{ft}OWA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) \oplus F_s.$$

5. *(Homogeneity). For any real number* $K \ge 0$

$$IVT - SFS_{ft}OWA(KF_{s_{11}},\ KF_{s_{12}},\ \ldots,\ KF_{s_{nm}}) = K\left(IVT - SFS_{ft}OWA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}})\right)$$

Proof. The proof is simple and follows from Theorem 3. □

4.3. Interval-Valued T-Spherical Fuzzy Soft Hybrid Aggregation $\left(IVT - SFS_{ft}HA\right)$ Operator

In this section, we will discuss interval-valued T-spherical fuzzy soft hybrid aggregation operator which can deal with both aspects like measuring the values of $IVT - SFS_{ft}Ns$ and also considering the ordered position by "SF" of $IVT - SFS_{ft}$ values.

Moreover, we will discuss the properties related to these operators.

Definition 15. *Let* $F_{s_{ij}} = \left(\left[\alpha^L{}_{ij},\ \alpha^U{}_{ij}\right],\ \left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right],\ \left[\gamma^L{}_{ij},\ \gamma^U{}_{ij}\right]\right)$ *for* $i = 1,\ 2,\ \ldots,\ n$ *and* $j = 1,\ 2,\ \ldots,\ m$, *be the family of* $IVT - SFS_{ft}Ns$, $\varpi = \{\varpi_1,\ \varpi_2,\ \ldots,\ \varpi_n\}$ *denote the weight vector of* e_i *experts and* $p = \{p_1,\ p_2,\ \ldots,\ p_m\}$ *denote the weight vector of parameters* s_j *with condition* $\varpi_i,\ p_j \in [0,\ 1]$ *and* $\sum_{i=1}^{n} \varpi_i = 1,\ \sum_{i=1}^{n} p_i = 1$. *Then* $IVT - SFS_{ft}HA$ *operator is the function defined by* $IVT - SFS_{ft}HA : \mathbb{Q}^n \to \mathbb{Q}$, *where* ($\mathbb{Q}$ *is the family of all* $IVT - SFS_{ft}Ns$)

$$IVT - SFS_{ft}HA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) = \oplus_{j=1}^{m} p_j\left(\oplus_{i=1}^{n} \varpi_i \widetilde{F}_{s_{ij}}\right).$$

Theorem 6. *Let* $F_{s_{ij}} = \left(\left[\alpha^L{}_{ij},\ \alpha^U{}_{ij}\right],\ \left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right],\ \left[\gamma^L{}_{ij},\ \gamma^U{}_{ij}\right]\right)$ *for* $i = 1,\ 2,\ \ldots,\ n$ *and* $j = 1,\ 2,\ \ldots,\ m$, *be the family of* $IVT - SFS_{ft}Ns$ *having weight vectors* $v = \{v_1,\ v_2,\ \ldots,\ v_n\}^T$

and $\mu = \{\mu_1, \mu_2, \ldots, \mu_n\}^T$ *with the condition* $v_i, \mu_j \in [0,1]$ *and* $\sum_{i=1}^{n} v_i = 1, \sum_{i=1}^{n} \mu_j = 1$. *Moreover, "n" represents the corresponding coefficient for the number of elements in the ith row, and the jth column connected with vectors* $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)^T$ *denotes the weight vector of* e_i *experts, and* $p = \{p_1, p_2, \ldots, p_m\}^T$ *denotes the weight vector of parameters* s_j *with condition* $\varpi_i, p_j \in [0, 1]$ *and* $\sum_{i=1}^{n} \varpi_i = 1, \sum_{i=1}^{n} p_i = 1$. *Then*

$$IVT - SFS_{ft}HA(F_{s_{11}}, F_{s_{12}}, \ldots, F_{s_{nm}}) = \oplus_{j=1}^{m} p_j \left(\oplus_{i=1}^{n} \varpi_i \widetilde{F}_{s_{ij}}\right)$$
$$= \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1 - \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(1 - (\widetilde{\alpha}^L{}_{ij})^q\right)^{\varpi_i} \right)^{p_j}}, \\ \sqrt[q]{1 - \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(1 - (\widetilde{\alpha}^U{}_{ij})^q\right)^{\varpi_i} \right)^{p_j}} \end{array} \right], \\ \left[\prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(\widetilde{\beta}^L{}_{ij}\right)^{\varpi_i} \right)^{p_j}, \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(\widetilde{\beta}^U{}_{ij}\right)^{\varpi_i} \right)^{p_j} \right], \\ \left[\prod_{j=1}^{m} \left(\prod_{i=1}^{n} (\widetilde{\gamma}^L{}_{ij})^{\varpi_i} \right)^{p_j}, \prod_{j=1}^{m} \left(\prod_{i=1}^{n} (\widetilde{\gamma}^U{}_{ij})^{\varpi_i} \right)^{p_j} \right] \end{array} \right) \tag{9}$$

where $\widetilde{F}_{s_{ij}} = nv_i\mu_j F_{s_{ij}}$ *denote the permutation of ith and jth largest object of the family of* $i \times j$ $IVT - SFS_{ft}Ns\widetilde{F}_{s_{ij}} = \left([\widetilde{\alpha}^L{}_{ij}, \widetilde{\alpha}^U{}_{ij}], \left[\widetilde{\beta}^L{}_{ij}, \widetilde{\beta}^U{}_{ij}\right], [\widetilde{\gamma}^L{}_{ij}, \widetilde{\gamma}^U{}_{ij}]\right)$.

Proof. The proof is similar to Theorem 1. □

Remark 3.

1. *Using* $q = 1$, *then established* $IVT - SFS_{ft}HA$ *operator will reduce to* $IVPFS_{ft}HA$ operator.
2. *Using* $q = 2$, *then established* $IVT - SFS_{ft}HA$ *operator will reduce to* $IVSFS_{ft}HA$ operator.
3. *If we neglect the obstinacy grade that is* $\beta_{ij} = 0$, *and using* $q = 2$, *the proposed* $IVT - SFS_{ft}HA$ *operator will reduce to* $IVP_yFS_{ft}HA$ operator.
4. *If we neglect the obstinacy grade that is* $\beta_{ij} = 0$, *and using* $q = 1$, *the proposed* $IVT - SFS_{ft}HA$ *operator will reduce to an interval-valued intuitionistic fuzzy soft hybrid average* $\left(IVIFS_{ft}HA\right)$ operator.
5. *Moreover, if we put only one parameter, that is* s_1 *(mean* $m = 1$*), then* $IVT - SFS_{ft}HA$ operator reduces to the interval-valued T-spherical fuzzy hybrid average IVT-SFHA operator.
6. *If* $v\mu = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$, *then the proposed* $IVT - SFS_{ft}HA$ *operator reduces to* $IVT - SFS_{ft}WA$ operator.
7. *If* $\varpi p = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$, *then proposed* $IVT - SFS_{ft}HA$ *operator reduces to* $IVT - SFS_{ft}OWA$ *operator.*

Hence it is clear that $IVPFS_{ft}HA$, $IVSFS_{ft}HA$, $IVP_yFS_{ft}HA$, $IVIFS_{ft}HA$, IVT-SFHA, $IVT - SFS_{ft}WA$ and $IVT - SFS_{ft}OWA$ operators are the special cases of $IVT - SFS_{ft}HA$ operator. The established work is more general.

Example 4. *Consider the family of* $IVT - SFS_{ft}Ns\ F_{s_{ij}} = \left([\alpha^L{}_{ij}, \alpha^U{}_{ij}], [\beta^L{}_{ij}, \beta^U{}_{ij}], [\gamma^L{}_{ij}, \gamma^U{}_{ij}]\right)$ *as given in Table 2 with weight vector* $v = \{0.17, 0.19, 0.12, 0.16, 0.36\}^T$ *and* $\mu = \{0.23, 0.2, 0.29, 0.28\}^T$ *and having the associated vector as* $\varpi = (0.23, 0.18, 0.1, 0.27, 00.22\ )^T$ *and* $p = \{0.23, 0.24, 0.18, 0.35\}^T$. *Then by using Equation (9) their score values are given in Table 4. The corresponding* $IVT - SFS_{ft}Ns\widetilde{F}_{s_{ij}} = \left([\widetilde{\alpha}^L{}_{ij}, \widetilde{\alpha}^U{}_{ij}], \left[\widetilde{\beta}^L{}_{ij}, \widetilde{\beta}^U{}_{ij}\right], [\widetilde{\gamma}^L{}_{ij}, \widetilde{\gamma}^U{}_{ij}]\right)$ *of the permutation of ith and jth* *largest object of the family of* $i \times j$ $IVT - SFS_{ft}Ns\widetilde{F}_{s_{ij}} = \left(\left[\widetilde{\alpha}^{L}{}_{ij}, \widetilde{\alpha}^{U}{}_{ij} \right], \left[\widetilde{\beta}^{L}{}_{ij}, \widetilde{\beta}^{U}{}_{ij} \right], \left[\widetilde{\gamma}^{L}{}_{ij}, \widetilde{\gamma}^{U}{}_{ij} \right] \right)$ *are given in Table 5.*

$$\widetilde{F}_{s_{ij}} = nv_i\mu_j F_{s_{ij}} = \left(\left[\begin{array}{c} \sqrt[q]{1 - \left(1 - \left(\alpha^{L}{}_{ij}{}^{q} \right)^{nv_i\mu_j} \right)}, \\ \sqrt[q]{1 - \left(1 - \left(\alpha^{U}{}_{ij}{}^{q} \right)^{nv_i\mu_j} \right)} \end{array} \right], \left[\left(\beta^{L}{}_{ij} \right)^{nv_i\mu_j}, \left(\beta^{U}{}_{ij} \right)^{nv_i\mu_j} \right], \left[\left(\gamma^{L}{}_{ij} \right)^{nv_i\mu_j}, \left(\gamma^{U}{}_{ij} \right)^{nv_i\mu_j} \right] \right). \quad (10)$$

Table 4. Tabular depiction of score values of $IVT - SFS_{ft}NS\widetilde{F}_{s_{ij}} = nv_i\mu_j F_{s_{ij}}$.

	s_1	s_2	s_3	s_4
x_1	(0.094904)	(0.041291)	(0.148938)	(0.101557)
x_2	(0.110053)	(0.091309)	(0.072026)	(0.129129)
x_3	(0.058903)	(0.058296)	(0.074272)	(0.077105)
x_4	(0.126681)	(0.038849)	(0.121334)	(0.108737)
x_5	(0.271802)	(0.186345)	(0.296199)	(0.27864)

Table 5. Tabular presentation of $\widetilde{F}_{s_{ij}} = nv_i\mu_j F_{s_{ij}}$.

	s_1	s_2	s_3	s_4
x_1	([0.03128, 0.09384], [0.04692, 0.12522], [0.06256, 0.0782])	([0.0136, 0.0408], [0.0272, 0.0544], [0.0136, 0.068])	([0.11832, 0.11832], [0.11832, 0.13804], [0.05916, 0.0986])	([0.05712, 0.0952], [0.07616, 0.0952], [0.0952, 0.0952])
x_2	([0.0874, 0.0874], [0.05244, 0.06992], [0.10488, 0.12236])	([0.0156, 0.0912], [0.0304, 0.1064], [0.0456, 0.0608])	([0.04408, 0.06612], [0.04408, 0.13224], [0.04408, 0.1102])	([0.04256, 0.12768], [0.06384, 0.119168], [0.08512, 0.09576])
x_3	([0.03312, 0.0552], [0.04416, 0.0552], [0.06624, 0.08832])	([0.0192, 0.0576], [0.0288, 0.0672], [0.0288, 0.04032])	([0.04176, 0.0696], [0.05568, 0.0696], [0.0696, 0.0696]).	([0.05376, 0.0672], [0.04032, 0.09408], [0.04032, 0.08064])
x_4	([0.0738, 0.11776], [0.05888, 0.08832], [0.05888, 0.075072])	([0.0128, 0.0384], [0.0128, 0.0384], [0.0256, 0.1152])	([0.07424, 0.11136], [0.05568, 0.11136], [0.03712, 0.11136])	([0.03584, 0.10752], [0.07168, 0.0896], [0.03584, 0.12544])
x_5	([0.19872, 0.23184], [0.09936, 0.1656], [0.09936, 0.19872])	([0.144, 0.1526], [0.144, 0.17568], [0.144, 0.20736])	([0.12528, 0.29232], [0.12528, 0.296496], [0.08352, 0.22968])	([0.2016, 0.24192], [0.12096, 0.28224], [0.2016, 0.28224])

Now, by using Equation (10), we get

$$IVT - SFS_{ft}HA(F_{s_{11}}, F_{s_{12}}, \ldots, F_{s_{nm}}) = \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1 - \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \left(\widetilde{\alpha}^{L}{}_{ij} \right)^{q} \right)^{\varpi_i} \right)^{p_j}}, \\ \sqrt[q]{1 - \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \left(\widetilde{\alpha}^{U}{}_{ij} \right)^{q} \right)^{\varpi_i} \right)^{p_j}} \end{array} \right], \\ \left[\prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(\widetilde{\beta}^{L}{}_{ij} \right)^{\varpi_i} \right)^{p_j}, \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(\widetilde{\beta}^{U}{}_{ij} \right)^{\varpi_i} \right)^{p_j} \right], \\ \left[\prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(\widetilde{\gamma}^{L}{}_{ij} \right)^{\varpi_i} \right)^{p_j}, \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(\widetilde{\gamma}^{U}{}_{ij} \right)^{\varpi_i} \right)^{p_j} \right] \end{array} \right)$$

$$= ([0.114491, 0.162759], [0.041894, 0.08105], [0.042285, 0.082046])$$

Theorem 7. *Let* $F_{s_{ij}} = \left(\left[\alpha^{L}{}_{ij}, \alpha^{U}{}_{ij} \right], \left[\beta^{L}{}_{ij}, \beta^{U}{}_{ij} \right], \left[\gamma^{L}{}_{ij}, \gamma^{U}{}_{ij} \right] \right)$ *for* $i = 1, 2, \ldots, n$ *and* $j = 1, 2, \ldots, m$*, be the family of* $IVT - SFS_{ft}Ns$ *having weight vectors* $v = \{v_1, v_2, \ldots, v_n\}^T$ *and* $\mu = \{\mu_1, \mu_2, \ldots, \mu_n\}^T$ *with condition* $v_i, \mu_j \in [0,1]$ *and* $\sum_{i=1}^{n} v_i = 1, \sum_{i=1}^{n} \mu_j = 1$*. Moreover, "n" represents the corresponding coefficient for the number of elements in ith row and jth column linked with vectors* $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)^T$ *denote the weight vector of* e_i *experts and* $p = \{p_1, p_2, \ldots, p_m\}^T$ *denote the weight vector of parameters* s_j *with condition*

$\varpi_i,\ p_j \in [0,\ 1]$ and $\sum_{i=1}^{n} \varpi_i = 1, \sum_{i=1}^{n} p_i = 1$. Then the $IVT - SFS_{ft}HA$ operator contains the subsequent properties

1. *(Idempotency). Let $F_{s_{ij}} = \widetilde{F'}_s$ for all $i = 1,\ 2,\ \ldots,\ n$ and $j = 1,\ 2,\ \ldots,\ m$, where $\widetilde{F'}_s = nv_i\mu_j F'_s$, then $IVT - SFS_{ft}HA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) = \widetilde{F'}_s$.*
2. *(Boundedness). If $\widetilde{F}^-_{s_{ij}} = \begin{pmatrix} \left\{\left[min_j min_i\left(\widetilde{\alpha}^L{}_{ij}\right),\ min_j min_i\left(\widetilde{\alpha}^U{}_{ij}\right)\right]\right\}, \\ \left\{\left[max_j max_i\left(\widetilde{\beta}^L{}_{ij}\right),\ max_j max_i\left(\widetilde{\beta}^U{}_{ij}\right)\right]\right\}, \\ \left\{\left[max_j max_i\left(\widetilde{\gamma}^L{}_{ij}\right),\ max_j max_i\left(\widetilde{\gamma}^U{}_{ij}\right)\right]\right\} \end{pmatrix}$ and $\widetilde{F}^+_{s_{ij}} = \begin{pmatrix} \left\{\left[max_j max_i\left(\widetilde{\alpha}^L{}_{ij}\right),\ max_j max_i\left(\widetilde{\alpha}^U{}_{ij}\right)\right]\right\}, \\ \left\{\left[min_j min_i\left(\widetilde{\beta}^L{}_{ij}\right),\ min_j min_i\left(\widetilde{\beta}^U{}_{ij}\right)\right]\right\}, \\ \left\{\left[min_j min_i\left(\widetilde{\gamma}^L{}_{ij}\right),\ min_j min_i\left(\widetilde{\gamma}^U{}_{ij}\right)\right]\right\} \end{pmatrix}$, then*
$$\widetilde{F}^-_{s_{ij}} \le IVT - SFS_{ft}HWA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) \le \widetilde{F}^+_{s_{ij}}.$$
3. *(Monotonicity). Let $F'_{s_{ij}} = \left(\left[\alpha'^L{}_{ij},\ \alpha'^U{}_{ij}\right],\ \left[\beta'^L{}_{ij},\ \beta'^U{}_{ij}\right],\ \left[\gamma'^L{}_{ij},\ \gamma'^U{}_{ij}\right]\right)$ be any other collection of $IVT - SFS_{ft}Ns$ for all $i = 1,\ 2,\ \ldots,\ n$ and $j = 1,\ 2,\ \ldots,\ m$ such that $\alpha^L{}_{ij} \le \alpha'^L{}_{ij}, \alpha^U{}_{ij} \le \alpha'^U{}_{ij}, \beta^L{}_{ij} \ge \beta'^L{}_{ij}, \beta^U{}_{ij} \ge \beta'^U{}_{ij}$ and $\gamma^L{}_{ij} \ge \gamma'^L{}_{ij}, \gamma^U{}_{ij} \ge \gamma'^U{}_{ij}$, then*
$$IVT - SFS_{ft}HA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) \le IVT - SFS_{ft}HA\left(F'_{s_{11}},\ F'_{s_{12}},\ \ldots,\ F'_{s_{nm}}\right).$$
4. *(Shift Invariance). If $F_s = \left(\left[\alpha^L,\ \alpha^U\right], \left[\beta^L,\ \beta^U\right],\ \left[\gamma^L,\ \gamma^U\right]\right)$ is another family of $IVT - SFS_{ft}Ns$, then*
$$IVT - SFS_{ft}HA(F_{s_{11}} \oplus F_s,\ F_{s_{12}} \oplus F_s,\ \ldots,\ F_{s_{nm}} \oplus F_s) = IVT - SFS_{ft}HA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}}) \oplus F_s.$$
5. ***(Homogeneity).** For any real number $K \ge 0$*
$$IVT - SFS_{ft}HA(KF_{s_{11}},\ KF_{s_{12}},\ \ldots,\ KF_{s_{nm}}) = K\Big(IVT - SFS_{ft}HA(F_{s_{11}},\ F_{s_{12}},\ \ldots,\ F_{s_{nm}})\Big)$$

Proof. The proof is simple and follows from Theorem 3. □

5. An Algorithm for MCDM Based on $IVT - SFS_{ft}$ Information

MCDM approach is a well-known and very effective technique for the selection of the best alternative among the given and this approach has been used in different fields of fuzzy sets theory for the selection of the best alternative. About an alternative, the decision makers keep many aspects in their mind, such as the flexibility of the alternative, benefits, different features, and drawbacks. After the evaluation of all these aspects, they could decide which alternative is best and reach the best result. In this section, we will propose a stepwise algorithm for MCDM under the environment of $IVT - SFS_{ft}Ns$.

Let $= \{x_1,\ x_2,\ x_3,\ \ldots,\ x_r\}$ be the set of "r" alternative, $D = \{D_1,\ D_2,\ D_3,\ \ldots,\ D_n\}$ be the sets of "n" senior experts with $E = \{s_1,\ s_2,\ s_3,\ \ldots,\ s_m\}$ which denotes the set of "m" parameters. Each alternative $x_l(l = 1,\ 2,\ 3,\ \ldots,\ r)$ has been evaluated by a team of "n" experts corresponding to their parameters s_j $(j = 1,\ 2,\ 3,\ \ldots,\ m)$. Suppose experts provide their evaluation in the shape of $IVT - SFS_{ft}Ns$, $F_{s_{ij}} = \left(\left[\alpha^L{}_{ij},\ \alpha^U{}_{ij}\right],\ \left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right],\ \left[\gamma^L{}_{ij},\ \gamma^U{}_{ij}\right]\right)$, for $i = 1,\ 2,\ \ldots,\ n$ and $j = 1,\ 2,\ \ldots,\ m$ having weight vector $\varpi = \{\varpi_1,\ \varpi_2,\ \ldots,\ \varpi_n\}$ and $p = \{p_1,\ p_2,\ \ldots,\ p_m\}$ of e_i experts and parameters s_j respectively with the condition that $\varpi_i,\ p_j \in [0,\ 1]$ and $\sum_{i=1}^{n} \varpi_i = 1, \sum_{i=1}^{n} p_i = 1$. The matrix $M = \left[F_{s_{ij}}\right]_{n\times m}$ denotes the overall information. After using the aggregation operator on the assessment value of the experts, the aggregated $IVT - SFS_{ft}N$ "ψ_l" for alternative $x_l(l = 1,\ 2,\ 3,\ \ldots,\ r)$ is given by $\psi_l = (\alpha_l,\ \beta_l,\ \gamma_l)$. Lastly, we will use the formula of score function for over aggregated $IVT - SFS_{ft}Ns$ for alternatives and rank them according to their order and choose the best result.

The stepwise algorithm for overall above discussion is given as follows:

Step 1. Accumulate the evaluation information of all experts for each alternative according to their parameters and arrange it to construct an overall decision matrix $M = \left[F_{s_{ij}}\right]_{n\times m}$ given by

$$M = \begin{bmatrix} \begin{pmatrix} [\alpha^L{}_{11},\ \alpha^U{}_{11}], \\ [\beta^L{}_{11},\ \beta^U{}_{11}], \\ [\gamma^L{}_{11},\ \gamma^U{}_{11}] \end{pmatrix} & \begin{pmatrix} [\alpha^L{}_{12},\ \alpha^U{}_{12}], \\ [\beta^L{}_{12},\ \beta^U{}_{12}], \\ [\gamma^L{}_{12},\ \gamma^U{}_{12}] \end{pmatrix} & \cdots & \begin{pmatrix} [\alpha^L{}_{1m},\ \alpha^U{}_{1m}], \\ [\beta^L{}_{1m},\ \beta^U{}_{1m}], \\ [\gamma^L{}_{1m},\ \gamma^U{}_{1m}] \end{pmatrix} \\ \begin{pmatrix} [\alpha^L{}_{21},\ \alpha^U{}_{21}], \\ [\beta^L{}_{21},\ \beta^U{}_{21}], \\ [\gamma^L{}_{21},\ \gamma^U{}_{21}] \end{pmatrix} & \begin{pmatrix} [\alpha^L{}_{22},\ \alpha^U{}_{22}], \\ [\beta^L{}_{22},\ \beta^U{}_{22}], \\ [\gamma^L{}_{22},\ \gamma^U{}_{22}] \end{pmatrix} & \cdots & \begin{pmatrix} [\alpha^L{}_{2m},\ \alpha^U{}_{2m}], \\ [\beta^L{}_{2m},\ \beta^U{}_{2m}], \\ [\gamma^L{}_{2m},\ \gamma^U{}_{2m}] \end{pmatrix} \\ \vdots & \vdots & & \\ \begin{pmatrix} [\alpha^L{}_{n1},\ \alpha^U{}_{n1}], \\ [\beta^L{}_{n1},\ \beta^U{}_{n1}], \\ [\gamma^L{}_{n1},\ \gamma^U{}_{n1}] \end{pmatrix} & \begin{pmatrix} [\alpha^L{}_{n2},\ \alpha^U{}_{n2}], \\ [\beta^L{}_{n2},\ \beta^U{}_{n2}], \\ [\gamma^L{}_{n2},\ \gamma^U{}_{n2}] \end{pmatrix} & \cdots & \begin{pmatrix} [\alpha^L{}_{nm},\ \alpha^U{}_{nm}], \\ [\beta^L{}_{nm},\ \beta^U{}_{nm}], \\ [\gamma^L{}_{nm},\ \gamma^U{}_{nm}] \end{pmatrix} \end{bmatrix}$$

Step 2. Normalize the given information by interchanging of cost type parameter into the benefit type parameter if it is needed. The formula is given below:

$$\rho_{ij} = \begin{cases} F^c{}_{s_{ij}},\ for\ cost\ type\ parameter \\ F_{s_{ij}}, for\ benefit\ type\ parameter \end{cases}$$

where $F^c{}_{s_{ij}} = \left(\left[\gamma^L{}_{ij},\ \gamma^U{}_{ij}\right],\ \left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right],\ \left[\alpha^L{}_{ij},\ \alpha^U{}_{ij}\right]\right)$ denote the complement of $F_{s_{ij}} = \left(\left[\alpha^L{}_{ij},\ \alpha^U{}_{ij}\right],\ \left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right],\ \left[\gamma^L{}_{ij},\ \gamma^U{}_{ij}\right]\right)$.

Step 3. Aggregate the $IVT-SFS_{ft}Ns$, $F_{s_{ij}} = \left(\left[\alpha^L{}_{ij},\ \alpha^U{}_{ij}\right],\ \left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right],\ \left[\gamma^L{}_{ij},\ \gamma^U{}_{ij}\right]\right)$ by using the proposed aggregation operators for each alternative $s_l(l = 1,\ 2,\ \ldots,\ r)$ to get the aggregated $IVT-SFS_{ft}Ns$ $\psi_l = \left(\left[\alpha^L{}_{ij},\ \alpha^U{}_{ij}\right],\ \left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right],\ \left[\gamma^L{}_{ij},\ \gamma^U{}_{ij}\right]\right)$.

Step 4. Calculate the score values for each "ψ_l" by using Definition 11.

Step 5. Organize the ranking result in explicit order for alternatives $x_l(l = 1,\ 2,\ 3,\ \ldots,\ r)$ and choose the preeminent result.

5.1. Application Steps for the Proposed Method

In this section, we will provide an example of the present work in detail to show its validity and advantages.

Let us have a team of experts on mobile phones consisting of five members $C = \{C_1,\ C_2,\ C_3, C_4\}$ with weight vectors $\varpi = \{0.28,\ 0.25,\ 0.23,\ 0.24\}$. The experts will give their information about the set of different mobile phones as alternatives consisting of four members $\{x_1 = Lenovo,\ x_2 = Samsung,\ x_3 = LG,\ x_4 = Apple\}$ having parameters $\{s_1 = Best\ audio\ and\ vidio\ features,\ s_2 = Long\ battery\ timing,\ s_3 = reasonable\ in\ price,$ $s_4 = Best\ camera\ features\}$. Let $p = \{0.29,\ 0.18,\ 0.22,\ 0.31\}$ denote the weight vectors of parameters "s_j" $(j = 1, 2,\ 3,\ 4,\ 5)$. Suppose all the experts provide their information in the form of $IVT-SFS_{ft}Ns$. Now we use the proposed algorithm for the selection of the best mobile phone.

By using $IVT-SFS_{ft}WA$ operators:

Step 1. The experts present their information of each alternative in the shape of $IVT-SFS_{ft}Ns$ according to their resultant parameters. This information is given in Tables 6–9 correspondingly.

Step 2. There is no requirement f or normalization of $IVT-SFS_{ft}$ matrix since all the parameters are of a similar kind.

Step 3. The information of each expert for each alternative x_i $(i = 1,\ 2,\ 3,\ 4)$ is aggregated by using Equation (1), so we have

$$\psi_1 = ([0.3927,\ 0.6938],\ [0.3072,\ 0.5096],\ [0.2770,\ 0.5323]),$$
$$\psi_2 = ([0.3797,\ 0.7396],\ [0.3002,\ 0.4690],\ [0.2464,\ 0.4950])$$

$$\psi_3 = ([0.4279,\ 0.7505],\ [0.2625,\ 0.4488],\ [0.3196,\ 0.5547]),$$
$$\psi_4 = ([0.4114,\ 0.7173],\ [0.3304,\ 0.4698],\ [0.2675,\ 0.4918])$$

Step 4. By using the formula of score function given in Definition 11, calculate the score values for each $\psi_i(i = 1,\ 2,\ 3,\ 4,\ 5)$ in step 3, i.e.,

$$Sc(\psi_1) = 0.6671,\ Sc(\psi_2) = 0.7154,$$
$$Sc(\psi_3) = 0.7285,\ Sc(\psi_4) = 0.7065$$

Table 6. $IVT - SFS_{ft}$ matrix for alternative x_1.

	s_1	s_2	s_3	s_4
C_1	$\begin{pmatrix}[0.3,\ 0.5],\\ [0.4,\ 0.6],\\ [0.2,\ 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.4,\ 0.8],\\ [0.2,\ 0.4],\\ [0.1,\ 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.5,\ 0.7],\\ [0.3,\ 0.6],\\ [0.3,\ 0.3]\end{pmatrix}$	$\begin{pmatrix}[0.2,\ 0.2],\\ [0.5,\ 0.5],\\ [0.2,\ 0.6]\end{pmatrix}$
C_2	$\begin{pmatrix}[0.1,\ 0.5],\\ [0.3,\ 0.6],\\ [0.4,\ 0.7]\end{pmatrix}$	$\begin{pmatrix}[0.4,\ 0.7],\\ [0.3,\ 0.6],\\ [0.5,\ 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.2,\ 0.8],\\ [0.3,\ 0.4],\\ [0.1,\ 0.3]\end{pmatrix}$	$\begin{pmatrix}[0.3,\ 0.4],\\ [0.3,\ 0.5],\\ [0.2,\ 0.6]\end{pmatrix}$
C_3	$\begin{pmatrix}[0.2,\ 0.7],\\ [0.4,\ 0.4],\\ [0.3,\ 0.5]\end{pmatrix}$	$\begin{pmatrix}[0.4,\ 0.5],\\ [0.2,\ 0.7],\\ [0.6,\ 0.7]\end{pmatrix}$	$\begin{pmatrix}[0.1,\ 0.9],\\ [0.3,\ 0.7],\\ [0.5,\ 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.3,\ 0.7],\\ [0.2,\ 0.3],\\ [0.3,\ 0.4]\end{pmatrix}$
C_4	$\begin{pmatrix}[0.4,\ 0.9],\\ [0.5,\ 0.6],\\ [0.3,\ 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.8,\ 0.8],\\ [0.2,\ 0.3],\\ [0.5,\ 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.3,\ 0.4],\\ [0.4,\ 0.8],\\ [0.4,\ 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.1,\ 0.4],\\ [0.2,\ 0.5],\\ [0.3,\ 0.8]\end{pmatrix}$

Table 7. $IVT - SFS_{ft}$ matrix for alternative x_2.

	s_1	s_2	s_3	s_4
C_1	$\begin{pmatrix}[0.3,\ 0.4],\\ [0.3,\ 0.5],\\ [0.2,\ 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.2,\ 0.7],\\ [0.1,\ 0.2],\\ [0.4,\ 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.7,\ 0.9],\\ [0.3,\ 0.3],\\ [0.4,\ 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.3,\ 0.8],\\ [0.4,\ 0.6],\\ [0.2,\ 0.2]\end{pmatrix}$
C_2	$\begin{pmatrix}[0.1,\ 0.6],\\ [0.3,\ 0.3],\\ [0.2,\ 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.4,\ 0.5],\\ [0.2,\ 0.7],\\ [0.6,\ 0.7]\end{pmatrix}$	$\begin{pmatrix}[0.1,\ 0.9],\\ [0.3,\ 0.7],\\ [0.5,\ 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.3,\ 0.5],\\ [0.4,\ 0.6],\\ [0.2,\ 0.6]\end{pmatrix}$
C_3	$\begin{pmatrix}[0.2,\ 0.2],\\ [0.5,\ 0.5],\\ [0.2,\ 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.4,\ 0.8],\\ [0.2,\ 0.4],\\ [0.1,\ 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.5,\ 0.7],\\ [0.3,\ 0.6],\\ [0.3,\ 0.3]\end{pmatrix}$	$\begin{pmatrix}[0.1,\ 0.5],\\ [0.3,\ 0.6],\\ [0.4,\ 0.7]\end{pmatrix}$
C_4	$\begin{pmatrix}[0.3,\ 0.7],\\ [0.2,\ 0.3],\\ [0.3,\ 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.4,\ 0.7],\\ [0.3,\ 0.6],\\ [0.5,\ 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.2,\ 0.8],\\ [0.3,\ 0.4],\\ [0.1,\ 0.3]\end{pmatrix}$	$\begin{pmatrix}[0.4,\ 0.9],\\ [0.5,\ 0.6],\\ [0.3,\ 0.4]\end{pmatrix}$

Table 8. $IVT - SFS_{ft}$ matrix for alternative x_3.

	s_1	s_2	s_3	s_4
C_1	$\begin{pmatrix}[0.4,\ 0.5],\\ [0.2,\ 0.7],\\ [0.6,\ 0.7]\end{pmatrix}$	$\begin{pmatrix}[0.1,\ 0.6],\\ [0.3,\ 0.3],\\ [0.2,\ 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.3,\ 0.8],\\ [0.4,\ 0.6],\\ [0.2,\ 0.2]\end{pmatrix}$	$\begin{pmatrix}[0.1,\ 0.4],\\ [0.2,\ 0.5],\\ [0.3,\ 0.8]\end{pmatrix}$
C_2	$\begin{pmatrix}[0.1,\ 0.9],\\ [0.3,\ 0.7],\\ [0.5,\ 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.8,\ 0.8],\\ [0.2,\ 0.3],\\ [0.5,\ 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.1,\ 0.9],\\ [0.3,\ 0.7],\\ [0.5,\ 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.3,\ 0.7],\\ [0.2,\ 0.3],\\ [0.3,\ 0.4]\end{pmatrix}$
C_3	$\begin{pmatrix}[0.7,\ 0.9],\\ [0.3,\ 0.3],\\ [0.4,\ 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.3,\ 0.4],\\ [0.4,\ 0.8],\\ [0.4,\ 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.2,\ 0.7],\\ [0.1,\ 0.2],\\ [0.4,\ 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.2,\ 0.7],\\ [0.4,\ 0.4],\\ [0.3,\ 0.5]\end{pmatrix}$
C_4	$\begin{pmatrix}[0.2,\ 0.7],\\ [0.4,\ 0.4],\\ [0.3,\ 0.5]\end{pmatrix}$	$\begin{pmatrix}[0.4,\ 0.7],\\ [0.3,\ 0.6],\\ [0.5,\ 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.3,\ 0.4],\\ [0.3,\ 0.5],\\ [0.2,\ 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.4,\ 0.8],\\ [0.2,\ 0.4],\\ [0.1,\ 0.6]\end{pmatrix}$

Table 9. $IVT - SFS_{ft}$ matrix for alternative x_4.

	s_1	s_2	s_3	s_4
C_1	$\begin{pmatrix} [0.1,\ 0.5], \\ [0.3,\ 0.6], \\ [0.4,\ 0.7] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.7], \\ [0.4,\ 0.4], \\ [0.3,\ 0.5] \end{pmatrix}$	$\begin{pmatrix} [0.5,\ 0.7], \\ [0.3,\ 0.6], \\ [0.3,\ 0.3] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.2], \\ [0.5,\ 0.5], \\ [0.2,\ 0.6] \end{pmatrix}$
C_2	$\begin{pmatrix} [0.2,\ 0.2], \\ [0.5,\ 0.5], \\ [0.2,\ 0.6] \end{pmatrix}$	$\begin{pmatrix} [0.8,\ 0.8], \\ [0.2,\ 0.3], \\ [0.5,\ 0.6] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.8], \\ [0.3,\ 0.4], \\ [0.1,\ 0.3] \end{pmatrix}$	$\begin{pmatrix} [0.1,\ 0.6], \\ [0.3,\ 0.3], \\ [0.2,\ 0.4] \end{pmatrix}$
C_3	$\begin{pmatrix} [0.2,\ 0.8], \\ [0.3,\ 0.4], \\ [0.1,\ 0.3] \end{pmatrix}$	$\begin{pmatrix} [0.3,\ 0.5], \\ [0.4,\ 0.6], \\ [0.2,\ 0.6] \end{pmatrix}$	$\begin{pmatrix} [0.7,\ 0.9], \\ [0.3,\ 0.3], \\ [0.4,\ 0.8] \end{pmatrix}$	$\begin{pmatrix} [0.1,\ 0.5], \\ [0.3,\ 0.6], \\ [0.4,\ 0.7] \end{pmatrix}$
C_4	$\begin{pmatrix} [0.1,\ 0.9], \\ [0.3,\ 0.7], \\ [0.5,\ 0.6] \end{pmatrix}$	$\begin{pmatrix} [0.4,\ 0.9], \\ [0.5,\ 0.6], \\ [0.3,\ 0.4] \end{pmatrix}$	$\begin{pmatrix} [0.3,\ 0.4], \\ [0.4,\ 0.8], \\ [0.4,\ 0.4] \end{pmatrix}$	$\begin{pmatrix} [0.3,\ 0.7], \\ [0.2,\ 0.3], \\ [0.3,\ 0.4] \end{pmatrix}$

Step 5. Rank the score values and select the best alternative. Hence, we obtain the ranking result as

$$Sc(\psi_3) > Sc(\psi_2) > Sc(\psi_4) > Sc(\psi_1)$$

Hence, from the above discussion, it is clear that "x_3" is the best alternative.

By using $IVT - SFS_{ft}OWA$ operators:

Step 1. Same as above.

Step 2. Same as above.

Step 3. The information of each expert for each alternative x_i $(i = 1,\ 2,\ 3,\ 4)$ is aggregated by using Equation (8), so we have

$$\begin{aligned}
\psi_1 &= ([0.4036,\ 0.7077],\ [0.3097,\ 0.5128],\ [0.2873,\ 0.5398]) \\
\psi_2 &= ([0.4001,\ 0.7451],\ [0.3141,\ 0.4783],\ [0.2695,\ 0.5048]) \\
\psi_3 &= ([0.4445,\ 0.7607],\ [0.2677,\ 0.4529],\ [0.3247,\ 0.5364]) \\
\psi_4 &= ([0.4263,\ 0.7323],\ [0.3337,\ 0.4771],\ [0.2737,\ 0.4997])
\end{aligned}$$

Step 4. By using the formula of score function given in Definition 11, calculate the score values for each $\psi_i(i = 1,\ 2,\ 3,\ 4,\ 5)$ in step 3, i.e.,

$$Sc(\psi_1) = 0.6790,\ Sc(\psi_2) = 0.7217, Sc(\psi_3) = 0.7456,\ Sc(\psi_4) = 0.7205$$

Step 5. Rank the score values and select the best alternative. Hence, we obtain the ranking result as

$$Sc(\psi_3) > Sc(\psi_2) > Sc(\psi_4) > Sc(\psi_1)$$

Hence, it is noted that the aggregated result for $IVT - SFS_{ft}OWA$ operator is the same as the result obtained for $IVT - SFS_{ft}WA$ operator. Hence "x_3" is the best alternative.

By using $IVT - SFS_{ft}HA$ operators:

Step 1. Same as above.

Step 2. Same as above.

Step 3. The information of each expert for each alternative x_i $(i = 1,\ 2,\ 3,\ 4)$ is to be aggregated by using Equation (8) with $v = \{0.21,\ 0.22,\ 0.23,\ 0.44\}^T$ and $\mu = \{0.25,\ 0.27,\ 0.28,\ 0.20\}^T$ be the weight vectors of $F_{s_{ij}} = \left(\left[\alpha^L{}_{ij},\ \alpha^U{}_{ij}\right],\ \left[\beta^L{}_{ij},\ \beta^U{}_{ij}\right],\ \left[\gamma^L{}_{ij},\ \gamma^U{}_{ij}\right]\right)$ Moreover, "n" represents the corresponding balancing coefficient for the number of elements in ith row and jth column. Let $\varpi = (0.28,\ 0.25,\ 0.23,\ 0.24)^T$ denote the weight vector of e_i experts and $p = \{0.29,\ 0.18,\ 0.22,\ 0.31\}^T$ denote the weight vector of parameters s_j, so we get

$$\begin{aligned}
\psi_1 &= ([0.4123,\ 0.7187],\ [0.3179,\ 0.5226],\ [0.2974,\ 0.5493]) \\
\psi_2 &= ([0.4225,\ 0.7323],\ [0.3224,\ 0.5055],\ [0.2767,\ 0.5225]) \\
\psi_3 &= ([0.4324,\ 0.7756],\ [0.3022,\ 0.4955],\ [0.3136,\ 0.5221]) \\
\psi_4 &= ([0.4355,\ 0.7141],\ [0.3454,\ 0.5061],\ [0.2898,\ 0.5167]).
\end{aligned}$$

Step 4. By using the formula of score function given in Definition 11, calculate the score values for each $\psi_i (i = 1, 2, 3, 4, 5)$ in step 3, i.e.,

$$Sc(\psi_1) = 0.6849,\ Sc(\psi_2) = 0.7095, Sc(\psi_3) = 0.7485,\ Sc(\psi_4) = 0.7006$$

Step 5. Rank the score values and select the best alternative. Hence, we obtain the ranking result as

$$Sc(\psi_3) > Sc(\psi_2) > Sc(\psi_4) > Sc(\psi_1)$$

Hence, it is noted that the aggregated result for $IVT - SFS_{ft}HA$ operator is the same as the result obtained for $IVT - SFS_{ft}WA$ and $IVT - SFS_{ft}OWA$ operator. Hence "x_3" is the best alternative.

5.2. Comparative Analysis

Here in this section, we will propose the comparative analysis of established work with other existing methods to prove the superiority of the present work. We will compare the present work with IVPFWA, IVPFOWA, IVPFHA, $IVPFS_{ft}WA$, IVSFWA, IVSFOWA, IVSFHA, $IVSFS_{ft}WA$, IVT-SFWA [36], IVT-SFOWA [36], IVT-SFHA [36], IVSFWAM [44], IVSFWGM [44], and $IVPFS_{ft}S$ [43].

Example 5. *A person plans to buy a house from a set of four alternatives* $= \{x_1,\ x_2,\ x_3,\ x_4\}$. *Let* $E = \{s_1 = beautiful,\ s_2 = reasonable\ price,\ s_3 = green\ surroundings,\ s_4 = suitable\ location\}$ *be a set of parameters. Let* $\varpi = \{0.25,\ 0.23,\ 0.24,\ 0.28\}$ *denote the weight vector of "*e_i*" experts and* $p = \{0.26,\ 0.20,\ 0.29,\ 0.25\}$ *denote the weight vector of "*s_j*" parameters. The experts provide their information in the form of* $IVPFS_{ft}Ns$ *as given in Table 10.*

Table 10. Information based on interval-valued picture fuzzy soft numbers.

	x_1	x_2	x_3	x_4
C_1	$\begin{pmatrix} [0,\ 0.1],\ [0.2,\ 0.3], \\ [0.3,\ 0.4] \end{pmatrix}$	$\begin{pmatrix} [0.1,\ 0.2], [0,\ 0.1], \\ [0.2,\ 0.4] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.5],\ [0,\ 0.1], \\ [0.2,\ 0.3] \end{pmatrix}$	$\begin{pmatrix} [0.3,\ 0.4],\ [0.1,\ 0.2], \\ [0.2,\ 0.3] \end{pmatrix}$
C_2	$\begin{pmatrix} [0.2,\ 0.4],\ [0,\ 0.1], \\ [0.2,\ 0.3] \end{pmatrix}$	$\begin{pmatrix} [0.1,\ 0.2],\ [0.2,\ 0.3], \\ [0.3,\ 0.4] \end{pmatrix}$	$\begin{pmatrix} [0,\ 0.1],\ [0.1,\ 0.3], \\ [0.4,\ 0.5] \end{pmatrix}$	$\begin{pmatrix} [0.1,\ 0.2],\ [0,\ 0.2], \\ [0.1,\ 0.3] \end{pmatrix}$
C_3	$\begin{pmatrix} [0.1,\ 0.2],\ [0.1,\ 0.2], \\ [0.2,\ 0.3] \end{pmatrix}$	$\begin{pmatrix} [0.1,\ 0.4],\ [0.1,\ 0.3], \\ [0,\ 0.2] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.3],\ [0.3,\ 0.5], \\ [0,\ 0.1] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.3],\ [0.1,\ 0.3], \\ [0.1,\ 0.2] \end{pmatrix}$
C_4	$\begin{pmatrix} [0.1,\ 0.3],\ [0,\ 0.1], \\ [0,\ 0.2] \end{pmatrix}$	$\begin{pmatrix} [0,\ 0.1],\ [0.1,\ 0.3], \\ [0.3,\ 0.5] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.3],\ [0.1,\ 0.2], \\ [0.2,\ 0.4] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.3],\ [0.1,\ 0.2], \\ [0,\ 0.1] \end{pmatrix}$

We use IVPFWA, IVPFOWA, IVPFHA, $IVPFS_{ft}WA$, IVSFWA, IVSFOWA, IVSFHA, $IVSFS_{ft}WA$, IVT-SFWA [36], IVT-SFOWA [36], IVT-SFHA [36], IVSFWAM [44], IVSFWGM [44], and $IVPFS_{ft}S$ [43] operators to compare with the present work and the evaluation results are shown in Table 11.

Table 11. Comparative study of different methods.

Methods	Score Values				Ranking Results
	x_1	x_2	x_3	x_4	
IVPFWA	0.4630	0.4885	0.5476	0.5094	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVPFOWA	0.4262	0.4687	0.5642	0.5471	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVPFHA	0.4160	0.4554	0.5866	0.5261	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
$IVPFS_{ft}WA$	0.4628	0.4857	0.5467	0.5021	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
$IVPFS_{ft}S$ [43]	0.2118	0.4650	0.6111	0.5132	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVSFWAM [44]	0.4157	0.4387	0.5342	0.5171	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVSFWGM [44]	0.3962	0.4287	0.5142	0.5071	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVSFWA	0.4840	0.4979	0.5703	0.5236	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVSFOWA	0.4520	0.4843	0.5837	0.5544	$\psi_3 > \psi_4 > \psi_2 > \psi_1$

Table 11. *Cont.*

Methods	Score Values				Ranking Results
	x_1	x_2	x_3	x_4	
IVSFHA	0.4462	0.4787	0.5742	0.5671	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
$IVSFS_{ft}WA$	0.4362	0.4578	0.5539	0.5237	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVT-SFWA [36]	0.4875	0.4986	0.5737	0.5253	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVT-SFOWA [36]	0.4565	0.4862	0.5859	0.5549	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVT-SFHA [36]	0.5033	0.5177	0.5684	0.5412	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
$IVT - SFS_{ft}WA$ Proposed work	0.4423	0.4821	0.5625	0.5563	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
$IVT - SFS_{ft}OWA$ Proposed work	0.4858	0.4921	0.5644	0.5334	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
$IVT - SFS_{ft}HA$ Proposed work	0.4041	0.4134	0.5789	0.5699	$\psi_3 > \psi_4 > \psi_2 > \psi_1$

From Table 11, we can see that we can use different methods to get different results under the same evaluation data. Notice that "x_3" is the best alternative in all cases that shows the validity of proposed work. Moreover, proposed operators can consider the parameterization structure while the operators given as IVPFWA, IVPFOWA, IVPFHA, IVSFWA, IVSFOWA, IVSFHA, IVT-SFWA [36], IVT-SFOWA [36], IVT-SFHA [36], IVSFWAM [44], IVSFWGM [44] cannot consider the parameterization structure. From the above analysis, it is clear that the present work is more general than existing methods.

Example 6. *A person plans to buy a house from a set of four alternatives* $= \{x_1,\ x_2,\ x_3,\ x_4\}$. *Let* $E = \{s_1 = beautiful,\ s_2 = reasonable\ price,\ x_3 = green\ surroundings,\ s_4 = location\ \}$. *Let* $\varpi = \{0.25,\ 0.23,\ 0.24,\ 0.28\}$ *denote the weight vector of "*e_i*" experts and* $p = \{0.26,\ 0.20,\ 0.29,\ 0.25\}$ *denote the weight vector of "*s_j*" parameters. The experts provide their information in the form of* $IVSFS_{ft}Ns$ *as given in Table 12.*

Table 12. Information based on interval-valued spherical fuzzy soft numbers.

	x_1	x_2	x_3	x_4
C_1	$\begin{pmatrix} [0,\ 0.1],\ [0.2,\ 0.7], \\ [0.3,\ 0.4] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.5],\ [0.4,\ 0.5], \\ [0.2,\ 0.3] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.5],\ [0.3,\ 0.6], \\ [0.2,\ 0.3] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.4],\ [0.1,\ 0.2], \\ [0.2,\ 0.6] \end{pmatrix}$
C_2	$\begin{pmatrix} [0.1,\ 0.4],\ [0.2,\ 0.4], \\ [0.1,\ 0.3] \end{pmatrix}$	$\begin{pmatrix} [0.3,\ 0.4],\ [0.5,\ 0.6], \\ [0.3,\ 0.5] \end{pmatrix}$	$\begin{pmatrix} [0.1,\ 0.3],\ [0.1,\ 0.5], \\ [0.4,\ 0.6] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.3],\ [0.1,\ 0.4], \\ [0.3,\ 0.4] \end{pmatrix}$
C_3	$\begin{pmatrix} [0.3,\ 0.4],\ [0.2,\ 0.4], \\ [0.1,\ 0.4] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.4],\ [0.1,\ 0.3], \\ [0.4,\ 0.6] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.3],\ [0.5,\ 0.7], \\ [0.3,\ 0.6] \end{pmatrix}$	$\begin{pmatrix} [0.3,\ 0.5],\ [0.4,\ 0.5], \\ [0.1,\ 0.2] \end{pmatrix}$
C_4	$\begin{pmatrix} [0,\ 0.4],\ [0.1,\ 0.3], \\ [0.1,\ 0.4] \end{pmatrix}$	$\begin{pmatrix} [0.2,\ 0.3],\ [0.1,\ 0.3], \\ [0.3,\ 0.5] \end{pmatrix}$	$\begin{pmatrix} [0.1,\ 0.4],\ [0.3,\ 0.5], \\ [0.4,\ 0.6] \end{pmatrix}$	$\begin{pmatrix} [0.1,\ 0.3],\ [0.3,\ 0.5], \\ [0.2,\ 0.4] \end{pmatrix}$

We still use IVPFWA, IVPFOWA, IVPFHA, $IVPFS_{ft}WA$, IVSFWA, IVSFOWA, IVSFHA, $IVSFS_{ft}WA$, IVT-SFWA [36], IVT-SFOWA [36], IVT-SFHA [36], IVSFWAM [44], IVSFWGM [44], and $IVPFS_{ft}S$ [43] to compare with the present work and the evaluation results are shown in Table 13.

It is clear from the above analysis that when decision makers provide information in the form of interval-valued spherical fuzzy soft numbers then the operator IVPFWA, IVPFOWA, IVPFHA, $IVPFS_{ft}WA$ operator and $IVPFS_{ft}S$ [43] fail to tackle that kind of information but on the other hand the proposed work along with IVSFWA, IVSFOWA, IVSFHA, $IVSFS_{ft}WA$, IVT-SFWA [36], IVT-SFOWA [36], IVT-SFHA [36], IVSFWAM [44], IVSFWGM [44] operators can handle this information. Moreover, it can be seen from Table 13 that all the ranking results are the same which shows the validity of the present work.

Table 13. Comparative study of different methods.

Methods	Score Values				Ranking Results
	x_1	x_2	x_3	x_4	
IVPFWA	Failed	Failed	Failed	Failed	Cannot be calculated
IVPFOWA	Failed	Failed	Failed	Failed	Cannot be calculated
IVPFHA	Failed	Failed	Failed	Failed	Cannot be calculated
$IVPFS_{ft}WA$	Failed	Failed	Failed	Failed	Cannot be calculated
$IVPFS_{ft}S$ [43]	Failed	Failed	Failed	Failed	Cannot be calculated
IVSFWAM [44]	0.4157	0.4387	0.5342	0.5171	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVSFWGM [44]	0.3962	0.4287	0.5142	0.5071	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVSFWA	0.4094	0.4330	0.5136	0.4370	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVSFOWA	0.3905	0.4206	0.5449	0.4632	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVSFHA	0.4062	0.4687	0.5542	0.5371	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
$IVSFS_{ft}WA$	0.4356	0.4467	0.5756	0.5123	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVT-SFWA [36]	0.4134	0.4356	0.5193	0.4398	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVT-SFOWA [36]	0.4565	0.4862	0.5859	0.5549	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
IVT-SFHA [36]	0.48033	0.4978	0.5945	0.5729	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
$IVT - SFS_{ft}WA$ present work	0.4423	0.4821	0.5625	0.5563	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
$IVT - SFS_{ft}OWA$ present work	0.4324	0.4534	0.5867	0.5655	$\psi_3 > \psi_4 > \psi_2 > \psi_1$
$IVT - SFS_{ft}HA$ present work	0.4858	0.4914	0.5792	0.5489	$\psi_3 > \psi_4 > \psi_2 > \psi_1$

Example 7. *A person plans to buy a house from a set of four alternatives* $= \{x_1,\ x_2,\ x_3,\ x_4\}$. *Let* $E = \{s_1 = beautiful,\ s_2 = reasonable\ price,\ x_3 = green\ surroundings,\ s_4 = location\ \}$. *Let* $\varpi = \{0.28,\ 0.25,\ 0.23,\ 0.24\}$ *denote the weight vector of "*e_i*" experts and* $p = \{0.26,\ 0.20,\ 0.29,\ 0.25\}$ *denote the weight vector of "*s_j*" parameters. We still use IVPFWA, IVPFOWA, IVPFHA,* $IVPFS_{ft}WA$*, IVSFWA, IVSFOWA, IVSFHA,* $IVSFS_{ft}WA$*, IVT-SFWA [36], IVT-SFOWA [36], IVT-SFHA [36], IVSFWAM [44], IVSFWGM [44], and* $IVPFS_{ft}S$ *[43] to compare with proposed work.*

It is clear that when a DM provides $\{[0.3,\ 0.8],\ [0.3,\ 0.6],\ [0.3,\ 0.9]\}$, then the methods given as IVPFWA, IVPFOWA, IVPFHA, $IVPFS_{ft}WA$, $IVPFS_{ft}S$ [43], IVSFWAM [44], IVSFWGM [44], IVSFWA, IVSFOWA, IVSFHA, $IVSFS_{ft}WA$ fail to handle this type of information because for this type of information $sum(0.8,\ 0.6,\ 0.9) \notin [0,1]$ and $sum\left(0.8^2,\ 0.6^2,\ 0.9^2\right) \notin [0,1]$. However, the proposed operators can handle such kinds of data along with the method given in [30]. Similarly, if data given in Tables 6–9 are considered, then all the above-given methods fail to handle all this information, while the present work along with the method given in [30] can easily handle this type of information. Hence, it is clear that the present work provides more space to DMs in making their decisions for MCDM problems. Hence, the present work is more general. For this, $IVT - SFS_{ft}Ns$ are aggregated and the overall decision matrix for different mobile phone brands $x_i;\ i = 1,\ 2,\ 3,\ 4$ by using WVs $\varpi = \{0.28,\ 0.25,\ 0.23,\ 0.24\}$ is given in Table 14. From Table 14, it is clear that all the information consists of $IVT - SFS_{ft}Ns$ and this information cannot be tackled by all the above-given methods, so we cannot calculate the score values for all the above given operators, while the presented operators can tackle this information along with the method given in [30] and also we can calculate the score values for all data given in Table 14. Now using this information, a comparative evaluation of all the above given aggregation operators with the present work is given together with their results in Table 15.

From Table 15, note that "x_3" is the best alternative, which shows the validity of the proposed work. Further, the characteristic evaluation of the present approach with all the above operators is given in Table 16. Hence, it is clear that IVPFWA, IVPFOWA, IVPFHA, IVSFWA, IVSFOWA, IVSFHA, IVSFWAM [44], IVSFWGM [44], IVT-SFWA [36], IVT-SFOWA [36], IVT-SFHA [36] cannot consider the parameterization structure. The main advantage of the present work is that it provides more space to DMs, generalizes many existing structures, and also considers parameterization structures to deal with real-life problems. Hence, the present work can be used in MCDM problems rather than using it for other operators in the $IVT - SFS_{ft}$ environment.

Table 14. Aggregated values of $IVT - SFS_{ft}Ns$ for $x_i; i = 1, 2, 3, 4$.

	x_1	x_2	x_3	x_4
C_1	$\left(\begin{array}{c}[0.3927, 0.6938], \\ [0.3072, 0.5096], \\ [0.2770, 0.5323]\end{array}\right)$	$\left(\begin{array}{c}[0.3797, 0.7396], \\ [0.3002, 0.4690], \\ [0.2646, 0.4950]\end{array}\right)$	$\left(\begin{array}{c}[0.4279, 0.7505], \\ [0.2625, 0.4488], \\ [0.3093, 0.5478]\end{array}\right)$	$\left(\begin{array}{c}[0.4477, 0.7173], \\ [0.3304, 0.5211], \\ [0.2675, 0.4918]\end{array}\right)$
C_2	$\left(\begin{array}{c}[0.4326, 0.6956], \\ [0.2967, 0.5130], \\ [0.2784, 0.5187]\end{array}\right)$	$\left(\begin{array}{c}[0.3867, 0.7491], \\ [0.2919, 0.4738], \\ [0.2745, 0.5023]\end{array}\right)$	$\left(\begin{array}{c}[0.4431, 0.7374], \\ [0.2606, 0.4453], \\ [0.3037, 0.5425]\end{array}\right)$	$\left(\begin{array}{c}[0.4477, 0.7228], \\ [0.3297, 0.5281], \\ [0.2698, 0.4793]\end{array}\right)$
C_3	$\left(\begin{array}{c}[0.4307, 0.7039], \\ [0.2930, 0.5167], \\ [0.2877, 0.5279]\end{array}\right)$	$\left(\begin{array}{c}[0.3760, 0.7512], \\ [0.2709, 0.4530], \\ [0.2790, 0.5305]\end{array}\right)$	$\left(\begin{array}{c}[0.4491, 0.7458], \\ [0.2674, 0.4528], \\ [0.3220, 0.5326]\end{array}\right)$	$\left(\begin{array}{c}[0.4393, 0.7308], \\ [0.3343, 0.5657], \\ [0.2742, 0.4831]\end{array}\right)$
C_4	$\left(\begin{array}{c}[0.4322, 0.7069], \\ [0.2991, 0.5086], \\ [0.2863, 0.5488]\end{array}\right)$	$\left(\begin{array}{c}[0.3731, 0.7240], \\ [0.2772, 0.4513], \\ [0.2708, 0.5255]\end{array}\right)$	$\left(\begin{array}{c}[0.4612, 0.7486], \\ [0.2689, 0.4537], \\ [0.3259, 0.5482]\end{array}\right)$	$\left(\begin{array}{c}[0.4386, 0.7326], \\ [0.3351, 0.5604], \\ [0.2729, 0.4978]\end{array}\right)$

Table 15. Comparative study of different methods.

Methods	**Score Values**				**Ranking Results**
	x_1	x_2	x_3	x_4	
IVPFWA	Failed	Failed	Failed	Failed	Cannot be calculated
IVPFOWA	Failed	Failed	Failed	Failed	Cannot be calculated
IVPFHA	Failed	Failed	Failed	Failed	Cannot be calculated
$IVPFS_{ft}WA$	Failed	Failed	Failed	Failed	Cannot be calculated
$IVPFS_{ft}S$ [43]	Failed	Failed	Failed	Failed	Cannot be calculated
IVSFWAM [44]	Failed	Failed	Failed	Failed	Cannot be calculated
IVSFWGM [44]	Failed	Failed	Failed	Failed	Cannot be calculated
IVSFWA	Failed	Failed	Failed	Failed	Cannot be calculated
IVSFOWA	Failed	Failed	Failed	Failed	Cannot be calculated
IVSFHA	Failed	Failed	Failed	Failed	Cannot be calculated
$IVSFS_{ft}WA$	Failed	Failed	Failed	Failed	Cannot be calculated
IVT-SFWA [36]	0.6535	0.7023	0.7132	0.7012	$\psi_3 > \psi_2 > \psi_4 > \psi_1$
IVT-SFOWA [36]	0.6577	0.7233	0.7566	0.7114	$\psi_3 > \psi_2 > \psi_4 > \psi_1$
IVT-SFHA [36]	0.6733	0.6977	0.7455	0.6845	$\psi_3 > \psi_2 > \psi_4 > \psi_1$
$IVT - SFS_{ft}WA$ Present work	0.6673	0.7172	0.7234	0.7066	$\psi_3 > \psi_2 > \psi_4 > \psi_1$
$IVT - SFS_{ft}OWA$ Present work	0.6776	0.7224	0.7435	0.7213	$\psi_3 > \psi_2 > \psi_4 > \psi_1$
$IVT - SFS_{ft}HA$ Present work	0.6877	0.7086	0.7466	0.7015	$\psi_3 > \psi_2 > \psi_4 > \psi_1$

Table 16. Characteristic evaluation of different methods.

Methods	**Fuzzy Data**	**Aggregate Parameter Data**
IVPFWA	Yes	No
IVPFOWA	Yes	No
IVPFHA	Yes	No
$IVPFS_{ft}WA$	Yes	Yes
$IVPFS_{ft}S$ [43]	Yes	Yes
IVSFWAM [44]	Yes	No
IVSFWGM [44]	Yes	No
IVT-SFWA [36]	Yes	No
IVT-SFOWA [36]	Yes	No
IVT-SFHA [36]	Yes	No
IVSFWA	Yes	No
IVSFOWA	Yes	No

Table 16. *Cont.*

Methods	Fuzzy Data	Aggregate Parameter Data
IVSFHA	Yes	No
$IVSFS_{ft}WA$	Yes	Yes
$IVT - SFS_{ft}WA$ Proposed work	Yes	Yes
$IVT - SFS_{ft}OWA$ Proposed work	Yes	Yes
$IVT - SFS_{ft}HA$ Proposed work	Yes	Yes

5.3. Scientifitic Decision of the Proposed Works

The idea of $IVT - SFS_{ft}S$ is an important technique to cope with complicated and uncertain information in real-life issues. The idea of $IVT - SFS_{ft}S$ is the mixture of two different ideas such as $IVT - SFS$ and $S_{ft}S$, which contains the grade of truth, abstinence, and falsity with a rule that the sum of the upper parts of the q-powers of all grades is restricted to unit interval. The advantages of the proposed $IVT - SFS_{ft}S$ are discussed below:

1. If we choose the value of $q = 2$, then the proposed $IVT - SFS_{ft}S$ is converted for interval-valued spherical fuzzy soft sets.
2. If we choose the value of $q = 1$, then the proposed $IVT - SFS_{ft}S$ is converted for interval-valued picture fuzzy soft sets.
3. If we choose the value of abstinence is zero, then the proposed $IVT - SFS_{ft}S$ is converted for interval-valued q-rung orthopair fuzzy soft sets.
4. If we choose the value of abstinence is zero with $q = 2$, then the proposed $IVT - SFS_{ft}S$ is converted for interval-valued Pythagorean fuzzy soft sets.
5. If we choose the value of abstinence is zero with $q = 1$, then the proposed $IVT - SFS_{ft}S$ is converted for interval-valued intuitionistic fuzzy soft sets.

Similarly, in future, we will extend the proposed work $IVT - SFS_{ft}S$ for the following ideas:

1. Interval-valued T-spherical hesitant fuzzy soft sets.
2. Interval-valued T-spherical hesitant fuzzy soft rough sets.
3. Interval-valued T-spherical fuzzy soft rough sets.
4. T-spherical hesitant fuzzy soft sets.
5. T-spherical hesitant fuzzy soft rough sets.

In future, this work will be used in the environment of image segmentation, pattern recognition, medical diagnosis, and determination of the dangers of brain cancers.

6. Conclusions

MCDM approach is a well-known and very effective technique for the selection of the best alternative among the given and this approach has been used in different fields of fuzzy sets theory. Aggregation operators are an effective tool to deal with fuzzy information and desirable results for real-life problems can be obtained by these means. Here in this paper, we have combined two notions, IVT-SFS and SS, to generate the new notion called $IVT - SFS_{ft}S$. It is a strong apparatus to deal with fuzzy information and also generalized many previous ideas such as $PFS_{ft}S$, $SFS_{ft}S$, $TT - SFS_{ft}S$, $IVPFS_{ft}S$ and $IVSFS_{ft}S$. Moreover, inspired by the parameterization property of soft set, we have established the operators such as $IVT - SFS_{ft}WA$, $IVT - SFS_{ft}OWA$ and $IVT - SFS_{ft}HA$ operators and also their properties are discussed in detail. An algorithm is developed and an application example is proposed to show the validity and superiority of the proposed work. Further, in comparative analysis, the established work is compared with another existing method to show the superiority of the present work.

In the future, one can combine $T - SFS_{ft}S$ and $IVT - SFS_{ft}S$ to introduce a new notion called cubic T-spherical fuzzy soft set $CT - SFS_{ft}S$. In addition, this notion can be used in many MCDM approaches and desirable results can be obtained. Moreover, numerous scholars have introduced the hybrid notion of rough set and other fuzzy sets theories and applied these notions to multi-attribute decision-making problems as given in [45–48]. Therefore, one can also use the established structure and rough set to introduce new hybrid notions like interval-valued T-spherical fuzzy soft rough set and soft rough interval-valued T-spherical fuzzy set, and then this notion can be used in many decision-making problems.

In future, we will extend the proposed idea to bipolar soft sets [49], complex T-spherical fuzzy sets [50,51], and complex neutrosophic sets [52]. This work will also be utilized in the environment of image segmentation [53], pattern recognition [54], medical diagnosis, and determination of the dangers of brain cancers.

Author Contributions: Conceptualization, T.M. and J.A.; methodology, J.A., D.M.; software, Z.A.; validation, T.M., J.A., Z.A. and D.P.; formal analysis, T.M.; investigation, J.A.; resources, D.P.; data curation, J.A.; writing—original draft preparation, J.A.; writing—review and editing, Z.A., D.P. and D.M.; visualization, T.M.; supervision, T.M.; project administration, T.M.; funding acquisition, D.M. All authors have read and agreed to the published version of the manuscript.

Funding: The APC was funded by German Research Foundation and the TU Berlin.

Data Availability Statement: No real data were used to support this study. The data used in this study are hypothetical and anyone can use them by just citing this article.

Acknowledgments: The authors wish to acknowledge the support received from German Research Foundation and the TU Berlin.

Conflicts of Interest: The authors declare that they have no conflict of interest.

Abbreviations

For the sake of clarity, the following table gives all the abbreviations used in this manuscript.

Abbreviations	**Complete Name**
MCDM	Multiple-criteria decision making
$S_{ft}S$	Soft set
$IVT-SFS$	Interval-valued T-spherical fuzzy set
$IVT-SFS_{ft}S$	Interval-valued T-spherical fuzzy soft set
DM	Decision-makers
$IVT-SFS_{ft}WA$	Interval-valued T-spherical fuzzy soft weighted averaging
$IVT-SFS_{ft}OWA$	Interval-valued T-spherical fuzzy soft ordered weighted averaging
$IVT-SFS_{ft}HA$	Interval-valued T-spherical fuzzy soft hybrid averaging

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