



# Article On Baryogenesis from a Complex Inflaton

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**Abstract:** We derive the particle asymmetry due to inflationary baryogenesis involving a complex inflaton, obtaining a different result to that in the literature. While asymmetries were found to be significantly smaller than previously calculated, in certain parameter regions, baryogenesis can still be achieved.

Keywords: baryogenesis; particle asymmetries; inflation

# 1. Introduction

Whilst the general principles behind the generation of particle asymmetries are well understood [1], the specific mechanism through which baryogenesis occurred remains a mystery. What is apparent is that the Standard Model alone does not give rise to the appropriate conditions to realise baryogenesis via the electroweak phase transition [2]. An interesting prospect is that the baryon asymmetry might be generated due to inflationary physics [3–12]. In this work, we re-examine the inflationary baryogenesis scenario proposed recently in [8,9]. We rederive the parametric form of the asymmetry, finding a significantly different result. Subsequently, we identify the parameter regions, which are simultaneously consistent with the cosmological evidence for inflation [13,14] and allow for successful baryogenesis. Moreover, we discuss issues related to effective field theory intuition and procedures for changing between dimensionful and dimensionless sets of variables, which are more generally applicable. Specifically, we highlight that consistent scaling of variables should typically be in terms of a single mass scale and thus equivalent to a change of units.

Hertzberg and Karouby [8,9] considered the possibility that the inflaton was a complex field

$$p(t) = \frac{1}{\sqrt{2}}\rho(t)e^{i\theta(t)} , \qquad (1)$$

which carries a conserved global quantum number (up to Planck scale,  $M_{\text{Pl}}$ , with effects that are expected to violate all continuous global symmetries). The requirements for generating a particle asymmetry in a given global charge is a period of out-of-equilibrium dynamics, together with violation of *C*, *CP*, and the associated global symmetry [1]. In the scenario at hand, the out-of-equilibrium dynamics are driven by inflation. Further, *C* and *CP* can be broken spontaneously due to the initial phase of the inflaton field  $\theta_i$  (reminiscent of the Affleck–Dine mechanism [4]). Finally, the violation of the inflaton global symmetry is sourced from small breaking terms in the potential. We shall take a simple quadratic potential (as used commonly in chaotic inflation [15]) supplemented by a single dimension- $\alpha$  operator (for  $\alpha \geq 3$ ), which breaks the U(1) global symmetry, and, following [8], we assume a potential of the form

$$V(\phi,\phi^*) = \frac{1}{2}m^2|\phi|^2 + \lambda \left(\frac{1}{\Lambda}\right)^{\alpha-4}(\phi^{\alpha} + \phi^{*\alpha}), \qquad (2)$$

where  $\lambda$  is a dimensionless coupling, and  $\Lambda$  has mass dimension one (deviating from the notation of [8]). As the latter term causes a perturbation from the quadratic potential, it is constrained to be small for successful inflation



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$$\left(\frac{\lambda}{2^{\frac{\alpha}{2}-1}\Lambda^{\alpha-4}}\right)\rho_i^{\alpha}\cos(\alpha\theta_i) \ll \frac{1}{2}m^2\rho_i^2 , \qquad (3)$$

where the subscript *i* indicates the value of a given parameter at some arbitrary initial time for which the inflaton's field value is  $\rho_i \sim M_{\text{Pl}}$ . Note that Equation (3) is required to be satisfied for all  $\theta_i$  in [8], but this seems too stringent a requirement. It follows that to avoid perturbing the quadratic potential it is required that

$$\lambda \ll \frac{2^{\frac{\alpha}{2}-1}\Lambda^{\alpha-4}m^2}{M_{\rm Pl}^{\alpha-2}\cos(\theta_i\alpha)}.$$
(4)

A natural measure of particle asymmetries is

$$A_{\infty} \equiv \frac{n - \bar{n}}{n + \bar{n}} , \qquad (5)$$

in terms of the number densities of particles n and antiparticles  $\bar{n}$ . This quantity is bounded by  $0 \le |A_{\infty}| \le 1$ . Extremal values correspond to equal populations  $n = \bar{n}$  for  $A_{\infty} = 0$ or a completely asymmetric population with vanishing n or  $\bar{n}$  for  $|A_{\infty}| = 1$ . Once an asymmetry is established in the inflaton charge, the inflaton can decay in a manner that transfers the asymmetry to baryons. An appropriate measure of the inflaton asymmetry at an early time is given by [8]

$$A_0 = \frac{m(n-\bar{n})}{\epsilon} , \qquad (6)$$

where  $\epsilon$  is the energy density and the subscripts 0 and  $\infty$  distinguish the asymmetry at early and late times. At late times, the energy density is determined by the non-relativistic gas of  $\phi$  and  $\phi^*$ ; thus,  $\epsilon = m(n + \bar{n})$  and the asymmetry reduces to the form of Equation (5).

It was argued in [8] that, by evaluating Equation (6), the late time asymmetry can be expressed as follows

$$A_{\infty}^{(\text{HK})} \sim -c_{\alpha} \lambda \left(\frac{M_{\text{Pl}}^{\alpha-2}}{m^2 \Lambda^{\alpha-4}}\right) \sin(\theta_i \alpha) .$$
<sup>(7)</sup>

The constant  $c_{\alpha}$  is estimated to be of the order [8]

$$c_{\alpha} \sim \left(\frac{2}{3}\right)^{\frac{\alpha}{2}} \alpha \Gamma_{\frac{1}{2}}\left(\frac{\alpha}{2}\right), \tag{8}$$

in terms of the incomplete  $\Gamma$ -function

Whilst the prospect of generating baryogenesis through the dynamics of a complex inflaton is rather elegant, the form of  $A_{\infty}^{(\text{HK})}$  raises some questions, in particular, whether Equation (7) can take values greater than unity. As discussed above, the coupling  $\lambda$  must satisfy Equation (4) to avoid perturbing the inflationary potential. Assuming  $\lambda$  saturates this bound (although it should also be perturbative  $\lambda \leq 1$ ), then  $|A_{\infty}^{(\text{HK})}|$  can take values in the range

$$0 \le |A_{\infty}^{(\mathrm{HK})}| < 2^{\alpha/2-1} c_{\alpha} \left(\frac{M_{\mathrm{Pl}}}{\rho_i}\right)^{\alpha-2} \tan(\theta_i \alpha) .$$
(9)

This allows values of  $A_{\infty}^{(\text{HK})}$  greater than unity for large  $\tan(\theta_i \alpha)$ , e.g., with  $\alpha = 5$  and  $\sin(5\theta_i) = -1$ , one finds

$$A_{\infty}^{(\mathrm{HK})} \sim 10^{12} \left(\frac{\lambda}{1}\right) \left(\frac{10^{16} \,\mathrm{GeV}}{\Lambda}\right) \left(\frac{10^{13} \,\mathrm{GeV}}{m}\right)^{2},\tag{10}$$

in contradiction with the definition of the asymmetry, as given in Equation (5). Further, from inspection of Equation (7), the scaling behaviour of  $A_{\infty}^{(\text{HK})}$ , seems counterintuitive as

it does not conform to expectations from effective field theory. The asymmetry receives sequentially larger contributions from operators with increasing mass dimension.

The purpose of this study was to rederive the form of the asymmetry due to a complex inflaton  $A_{\infty}$ . We obtained a result that differs in form from Equation (7) and that satisfies both  $|A_{\infty}| \leq 1$  and effective field theory considerations. In particular, Section 3 gives a careful account of the differences between the derivation here and that of [8], along with arguments in favour of our approach.

#### 2. The Inflaton Asymmetry

The asymmetry  $A_0$  can be expressed in terms of the overall charge difference  $\Delta N_{\phi}$ , which is related to the particle asymmetry per comoving volume  $n - \bar{n}$  as follows

$$A_0 = \frac{m(n-\bar{n})}{\epsilon} = \frac{m}{\epsilon} \left(\frac{\Delta N_{\phi}}{V_{\rm co}a^3}\right),\tag{11}$$

where  $V_{co}$  is the comoving volume, and a(t) is the scale factor. By examining the evolution of the number of inflatons  $N_{\phi}$  relative to the number of anti-inflatons  $N_{\bar{\phi}}$ , assuming an FRW metric, one obtains

$$\Delta N_{\phi} \equiv N_{\phi} - N_{\bar{\phi}} = i V_{\rm co} a^3 (\phi^* \dot{\phi} - \dot{\phi}^* \phi) , \qquad (12)$$

Following the first steps of [8]; we start from the equation of motion (EoM) for  $\phi$  given by

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + \lambda\alpha \left(\frac{1}{\Lambda}\right)^{\alpha-4} \phi^{*\alpha-1} = 0 , \qquad (13)$$

where  $H = \dot{a}/a$  is the Hubble parameter. The final charge difference  $\Delta N_{\phi}(t_f)$  can be found by taking the time derivative of Equation (12) and using the EoM, to obtain

$$\Delta N_{\phi}(t_f) \simeq \Delta N_{\phi}(t_i) + i\lambda \left(\frac{1}{\Lambda}\right)^{\alpha-4} \alpha V_{\rm co} \int_{t_i}^{t_f} a(t)^3 [\phi(t)^{\alpha} - \phi^*(t)^{\alpha}].$$

Moreover, as any initial asymmetry is likely erased via inflation:  $\Delta N_{\phi}(t_i) \simeq 0$ . Thus, to  $\mathcal{O}(\lambda)$  this gives

$$\Delta N_{\phi}(t_f) \simeq -\left(\frac{1}{\Lambda}\right)^{\alpha-4} \frac{\lambda \alpha V_{\rm co}}{2^{\frac{\alpha}{2}-1}} \int_{t_i}^{t_f} a(t)^3 \rho(t)^{\alpha} \sin(\alpha \theta(t)). \tag{14}$$

Since at zeroth order in  $\lambda$  the argument does not evolve, we can take  $\theta(t) = \theta_i$ . Moreover, at zeroth order in  $\lambda$ , the radial component  $\rho$  is a real valued function satisfying

$$\ddot{o} + 3H\dot{\rho} + m^2\rho = 0 , \qquad (15)$$

with the associated Friedmann equation

$$H^{2} = \frac{1}{6M_{\rm Pl}^{2}} \left( \dot{\rho}^{2} + m^{2} \rho^{2} \right) \equiv \frac{\epsilon}{3M_{\rm Pl}^{2}} \,. \tag{16}$$

Then, working at lowest order, one can express  $\Delta N_{\phi}(t_f)$  in terms of the radial component to obtain [8]

$$\Delta N_{\phi}(t_f) \simeq -\lambda \left(\frac{1}{\Lambda}\right)^{\alpha-4} \frac{V_{\rm co}\alpha}{2^{\frac{\alpha}{2}-1}} \sin(\theta_i \alpha) I(t_i, t_f) , \qquad (17)$$

with

$$I(t_i, t_f) = \int_{t_i}^{t_f} dt \, a(t)^3 \rho(t)^{\alpha} \,. \tag{18}$$

It follows that the asymmetry can be expressed as follows

$$A_0 = -\frac{m}{a^3 \epsilon} \left(\frac{1}{\Lambda}\right)^{\alpha - 4} \lambda \frac{\alpha}{2^{\frac{\alpha}{2} - 1}} \sin(\theta_i \alpha) I(t_i, t_f) .$$
<sup>(19)</sup>

Here is where our derivation differs crucially from [8] (in the next section, we explore this difference in detail). We make a change of variables such that everything is measured in units of inflaton mass m

$$\tau \equiv mt, \qquad \hat{\rho} \equiv \frac{\rho}{m}, \qquad \hat{H} \equiv \frac{H}{m}.$$
(20)

Thus, each of the rescaled quantities is dimensionless (reminiscent of the  $M_{\text{Pl}}$ -units sometimes employed). The scaling leads to a dimensionless version of Equation (15)

$$\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}\hat{\rho} + 3\hat{H}\frac{\mathrm{d}}{\mathrm{d}\tau}\hat{\rho} + \hat{\rho} = 0.$$
<sup>(21)</sup>

The corresponding dimensionless Friedmann equation is  $\hat{H}^2 = \frac{\hat{\epsilon}}{3}$  in terms of  $\hat{\epsilon} \equiv \epsilon/(mM_{\rm Pl})^2$ . Following the notation of [8], we introduce

$$f_{\alpha} = \frac{\alpha}{2^{\frac{\alpha}{2}-1}} \frac{1}{a^3 \hat{\epsilon}} \,\bar{I}(\tau_i, \tau_f) \,, \tag{22}$$

in terms of the scaled quantity

$$\bar{I}(\tau_i, \tau_f) = \int_{\tau_i}^{\tau_f} \mathrm{d}\tau \ a(\tau)^3 \hat{\rho}(\tau)^{\alpha} \ .$$
(23)

It follows that Equation (19) can be rewritten as

$$A_0 = -\lambda f_{\alpha} \left( \frac{m^{\alpha - 2}}{M_{\rm Pl}^2 \Lambda^{\alpha - 4}} \right) \sin(\theta_i \alpha) .$$
(24)

To obtain the late time asymmetry, we should evaluate  $f_{\alpha}$  in the limit  $\tau_i, \tau_f \to \pm \infty$ . Then, making the replacement  $f_{\alpha} \to c_{\alpha} = f_{\alpha}(\tau_i \to -\infty, \tau_f \to \infty)$  gives our main result

$$A_{\infty} = -c_{\alpha}\lambda\left(\frac{m^{\alpha-2}}{M_{\rm Pl}^2\Lambda^{\alpha-4}}\right)\sin(\theta_i\alpha) , \qquad (25)$$

where the coefficients  $c_{\alpha}$  are given parametrically in Equation (8). Observe that  $A_{\infty}$  above is distinct from Equation (7); the asymmetry is bounded  $|A_{\infty}| \leq 1$ , and contributions are suppressed for increasing  $\alpha$ , as expected from effective-field-theory considerations.

## 3. Changing Variables

We next discuss the differences between Equation (25), derived here, and Equation (7), from [8]. Notably, the derivation of [8] introduces a number of dimensionful arbitrary scaling constants; here, we highlight that these scalings cannot be independent and highlight that a proper choice for these scalings lead one to the result of Equation (25). Specifically, in [8], the following dimensionless EoM is considered

$$\frac{\mathrm{d}^2}{\mathrm{d}\bar{\tau}^2}\bar{\rho} + 3\bar{H}\frac{\mathrm{d}}{\mathrm{d}\bar{\tau}}\bar{\rho} + \bar{\rho} = 0 , \qquad (26)$$

where a change of variables different to Equation (20) is used, and variables with mass dimension are not scaled by a single mass scale. To see why the change of variables used in [8] runs into difficulties, consider the general scaling

$$\bar{\tau} \equiv M_t t , \qquad \bar{\rho} \equiv \frac{\rho}{M_{\rho}} , \qquad \bar{H} \equiv \frac{H}{M_H} .$$
(27)

Then rescaling Equation (26) one obtains

$$\ddot{\rho} + 3H \frac{M_t}{M_H} \dot{\rho} + M_t^2 \rho = 0.$$
(28)

Requiring that Equation (15) is recovered from Equation (28) fixes  $M_t = M_H = m$ . However, this does not specify  $M_{\rho}$ . Moreover, as we show shortly,  $M_{\rho}$  appears explicitly in the form of  $A_{\infty}$  and so can not be chosen arbitrarily. In [8], the identification  $M_{\rho} = M_{\text{Pl}}$  is made, causing a problem, which we address below.

Without specifying  $M_{\rho}$  we now rederive the form of the asymmetry  $A_{\infty}$ . Starting from Equation (19)

$$A_0 = -\frac{m}{a^3 \epsilon} \left(\frac{1}{\Lambda}\right)^{\alpha - 4} \lambda \frac{\alpha}{2^{\frac{\alpha}{2} - 1}} \sin(\theta_i \alpha) \ I \ . \tag{29}$$

Recall from Equations (18) and (23) the definitions of I and  $\overline{I}$ , with the scaling factor  $M_{\rho}$  unspecified these can be related by  $\overline{I} = \frac{m}{M_{\rho}^{\alpha}}I$ . Thus in terms of  $f_{\alpha}$ , cf. Equation (22), we obtain

$$A_0 = -\left(\frac{\hat{\epsilon}}{\epsilon}\right) \lambda f_\alpha \left(\frac{1}{\Lambda}\right)^{\alpha-4} M_\rho^\alpha \,\sin(\theta_i \alpha) \,. \tag{30}$$

Using  $\hat{\epsilon} \equiv \epsilon / (mM_{\rm Pl})^2$  and replacing  $f_{\alpha}$  with  $c_{\alpha}$  we obtain

$$A_{\infty} = -\lambda c_{\alpha} \frac{M_{\rho}^{\alpha}}{m^2 M_{\rm Pl}^2 \Lambda^{\alpha-4}} \,\sin(\theta_i \alpha) \,. \tag{31}$$

Given that the asymmetry changes with  $M_{\rho}$  signals, this scale can not be chosen arbitrarily. Observe that taking  $M_{\rho} = M_{\text{Pl}}$  gives the result of [8], as quoted in Equation (7), and, for  $M_{\rho} = m$ , we recovered Equation (25), as expected.

We already argued that an appropriate scaling amounts to a simple change of units. We next give an explicit argument in support of this approach. Whilst previously we have rescaled to obtain a dimensionless EoM for  $\rho$ , we now consider the EoM for  $\phi$ 

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + \lambda\alpha \left(\frac{1}{\Lambda}\right)^{\alpha-4} \phi^{*\alpha-1} = 0.$$
(32)

The validity of a set of scalings is independent of  $\alpha$ , and the root of the problem is most transparent for  $\alpha = 4$ . We consider again a general rescaling, as in Equation (27), with  $\bar{\phi} = \phi/M_{\rho}$ . The desired form of the rescaled EoM is

$$\frac{\mathrm{d}^2}{\mathrm{d}\bar{\tau}^2}\bar{\phi} + 3\bar{H}\frac{\mathrm{d}}{\mathrm{d}\bar{\tau}}\bar{\phi} + \bar{\phi} + 4\lambda\bar{\phi}^{*3} = 0. \tag{33}$$

Applying the parameter scalings, we obtain

$$\ddot{\phi} + 3H \frac{M_t}{M_H} \dot{\phi} + M_t^2 \phi + 4\lambda \left(\frac{M_t^2}{M_\rho^2}\right) \phi^{*3} = 0 , \qquad (34)$$

and, to recover Equation (32) with  $\alpha = 4$ , we require  $m = M_{\rho} = M_t = M_H$ . Finally, consider the case of general  $\alpha$ , where the appropriate dimensionless version of Equation (32) is

$$\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}\bar{\phi} + 3\hat{H}\frac{\mathrm{d}}{\mathrm{d}\tau}\bar{\phi} + \bar{\phi} + \lambda\alpha \left(\frac{1}{\bar{\Lambda}}\right)^{\alpha-4}\bar{\phi}^{*\alpha-1} = 0.$$
(35)

We rescale all dimensionful quantities (including  $\Lambda$ ) by a single scale *m*, except for  $\phi$ , which we leave unspecified, in order to show that this fixes the scaling factor for  $\phi$ 

$$\tau \equiv mt$$
,  $\bar{\phi} \equiv \frac{\phi}{M_{\rho}}$ ,  $\hat{H} \equiv \frac{H}{m}$ ,  $\hat{\Lambda} \equiv \frac{\Lambda}{m}$ . (36)

Applying Equation (36) to (35), we obtain

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + \left(\frac{m}{\Lambda}\right)^{\alpha-4} \left(\frac{m^2}{M_{\rho}^{\alpha-2}}\right) \lambda \alpha \phi^{*\alpha-1} = 0.$$

To recover Equation (32), we again require  $m = M_{\rho}$ . Hence, consistent scalings should typically be in terms of a single mass scale and thus be equivalent to a change of units.

## 4. The Baryon Asymmetry

The asymmetry in baryons is typically given as follows

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$$\eta_b = \frac{n_b - \bar{n}_b}{n_\gamma} \simeq 6 \times 10^{-10} \,. \tag{37}$$

This is similar to Equation (5), but here the difference between baryons  $n_b$  and antibaryons  $\bar{n}_b$  is normalised relative to the photon number density  $n_{\gamma}$ . In the previous section, we derived the parametric form of the asymmetry in the inflaton global charge  $A_{\infty}$ , which is related to  $\eta_b$  via

$$\eta_b \sim g_*^{\frac{3}{4}} A_\infty \left(\frac{\sqrt{\Gamma M_{\rm Pl}}}{m}\right) \,. \tag{38}$$

We calculated here the magnitude of  $\eta_b$ , which arises due to the inflaton asymmetry, and to identify parameter regions in which the observed baryon asymmetry can be realised.

As only quarks carry baryon number in the Standard Model, the first gauge and Lorentz invariant baryon number violating operator is  $\phi QQQL$ ; however, mild extensions of the Standard Model can alter this. Incidentally, leptogenesis might be accomplished with lower-dimension operators. We do not pursuit these possibilities further but hope to return to them in a future publication. For our purposes, we simply suppose that the inflaton decays dominantly via a dimension-*p* operator, suppressed by a scale *M*. Thus, the decay rate is parametrically

$$\Gamma_{\phi} \sim m \left(\frac{m}{M}\right)^{2(p-4)}.$$
(39)

It would be quite natural to identify M with  $\Lambda$ , but, for the moment, we maintain the more general possibility that these scales are distinguished. Substituting the forms of  $\Gamma_{\phi}$  and  $A_{\infty}$ , and assuming that each inflaton decay violates baryon number by one unit (as with  $\phi QQQL$ ), the resultant baryon asymmetry is given by

$$\eta_b \sim -c_\alpha \lambda g_*^{\frac{3}{4}} \left(\frac{m}{M}\right)^{p-4} \left(\frac{m}{\Lambda}\right)^{\alpha-4} \left(\frac{m}{M_{\rm Pl}}\right)^{\frac{3}{2}} \sin(\theta_i \alpha). \tag{40}$$

Further, up to a dependence on the number of e-folds of inflation N, the observed value [13] of the squared amplitude of density fluctuations  $\Delta_R^2 \approx 2.45 \times 10^{-9}$  fixes the inflaton mass in models of single-field slow-roll inflation

$$m \simeq \frac{\sqrt{6\pi\Delta_R M_{\rm Pl}}}{N} \simeq 1.5 \times 10^{13} \,\,{\rm GeV}\left(\frac{60}{N}\right)\,. \tag{41}$$

Notably, the observed baryon asymmetry can be reproduced with this value of inflaton mass. For a dimension 5 breaking operator ( $\alpha = 5$ ) and a dimension 7 transfer operator (p = 7) generated at the scale  $\Lambda = M$ , with  $\sin 5\theta_i \approx 1$ , the observed  $\eta_b$  is obtained for

$$\eta_b \sim 10^{-9} \left(\frac{\lambda}{1}\right) \left(\frac{m}{1.5 \times 10^{13} \text{GeV}}\right)^{\frac{11}{2}} \left(\frac{5 \times 10^{13} \text{GeV}}{\Lambda}\right)^4 \tag{42}$$

where we took  $g_* \simeq 100$ .

In [8], it is argued, using the form of  $A_{\infty}^{(\text{HK})}$ , that  $\eta_b \sim 10^{-9}$  can be obtained for  $M = \Lambda \sim 10^{16}$  GeV. As a result, decays via  $\phi QQQL$  are subdominant to dimension five U(1)-violating  $M_{\text{Pl}}$ -suppressed decays, and the asymmetry is erased unless some symmetry forbids these operators. However, using instead the form of  $A_{\infty}$  from Equation (25), such *ad hoc* symmetries are no longer necessary.

We conclude that realistic values of the baryon asymmetry can in principle be generated. It remains to examine the requirements of inflation, as given in Equations (3) and (4). For general  $\theta_i$ , this constraint is highly problematic, as it implies  $\lambda \ll 1$ ; e.g., with  $\sin(\alpha \theta_i) \sim \frac{1}{\sqrt{2}}$  and  $\alpha = 5$ , it is required that

$$\lambda \ll \frac{\Lambda m^2}{M_{\rm Pl}^3} \simeq 10^{-14} \left(\frac{m}{10^{13} \,{\rm Gev}}\right)^2 \left(\frac{\Lambda}{10^{14} \,{\rm Gev}}\right).$$
 (43)

Such values of  $\lambda$  are typically too small to realise the observed baryon asymmetry. In the example studied in Equation (42) this leads to baryon asymmetries  $\eta_b \ll 10^{-23}$ .

However, observe that Equation (4) is trivially satisfied for  $\cos(\alpha\theta_i) \approx 0$  (also note that, in this case,  $|A_{\infty}|$  is maximal, as  $\sin(\alpha\theta_i) \approx \pm 1$ ). Thus, for special values of  $\theta_i$ , inflationary cosmology is unperturbed. Note also that forms of Equations (3) and (25) can vary if the symmetry violating operator is changed (e.g.,  $\Lambda^{4-\alpha}\phi^{\alpha-1}\phi^* + \text{c.c.}$ ). Thus, so can the values of  $\theta_i$  for which Equation (3) is automatically satisfied.

It would be interesting to investigate whether there are mechanisms that can fix  $\theta_i$  at these distinguished values. From an alternative perspective, given that prior to inflation the field  $\phi$  takes different values of  $\theta_i$  in different local patches, one of which subsequently inflates to form the visible universe, this might allow for an anthropic explanation.

#### 5. Conclusions

Our main result is the expression of  $A_{\infty}$  given in Equation (25), the magnitude of the particle asymmetry expected due to a complex inflaton. The form of the asymmetry is characteristically different from  $A_{\infty}^{(HK)}$  derived in [8,9], as quoted in Equation (7). Arguments were presented for why we believe the asymmetry derived here is correct. Using the form of  $A_{\infty}$  derived in Equation (25), we identified parameter regions in which an appropriate baryon asymmetry can be generated without perturbing the quadratic potential that drives inflation. In particular, we argued that, for the simple model studied, inflationary cosmology and the observed baryon asymmetry can only be simultaneously reproduced for special values of  $\theta_i$ .

The calculations presented here have involved purely perturbative inflationary processes; however, there have been some efforts [16,17] to examine analogous baryogenesis scenarios involving preheating [18,19] and oscillons [20–23]. As these non-perturbative calculations are distinct, the results of [16,17] are likely unaffected by the issues discussed here. On the other hand, some model-building considerations explored in [8], and certain subsequent studies, e.g., [24,25], may need to be re-examined.

The possibility of realising baryogenesis via a complex inflaton is quite elegant, especially in its minimality. The purpose of this study was to compute the expected magnitude of asymmetries generated in this manner for simple models of inflation, which is a crucial step towards building more elaborate scenarios. We leave the myriad of model building opportunities for future work. Funding: This research received no external funding.

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