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Rediscovering the Scaling Law of Urban Land from a Multi-Scale Perspective—A Case Study of Wuhan

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Abstract: The law of urban scaling implies that there is a universally applicable nonlinear scaling relationship between population size and urban indicators, which is a method of quantitative analysis that can reflect the growth law and internal logic of the urban system. However, most present research is conducted at the municipal scale, and studies of scaling law in the inner-city system are scarce, especially from the perspective of compact urban form development. The goal of this paper is to discover the scaling law within urban systems from a multi-scale perspective. Through the empirical analysis of Wuhan, this paper examines the internal scale law of the urban system from the municipal and district scales. Moreover, we use the landscape expansion index to perform spatial autocorrelation analysis. In this way, we assess the relationship between the compactness of urban morphological development and the urban scaling law. The results indicate that the temporal scaling law on the city scale has a more significant linear law than the single-year scaling law. The analysis also shows the scaling law relationship within the inner-city system. Nevertheless, there is a deviation between the temporal scaling law and the cross-section scaling law. Namely, the time series development of a district does not follow the section scaling law of the urban system. Furthermore, the urban scaling law shows a negative correlation with the compactness of the urban form development. It is crucial to understand the current economic development and resource endowment of an urban system in the urbanization process, as it significantly contributes to urban development and regional coordinated planning.

Keywords: urban internal system; scaling law; multiscale; urban science

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1. Introduction

Since its reform, China's economic development has led to a rapid increase in the rate of urbanization, increasing from 17.92% in 1978 to 63.89% in 2021 [1]. At present, the total number of cities has reached 687, while the urban population has grown from 170 million in 1978 to 902 million in 2021. By 2035, the urbanization rate is expected to reach 75% to 80%, with nearly 400 million new urban residents [2]. Nevertheless, extensive and rapid urbanization has caused several problems for China's urban development, namely the intensification of human–land conflicts, the disharmony between social and economic growth and the carrying capacity of resources and the environment, and regional imbalances [3]. Even though urbanization cannot be avoided, it requires finding a balance of urban development in urban scientific research. At present, China is vigorously promoting a new type of urbanization, giving a leading role to central cities and urban circles that promote regional coordinated development and improve the quality of urbanization. As a typical complex system, the city has interrelated and relatively independent subunits, with similar social and interactive networks [4]. The urban population is the basic and most critical component within the urban system. It is also a key determinant of urban production activities, with urban indicators being reliable indicators of urban development [5,6]. A significant number of empirical studies have noted the scaling law relationship between population and urban indicators. The law refers to the quantitative analysis method that reflects the growth law and the internal logic of the urban system [1,5,7,8]. It has already attracted much attention and application in the field of new urban science.

Bettencourt et al. [8] were the first to discuss the concept of the scaling law in urban systems. Since then, a number of studies have focused on the importance of urban population size in urban systems, focusing mainly on the concept of urban scaling law [7,9,10], causes and mechanism of formation [11–15], verification [16–21], application [22–25], and questioning [26–30]. The abovementioned research was conducted at the municipal scale. Thus, academic circles generally agree that the urban scaling law appears to show a nonlinear scaling relationship between the urban population scale and different urban indicators during the process of urban expansion.

The process of urbanization in China has unique characteristics, with uneven regional resource endowment and distribution that hinders the integrated development of cities. Understanding the relationship between urban indicators and the size of an urban population is crucial for grasping a city's size, its state of urban economic development, and to promoting the development of inter-regional macroeconomic co-urbanization. In addition, the urban scaling law reflects the results of many indicators such as nature, economy, and policy. It also illustrates the process of urban evolution, reflects the future trend of the city's development, and deepens people's understanding of China's urbanization from the spatial scale. In addition, research on the urban scaling law in China is still relatively new. At present, it is mostly focused on the scaling relationship between the built-up urban area and the population scale, with most studies focusing on the municipal scale. Therefore, little is known about the internal scaling law on the micro scale, with an even more obvious lack of multi-scale research and comparative analysis. The interior of the city can be divided into smaller units, such as districts and patches.

As far as this study is aware, there are so far only studies of the internal scaling law of Shanghai [7] at the district scale. Furthermore, this particular research analyzed the temporal and cross-sectional scaling law from a single district scale perspective. Thus, with the exception of municipal and district scales, existing research is yet to address the patch scale. The patch is the most basic component unit of a landscape pattern. It is a spatial entity that differs significantly in nature or appearance from its surroundings, based on scale effects. By observing the evolution of the patch, we may be able to analyze the change in the compactness of the urban form. Various studies have examined the correlation between urban form compactness and spatial vitality [31], transportation and commuting [32–34], and air quality [35,36]. The abovementioned research leads us to perform a quantitative study of the relationship between the landscape sprawl index and the scalar law of different urban elements at the patch scale [37,38]. There is still a lack of empirical research on whether urban internal units adhere to the scaling law. Based on the fitting results of the scaling exponent, we are able to examine the current economic development and resource endowment of cities, and thus provide a scientific basis for regionally coordinated development.

Due to the research gaps discussed above, the research objectives of this study are as follows: (1) Firstly, we examine the difference between the time series scaling law of municipal cities and the theoretically expected single time point scaling law; (2) secondly, the paper analyzes whether the scaling law also exists in the urban system; (3) we analyze the relationship and characteristics of the temporal scaling law and the cross-sectional scaling law in each region; and (4) lastly, we aim to examine the correlation between the compactness of urban form development and the urban scale law. The interconnectedness between the objectives is provided in Figure 1.

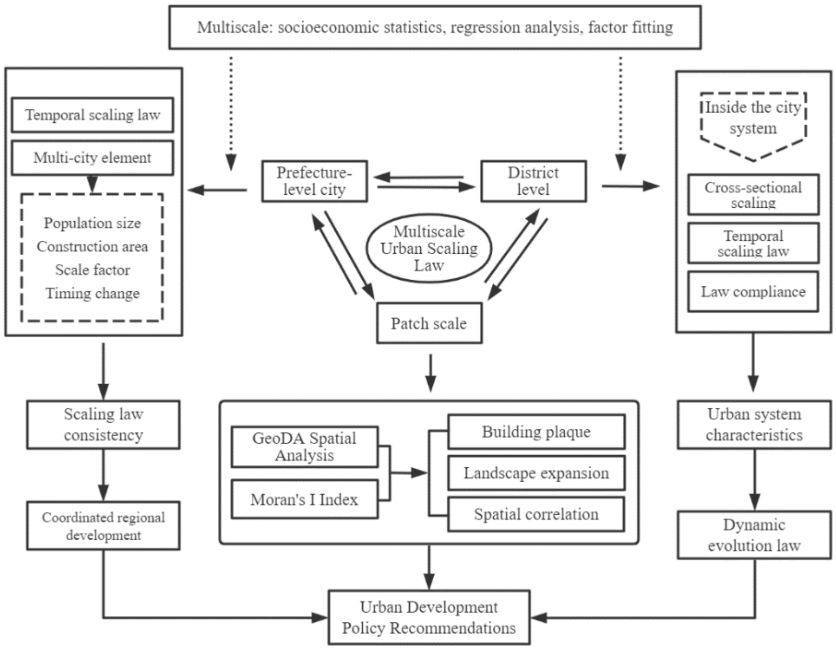

Figure 1. Flowchart of the study.

The process flow diagram expresses the basic idea of this research. Socio-economic statistical data of the city of Wuhan and each municipal district from 2005 to 2020 were collected, and the regression analysis method for calculating the scaling law exponent was determined. At the municipal scale, the temporal scaling law of the main urban indicators of Wuhan is being studied. The municipal city time series scalar law accordance with the theoretically expected single time point scalar law is checked. Therefore, the existing conclusions of the urban scaling law can be used to guide regional coordinated development. On the district scale, we first test whether the same scaling law exists within the inner-city system, and then calculate the cross-sectional scaling law and the temporal scaling law for 13 municipal districts. Finally, we focus on whether the temporal scaling law follows the cross-sectional scaling law in order to investigate the characteristics and evolution of the inner-city system. On the patch scale, based on GeoDA software, we conduct a bivariate global spatial autocorrelation analysis of the temporal scaling exponent and the landscape expansion index of each municipal district. Moran's I, the autocorrelation index, is then used to verify the relationship between the urban scaling law and the compactness of urban form development. Finally, based on the results of the multi-scale urban scaling law, we put forward policy suggestions for urban development.

Compared to existing research, this paper has a number of contributions. Firstly, it contributes to the understanding of spatial scale innovation. Spatial scale is a measure of the spatial size used to conduct research and is one of the key research issues in urban geography. The existence of spatial correlations within complex urban systems means that the results of detailed analyses of research objects will vary across spatial scales. At present, the urban scaling law does not include the patch scale. Based on remote sensing images and big data, the landscape expansion index is introduced in order to investigate the relationship between the compactness of urban form development and the urban scaling law. Secondly, the results of this study will provide a better theoretical understanding of the urban scaling law. Namely, this paper selects various urban indicators from the city, district, and patch scale in order to support the existing findings of the urban scaling law. At the same time, we are taking an innovative perspective to deepen the dynamic analysis of the scaling law within the inner-city system. The study also provides a theoretical basis for understanding the relationship between the urban scaling law and the temporal development of a city.

Wuhan is strategically important for the development of central China. Based on empirical analysis of Wuhan, this paper studies the urban scale law from the multi-scale perspective of the city, district, and patch. Based on the scaling law relationship between the urban population scale and the urban indicators, this study aims to understand the dynamics of the urban population development scale and economic development trend, guide regional coordinated development, allocate urban resources and new construction, as well as quantify the development and evolution law of the urban system.

2. Method and Data

2.1. Urban Scaling Law

The urban scaling law was introduced into the urban system from biological research. Foreign scholars used data from different cities around the world, such as European cities, Indian cities, Mexico, and former Spanish settlements. The abovementioned research confirmed the universality of the urban scaling law in different regions and periods from the municipal and settlement scale. Bettencourt et al. [8] stated that urbanization has increased the pace of urban life with increasing population. By comparing biological and urban systems and using data on relevant urban infrastructure, personal needs, and socio-economic indicators, the urban scale law is verified. In other words, the urban indicators and the urban population scale are expressed as a power law function of the scaling exponent β. Based on the theoretical and empirical evidence, scholars divide the relationship between the scaling exponent and 1 into urban indicators and further divide it into super-linear, sub-linear, and linear, thus establishing a systematic framework for urban scaling law. The power law function formed between urban indicators and the urban population size is presented in Formula (1) as follows:

$$Y(t) \; = \; Y_0 N(t)^{\beta} \tag{1}$$

In the formula above, $N(t)$ represents the urban population size at time t, while $Y(t)$ stands for the urban indicators corresponding to the urban population size at time t. Next, Y_0 is a standardized constant, while the index β is the urban scaling law. The size of β and 1 is used to measure the relationship between the urban population size and urban indicators. It is divided into the following three types [8]: (1) Linear relationship ($\beta \approx 1$), in which urban indicators (e.g., number of jobs, household water) are related to individual needs. In general, the increase in indicators is basically consistent with the population increase; (2) sub-linear relation ($\beta < 1$), in which the indicators (e.g., total road length, number of gas stations) are related to urban infrastructure. The increase in indicators is less than the increase in population size, which reflects economies of scale. The relatively large population of cities allows residents to share infrastructure. For example, city A has 3 times the population of city B, but city A does not need 3 times the total length of roads of city B; and (3) super-linear relation ($\beta > 1$), in which the growth rate of indicators (e.g., GDP, local general public budget revenue, and knowledge output) interacting with urban social and economic development is greater than that of the urban population. Social interaction grows super-linearly with population growth, reflecting the characteristics of the urban economic agglomeration effect. In this study, we simultaneously considered the logarithms on both sides of Equation (1), and applied the least square method to obtain the "scaling law" regression equation:

$$logY(t) \; = \; \beta logN(t) + logY_0 \tag{2}$$

After taking the logarithms of the city and population size indicators, a linear fitting was performed to obtain the scale exponent β. The least square method for linear fitting is simple and easy to operate. It is the most commonly used fitting method in the study of the urban scaling law.

2.2. Landscape Expansion Index

The landscape expansion index (LEI) was proposed by Liu Xiaoping [39] and is used to quantitatively describe the different types of dynamic expansion and spatial pattern distribution, thus reflecting information on the spatial layout and dynamic changes of a landscape. The calculation process requires the following steps: Firstly, information on the expansion of urban construction land in different time periods is extracted from multi-temporal remote sensing data. Then, a buffer zone is established using the GIS spatial analysis function. Lastly, the urban landscape expansion index is calculated [40,41]. The degree of dispersion of the new patch can be obtained from the intersection area of the new patch buffer and the old patch. The higher the index value, the more compact the location relationship between the newly expanded patch and the original city. This provides a more intuitive spatial meaning and can well explain the spatial and temporal pattern evolution of urban land expansion. The expression formula is as follows:

$$LEI = \frac{A_0}{A_0 + A_V} \times 100 \quad (3)$$

In Formula (3), LEI represents the landscape expansion index of the newly added patch, while A_0 represents the intersection area of the newly added patch buffer and the old patch. Next, A_V indicates the intersection area of the newly added patch buffer and other patches, except for the old patch. The LEI can range from 0 to 100, and this study sets the LEI thresholds for three expansion types [39]: (1) when $0 < LEI < 2$, it is an enclave expansion; (2) when $2 \leq LEI \leq 50$, it is an edge expansion; (3) and when $50 < LEI \leq 100$, it is a filling expansion. Marginal sprawl and enclave sprawl represent the urban "diffusion" process that usually results in a more discrete urban form, while infill sprawl leads to a more compact urban form by reducing intra-city voids.

Spatial autocorrelation analysis originated from biometrics, and it is now one of the basic methods for analyzing the statistical distribution of spatial data [42]. GeoDA is an open-source software designed to analyze spatial autocorrelation. Using bivariate Moran′s I analysis, it explores the spatial correlation and dependence of two elements. The LEI examined in this paper has spatial characteristics on the patch scale. Compared to Pearson's test, bivariate Moran's I has a higher efficiency and applicability rate. Thus, this study used GeoDA software to analyze both the temporal scaling exponent of each district and the LEI of each district from 2005 to 2020. Furthermore, the study conducted a bivariate global spatial autocorrelation analysis between them. Moran's I verified the relationship between the urban scaling law and the compactness of urban form development.

2.3. Study Area and Data Source

The research period of this study is from 2005 to 2020. The research area is Wuhan, a Chinese megalopolis with a population of over 13 million. The city is an important transportation hub, industrial, scientific, and educational base, as well as a crucial point of economic development in central China. The city has 13 municipal districts under its jurisdiction, including the Jiang'an District, Jianghan District, Qiaokou District, Hanyang District, Wuchang District, Qingshan District, Hongshan District, Caidian District, Jiangxia District, Huangpi District, Xinzhou District, Dongxihu District, and HanNan District. In addition, there are several functional areas of Wuhan. To unify the analysis of research units and urban indicators, the statistical data of urban indicators of the Wuhan Economic and Technological Development Zone is merged into HanNan District. Furthermore, data for the East Lake High-tech Development Zone is merged into the Hongshan District, and data of the Wuhan New Chemical Industry Zone is merged into the Qingshan District.

This paper defines the urban population as the residential population in the urban area of the city. The statistical scope of the remaining urban indicators data is defined as the municipal area within the statistical yearbook. Taking into account statistical limitations and factors such as representativeness of indicators and data availability, 18 urban indicators, including urban construction land and residential land, were selected at the municipal scale. At the regional scale, eight urban indicators such as GDP and local general public budget revenue were selected. Data regarding urban indicators are from the "*China Urban Construction Statistical Yearbook*", the "*Hubei Statistical Yearbook*", the "*Wuhan Statistical Yearbook*", the "*Wuhan Yearbook*", as well as from the statistical yearbooks and bulletins of social and national economic development of various districts.

3. Results

3.1. Temporal Scaling Law of Urban Indicators in Wuhan

At the municipal scale, studying the temporal scaling relationship between major urban indicators and urban population size can help measure the inherent time series growth law of China's urban system. Taking the regional GDP, the local general public budget revenue, and the urban population size as examples, a linear fitting regression is performed on the double logarithm of urban indicators and population size (Figure 2). The ß value of the GDP scale exponent is 2.953 and the goodness-of-fit R^2 is 0.866, while the p value is less than 0.01. This indicates that if the population is twice its original size, GDP will increase by 7.744 ($2^{2.953}$) times. Furthermore, the local general public budget revenue scale exponent ß is 3.461, while the goodness-of-fit R^2 is 0.793 and the p value is less than 0.01. In other words, if the population is twice the original size, the local general public budget revenue will increase 11.012 ($2^{3.461}$) times. In the double logarithmic coordinate system, the two urban indicators show a significant linear correlation with population size, and a solid fitting effect. With respect to social economy, there is a super-linear scale relationship between GDP and population size, reflecting the strong agglomeration effect of social economy in Wuhan during the period of 2005–2020. In China's rapid urbanization process, such a relationship also suggests a significant advantage for larger cities when it comes to economy. Moreover, in terms of public finance, the temporal scaling exponent of local general public budget revenue is close to 3.5. This result indicates that the exponent in question is in a super-linear relationship with the population size, which even exceeds the GDP scale exponent. As the public finance scale exponent has the maximum value of all major urban indicators studied, it suggests the presence of the phenomenon of urban fiscal polarization, which is peculiar to urbanization in China. The growth rate of fiscal revenues in larger cities is increasing from year to year and is significantly higher than the increase in population.

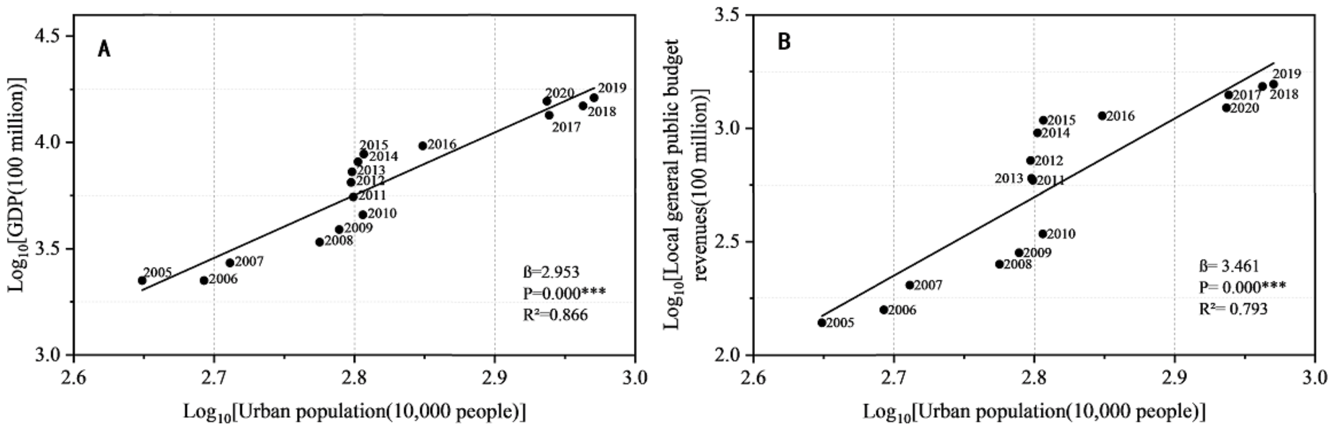

Figure 2. Temporal scaling laws of GDP and local general public budget revenues. (**A**) Temporal scaling law between GDP and urban population in Wuhan from 2005 to 2020; (**B**) Temporal scaling law between local general public budget revenues and urban population in Wuhan from 2005 to 2020; *** $p < 0.001$.

For the temporal scaling exponent of each indicator and the urban population size (Figure 3) within the same linear fitting processing, the average goodness-of-fit R^2 is measured to be 0.689, while the p values are less than 0.05. These results show that there is a significant scaling relationship between urban indicators and population size, and it can be used to reveal simple laws in the evolution of complex urbanization. Except the negative β index of industrial wastewater discharge, the β index of other indicators is positive and shows a positive correlation with population size.

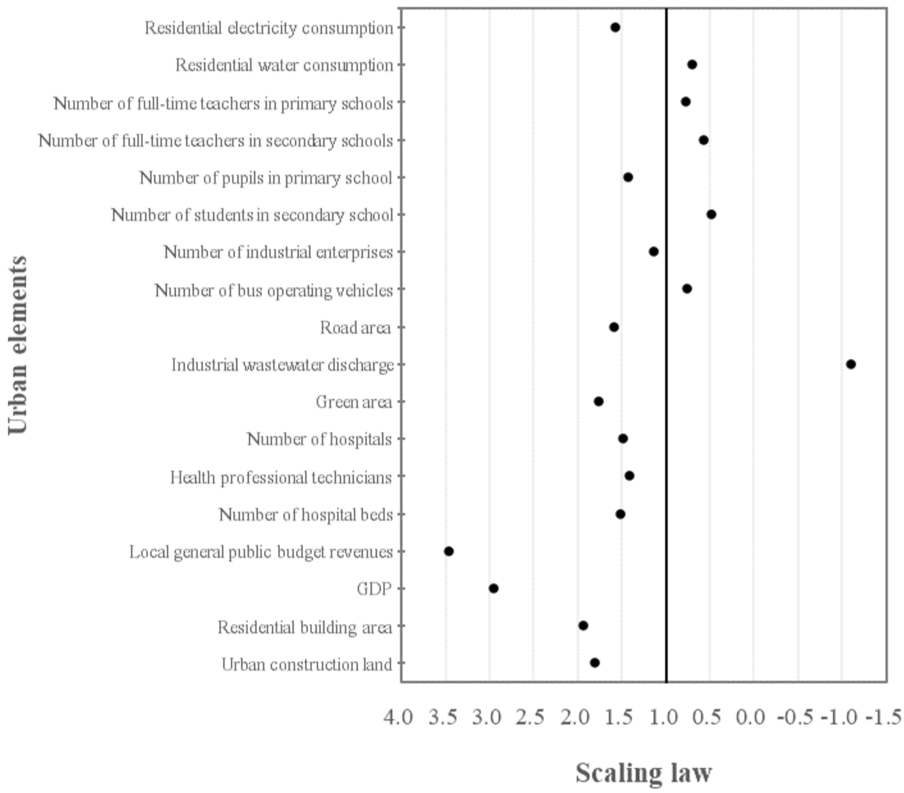

Figure 3. Temporal scaling laws of urban indicators in Wuhan.

In terms of land consumption, the β scale factor is between 1.8 and 1.9 for the urban construction land area and the residential construction land area, which shows a significant super-linear relationship with population size. This reveals the high demand for construction land and residential land in China. As the population size of an area increases, so will the required residential building area. Green spaces, roads, and population size form a super-linear relationship. This suggests that during urban expansion there is an adequate supply of land for roads, which makes for good transportation accessibility in large cities. In addition, against the background of high-density urban construction, a substantial increase in urban green area shows the remarkable importance of ecological greening in large cities.

In terms of educational indicators, the increase in the number of teachers in schools is smaller than the population growth. Moreover, the temporal scaling exponents of the number of full-time middle school teachers and full-time primary school teachers are both on the right side of the $\beta = 1.0$ indicator line. Namely, secondary schools have the lowest number of full-time teachers. This fact highlights the current issue of teacher shortages in larger cities, which in turn causes competition for educational resources. This leads to vicious competition for urban education resources, and leads to allocation and utilization of other resources, such as school district housing supply, zoning of schools for school-age children, etc. There is a significant difference in the temporal scaling law between primary and secondary school students. The temporal scaling exponent of the number of students in middle school is the same as the theoretical expectation, i.e., β is less than 0.5. On the contrary, the number of students enrolled in elementary school has a super-linear scalar relationship with the urban population size. These findings relate to the mandatory nine-year study period defined by the state, the impact of the two-child policy, and the Wuhan economic agglomeration, which drives foreign resident population. Thus, the number of primary school students has increased, increasing the need for educational resources in larger cities as well.

With respect to medical facilities, the temporal scaling exponent β of the number of hospital beds, practicing physicians, and hospitals is between 1.4 and 1.5. These findings indicate a super-linear relationship between these factors and population size, which is contrary to theoretical expectations ($\beta < 1$). These results also reflect the economic development of large Chinese cities, with a massive increase in medical resources and a concentration of high-quality medical resources in large cities.

In relation to infrastructure, the number of buses within a city shows a sublinear relationship with the population size, indicating that the development of bus transportation in large cities is adequate and at the level of demand. However, as subway travel in larger cities is more convenient in practice, the bus increase rate is actually lower than the population growth rate.

In terms of industrial enterprises, there is a negative super-linear relationship between industrial wastewater discharge and the population size, and a positive super-linear relationship between the number of industrial enterprises and the population size. This reflects the "structural adjustment and pollution reduction" in the industries of big cities with a declining trend in industrial wastewater discharge.

In relation to personal needs, the residents' water consumption shows a sub-linear relationship, while electricity consumption shows a super-linear relationship. In the development of a city, with the increase of population, the consumption of water resources for domestic use also increases greatly. However, it is maintained in relative balance with the population increase. On the contrary, the residents' electricity consumption shows a significant linear increase. These findings indicate that there is significant consumer demand for electricity in larger cities, with supply and demand for electricity resources being greater than demand for water resources.

3.2. Scaling Law of Urban Indicators for Wuhan Districts

3.2.1. The Cross-Sectional Scaling Law for Each District

In order to explore the scaling law of the internal units of an urban system at the district level, this paper selects eight representative urban indicators.

The scaling law of major urban indicators was calculated for 13 districts in Wuhan from 2005 to 2020 (Figure 4). The results are classified as follows: (1) $\beta \approx 1$: the cross-sectional scalar exponents in the number of hospital beds, health technicians, and retail sales of social consumer goods changed more moderately. Before 2014, the exponent increase rate was higher than the rate of population growth. This showed a fluctuating trend of small amplitude and a weakening of the scale economies. However, after 2014, overall scale indicators exhibited a downward trend. Thus, the scale economy of urban indicators is strengthened, with the possibility of causing tensions among related resource indicators; (2) $0.5 < \beta < 1$: the cross-sectional scaling exponent of GDP fluctuated around 0.7, which was not in line with theoretical expectations. Based on the cross-sectional scale, this measurement indicates that the economic output of each district had a strong agglomeration effect. Nevertheless, the trend of socio-economic development within each district differs, while the internal distribution of economic resources remains unbalanced; (3) $0.4 < \beta < 1.2$: the cross-sectional scaling exponent of investments in real estate development varied greatly. It reached its minimum of 0.485 in 2010 and its maximum of 1.144 in 2017. In general, this exponent shows a downward trend from 2005 to 2010, and an upward trend after 2010, as more capital accumulated in the real estate economy. Furthermore, this exponent points to the fact that government real estate policies in larger cities are variable and regulatory, as the real estate market differs between districts due to their geographical location, political status, social economy, and other factors; and (4) $\beta \approx 0.5$: the cross-sectional scaling exponents of local general public budget revenue, total output value of the construction industry, and added value of the construction industry were stable with the value around 0.5 from 2005 to 2015. Moreover, these exponents showed a significant upward trend after 2016. In addition, the scale exponents of total output value and added value of the construction industry were both higher than 1 after 2016. This indicates a growth trend identical to that of investment in real estate development and reflects the trend of rapid development within the real estate market. It is worth noting that all the scale exponents of urban indicators showed a small decline in the period 2019–2020 due to the impact of the COVID-19 pandemic.

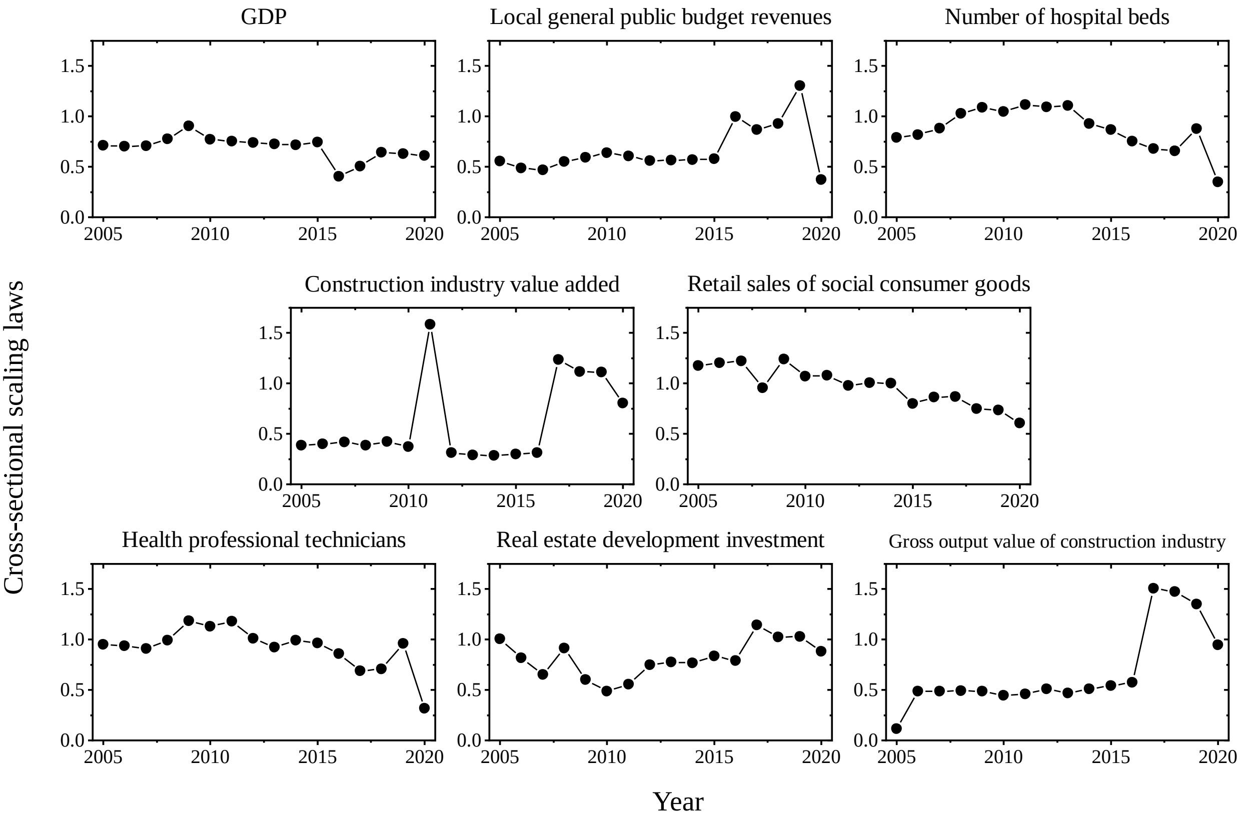

Figure 4. Cross-sectional scaling law of each region.

3.2.2. Temporal Scaling Law of Each District

In order to calculate the temporal scaling law of the main urban indicators for each district from 2005 to 2020, a temporal scale analysis was carried out in 13 Wuhan districts (Figure 5). With the exception of the Caidian District and the Jiangxia District, the temporal scaling exponent for health technology was less than 1, while the other urban indicators' temporal scaling exponent was found to be greater than 1. Comparing the temporal scaling exponent of zoning with the sectional scaling exponent of zoning indicates that the span of the Y-axis values and the difference between maximum and minimum values are large. It also points to the fact that the linear relationship between different urban indicators and population size differs significantly for each municipal district. In the example of GDP, the temporal scaling exponent of the Xinzhou District was 21.17, while the temporal scaling exponent of the Qiaokou District was 1.98. The GDP of each district has a super-linear relationship with the population size. In contrast, the cross-sectional scaling exponent of Wuhan's GDP from 2005 to 2020 was less than 1, indicating a sublinear relationship. This shows that there are fundamental differences between temporal scaling laws and cross-sectional scaling laws within the urban system, and that the linear relationship between urban indicators and population differs across different spatial and temporal scales. Due to the impact of the epidemic in 2020, the cross-sectional scaling exponents have shown a downward trend. However, the temporal scaling exponents of each district were not affected. In general, these indicators and population size continued to show a super-linear relationship, reflecting the development trend and law of each district over time.

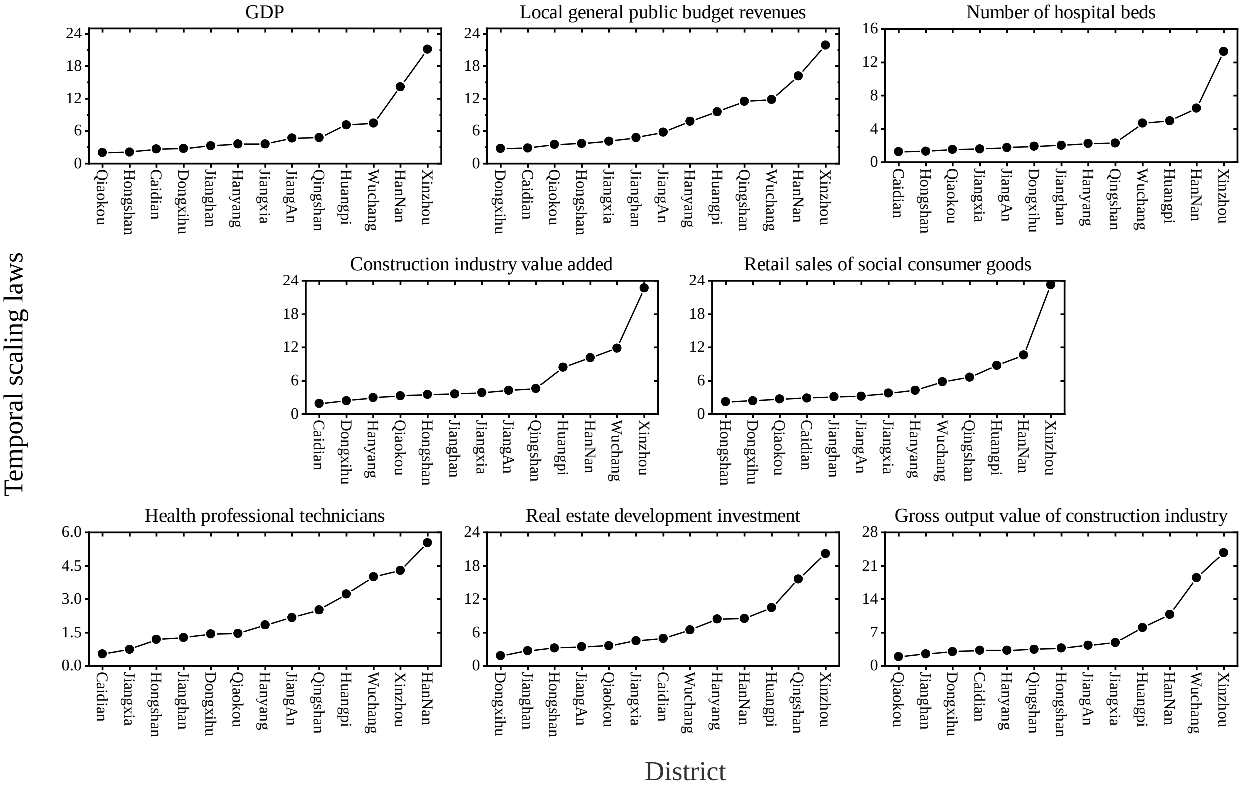

Figure 5. Temporal scaling law of each region.

The temporal scaling exponents of other urban indicators were further analyzed. Therefore, indicators such as medical facilities, hospital beds, and health technical personnel showed little difference. For example, in the Xinzhou District, the hospital beds indicator was at a maximum of 13.28, while in the HanNan District, the health technical personnel exponent was at a maximum of 5.23. Compared to other economic indicators, all indicators of medical facilities and population size show a relatively small growth rate. The temporal scaling exponents of other economic indicators fluctuated between 1.0 and 24.0. However, the values of most municipal districts ranged between 1.0 and 6.0, thus showing the strong effect of economic agglomeration for larger cities and the trend of prosperous development over time. Nevertheless, the analysis of temporal scaling exponents showed that the economic scaling exponent remained stable at around 0.5 before 2015, showing an upward trend only after 2016. These findings further support the assumption that the development law of a single municipal district differs from the cross-sectional development. Furthermore, this indicates that the sequential development of a single district does not follow the cross-sectional scaling law of the urban system.

3.3. Correlation Analysis between Landscape Expansion Index and Scaling Law

At the patch scale, based on an in-depth understanding of the scaling law relationship between the urban population size and urban indicators, we first calculated the temporal scaling exponents and the *LEI* of each district from 2005 to 2020, and then conducted a spatial autocorrelation analysis. In this way, the relationship between the compactness of urban form development and the urban scaling law is checked, and the development of law within the urban system can be obtained.

First, data on land use between 2005 and 2020 helped to identify new patches of construction land in each district. To understand urban development in each Wuhan district, the landscape expansion index for each district (*GLEI*) was calculated [43] (Figure 6). The lowest *GLEI* value was in the HanNan District, measured at 3.56, while the highest *GLEI* value was in the Jianghan District, measured at 47.25. These findings indicate that the Jianghan District has the most compact urban form development, while the HanNan District is the least compact. Secondly, GeoDa tested the spatial autocorrelation between the temporal scaling exponents of urban indicators and *GLEI*. The Moran's I index of urban indicators was also calculated [44,45] (Table 1). The Moran's I index of the eight major urban indicators was negative, and the p values of the other indicators were less than 0.1, except for the health technician factor. This indicates that the temporal scaling exponent is negatively correlated with *GLEI*. Taking as an example the number of hospital beds in the HanNan District, which has the least compact urban form, this indicator had a temporal scaling exponent of 6.47. In other words, it was neither compact nor did it have economy of scale. On the other hand, in the Jianghan District, which had the most compact urban form, the temporal scaling exponent of hospital beds was 1.99, indicating a compact economy of scale. The degree of compactness of the urban form had a significantly negative impact (19.67%) on the scaling exponent of hospital beds. Namely, when *GLEI* is higher, the urban form development is more compact. Furthermore, the scaling exponent of urban indicators is smaller, and the scale economy of social development is more significant. Finally, a higher *GLEI* and a smaller scaling exponent cause more efficient distribution and circulation of various resources within the system.

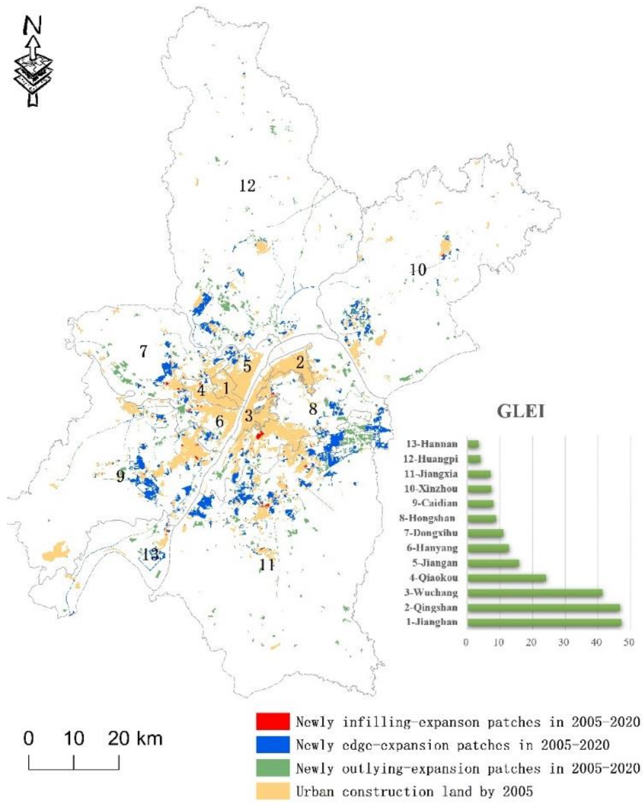

Figure 6. Distribution of new types of expansion patches in each district of Wuhan and calculated GLEI.

Table 1. Results of the spatial autocorrelation test.

	GDP	Local General Public Budget Revenues	Number of Hospital Beds	Health Professional Technicians	Real Estate Development Investment	Gross Output Value of Construction Industry	Construction Industry Value Added	Retail Sales of Social Consumer Goods
Moran's I	−0.2207	−0.1697	−0.1967	−0.1213	−0.1687	−0.161	−0.1638	−0.223
p	0.05	0.10	0.05	0.12	0.08	0.08	0.10	0.07

4. Comparative Analysis and Policy Implications of Scaling Law

When the city-level temporal scale law is compared with previous research, it appears to be in line with the sub-linear, linear, and super-linear law of Bettencourt et al. However, due to the temporal span, the urban indicators are shown to be more sensitive in correlation with population size.

In terms of social economy, the gross regional product, local general public budget revenue, and population size should form a super-linear relationship, with the β exponent above 1.3. The temporal scaling exponents of Wuhan's two indicators are 2.953 and 3.461, while the β value is significantly higher than 1.3. This indicates a strong agglomeration effect for larger cities undergoing urbanization, rapid development, and a sharp increase in the size of urban population. Simultaneously, it has a super-linear relationship with the high exponent, on the temporal scaling law.

When the results of the zoning temporal scaling law and cross-sectional scaling law are compared with the internal scaling law of Shanghai [7], it seems that the scaling law is present in the urban system, while the cross-sectional and temporal scaling exponents do not appear to follow the same law. Due to differences between the sampled cities and some statistical factors, the scale exponents of some urban indicators still differ. In terms of the cross-sectional scaling law, the cross-sectional scaling exponent of Shanghai's GDP from 2005 to 2017 is greater than 2, while in Wuhan it is around 0.7. This may be due to the fact that in 2017, the permanent population of Shanghai exceeded 24 million, while in Wuhan it was only 8.68 million. Therefore, the difference in the urban population size results in the difference in the scale exponents. As the urbanization process is closely related to the Chinese leadership, the urbanization rate and the major differences between provinces and cities also depend on the government. Therefore, cities with a larger population have a more significant linear relationship with the growth rate of urban indicators when the scale exponent value is higher. With respect to the temporal scaling law, the temporal scaling exponents of the GDP of each district in Shanghai are between -4 and 15. On the other hand, in Wuhan they are between 1.9 and 22, making the change of the exponents of each district greater in Wuhan than in Shanghai. These results are due to the different resource endowments and population scale of each district within the urban system. Each district's urban indicators are unevenly distributed, resulting in significant temporal differences in scaling exponents between districts.

Rapid urbanization in China differs from that in developed countries, making the law of the urban scale in China quite unique. Namely, the agglomeration of the urban population encourages economies of scale and the agglomeration of other indicators. The crucial difference in the scale exponent results also reflects the imbalance of resources both between and within Chinese cities. Based on the scale exponent results, this paper proposes a number of policies for regional coordinated development. First, Shanghai is a mega-city, while Wuhan is a super-city. Nevertheless, both are among the most prominent Chinese cities in terms of urban population and economic development. However, in terms of the cross-sectional scaling exponent of GDP, Shanghai is much larger than Wuhan, indicating significant differences in China's urban development. To resolve this discrepancy, policies should promote urban–rural integration and a more equitable and efficient flow of resources. Moreover, they should play a radiant role for big cities, while working to increase the population size and urban indicators of small and medium-sized cities. Secondly, even though larger cities have enough tax revenues to ensure infrastructural development, their scale exponent is still far smaller than the socio-economic indicator. Therefore, to ensure the scale exponent and efficient infrastructural development, it is necessary to focus on different infrastructure indicators. Furthermore, as China's urban construction occupies a large amount of land, the scale exponent of land use is large. Therefore, it is important to pay attention to the construction of infrastructure in order to ensure economies of scale, as well as the efficient operation of facilities. Thirdly, the urban scale law has a significant negative correlation with the compactness of urban form development. Namely, from 2005 to 2020, $GLEI$ values in Wuhan districts ranged from 0 to 50, indicating marginal expansion.

It is important to pay attention to the outward expansion of the city and strengthen the leading position of urban planning in urban construction. Then, planning can be used as a blueprint to promote compact urban development, making economies of scale of social development increasingly visible.

5. Discussion and Conclusions

By examining the Chinese city of Wuhan, this study explores the scale law of urban population size and urban indicators from the city, district, and patch levels. Research shows that China's urban development is in line with the characteristics of the scalar law. However, rapid urbanization in China has led to imbalances in resource endowments and urban development of cities with different population sizes. The numerical difference in the scale exponent between cities is large and sensitive. On the other hand, the increase in socio-economic benefits from population clustering in Chinese cities is more significant, and the efficiency of the use of urban infrastructure indicators is becoming apparent. The results have several implications. First, there is an evident scale law relationship within the inner-city system. At the municipal district level, the city is divided into district-level administrative units. This is done to improve the reliability of statistical data and to clearly define internal urban indicators and population size. In the process of urban expansion, there is a correlation between the population size and the indicators of the metropolitan system and the inner-city system. Secondly, the temporal scale law and the cross-section scale law did not adhere to theoretical predictions. The generally accepted scaling law relationship is expressed as a power law function of the scaling exponent β. Furthermore, the index β of infrastructure elements is sub-linear with values around 0.8, the index β of individual needs is linear and measured to be 1, and the index β of socio-economic relatedness is super-linear and measured at around 1.2. Nevertheless, the temporal and cross-sectional scaling laws differ significantly from the established scaling law relationships in time and space. These discrepancies suggest that the scaling laws within the urban system theoretically follow the meaning of the exponent β. However, the interpretation of its β should be based on the development process within a single urban system. Thirdly, the temporal scale law at the city level shows a more significant linear law than the single year scale exponent. In long-term urban development, the law of temporal scale is more sensitive to urban population growth and indicator density growth. Fourthly, there are fundamental differences between the temporal scale law and the cross-sectional scale law in the urban system. Furthermore, the temporal development of the city does not follow the cross-sectional scale law of the urban system. Namely, the temporal scale law of the district reflects the individual development of the inner city, while the cross-sectional scale law indicates the overall development trajectory of the city in a particular time node. The cross-sectional exponent is more stable than the temporal exponent. Lastly, there is a significant negative correlation between the urban scale law and the development of urban form compactness. In other words, as urban morphological development becomes more compact, the smaller the urban scaling exponent is, the more prominent the scale economy of social development becomes.

This paper studies the urban internal scaling law on multiple levels. However, the sample size does not sufficiently represent the characteristics of the entire Chinese urban system. Examination of the cross-sectional and temporal scaling law of each district in Wuhan is limited by the current statistical caliber and the quality of statistical data. Therefore, the number of indicators in cities at the district level is smaller than the number of indicators on the city level, hindering the complete unification of the research indicators. A comparative study at the district and municipal scales needs to be further explored.

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References

1. Jiao, L.M.; Lei, W.Q.; Xu, G.; Xu, Z.B.; Zhou, Z.Z. Scaling Law and Spatial-Temporal Characteristics of Scaling Factors in Chinese Cities. *Acta Geogr. Sin.* **2020**, *75*, 2744–2758.
2. Wang, K.; Lin, C.H.; Wu, C.Y. Trends and Planning Options after 60% Urbanization rate in China. *Urban Plan.* **2020**, *44*, 9–17.
3. Liu, R.Z. Urbanization in China: Problems, Reflections and Transformation. *J. Zhengzhou Univ. (Philos. Soc. Sci. Ed.)* **2013**, *46*, 68–72.
4. Berry, B.J.L. Cities as systems within systems of cities. *Pap. Reg. Sci.* **1964**, *13*, 146–163. [CrossRef]
5. Bettencourt, L.M.; Lobo, J.; Strumsky, D.; West, G.B. Urban scaling and its deviations: Revealing the structure of wealth, innovation and crime across cities. *PLoS ONE* **2010**, *5*, e13541. [CrossRef] [PubMed]
6. Schläpfer, M.; Bettencourt, L.M.; Grauwin, S.; Raschke, M.; Claxton, R.; Smoreda, Z.; West, G.B.; Ratti, C. The scaling of human interactions with city size. *J. R. Soc. Interface* **2014**, *11*, 20130789. [CrossRef]
7. Xu, G.; Xu, Z.; Gu, Y.; Lei, W.; Pan, Y.; Liu, J.; Jiao, L. Scaling laws in intra-urban systems and over time at the district level in Shanghai, China. *Phys. A Stat. Mech. Appl.* **2020**, *560*, 125162. [CrossRef]
8. Bettencourt, L.M.; Lobo, J.; Helbing, D.; Kühnert, C.; West, G.B. Growth, innovation, scaling, and the pace of life in cities. *Proc. Natl. Acad. Sci. USA* **2007**, *104*, 7301–7306. [CrossRef]
9. Fu, J.C.; Li, G.; Zhao, H.; Zhang, J.Y. Allometric relationship between urban population and built-up area in China: A case study of 652 cities. *China Land Sci.* **2015**, *29*, 46–53.
10. Lei, D.; Hao, W.; Hongrui, Z. The definition of city boundary and scaling law. *Acta Geogr. Sin.* **2017**, *72*, 213–223.
11. West, G. *Scale: The Universal Laws of Growth, Innovation, Sustainability, and the Pace of Life in Organisms, Cities, Economies, and Companies; Environment and Planning B: Urban Analytics and City Science*; Penguin Press: New York, NY, USA, 2016.
12. Yakubo, K.; Saijo, Y.; Korošak, D. Superlinear and sublinear urban scaling in geographical networks modeling cities. *Phys. Rev. E* **2014**, *90*, 22803. [CrossRef] [PubMed]
13. Zhang, J.; Li, X.; Wang, X.; Wang, W.X.; Wu, L. Scaling behaviours in the growth of networked systems and their geometric origins. *Sci. Rep.* **2015**, *5*, 9767. [CrossRef] [PubMed]
14. Ribeiro, F.L.; Meirelles, J.; Ferreira, F.F.; Neto, C.R. A model of urban scaling laws based on distance dependent interactions. *R. Soc. Open Sci.* **2017**, *4*, 160926. [CrossRef] [PubMed]
15. Li, R.; Dong, L.; Zhang, J.; Wang, X.; Wang, W.X.; Di, Z.; Stanley, H.E. Simple spatial scaling rules behind complex cities. *Nat. Commun.* **2017**, *8*, 1841. [CrossRef]
16. Bettencourt, L.M.; Lobo, J. Urban scaling in Europe. *J. R. Soc. Interface* **2016**, *13*, 20160005. [CrossRef]
17. Sahasranaman, A.; Bettencourt, L.S.M.A. Urban geography and scaling of contemporary Indian cities. *J. R. Soc. Interface* **2019**, *16*, 20180758. [CrossRef]
18. Cesaretti, R.; Lobo, J.; Bettencourt, L.M.; Ortman, S.G.; Smith, M.E. Population-Area Relationship for Medieval European Cities. *PLoS ONE* **2016**, *11*, e0162678. [CrossRef]
19. Ortman, S.G.; Cabaniss, A.H.; Sturm, J.O.; Bettencourt, L.M. Settlement scaling and increasing returns in an ancient society. *Sci. Adv.* **2015**, *1*, e1400066. [CrossRef]
20. Lobo, J.; Bettencourt, L.M.; Smith, M.E.; Ortman, S. Settlement scaling theory: Bridging the study of ancient and contemporary urban systems. *Urban Stud.* **2020**, *57*, 731–747. [CrossRef]
21. Zünd, D.; Bettencourt, L.M. Growth and development in prefecture-level cities in China. *PLoS ONE* **2019**, *14*, e0221017. [CrossRef]
22. Alves, L.G.; Ribeiro, H.V.; Lenzi, E.K.; Mendes, R.S. Distance to the Scaling Law: A Useful Approach for Unveiling Relationships between Crime and Urban Metrics. *PLoS ONE* **2013**, *8*, e69580. [CrossRef]
23. Ribeiro, H.V.; Hanley, Q.S.; Lewis, D. Unveiling relationships between crime and property in England and Wales via density scale-adjusted metrics and network tools. *PLoS ONE* **2018**, *13*, e0192931. [CrossRef] [PubMed]
24. Lobo, J.; Bettencourt, L.M.; Strumsky, D.; West, G.B. Urban Scaling and the Production Function for Cities. *PLoS ONE* **2013**, *8*, e58407. [CrossRef] [PubMed]
25. Jiao, L.; Xu, Z.; Xu, G.; Zhao, R.; Liu, J.; Wang, W. Assessment of urban land use efficiency in China: A perspective of scaling law. *Habitat Int.* **2020**, *99*, 102172. [CrossRef]
26. Leitao, J.C.; Miotto, J.M.; Gerlach, M.; Altmann, E.G. Is this scaling nonlinear? *R. Soc. Open Sci.* **2016**, *3*, 150649. [CrossRef]

27. Arcaute, E.; Hatna, E.; Ferguson, P.; Youn, H.; Johansson, A.; Batty, M. Constructing cities, deconstructing scaling laws. *J. R. Soc. Interface* **2015**, *12*, 20140745. [CrossRef]
28. Emanuele, S.; Vishal, S. Rich and Poor Cities in Europe. An Urban Scaling Approach to Mapping the European Economic Transition. *PLoS ONE* **2016**, *11*, e159465.
29. Gudipudi, R.; Rybski, D.; Lüdeke, M.K.; Zhou, B.; Liu, Z.; Kropp, J.P. The efficient, the intensive, and the productive: Insights from urban Kaya scaling. *Appl. Energy* **2019**, *236*, 155–162. [CrossRef]
30. Muller, N.Z.; Jha, A. Does environmental policy affect scaling laws between population and pollution? Evidence from American metropolitan areas. *PLoS ONE* **2017**, *12*, e0181407. [CrossRef]
31. Fang, R.; He, Q.S.; Xie, P.; Cairen, Z.M. Spatial dynamics of enclave patches in Urban agglomerations: A case study of Three Major Urban agglomerations in China. *Geogr. Inf. World* **2021**, *28*, 40–45.
32. Aulia, P.L.; Taki, H.M.; Wartaman, A.S. Impact of transportation on urban compactness index in South Tangerang City, Indonesia. *IOP Conf. Series. Earth Environ. Sci.* **2021**, *737*, 12052. [CrossRef]
33. Su, Q. Urban Spatial Expansion, Urban Compactness, and Average Travel Demand in the US Urbanized Areas. *Int. J. Reg. Dev.* **2020**, *7*, 1. [CrossRef]
34. Zia, M.M.; Jeonghyun, R. Effects of Urban Compactness and Residential Density on Trip Generation: Focusing on Work Trips in Seoul, Korea. *J. Korean Soc. Transp.* **2017**, *35*, 1–10.
35. Xu, C.; Haase, D.; Su, M.; Yang, Z. The impact of urban compactness on energy-related greenhouse gas emissions across EU member states: Population density vs. physical compactness. *Appl. Energy* **2019**, *254*, 113671. [CrossRef]
36. Yuan, M.; Huang, Y.; Shen, H.; Li, T. Effects of urban form on haze pollution in China: Spatial regression analysis based on PM2.5 remote sensing data. *Appl. Geogr.* **2018**, *98*, 215–223. [CrossRef]
37. Wei, W.; Liping, X.; Min, Z.; Minghao, O.; Haiyue, F. Response of landscape index grain-size effects in different patch types: A case study of Wuxi city. *Acta Ecol. Sin.* **2016**, *36*, 2740–2749.
38. Fan, C.; Myint, S. A comparison of spatial autocorrelation indices and landscape metrics in measuring urban landscape fragmentation. *Landsc. Urban Plan.* **2014**, *121*, 117–128. [CrossRef]
39. Liu, X.; Li, X.; Chen, Y.; Qin, Y.; Li, S.; Chen, M. Landscape Expansion Index and Its Applications to Quantitative Analysis of Urban Expansion. *Acta Geogr. Sin.* **2009**, *64*, 1430–1438.
40. Qingsong, H.E.; Mingjun, W.; Zhuoma, C. Analysis of Urban Compactness Based on Multi-dimensional Landscape Expansion Index. *Geomat. World* **2021**, *28*, 58–65.
41. He, Q.; Song, Y.; Liu, Y. Diffusion or coalescence Urban growth pattern and change in 363 Chinese cities from 1995 to 2015. *Sustain. Cities Soc.* **2017**, *35*, 729–739. [CrossRef]
42. Moran, P.A.P. Notes on Continuous Stochastic Phenomena Author. *Biometrika* **1950**, *37*, 17–23. [CrossRef] [PubMed]
43. Jiao, L.; Mao, L.; Liu, Y. Multi-order Landscape Expansion Index: Characterizing urban expansion dynamics. *Landsc. Urban Plan.* **2015**, *137*, 30–39. [CrossRef]
44. Zhang, S.L.; Zhang, K. Comparative study on Moran index and G coefficient of global spatial autocorrelation. *J. Sun Yat-Sen Univ. (Nat. Sci. Ed.)* **2007**, *4*, 93–97.
45. Xiong, Y.; Bingham, D.; Braun, W.J.; Hu, X.J. Moran's I statistic-based nonparametric test with spatio-temporal observations. *J. Nonparametric Stat.* **2019**, *31*, 244–267. [CrossRef]