Table S1. XRF characterization of red mud (wt %).

Composition of red mud	CaO	Al_2O_3	SiO_2	Na_2O	TiO_2	Fe_2O_3	MnO_2
Proportion	5.05	25.37	12.23	7.65	4.54	39.32	0.12

Table S2. Scale method.

Intensity of Importance	Comparison between A_i and A_j
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2，4，6，8	Intermediate values

Table S3. Construction of pairwise comparison matrix.

Index	A_1	A_2	A_3	……	A_n
A_1	b_{11}	b_{12}	b_{13}	……	b_{1n}
A_2	b_{21}	b_{22}	b_{23}	……	b_{2n}
A_3	b_{31}	b_{32}	b_{33}	……	b_{3n}
……	……	……	……	……	……
A_n	b_{n1}	b_{n2}	b_{n3}	……	b_{nn}

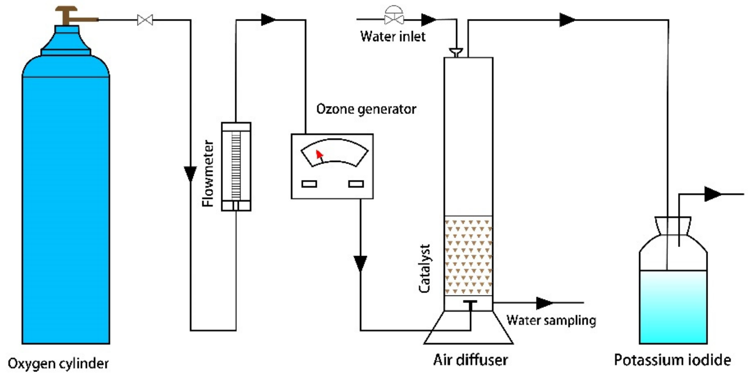

Figure. S1. Ozone catalytic reaction flow chart.

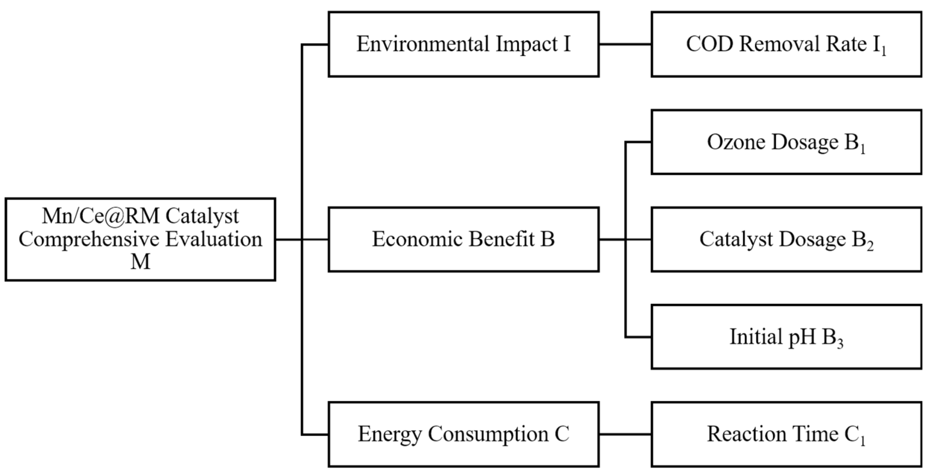

Figure. S2. Comprehensive evaluation index system of Mn/Ce@RM catalyst.

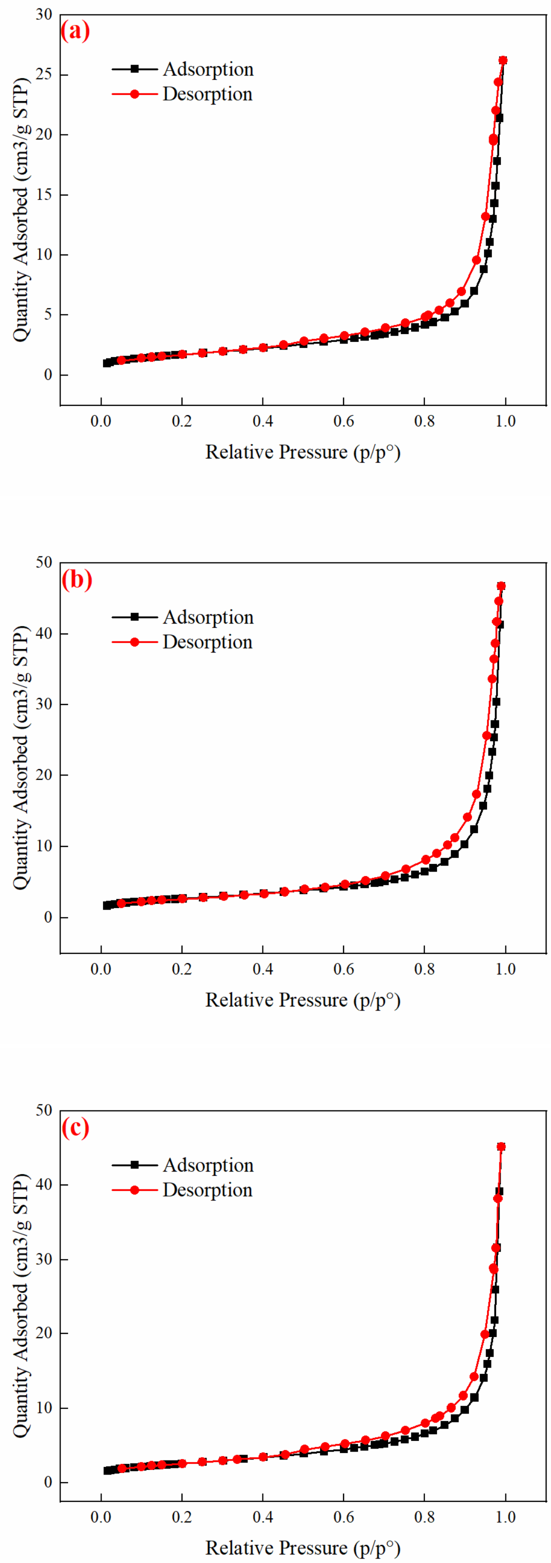

Figure S3. BET characterization results: (**a**)Red mud blank sample, (**b**)Optimal Mn/Ce@RM catalyst sample, (**c**) Mn/Ce@RM catalyst reuse 25 times.

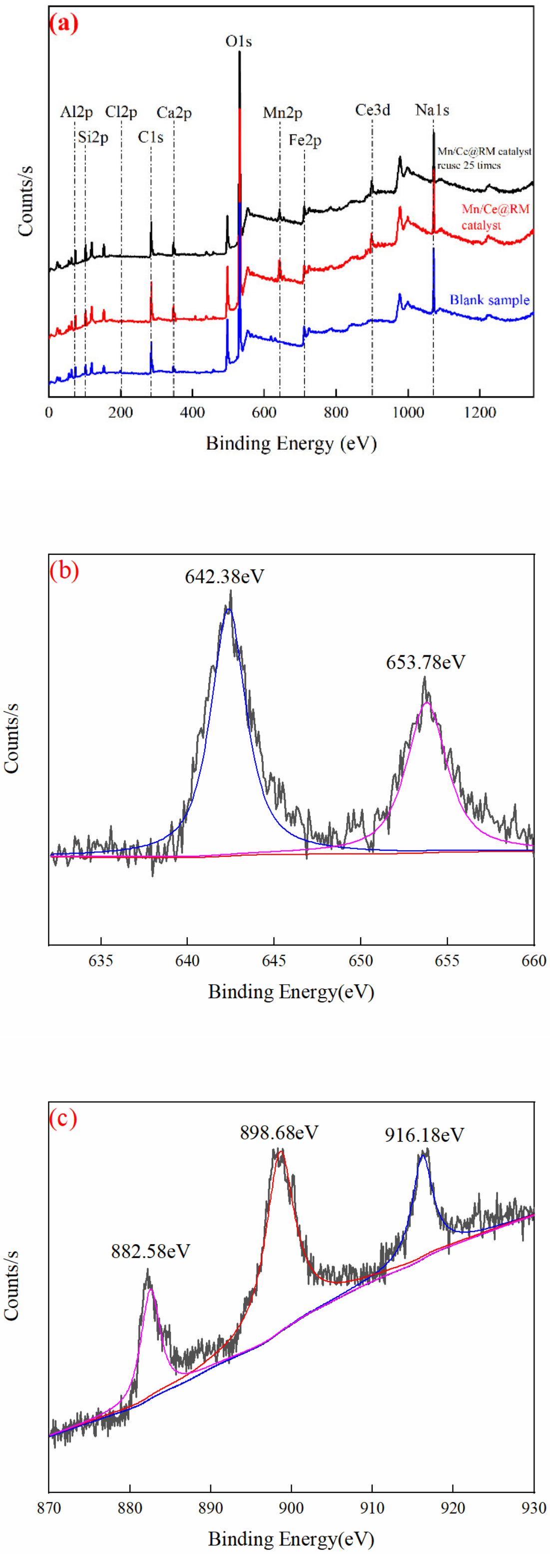

Figure.S4. XPS characterization results: (**a**) XPS full spectrum, (**b**) Mn peak fitting, (**c**) Ce peak fitting.

TEXT S1 The calculation of index weight and consistency check, the calculation of index membership, and the structure of factor evaluation set R.

2.5.3 Calculation of index weight and consistency test

(1) Calculation of weight vector

Calculation of index weight W_i is shown in functions 1 to 3：

$$B_i = \prod_{j=1}^{n} b_{ij} (i = 1,2,3,\ldots,n) \tag{1}$$

$$W_i = (B_i)^{\frac{1}{n}} \tag{2}$$

$$W_i' = \frac{W_i}{\sum_{j=1}^{n} W_j} \tag{3}$$

$W_e = [W_1', W_2', W_3', \ldots, W_n']^T$ is the desired weight vector

(2) Consistency test

In order to ensure the credibility of the fuzzy analytic hierarchy process of Mn/Ce@RM catalyst, the weights need consistency test, as shown in functions 4 to 6:

$$\lambda_{\max} = \sum_{i=1}^{n} \frac{(AW_e)}{nW_i} = \frac{1}{n} \sum_{i=1}^{n} \frac{\sum_{j=1}^{n} b_{ij} W_j}{W_i'} \tag{4}$$

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{5}$$

$$CR = \frac{CI}{RI} \times 100\% \le 0.1 \tag{6}$$

RI is the mean consistency index, and its value is shown in Table S4.

Table S4. The value standard of RI.

n	1	2	3	4	5	6	7	8	9
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45

The consistency test is passed when $CI \le 0.1$. Use the method above to calculate the five indicators selected in this article and rank them according to the importance of each indicator in this evaluation level relative to the indicators in the previous level. The index ranking results are shown in Tables S5-S9.

Table S5. Judgment matrix of evaluation layer to target layer and single ranking result.

M	I	B	C	Weigh W_i
Environmental impact I	1	1/3	1	0.200
Economic benefit B	3	1	3	0.600
Energy consumption C	1	1/3	1	0.200

Table S6. Judgment matrix and single ranking result of index layer's impact on Environment.

I	I_1	weight W_i
COD removal rate I_1	1	1.000

Table S7. The judgment matrix of economic benefit in index level and the result of single rank.

B	B_1	B_2	B_3	weight W_i
Ozone dosage B_1	1	1	1	0.333
Catalyst dosage B_2	1	1	1	0.334
Initial pH B_3	1	1	1	0.333

Table S8. Judgment matrix and single ranking result of energy consumption in index layer.

C	C_1	weight W_i
Reaction time C_1	1	1.000

Table S9. The total ranking weight of each index layer.

Goal	Criteria	Sub-criteria	Group ranking weight	Total ranking weight
Mn/Ce@RM Catalyst Comprehensive Evaluation M	Environmental impact I (0.200)	COD removal rate I_1	1.000	0.200
	Economic benefit B (0.600)	Ozone dosage B_1	0.333	0.200
		Catalyst dosage B_2	0.334	0.200
		Initial pH B_3	0.333	0.200
	Energy consumption C (0.200)	Reaction time C_1	1.000	0.200

2.5.4 Calculation of degree of membership

According to the index system determined above, the set of evaluation factors are:

U={Environmental impact u_1, Economic benefit u_2, Energy consumption u_3},

u_1={COD removal rate u_{11}},

u_2={Ozone dosage u_{21}, Catalyst dosage u_{22}, Initial pH u_{23}},

u_3={Reaction time u_{31}}

According to the comprehensive evaluation index weight of Mn/Ce@RM catalyst determined by the AHP above, the weight set W is:

$$W = [0.200, 0.200, 0.200, 0.200, 0.200]$$

The five indexes selected in this article are classified according to quantitative index and qualitative index, which are shown in Table S10.

Table S10. Comprehensive evaluation index classification of Mn/Ce@RM based ozone catalyst.

Classification	Index
Quantitative index	COD removal rate I_1 Ozone dosage B_1 Catalyst dosage B_2
Qualitative index	Reaction time C_1 Initial pH B_3

For benefit index (the larger index value, the better evaluation) of quantitative indexes, use function 7 to determine the degree of membership:

$$\mu(a) = e^{-2.1(a_j^{\max} - a_{ij})/(a_j^{\max} - a_j^{\min})} \tag{7}$$

Where, a_{ij} —— The j-th index of the i-th plan;

$a_{\mathrm{j}}^{\max}$ —— The maximum value of the j-th index;

a_{j}^{min} —— The minimum value of the j-th index

For cost index (the smaller index value, the better evaluation) of quantitative indexes, use function 8 to determine the degree of membership:

$$\mu(a) = e^{-2.1(a_{ij} - a_j^{min})/(a_j^{\max} - a_j^{\min})} \tag{8}$$

Where, a_{ij} —— The j-th index of the i-th plan;

$a_{\mathrm{j}}^{\max}$ —— The maximum value of the j-th index;

a_j^{min} —— The minimum value of the j-th index

Since qualitative index cannot be directly numerically valued, the ten-point system is used to assign values to qualitative index. Table S11 shows specific assignment standards. The value obtained after the assignment is a benefit index. Therefore, function 7 is used to calculate the degree of membership.

The experimental results of Mn/Ce@RM catalytic ozonation degrading biochemical tail water are summarized in Table S12. According to the data standardization method introduced above, the five evaluation indexes are calculated, and the calculation results are as follows:

Table S11. Qualitative index evaluation standard.

Qualitative index	Extremely reasonable	More reasonable	Medium	Less reasonable	Totally unreasonable
Initial pH B_3	8-10	6-8	4-6	2-4	0-2

Table S12. Summary of experimental results.

Operating condition	Environmental impact I	Economic benefit B			Energy consumption C
	COD removal rate I_1	Ozone dosage B_1	Catalyst dosage B_2	Initial pH B_3	Reaction time C_1
	%	g/h	g/L	/	min
A_1	72.33	2.0	50	7.3 (原水)	80
A_2	73.45	2.0	50	7.3 (原水)	100
A_3	73.49	2.0	50	7.3 (原水)	120
A_4	73.51	2.0	50	7.3 (原水)	140
B_1	66.47	1.6	50	7.3 (原水)	80
B_2	75.68	2.0	50	7.3 (原水)	80
B_3	76.38	2.4	50	7.3 (原水)	80
C_1	67.84	2.0	37.5	7.3 (原水)	80
C_2	76.91	2.0	50	7.3 (原水)	80
C_3	79.95	2.0	62.5	7.3 (原水)	80
D_1	65.48	2.0	62.5	5	80
D_2	76.97	2.0	62.5	7	80
D_3	84.96	2.0	62.5	9	80

(1) COD removal rate I_1

Use function 7 to calculate degree of membership of COD removal rate, and the degree of membership function is $\mu(a)=e^{-2.1(0.8496-a_{ij})/0.1948}$. According to Table S12, the calculation results are as follows:

$$(a_1)_{13\times1}=\begin{bmatrix}0.2563,0.2891,0.2904,0.2910,0.1362,0.3677,0.3966,\\0.1579,0.4199,0.5827,0.1225,0.4226,1.0000\end{bmatrix}^T$$

(2) Ozone dosage B_1

Use function 8 to calculate degree of membership of ozone dosage, and the degree of membership function is $\mu(a)=e^{-2.1(a_{ij}-1.6)/0.8}$. According to Table S12, the calculation results are as follows:

$$(a_2)_{13\times1}=\begin{bmatrix}0.3499,0.3499,0.3499,0.3499,1.0000,0.3499,0.1225,\\0.3499,0.3499,0.3499,0.3499,0.3499,0.3499\end{bmatrix}^T$$

(3) Catalyst dosage B_2

Use function 8 to calculate degree of membership of catalyst dosage, and the degree of membership function is $\mu(a)=e^{-2.1(a_{ij}-37.5)/25}$. According to Table S12, the calculation results are as follows:

$$(a_3)_{13\times1}=\begin{bmatrix}0.3499,0.3499,0.3499,0.3499,0.3499,0.3499,0.3499,\\1.0000,0.3499,0.1225,0.1225,0.1225,0.1225\end{bmatrix}^T$$

(4) Initial pH B_3

Use function 7 to calculate degree of membership of initial pH, and the degree of membership function is $\mu(a)=e^{-2.1(9-a_{ij})/5}$. According to Table S12, the calculation results are as follows:

$$(a_4)_{13\times1}=\begin{bmatrix}0.6570,0.6570,0.6570,0.6570,0.6570,0.6570,0.6570,\\0.6570,0.6570,0.6570,0.1225,0.2837,1.0000\end{bmatrix}^T$$

(5) Reaction time C_1

Use function 8 to calculate degree of membership of reaction time, and the degree of membership function is $\mu(a)=e^{-2.1(a_{ij}-80)/60}$. According to Table S12, the calculation results are as follows:

$$(a_5)_{13\times1}=\begin{bmatrix}1.0000,0.4966,0.2466,0.1225,1.0000,1.0000,1.0000,\\1.0000,1.0000,1.0000,1.0000,1.0000,1.0000\end{bmatrix}^T$$

2.5.5 Factor evaluation set R

From the degree of membership value of the five indexes in the previous section, the single-factor evaluation set of Mn/Ce@RM catalyst can be obtained:

$$R_1=[0.2563,0.3499,0.3499,0.6570,1.0000]$$

$$R_2=[0.2891,0.3499,0.3499,0.6570,0.4966]$$

$$R_3=[0.2904,0.3499,0.3499,0.6570,0.2466]$$

$$R_4=[0.2910,0.3499,0.3499,0.6570,0.1225]$$

$$R_5=[0.1362,1.0000,0.3499,0.6570,1.0000]$$

$$R_6 = [0.3677, 0.3499, 0.3499, 0.6570, 1.0000]$$

$$R_7 = [0.3966, 0.1225, 0.3499, 0.6570, 1.0000]$$

$$R_8 = [0.1579, 0.3499, 1.0000, 0.6570, 1.0000]$$

$$R_9 = [0.4199, 0.3499, 0.3499, 0.6570, 1.0000]$$

$$R_{10} = [0.5827, 0.3499, 0.1225, 0.6570, 1.0000]$$

$$R_{11} = [0.1225, 0.3499, 0.1225, 0.1225, 1.0000]$$

$$R_{12} = [0.4226, 0.3499, 0.1225, 0.2837, 1.0000]$$

$$R_{13} = [1.0000, 0.3499, 0.1225, 1.0000, 1.0000]$$

Then the multi-factor judgment set R of the comprehensive evaluation of Mn/Ce@RM catalyst is:

$$R = [R_1, R_2, \cdots R_{13}]^T$$

Combine the weight vector W obtained above with the multi-factor judgment set R to obtain the fuzzy decision vector A:

$$\begin{aligned} A &= W \times R = [0.200, 0.200, 0.200, 0.200, 0.200] \times R \\ &= \begin{bmatrix} 0.5226, 0.4285, 0.3788, 0.3541, 0.6286, 0.5449, 0.5052, \\ 0.6330, 0.5554, 0.5424, 0.3435, 0.4357, 0.6945 \end{bmatrix} \end{aligned}$$

Table 5 shows the scores and rankings of each operating condition in this comprehensive evaluation.