

**Supplementary Materials for**  
**On the Evolution of the Optimal Design of WDS: Shifting towards the Use of a Fractal Criterion**

Juan Saldarriaga <sup>1</sup>, Camilo Salcedo <sup>2</sup>, María Alejandra González <sup>1</sup>, Catalina Ortiz <sup>3</sup>, Federico Wiesner <sup>3</sup>  
and Santiago Gómez <sup>3</sup>

<sup>1</sup> Department of Civil and Environmental Engineering, Universidad de los Andes, Bogotá 111711, Colombia

<sup>2</sup> Department of Civil & Architectural Engineering & Mechanics, University of Arizona, Tucson, AZ 85721, USA

<sup>3</sup> Water Supply and Sewer Systems Research Center (CIACUA), Universidad de los Andes, Bogotá 111711, Colombia

**OPUS/NSGA-II RESULTS**

To determine the fractal dimension of each of the four analyzed networks, the results obtained in the pareto front of the OPUS/NSGA-II methodology were implemented. Initially, three points of the net were used, the ones at each end and its *knee*. The names used in the paper for each point are: min(C), min(NRI), knee(C), knee(C), max(C), max(NRI).

Within the procedure performed to obtain the Pareto Front of the OPUS/NSGA-II methodology, different iterations were carried out. For Hanoi and Fossolo 30 iterations were performed, while for Balerna and Modena one iteration was performed. The tables S1, S2, S3 and S4 present the three points (min(C), min(NRI), knee(C), knee(C), max(C), max(NRI)) of the Pareto Front that were chosen for each iteration. This choice of points was made with a procedure that is described in detail in the paper.

**Table S1.** Hanoi, [min(C), min(NRI)], [knee(C), knee(C)], [max(C), max(NRI)] individual choices for analysis of optimal WDS configurations obtained through the OPUS/NSGA-II Methodology.

No. of iterations	Hanoi: Sample points characteristics					
	1. [Min (C), Min (NRI)]		2. [Knee (C), Knee(NRI)]		3. [Max (C), Max (NRI)]	
	Cost (\$)	NRI (—)	Cost (\$)	NRI (—)	Cost (\$)	NRI (—)
<b>1</b>	6604006.5000	0.2508	7529688.7500	0.3145	10969798.0000	0.3538
<b>2</b>	6487837.5000	0.2433	7273868.2500	0.3047	10969798.0000	0.3538
<b>3</b>	6412326.5000	0.2228	7176011.0000	0.2993	10969798.0000	0.3538
<b>4</b>	6457547.0000	0.2417	7273595.7500	0.3048	10969798.0000	0.3538
<b>5</b>	6426680.5000	0.2344	7307810.5000	0.3055	10969798.0000	0.3538
<b>6</b>	6320244.5000	0.2118	7132225.5000	0.2976	10969798.0000	0.3538
<b>7</b>	6354539.5000	0.2307	7194142.7500	0.3000	10969798.0000	0.3538
<b>8</b>	6381262.0000	0.2302	7219875.7500	0.3017	10969798.0000	0.3538
<b>9</b>	6501110.0000	0.2460	7280724.0000	0.3050	10969798.0000	0.3538
<b>10</b>	6499414.5000	0.2408	7282336.2500	0.3047	10969798.0000	0.3538
<b>11</b>	6449511.0000	0.2437	7299840.2500	0.3058	10969798.0000	0.3538
<b>12</b>	6307711.5000	0.2175	7217912.7500	0.3004	10969798.0000	0.3538
<b>13</b>	6081151.0000	0.1756	6911245.5000	0.2838	10969798.0000	0.3538

No. of iterations	Hanoi: Sample points characteristics					
	1. [Min (C), Min (NRI)]		2. [Knee (C), Knee(NRI)]		3. [Max (C), Max (NRI)]	
	Cost (\$)	NRI (—)	Cost (\$)	NRI (—)	Cost (\$)	NRI (—)
14	6461788.5000	0.2340	7283245.5000	0.3045	10969798.0000	0.3538
15	6553108.5000	0.2534	7362807.2500	0.3086	10969798.0000	0.3538
16	6437176.0000	0.2248	7201723.7500	0.3006	10969798.0000	0.3538
17	6553108.5000	0.2537	7405071.7500	0.3106	10969798.0000	0.3538
18	6371998.5000	0.2196	7196055.7500	0.3007	10969798.0000	0.3538
19	6487837.5000	0.2433	7254115.5000	0.3042	10969798.0000	0.3538
20	6370242.5000	0.2253	7181676.5000	0.2998	10969798.0000	0.3538
21	6448901.5000	0.2437	7311294.2500	0.3063	10969798.0000	0.3538
22	6448901.5000	0.2435	7321335.2500	0.3069	10969798.0000	0.3538
23	6462510.0000	0.2410	7284895.0000	0.3055	10969798.0000	0.3538
24	6437584.0000	0.2428	7343579.7500	0.3075	10969798.0000	0.3538
25	6487837.5000	0.2433	7288509.7500	0.3055	10969798.0000	0.3538
26	6439320.5000	0.2223	7193271.2500	0.3002	10969798.0000	0.3538
27	6428073.0000	0.2384	7242831.7500	0.3033	10969798.0000	0.3538
28	6428073.0000	0.2384	7242831.7500	0.3033	10969798.0000	0.3538
29	6492103.5000	0.2442	7213946.0000	0.3018	10969798.0000	0.3538
30	6604006.5000	0.2508	7531605.2500	0.3145	10969798.0000	0.3538

**Table S2.** Balerma, [min(C), min(NRI)], [knee(C), knee(C)], [max(C), max(NRI)] individual for analysis of optimal WDS configurations obtained through the OPUS/NSGA-II Methodology

No. Of iterations	Balerma: Sample points characteristics					
	1. [Min (C), Min (NRI)]		2. [Knee (C), Knee(NRI)]		3. [Max (C), Max (NRI)]	
	Cost (\$)	NRI (—)	Cost (\$)	NRI (—)	Cost (\$)	NRI (—)
1	2288460.00	0.4312	2807753.8750	0.7189	13191652.0000	0.8913

**Table S3.** Modena, [min(C), min(NRI)], [knee(C), knee(C)], [max(C), max(NRI)] individual for analysis of optimal WDS configurations obtained through the OPUS/NSGA-II Methodology

No. Of iterations	Modena: Sample points characteristics					
	1. [Min (C), Min (NRI)]		2. [Knee (C), Knee(NRI)]		3. [Max (C), Max (NRI)]	
	Cost (\$)	NRI (—)	Cost (\$)	NRI (—)	Cost (\$)	NRI (—)
1	2613550	0.3607	3089301.75	0.7302	6731936	0.9073

**Table S4.** Fossolo, [min(C), min(NRI)], [knee(C), knee(C)], [max(C), max(NRI)] individual for analysis of optimal WDS configurations obtained through the OPUS/NSGA-II Methodology

No. of iterations	Fossolo: Sample points characteristics					
	1. [Min (C), Min (NRI)]		2. [Knee (C), Knee(NRI)]		3. [Max (C), Max (NRI)]	
	Cost (\$)	NRI (—)	Cost (\$)	NRI (—)	Cost (\$)	NRI (—)
1	22278.7773	0.2590	41792.7383	0.6831	1661922.5000	0.99979

No. of iterations	Fossolo: Sample points characteristics					
	1. [Min (C), Min (NRI)]		2. [Knee (C), Knee(NRI)]		3. [Max (C), Max (NRI)]	
	Cost (\$)	NRI (—)	Cost (\$)	NRI (—)	Cost (\$)	NRI (—)
2	24819.5234	0.3347	43982.4063	0.7094	1661922.5000	0.99979
3	22782.4668	0.3545	41723.3926	0.7133	1661922.5000	0.99979
4	22784.1289	0.3345	43877.1387	0.7050	1661922.5000	0.99979
5	23001.8652	0.3263	43234.4063	0.7053	1661922.5000	0.99979
6	23144.8340	0.3059	44846.5137	0.6993	1661922.5000	0.99979
7	22514.2988	0.3153	45468.7402	0.7096	1661922.5000	0.99979
8	22889.5215	0.3015	42006.8066	0.6814	1661922.5000	0.99979
9	23046.9844	0.2948	41504.4355	0.6952	1661922.5000	0.99979
10	22701.6426	0.2964	44029.0078	0.7012	1661922.5000	0.99979
11	22903.2773	0.3140	45237.7930	0.7106	1661922.5000	0.99979
12	22284.8691	0.3159	41967.5898	0.6962	1661922.5000	0.99979
13	23262.5586	0.3269	44075.3105	0.7136	1661922.5000	0.99979
14	23187.9453	0.2860	40568.4707	0.6802	1661922.5000	0.99979
15	22992.4570	0.2990	43738.6582	0.6958	1661922.5000	0.99979
16	25038.6211	0.2717	44455.0625	0.6970	1661922.5000	0.99979
17	22314.6074	0.3194	41204.8184	0.6991	1661922.5000	0.99979
18	21410.6426	0.3019	42535.8926	0.7008	1661922.5000	0.99979
19	23107.8438	0.3376	43373.7129	0.7069	1661922.5000	0.99979
20	22966.4082	0.2979	41629.5879	0.6849	1661922.5000	0.99979
21	22584.7012	0.3216	41118.9160	0.6928	1661922.5000	0.99979
22	22917.1523	0.3184	42410.4395	0.6947	1661922.5000	0.99979
23	23045.4355	0.3388	47027.4727	0.7212	1661922.5000	0.99979
24	21776.3047	0.2967	42489.9980	0.6855	1661922.5000	0.99979
25	24081.7852	0.3637	46000.4492	0.7212	1661922.5000	0.99979
26	23921.1797	0.2995	46846.4160	0.7202	1661922.5000	0.99979
27	23495.2090	0.2881	39232.2949	0.6811	1661922.5000	0.99979
28	22525.9102	0.3208	41567.8555	0.6946	1661922.5000	0.99979
29	23578.1367	0.3308	42873.8730	0.7002	1661922.5000	0.99979
30	24212.9160	0.3738	47959.7793	0.7220	1661922.5000	0.99979

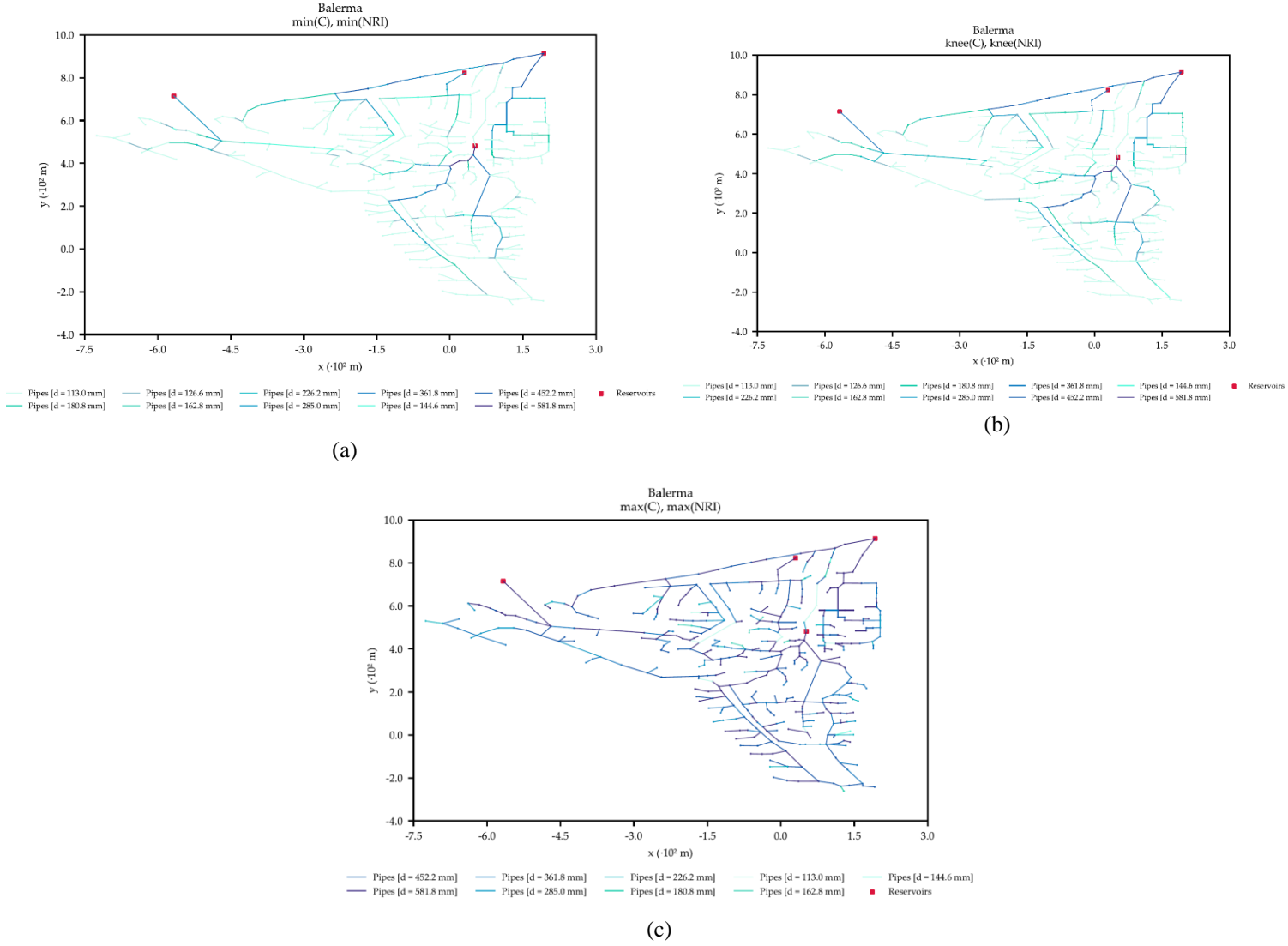
After selecting the three points of each network, these were plotted to determine the characteristics of the WDS.

Figures S1, S2, S3 and S4 shows the pipe diameter in the final WDS configuration obtained through OPUS/NSGA-II for each network.

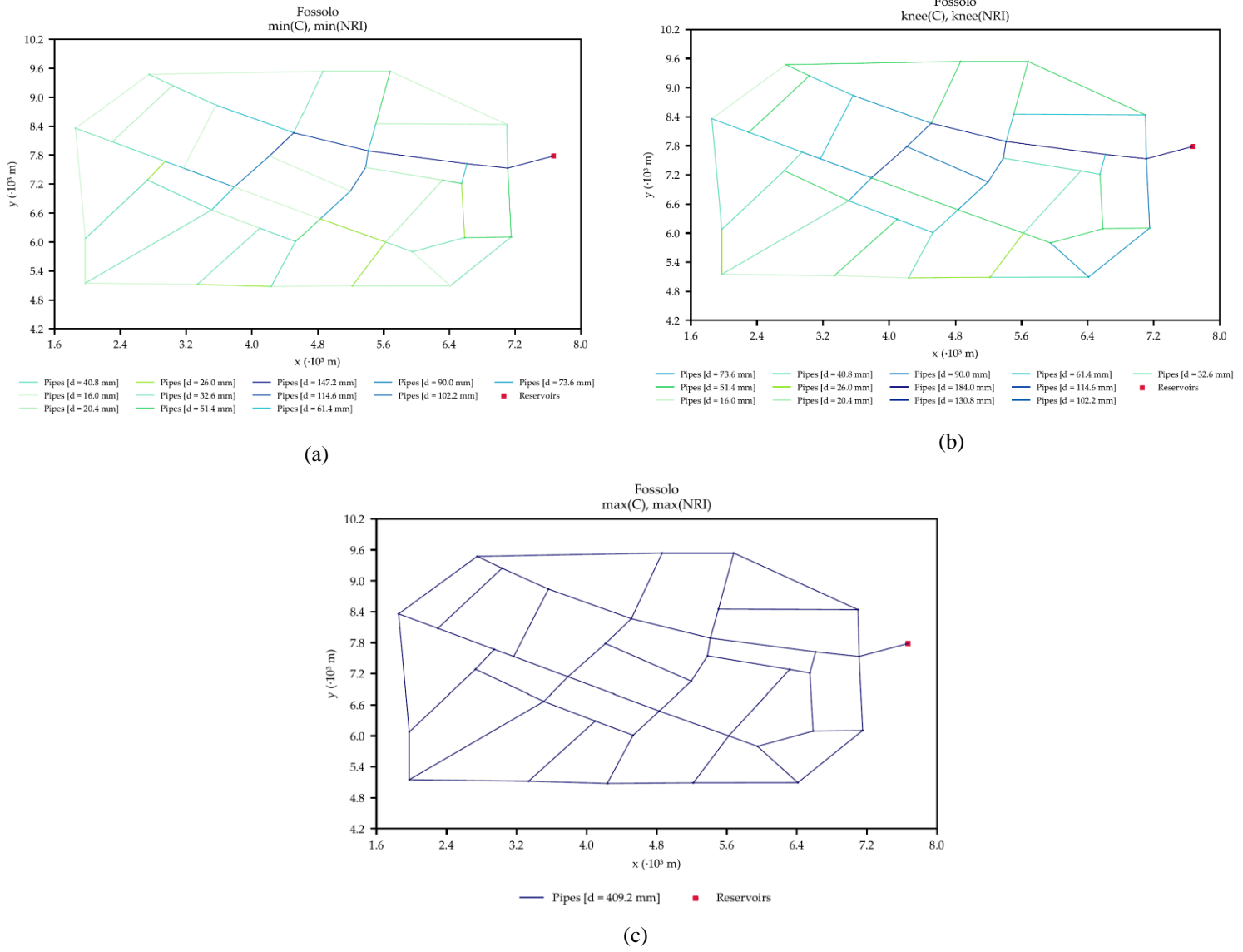
**Figure S1.** Hanoi pipe diameter choices in the final WDS configuration obtained through OPUS/NSGA II for the (a) [min(C), min (NRI)] (b) [knee(C), knee(C)] and (c) [max(C), max (NRI)]



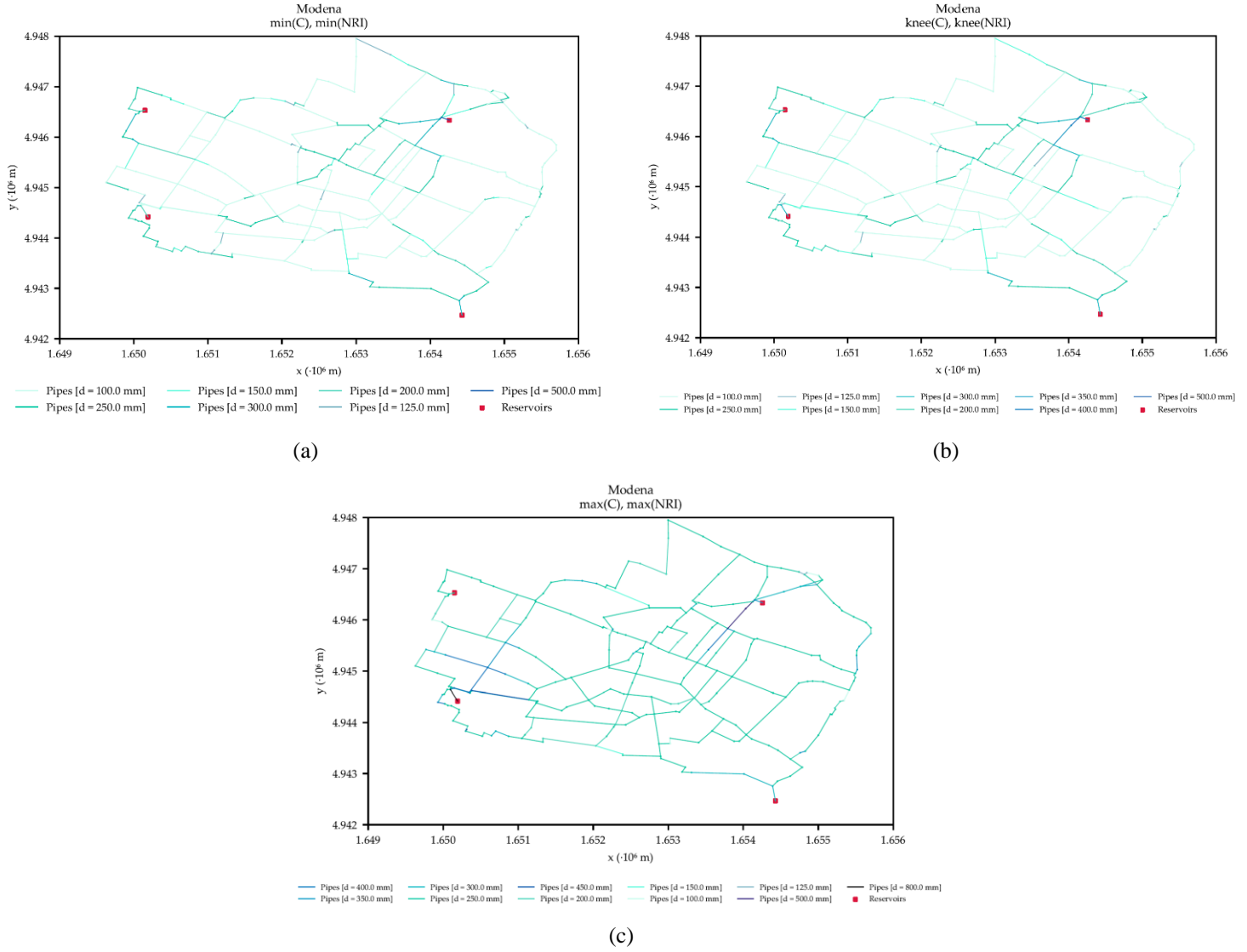
**Figure S2.** Balerna pipe diameter choices in the final WDS configuration obtained through OPUS/NSGA II for the  
(a)  $[\min(C), \min(NRI)]$  (b)  $[\text{knee}(C), \text{knee}(NRI)]$  and (c)  $[\max(C), \max(NRI)]$



**Figure S3.** Fosolo pipe diameter choices in the final WDS configuration obtained through OPUS/NSGA II for the (a)  $[\min(C), \min(NRI)]$  (b)  $[\text{knee}(C), \text{knee}(NRI)]$  and (c)  $[\max(C), \max(NRI)]$



**Figure S4.** Modena pipe diameter choices in the final WDS configuration obtained through OPUS/NSGA II for the (a) [min(C), min (NRI)] (b) [knee(C), knee(C)] and (c) [max(C), max (NRI)]



## WATER DISTRIBUTION SYSTEM CLASSIFICATION

After determining the fractal analysis for the three points of each network, the possibility of a relationship between the values found and the type of network was studied. In addition, we wanted to check if the proposed methodology worked better for any specific type of network. For this reason, a methodology for the characterization of distribution systems was implemented. The paper describes the methodology, and the table S5 details the values obtained for each parameter.

**Table S5.** Values of each parameter for the WDS classification.

<b>WDS</b>	<b>Point</b>	<b>Nodes (No.)</b>	<b>Links (No.)</b>	<b>e<sub>b</sub></b>	<b>e<sub>r</sub></b>	<b>LD</b>	<b>AND</b>	<b>MC</b>	<b>BI</b>
Hanoi	min(C), min (NRI)	32	34	7	9	0.0685	2.1250	0.0508	0.4375
	knee(C), knee(C)	32	34	7	9	0.0685	2.1250	0.0508	0.4375
	max(C), max (NRI)	32	34	7	9	0.0685	2.1250	0.0508	0.4375
Balerna	min(C), min (NRI)	447	454	292	87	0.0046	2.0313	0.0090	0.7704
	knee(C), knee(C)	447	454	292	87	0.0046	2.0313	0.0090	0.7704
	max(C), max (NRI)	447	454	292	87	0.0046	2.0313	0.0090	0.7704
Fossolo	min(C), min (NRI)	37	58	1	57	0.0871	3.1351	0.3188	0.0172
	knee(C), knee(C)	37	58	1	57	0.0871	3.1351	0.3188	0.0172
	max(C), max (NRI)	37	58	1	57	0.0871	3.1351	0.3188	0.0172
Modena	min(C), min (NRI)	272	317	4	117	0.0086	2.3309	0.0853	0.0331
	knee(C), knee(C)	272	317	4	117	0.0086	2.3309	0.0853	0.0331
	max(C), max (NRI)	272	317	4	117	0.0086	2.3309	0.0853	0.0331