Table S1. Reviews of list of introduced formulas for computing the exchange flow.

Reference	Exchange flow equation	The equivalent of the exchange coefficient	Unit of Coe.	The value of Coe.	model	Other conditions
Bauer et al. [1]	$Q_{ex} = \sigma A_F K_m \Delta h$	$\sigma A_F K_m$	$[L^2T^{-1}]$	1.0×10^{-8} m^2/s^{-1} to 1.0×10^{-4} m^2/s^{-1}	The model Carbonate Aquifer Void Evolution (CAVE)	
Cornaton and Perrochet [2]	$q_{ex} = -\alpha_{ex}\,[H_m(t) - H_c(t)]\,\frac{r_c}{2}$ $q_{ex} = -K_m \frac{H_m(t) - H_c(t)}{\varepsilon r_m}$	exchange coefficient 3D $\alpha_{ex} = \frac{2\pi}{\varepsilon}\frac{K_m}{\sqrt{\beta}}$	$[L^{-1}T^{-1}]$			$\beta = \left(\frac{r_m}{r_c}\right)^2$
		exchange coefficient 1D $\alpha_{ex0} = \pi\, r_c^2$	$[L\,T^{-1}]$			
Bauer et al. [3]	$q_{ex} = \sigma A_F K_F (h_F - h_c)$	$\alpha_{ex} = \sigma A_F K_F$	$[L^2T^{-1}]$	1.0×10^{-7} m^2/s^{-1}	The model Carbonate Aquifer Void Evolution (CAVE)	
Wang [4]	$q_{ex,i} = \alpha_{ex,i}(h_{m,i} - h_{c,i})$	σAK discrete exchange coefficient	$[L^2T^{-1}]$		A new modification of the original CCPF model	
		σ	$[L^{-1}]$			
		$\tilde{\alpha}_{ex} = \sigma\pi dK$ continuum exchange coefficient	$[L\,T^{-1}]$			
Reimann et al. [5]	$q_{ex} = \frac{K'}{b'} P_{ex}\,\Delta h_{ex}$	$\frac{K'}{b'}$	$[1/T]$	1.0×10^{-5} Per second	MODBRANCH	q_{ex} equal to exchange fow per unit length of conduit $[L^2/T]$
Kordilla et al. [6]	$\Gamma_{ex} = \alpha_{ex} K_i k_{ri}\,(\psi_c - \psi_m)$ volumetric fluid exchange rate per unit volume	$\alpha_{ex} = \frac{\beta}{\sigma^2}\gamma w$	$[L^{-2}]$	Determined by calibration	Hydro GeoSphere model	Modified Richard's equation to calculation exchange flow
Saller et al. [7]	$Q_{ex} = \alpha_{ex}(h_n - h_{ijk})$	α_{ex}	$[L^2T^{-1}]$		CFP Mode 1 equivalent porous medium (EPM)	

Table S1. Continue of Table S1.

Reference	Exchange flow equation	The equivalent of the exchange coefficient	Unit of Coe.	The value of Coe.	model	Other conditions
Xu et al. [8]	$Q_{ex} = ck_{\alpha\beta\gamma}\,(h_i\text{-}h_{\alpha\beta\gamma})$ Used for flow	$Ck_{\alpha\beta\gamma}$	$[L^2T^{-1}]$		Conduit Flow Process (CFP_{v2})	
	$K_{ex} = \frac{\Gamma_i C_{m.i}}{V_{m.i}}$ Used for solute				UMT3D	
Hubinger et al. [9]			$[L^2T^{-1}]$	1.0×10^{-6} m^2/s^{-1}	CAVE	
Giese et al. [10]	$Q_{exi}=\alpha_{ex}(h_c\text{-}h_m)$	α_{ex}	$[L^2T^{-1}]$	0.031 m^2/s^{-1}		Linear and steady exchange
Basha and Zoghbi [11]	$q_*= -k_{s_*} d_* \frac{dh_{a^*}}{dy_*}$	The interface height, nodal spacing, and hydraulic conductivity are lumped together in a parameter defined as the exchange coefficient			Analytical Method of Laplace transform	

Table S2. The characteristics of the spring hydrograph in the scenario A1.

	Exchange coefficient (m^2/s)	10^{-2}	10^{-3}	10^{-4}	10^{-5}
	T_p (min)	110	125	110	90
	Q_p (m^3/min)	1.12	0.55	0.10	0.044
	V_b (m^3)	2299.20	2312.50	1810.50	436.08
CFR	Duration of occurrence of Q_{MC-} discharge (Point recharge > Q_{MC-}) (min)				
CFR	Duration of occurrence of Q_{MC+} discharge (Point recharge > Q_{MC+}) (min)				90-130
CFR	β (m^3/min^2)				0.07×10^{-2}
CIFR and /or MRFR	Duration of occurrence of Q_{MC+} discharge (Point recharge < Q_{MC+}) (min)	110-50000	125-50000	110-50000	130-50000
CIFR and /or MRFR	α $(1/min)$	6.9×10^{-3} 1.3×10^{-3} 3.0×10^{-4} 5.6×10^{-5}	3.2×10^{-3} 9.7×10^{-4} 2.9×10^{-4} 5.1×10^{-5}	1.7×10^{-3} 2.1×10^{-4} 9.0×10^{-5} 2.4×10^{-5}	7.1×10^{-3} 3.6×10^{-6}

Table S3. The characteristics of the spring hydrograph in scenario B1.

	Exchange coefficient (m^2/s)	10^{-2}	10^{-3}	10^{-4}	10^{-5}
	T_p (min)	100	95	95	95
	Q_p (m^3/min)	15.14	16.90	19.04	19.49
	V_b (m^3)	2431.00	2411.50	1829.50	450.34
CFR	Duration of occurrence of Q_{MC-} discharge (Point recharge > Q_{MC-}) (min)	100-115	95-120	95-125	95-125
	Duration of occurrence of Q_{MC+} discharge (Point recharge > Q_{MC+}) (min)	115-140	120-150	125-165	125-190
	β (m^3/min^2)	0.23	0.29	0.31	0.32
CIFR and /or MRFR	Duration of occurrence of Q_{MC+} discharge (Point recharge < Q_{MC+}) (min)	140-50000	150-50000	165-50000	190-50000
	α $(1/min)$	2.4×10^{-2} 3.9×10^{-3} 1.1×10^{-3} 3.3×10^{-4} 5.6×10^{-5}	3.6×10^{-2} 3.1×10^{-3} 8.7×10^{-4} 3.0×10^{-4} 5.1×10^{-5}	4.0×10^{-2} 6.0×10^{-4} 1.1×10^{-4} 2.4×10^{-5}	2.3×10^{-2} 3.6×10^{-6}

Table S4. The characteristics of spring hydrograph in response to changes of conduit diameter in scenario A2.

	Diameter (m)	0.5	0.4	0.3	0.2	0.1
	T_p (min)	125	125	125	125	125
	Q_p (m^3/min)	0.55	0.54	0.49	0.32	0.098
	V_b (m^3)	2312.65	2313.60	2317.50	2325.97	1952.35
CFR	Duration of occurrence of Q_{MC-} discharge (Point recharge > Q_{MC-}) (min)					
	Duration of occurrence of Q_{MC+} discharge (Point recharge > Q_{MC+}) (min)					
	β (m^3/min^2)					
CIFR and /or MRFR	Duration of occurrence of Q_{MC+} discharge (Point recharge < Q_{MC+}) (min)	125- 50000	125- 50000	125-50000	125- 50000	125- 50000
	α $(1/\ min)$	3.2×10^{-3} 9.7×10^{-4} 2.9×10^{-4} 5.1×10^{-5}	3.1×10^{-3} 9.5×10^{-4} 2.9×10^{-4} 5.0×10^{-5}	2.6×10^{-3} 8.8×10^{-4} 2.8×10^{-4} 5.0×10^{-5}	1.5×10^{-3} 5.6×10^{-4} 2.3×10^{-4} 4.9×10^{-5}	1.5×10^{-3} 2.7×10^{-4} 8.7×10^{-5} 2.3×10^{-5}

Table S5. The characteristics of spring hydrograph in response to changes in conduit diameter in scenario B2.

	Diameter (m)	0.5	0.4	0.3	0.2	0.1
	T_p (min)	95	95	95	100	105
	Q_p (m^3/min)	16.90	13.94	10.11	5.17	0.62
	V_b (m^3)	2411.30	2534.85	2753.15	2813.99	2813.99
CFR	Duration of occurrence of Q_{MC-} discharge (Point recharge > Q_{MC-}) (min)	95-120	95-125	95-135	100-145	105-160
CFR	Duration of occurrence of Q_{MC+} discharge (Point recharge > Q_{MC+}) (min)	125-150	125-150	135- 155	145-150	
CFR	β (m^3/min^2)	0.29	0.22	0.15	0.07	0.003
CIFR and /or MRFR	Duration of occurrence of Q_{MC+} discharge (Point recharge < Q_{MC}) (min)	155- 50000	155- 50000	155-50000	155- 50000	160-50000
CIFR and /or MRFR	α $(1/min)$	3.6×10^{-2} 3.1×10^{-3} 8.7×10^{-4} 3.0×10^{-4} 5.1×10^{-5}	2.9×10^{-2} 3.8×10^{-3} 1.0×10^{-3} 3.3×10^{-4} 5.1×10^{-5}	2.1×10^{-2} 3.7×10^{-3} 1.1×10^{-3} 3.9×10^{-4} 5.1×10^{-5}	1.5×10^{-2} 2.9×10^{-3} 9.2×10^{-4} 3.6×10^{-4} 2.1×10^{-5}	3.6×10^{-3} 1.5×10^{-3} 7.0×10^{-4} 2.8×10^{-4} 2.1×10^{-5}

Table S6. The spring hydrograph changes influenced by matrix hydraulic conductivity changes in scenario A3.

	Hydraulic conductivity (m/s)	10^{-3}	10^{-4}	10^{-5}	10^{-6}
	T_p (min)	150	135	125	110
	Q_p (m^3/min)	0.87	0.76	0.55	0.33
	V_b (m^3)	2424.35	2438.15	2312.50	940.250
CFR	Duration of occurrence of Q_{MC-} discharge (Point recharge > Q_{MC-}) (min)				
CFR	Duration of occurrence of Q_{MC+} discharge (Point recharge > Q_{MC+}) (min)				
CFR	β (m^3/min^2)				
CIFR and /or MRFR	Duration of occurrence of Q_{MC+} discharge (Point recharge < Q_{MC+}) (min)	150-50000	135-50000	125- 50000	110- 50000
CIFR and /or MRFR	α $(1/min)$	3.5×10^{-4}	6.5×10^{-4} 2.6×10^{-4}	3.2×10^{-3} 9.7×10^{-4} 2.9×10^{-4} 1.3×10^{-5}	7.7×10^{-3} 1.3×10^{-3} 2.8×10^{-4} 1.3×10^{-5}

Table S7. The spring hydrograph changes influenced by matrix hydraulic conductivity changes in scenario B3.

	Hydraulic conductivity (m/s)	**10^{-3}**	**10^{-4}**	**10^{-5}**	**10^{-6}**
	T_p (min)	95	95	95	95
	Q_p (m^3/min)	16.27	16.44	16.90	17.54
	V_b (m^3)	2600.20	2585.00	2411.34	980.55
CFR	Duration of occurrence of Q_{MC-} discharge (Point recharge > Q_{MC-}) (min)	95-125	95-125	95-120	95-110
CFR	Duration of occurrence of Q_{MC+} discharge (Point recharge > Q_{MC+}) (min)	130- 155	130- 155	125- 150	110- 155
CFR	β (m^3/min^2)	0.27	0.28	0.29	0.29
CIFR and /or MRFR	Duration of occurrence of Q_{MC+} discharge (Point recharge <Q_{MC+}) (min)	155-50000	160-50000	155-50000	160-50000
CIFR and /or MRFR	α $(1/min)$	2.6×10^{-2} 4.8×10^{-4} 3.5×10^{-4}	2.7×10^{-2} 1.3×10^{-3} 4.7×10^{-4} 2.5×10^{-4}	3.6×10^{-2} 3.1×10^{-3} 8.7×10^{-4} 3.0×10^{-4} 5.0×10^{-5}	5.3×10^{-2} 3.9×10^{-3} 1.2×10^{-3} 2.8×10^{-4} 1.2×10^{-5}

Appendix A: Nomenclature

A_F, A [L^2]: The exchange surface between the conduit and the fissured matrix

$Ck_{\alpha\beta\gamma}$ [L^2T^{-1}]: Pipe conductance in MODFLOW cell $\alpha\beta\gamma$

$C_{m,i}$ [ML^{-3}]: Concentration in the porous matrix at conduit node i

d [L]: The height of the conduit interface

h_C [L]: The head in the conduit

h_F [L]: The head in the fissured system

h_i, h_n [L]: The head in the conduit node i or n

$h_{\alpha\beta\gamma}$, h_{ijk} , h_m [L]: The head in the matrix cell

h_a [L]: the height of the water table measured from the same base level as for the conduit

K_F, K, K_m [L T^{-1}]: The hydraulic conductivity of the matrix or fissured system

K_i [L T^{-1}]: the hydraulic conductivity of the interface between matrix and conduit (e.g. sediments)

k_{ri} [-]: the relative permeability of the interface between matrix and conduit

k_s [L T^{-1}]: The saturated hydraulic conductivity

K_{ex} [$ML^{-3}T^{-1}$]: Advective exchange rate between a conduit and matrix

P_{ex} [L]: exchange perimeter

q_{ex} [L^2/T]: lateral outflow per unit length of the channel (Reimann *et al.*, 2011)

Q_{ex}, Γ_i [L^3T^{-1}]: the volumetric exchange rate between the conduit and the matrix

r_c [L]: The conduit radii

r_m [L]: The matrix radii

$V_{m,i}$ [L^3]: The volume of the respective cell

y [L]: The horizontal lateral coordinate

σ [L^{-1}]: Parameter that depends on conduit geometry and is as inverse fissure spacing

σ [L^{-1}]: The distance between the centre of a matrix block and the adjacent fracture or conduit (Kordilla *et al.,* 2012)

β: A geometry factor (3 for rectangular matrix blocks, 15 for spheres)

γ_w: An empirical coefficient usually set to 0.4

ψ_c, ψ_m: the pressure heads in the conduit and matrix, respectively

ε: A factor multiplying r_m (the distance allowing to evaluate the gradient being unknown a priori)

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