# A Method for Estimating the Velocity at Which Anaerobic Metabolism Begins in Swimming Fish 

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#### Abstract

Anaerobic metabolism begins before fish reach their critical swimming speed. Anaerobic metabolism affects the swimming ability of fish, which is not conducive to their upward tracking. The initiation of anaerobic metabolism therefore provides a better predictor of flow barriers than critical swimming speed. To estimate the anaerobic element of metabolism for swimming fish, the respiratory metabolism and swimming performance of adult crucian carp (Carassius auratus, mass $=260.10 \pm 7.93$, body length $=19.32 \pm 0.24$ ) were tested in a closed tank at $20 \pm 1^{\circ} \mathrm{C}$. The swimming behavior and rate of oxygen consumption of these carp were recorded at various swimming speeds. Results indicate (1) The critical swimming speed of the crucian carp was $0.85 \pm 0.032 \mathrm{~m} / \mathrm{s}$ ( $4.40 \pm 0.16 \mathrm{BL} / \mathrm{s}$ ). (2) When a power function was fitted to the data, oxygen consumption, as a function of swimming speed, was determined to be $A M R=131.24+461.26 U_{s}^{1.27}\left(R^{2}=0.948, p<0.001\right)$ and the power value (1.27) of $U_{s}$ indicated high swimming efficiency. (3) Increased swimming speed led to increases in the tail beat frequency. (4) Swimming costs were calculated via rate of oxygen consumption and hydrodynamic modeling. Then, the drag coefficient of the crucian carp during swimming was calibrated ( $0.126-0.140$ ), and the velocity at which anaerobic metabolism was initiated was estimated ( $0.52 \mathrm{~m} / \mathrm{s}$ ), via the new method described herein. This study adds to our understanding of the metabolic patterns of fish at different swimming speeds.


Keywords: oxygen consumption rate; drag coefficient; drag; anaerobic metabolism

## 1. Introduction

Construction of sluices and dams brings huge benefits in terms of flood control, irrigation and power generation, but also destroys the natural connectivity of rivers and lakes, blocking fish migration [1-3]. Fishways play a role in repairing this ecological damage, and successful fishways are essential for fish migration, which they assist via removing or minimizing the effects of physical barriers to movement such as a hydroelectric dams [4]. However, the flow velocity in fishways is often high, and thus they may continue to present hydraulic barriers to fish [5]. It is therefore critical to the improvement of fishway efficiency to formulate a standardized approach to the measurement of the hydraulic barrier that they present to fish.

The research focus differs depending on the stage of the fishway, and can be broadly categorized as concerning either design or operation. During the design phase, the main considerations are the hydraulic conditions within the fishway, and the control of fishway hydraulic parameters such as water velocity, intensity of turbulence, and energy dissipation
rate in the pool, in order to minimize the hydraulic barriers presented to the upstream movement of fish [6-10]. After construction of a fishway, prototype observations can be conducted through fish release experiments to further identify flow barriers and improve the efficiency of fish passage. However, although many studies on fishway hydraulics have been conducted, the efficiency of fishways remains relatively low [11,12]. In order to improve fishway efficiency, interdisciplinary research is necessary [4]. Through study of the swimming abilities and respiratory metabolism of fish, fish biologists have divided fish swimming speeds into three categories: sustained, burst, and prolonged [13], of which the prolonged swimming speed has been measured most often in laboratory testing [14]. The main subcategory adopted has been the critical swimming speed, first defined and employed by Brett [15], which can be defined as the highest swimming speed that a fish can maintain for the time period being tested [16]. Research on respiratory metabolism is often conducted using respirometer testing [15,17], and fish energy consumption data can be obtained from measurements of their rate of oxygen consumption.

The adaptation of fish to their surrounding fluid environment can be understood from the perspective of energy consumption [18-24]. However, using hydrodynamic modelling to calculate swimming cost, Thiem et.al [25] did not find a difference in the rate of energy expenditure between sturgeon (Acipenser fulvescens) that were successful or unsuccessful in achieving fishway passage, even though the successful fish exhibited higher costs of transport ( 42.75 versus $25.85 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~m}^{-1}$, respectively). Since energy expenditure metrics did not appear to be predictive of successful fishway passage, these researchers were led to conclude that other endogenous or exogenous factors were influencing passage success. In the drag force hydrodynamic model, it is generally assumed that the drag coefficient of a fish is identical to that of an equivalent rigid body [19,26], an assumption that may be invalid while fish are swimming. Moreover, the estimation of energy consumption via oxygen consumption does not include the energy provided by anaerobic metabolism during exercise, and many studies show that fish enter anaerobic metabolism before reaching critical swimming speed [27-29]. The energy contribution of anaerobic metabolism is difficult to estimate from the swimming speed of live fish.

In fishway design, the critical speed model is often quite simply applied, with fish assumed to fail passage if the water velocity within the fishway exceeds their critical swimming speed [30]. However, research results regarding respiratory metabolism have rarely been applied to fishways. Rather, hydrodynamic models have been used to calculate the drag force of fish, and estimate the energy needed to overcome this drag within the fishway. Through cross-disciplinary work combining fish biology and fish hydraulics, this paper aims to establish a relationship between respiratory metabolism and fish swimming drag force calculations. To obtain the necessary metabolic parameters in relation to swimming speed, estimating the rate of oxygen consumption for series of increasing velocities within a closed tank, which provides ample space for free swimming, has an important advantage: specifically, this avoids the reduced sensitivity to changes in oxygen levels that occur if the volume of circulating water is excessively large.

## 2. Materials and Methods

### 2.1. Respirometer

Fish were tested in a flume-type swimming respirometer (SW10200, Loligo Systems, Denmark), volume 90 L, with a 28 L rectangular swim chamber ( $0.70 \mathrm{~m} \times 0.20 \mathrm{~m} \times 0.20 \mathrm{~m}$ ). The respirometer was submerged in the tank (Figure 1). The maximum water velocity possible in the chamber was $1.50 \mathrm{~m} / \mathrm{s}$. A flow rectifier maintained smooth flow within the chamber, while a grid was placed at the end of the chamber to prevent fish being swept away. Water velocity in the respirometer was controlled by a propeller driven by a variablespeed motor; this was calibrated using a LaVision Particle Image Velocimetry system (PIV). The respirometer could be flushed with a submersible pump in order to maintain the dissolved oxygen $(D O)$ level within it. The respirometer was equipped with a high-speed video camera (Peiport FC-01, 200 HZ frame rate, $100 \mathrm{~cm} \times 65 \mathrm{~cm}$ sampling window) set
above the swim chamber to record fish swimming behavior and tail beat frequency. During the test, the respirometer was sealed, and its internal dissolved oxygen and temperature levels were monitored using a multi-parameter probe (HACH HQ30d, USA).


Figure 1. Swim tunnel respirometer (SW10200, Loligo Systems, Denmark).

### 2.2. Experimental Design

Fish. Crucian carp ( $n=31$, body mass $=260.10 \pm 7.93$ g, total length $=24.73 \pm 0.25 \mathrm{~cm}$, spawning period) were obtained from a fish farm in Wuhan, Hubei, China. The fish were immediately transferred to laboratory aquaria (diameter 2.0 m , height 0.8 m ), where they acclimated for at least two days. They were fed daily and any remaining food was removed after 1 hr . Testing took place from 1 July to 10 August 2019 and was carried out in the Key Laboratory of Ecological Impacts of Hydraulic-Projects and Restoration of Aquatic Ecosystem of the Ministry of Water Resource, Wuhan, China, with room temperature controlled by air conditioning. The test fish were kept temporarily in the pool on site for 2 d and not fed for the 24 h period prior to being transferred, in water, to the respirometer [15]. The duration of each test was around 6 h . Dissolved oxygen $(D O)$ within the respirometer was maintained above $6.0 \mathrm{mg} / \mathrm{L}$ [29], and the water temperature maintained at $20( \pm 1)^{\circ} \mathrm{C}$. When the $D O$ fell below $6.0 \mathrm{mg} / \mathrm{L}$, testing was interrupted and the water was aerated [17]. Sample numbers and corresponding water velocities are shown in Table 1.

Table 1. Number of samples at each stage. When the velocity ( $\mathrm{m} / \mathrm{s}$ ) reaches the critical swimming speed, the number of samples naturally decreases.

| Number of Samples | $\begin{gathered} 0.15 \\ (\mathrm{~m} / \mathrm{s}) \end{gathered}$ | $\begin{gathered} 0.30 \\ (\mathrm{~m} / \mathrm{s}) \end{gathered}$ | $\begin{gathered} 0.45 \\ (\mathrm{~m} / \mathrm{s}) \end{gathered}$ | $\begin{gathered} 0.60 \\ (\mathrm{~m} / \mathrm{s}) \end{gathered}$ | $\begin{gathered} 0.75 \\ (\mathrm{~m} / \mathrm{s}) \end{gathered}$ | $\begin{gathered} 0.90 \\ (\mathrm{~m} / \mathrm{s}) \end{gathered}$ | $\begin{gathered} 1.05 \\ (\mathrm{~m} / \mathrm{s}) \end{gathered}$ | $\begin{gathered} 1.20 \\ (\mathrm{~m} / \mathrm{s}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $n$ | 31 | 31 | 31 | 31 | 25 | 14 | 7 | 2 |

Cameras. The PIV laser was placed horizontally emitting a light surface parallel to the bottom surface of the device, in order to ensure that the two horizontally-oriented CCD cameras could photograph the test plane illuminated by the laser; the details were described in Tarrade's paper [31]. The high-speed video camera was set above the chamber to record the swimming behavior of the fish (video was recorded during the test and the tail beat frequency could be obtained through frame-by-frame analysis by ProAnaly software). It was ensured that the three cameras were capturing the same plane. The requisite supplementary light sources were also set up alongside the test device (Figure 2).


Figure 2. Experimental equipment layout (layout drawings were not drawn to scale).
Calibration. The relationship between the frequency setting of the variable-speed motor, $x$, and water velocity, y , was calibrated using the LaVision Particle Image Velocimetry system (PIV). The equation $y=0.03788 x\left(R^{2}=0.9978\right)$ fitted this data. (See Appendix A.) During testing, the rotation speed of the motor was adjusted using its frequency setting, in order to control the water velocity in the swim chamber.

Stepped Velocity Testing. Stepped velocity tests were conducted to estimate the critical swimming speed $\left(U_{\text {crit }}\right)$, oxygen consumption rate (AMR), cost of transport (COT), and tail beat frequency (TBF) of the crucian carp; these tests were repeated $n$ times (Table 1). During each test, water velocity was increased by $0.15 \mathrm{~m} / \mathrm{s}$, starting at $0.15 \mathrm{~m} / \mathrm{s}$, at $25-\mathrm{min}$ intervals until the test fish was fatigued. At the same time, the $D O(\mathrm{mg} / \mathrm{L})$ and temperature in the respirometer were recorded every 5 min for 25 min . A fish was considered as fatigued when it stopped at the end of the test area, lightly patted the downstream wall for 20 s and still could not swim [32]. After testing was complete, the body length (cm), total length (cm), and body mass (g) of the fish were measured.

### 2.3. Morphometric and Statistical Analysis

The critical swimming speed $U_{\text {crit }}$ was calculated using the flow velocities and step intervals recorded during the test using Equation (1) [14]:

$$
\begin{equation*}
U_{c r i t}=v+\frac{t}{\Delta t} \Delta v \tag{1}
\end{equation*}
$$

where $t(\mathrm{~min})$ is the time elapsed at fatigue velocity, $\Delta t(\mathrm{~min})$ is the prescribed interval time (here 25 min ). $\Delta v(\mathrm{~m} / \mathrm{s})$ is the velocity increment (here $0.15 \mathrm{~m} / \mathrm{s}$ ), and $v(\mathrm{~m} / \mathrm{s})$ is the final velocity recorded before the fish became fatigued. It was ensured that the crosssectional area of the test fish was less than $10 \%$ of the cross-sectional area of the test area. This prevented it causing a blocking effect, and eliminated the need for a correction in Equation (1) [33].

Oxygen consumed per hour per kilogram of body mass (AMR) was obtained using Equation (2) [34]:

$$
\begin{equation*}
A M R=(S t-S b) \times 60 \times \mathrm{Vol} / \mathrm{m} \tag{2}
\end{equation*}
$$

where $A M R$ is the oxygen consumption rate of the standard weight of the test fish ( $\mathrm{mg} \mathrm{O} \mathrm{O}_{2} \cdot \mathrm{~h}^{-1} \cdot \mathrm{~kg}^{-1}$ ); St is the value of the slope of dissolved oxygen with respect to time with the test fish swimming (apparent oxygen consumption $\mathrm{mg} \mathrm{O}_{2} \cdot \mathrm{~min}^{-1} \cdot \mathrm{~L}^{-1}$ ); Sb is the absolute value of the slope of dissolved oxygen per unit time in the absence of a test fish (bacterial oxygen consumption $\mathrm{mg} \mathrm{O}_{2} \cdot \mathrm{~min}^{-1} \cdot \mathrm{~L}^{-1}$ ); 60 is the time constant ( $\mathrm{s} / \mathrm{min}$ ); Vol is
the total volume ( L ) of the experimental water system ( 90 L in this study), and $m$ is the body mass of the fish (kg).

The cost of transport (COT) $\left(\mathrm{J} \cdot \mathrm{kg}^{-1} \cdot \mathrm{~m}^{-1}\right)$ is the energy expended in swimming, which was obtained using Equation (3) [25]:

$$
\begin{equation*}
\mathrm{COT}=\frac{A M R}{U_{s}} \times 13.54 \tag{3}
\end{equation*}
$$

where $U_{s}$ is swimming speed $\left(\mathrm{m} \cdot \mathrm{s}^{-1}\right)$ and 13.54 is the oxycaloric value $\left(\mathrm{J} \cdot \mathrm{mg} \mathrm{O}_{2}{ }^{-1}\right)$ [35].
Vogel [26] proposed that the drag of a fish moving through water can be determined using Equation (4) [21,24]:

$$
\begin{equation*}
F=0.5 C_{d} \rho A_{s} U_{s}^{2} \tag{4}
\end{equation*}
$$

where $C_{d}$ is the drag coefficient; $\rho$ is the density of water $\left(1000 \mathrm{~kg} \cdot \mathrm{~m}^{-3}\right) ; A_{s}$ is the wetted surface area of the fish, which can be calculated using Equation (5); and $U_{s}$ is the swimming speed. $C_{d}$ is composed of two factors, the frictional drag coefficient $C_{f}$ and pressure drag coefficient $C_{p} . C_{d}$ can be obtained using Equations (6)-(8) [19,20]:

$$
\begin{gather*}
A_{s}=\alpha L^{\beta}  \tag{5}\\
C_{d}=C_{f}+C_{p} \approx 1.2 C_{f}  \tag{6}\\
C_{f}=0.074 R e^{-0.2}  \tag{7}\\
C_{d}=0.088 R e^{-0.2} \tag{8}
\end{gather*}
$$

where $R e$ is the Reynolds number, $R e=L U_{s} / v$ [33] ( $v$ is the kinematic viscosity of water); $\alpha$ and $\beta$ are empirical constants, $\alpha=0.465, \beta=2.11[20]$; and $L$ is the body length of the fish.

The metabolic rate of the fish was modelled as being divided into aerobic and anaerobic metabolism. Since the only variable in the test is swimming speed, the total mechanical power caused by differences in speed can be obtained from Equation (9):

$$
\begin{equation*}
P_{s}=\left(A M R-A M R_{0}\right) \times m \times 13.54 / 3600 \tag{9}
\end{equation*}
$$

where $A M R_{0}\left(\mathrm{mg} \mathrm{O}_{2} \cdot \mathrm{~h}^{-1} \cdot \mathrm{~kg}^{-1}\right)$ is the oxygen consumed per hour per kilogram of body mass in still water.

The drag on the fish is $F$ and thus the mechanical power needed to overcome the drag is $F U_{s}$. The energy efficiency of fish swimming under aerobic metabolism can be obtained using Equation (10) [36]:

$$
\begin{equation*}
\eta=\frac{P_{d}}{P_{s}} \times 100 \% \tag{10}
\end{equation*}
$$

where the $P_{d}(\mathrm{~W})$ is the power of the drag, $P_{d}=F U_{s}$.

### 2.4. Data Analysis

Linear regression analysis, calculated with Origin 8.5 statistical software, was applied. The drag and the energy efficiency data are expressed as means; other data (body length, total length, body mass, critical swimming speed, oxygen consumption rate, cost of transport, tail beat frequency) are expressed as means $\pm$ standard errors (mean $\pm$ SEM). The error bars in the figures below represent standard errors. The significance level for F-tests was set as $p<0.001$.

## 3. Results

### 3.1. Swimming Kinematics

The crucian carp tested under $20 \pm 1{ }^{\circ} \mathrm{C}$ water temperature conditions had a total length of $24.73 \pm 0.25 \mathrm{~cm}$, body length of $19.32 \pm 0.24 \mathrm{~cm}$, and body mass of $260.10 \pm 7.93 \mathrm{~g}$. Their $U_{\text {crit }}$ was $0.85 \pm 0.032 \mathrm{~m} / \mathrm{s}(4.40 \pm 0.16$ body lengths, BL/s). Correlation between swimming speed $U_{s}$ and tail beat frequency $T B F$ are shown in Figure 3. The equation

TBF $=0.82+5.2 U_{s}\left(R^{2}=0.977, p<0.001\right)$ was fitted to this data. For the highest water velocities, in excess of $U_{c r i t}$, the fit was poorer than for lower velocities.


Figure 3. Tail beat frequency (TBF, mean $\pm$ SEM) as a function of swimming speed (linear function: $T B F=0.82+5.2 U_{s}, R^{2}=0.977, p<0.001$ ).

### 3.2. Relationship Between Oxygen Consumption and Water Velocity

The relationship between rate of oxygen consumption and swimming speed (Figure 4) was estimated using Equation (2), $A M R=131.24+461.26 U_{s}{ }^{1.27}\left(R^{2}=0.948, p<0.001\right)$. For the highest water velocities, in excess of $U_{\text {crit }}$, the fit was poorer than for lower velocities. The rate of oxygen consumption in still water was $131.24 \pm 13.44 \mathrm{mg} \mathrm{O} 2 \cdot \mathrm{~h}^{-1} \cdot \mathrm{~kg}^{-1}$. As water velocity increased, the rate of oxygen consumption also increased, varying over velocity intervals. However, the rate of increase varied over velocity intervals. With water velocity at $1.2 \mathrm{~m} / \mathrm{s}$, the rate of oxygen consumption reached its maximum, at $621.92 \pm$ $111.74 \mathrm{mg} \mathrm{O}_{2} \cdot \mathrm{~h}^{-1} \cdot \mathrm{~kg}^{-1}$.


Figure 4. Oxygen consumption rate ( $A M R$, mean $\pm$ SEM) as a function of swimming speed (power function: $\left.A M R=131.24+461.26 U_{s}^{1.27}, R^{2}=0.948, p<0.001\right)$.

The relationship between COT and swimming speed, $U_{s}$, is shown in Figure 5. Based on the relationship between COT and $A M R$, the relationship between COT and $U_{s}$ was calculated using Equation (3), as $C O T=0.49 U_{s}{ }^{-1}+1.73 U_{s}{ }^{0.27}\left(R^{2}=0.594, p<0.001\right)$.


Figure 5. Cost of transport (COT, mean $\pm$ SEM) as a function of swimming speed (function: COT $=0.49 U_{s}{ }^{-1}+1.73 U_{s}^{0.27}, R^{2}=0.594, p<0.001$ ).

### 3.3. Drag Calculation Based on Hydrodynamic and Energy Efficiency

The relationship between drag and swimming speed is shown in Figure 6. As the speed of the water increases, drag increases, with data fitting the equation $F=0.0055+0.13$ Us ( $R^{2}=0.998, p<0.001$ ).


Figure 6. Drag, $F$, and efficiency, $\eta$, of crucian carp at different swimming speeds. Drag of fish ( $F$, mean) as function of swimming speed (Function: $F=0.0055+0.13 U s, R^{2}=0.998, p<0.001$, black solid line). Trend of $F$ relative to $U s$ (black dotted line). Trend of $\eta$ relative to $U_{s}$ (blue solid line).

Based on Equations (4)-(10), the estimated relationship between crucian carp energy efficiency and swimming speed is as shown in Figure 6. Energy efficiency increased with swimming speed, over the range 7.25-41.13\%.

## 4. Discussion

Hydraulic factors in the fishway, such as water velocity, energy and intensity of turbulence, unit volume dissipation rate and strain rate, are important determinants of whether fish successfully migrate upstream [37,38]. Fish metabolism is also affected by many ecological factors [39,40]. Stimulation caused by changing hydraulic factors in fish courses will often lead to fish exhibiting behavioral responses, and these behavioral changes can cause significant metabolic changes, which in turn cause changes in energy requirements [41].

### 4.1. Swimming Kinematics

Critical swimming speed $U_{\text {crit }}$ is an important parameter used in fishway design [42]. The $U_{\text {crit }}$ is related to the duration and speed increments used during testing [34]. In this study, with water temperature at $21 \pm 1^{\circ} \mathrm{C}, 25 \mathrm{~min}$ duration stages, and $0.15 \mathrm{~m} / \mathrm{s}$ speed increments, the critical speed for crucian carp of body length $19.32 \pm 0.24 \mathrm{~cm}$ was $0.85 \pm 0.032 \mathrm{~m} / \mathrm{s}(4.40 \pm 0.16 \mathrm{BL} / \mathrm{s})$. The critical swimming speed of crucian carp determined by Li et al. [43] was $1.19 \pm 0.024 \mathrm{~m} / \mathrm{s}(5.91 \pm 0.09 \mathrm{BL} / \mathrm{s}$, with body length of $20.0 \pm 0.80 \mathrm{~cm}$ ). The fish used in both this and Li's study were crucian carp of similar body length, but their source, habitat/environment and living habits were quite different. The crucian carp used in this study came from a fishing ground in the Wuhan area; those used in Li's study came from a fishing ground in the Harbin area, and had higher critical swimming speeds.

At a stable swimming speed, fish use their bodies and tail fins to generate a propulsive force to further increase swimming speed. Previous studies [17,32,44,45] have found that the tail beat frequency of many bony fish species positively correlated with swimming speed, and a similar trend relationship was obtained for crucian carp in this study (Figure 3). The deviation from the linear relationship and increased standard deviation apparent at higher velocities may be related to the significantly reduced sample sizes available at those velocities (Table 1).

### 4.2. The Relation Between Metabolic Rate and Speed

The relationship between the oxygen consumption rate of fish and their swimming speed is often fitted using relationships with four functional forms [46-48]: linear, exponential, logarithmic and power functions. In the hydraulic model, the mechanical energy required to overcome the drag force of a swimming fish is a power function of its velocity. In addition, a review of related literature indicated that the power function model was the most commonly used, and so this was adopted for fitting in this study. In the power function model, $A M R=\mathrm{a}+\mathrm{b} U_{s}{ }^{\mathrm{c}}$, the velocity index, c , is typically in the range 1.1-3.3 [49]. In this study, the velocity index for the crucian carp was 1.27 , reflecting their energy efficiency during aerobic exercise. (The higher the index, the lower the swimming energy efficiency, and vice versa.) In Beamish's [50] research on the velocity indexes of several bony fish species, their mean value was 2.3. The smaller speed index determined in this study indicates a higher energy efficiency.

The energy cost per unit distance, $C O T$, is a reflection of the energy efficiency of the exercise process as a whole; the lower its value, the higher the energy efficiency. In this study, the energy cost per unit distance of crucian carp showed a tendency to decrease with increasing swimming speed because of the gradually decreasing contribution of basal metabolism to the total oxygen consumption rate [51]. This result is similar to that of Cai et al. [17].

If the metabolic mode during exercise is purely aerobic, then total energy consumption can be calculated using the rate of oxygen consumption measured during testing. But many studies [29,52,53] have shown that anaerobic metabolism begins while fish are swimming below the critical swimming speed. As a result, energy consumption calculated based upon oxygen consumption will tend to underestimate total energy consumption. However, the mechanical energy necessary for fish to overcome drag can be calculated
using hydrodynamic models. The energy provided by anaerobic metabolism can then be obtained by subtracting a measure of energy consumption based on aerobic metabolism from the estimated mechanical energy required to overcome drag.

### 4.3. Drag Coefficient and Drag Force

At higher Re numbers the dynamic forces of the fluid gain importance, and bodies moving through fluids must be streamlined to reduce drag [51]. The drag force on a steadily moving fish can be calculated using the hydrodynamic model, $F=0.5 C_{d} \rho A_{s} U_{s}$. At low values of $\operatorname{Re}, A_{s}$ is usually based on frontal area whereas at higher values of $R e$, total wetted area is commonly used [51]. In this test, with $3476<R_{e}<59583$, the decision was taken to use total wetted area, calculated using Equation (5), which was formulated to describe the wetted area of a salmonid [33]. The drag coefficient $C_{d}$ is a complicated function of the Re number and its value depends on the flow conditions surrounding a body [33]. Under turbulent conditions, $C_{d}$ gradually decreases as $R e$ increases, approximately in proportion with $R e^{-0.2}$.

Using drag forces at various speeds calculated according to the hydrodynamic model, the estimated maximum energy efficiency, $\eta$, of the fish was only $41.13 \%$ (Figure 6). This is inconsistent with the observed energy efficiency of fish in nature, which exceeds $90 \%$ [36], implying that either the wetted surface area, $A_{s}$, or the drag coefficient, $C_{d}$, used in the calculation was too small, resulting in excessively low energy efficiency $\eta$. However, the incorrectness of $A_{s}$ was excluded by using the measured surface area of the fish.

According to the law of conservation of energy, calibration of $C_{d}$ was then possible. The calibration principle follows the boundary layer separation theory of hydraulics under high Reynolds number values, i.e., for $103<\operatorname{Re}<2 \times 105, C_{d}$ can be regarded as a constant, the position of the separation point remains basically constant, and $F \propto U_{\infty}{ }^{2}$ [54]. With the swimming attitude at different water velocities remaining basically unchanged, along with the angle of attack, the drag coefficient $C_{d}$ can be treated as a constant. For the range of $R e$ numbers mentioned above, $\operatorname{drag} F$ will be proportional to $U_{s}{ }^{2}$, and the power needed to overcome this drag will therefore be proportional to $U_{s}{ }^{3}$. During exercise entailing purely aerobic respiration, it can be assumed that $\eta=90 \%-100 \%\left(P_{s}=0.9-1.0 P_{d}\right)$, and fitting of the relationship between $P_{s}$ and $U_{s}$ should assume proportionality to the third power of the latter; that is, $P_{d}=k U_{s}{ }^{3}=90 \% P_{s}$ and $P_{d}=k U_{s}{ }^{3}=100 \% P_{s}$ ( $k$ is a constant). According to Equations (4) and (10), a formula for $C_{d}$ can then be derived via inversion: $C_{d}=2 k / \rho A_{s}$.

Studies have demonstrated that the energy efficiency of fish increases rapidly until around $70 \%$ of the critical swimming speed has been achieved [29,52,53], and that anaerobic metabolism starts at approximately $80 \%$ of the critical swimming speed [28,29]. Therefore, we assumed that while water velocity remained below $0.45 \mathrm{~m} / \mathrm{s}$ (less than $60 \% U_{\text {crit }}$ ) during testing, fish were relying purely upon aerobic respiration, and the oxygen consumption rate could be used to estimate the total energy consumption of fish at these swimming speeds. Power for low-flow aerobic swimming was then fitted (Figure 7) resulting in the equations $P_{s}=1.718 U_{s}{ }^{3}\left(P_{s}=0.9 P_{d}, R^{2}=0.953, p<0.001\right)$, and $P_{s}=1.910 U_{s}^{3}\left(P_{s}=1.0 P_{d}, R^{2}=0.954\right.$, $p<0.001$ ); that is, $k=1.718-1.910$. From this, the drag coefficient, $C_{d}=0.126-0.140$ can be derived, with a value larger than the estimate obtained using Equation (8) (Table 2), but smaller than the drag coefficient (0.45) [54] for flow past a round sphere at the corresponding Reynolds number. Considering that the bodies of fish are streamlined, it is reasonable that their drag coefficients are smaller than that of a sphere.


Figure 7. Power of crucian carp at different swimming speeds. The value of $P_{S}$ (black solid line) was obtained using Equation (9). $P_{d}$ (blue dotted line) were fitted using Equation $P_{d}=k U_{s}{ }^{3}$ using oxygen consumption rate data for swimming speeds not greater than $0.45 \mathrm{~m} / \mathrm{s}\left(P_{s}=0.9 P_{d}, P_{s}=1.718 U_{s}{ }^{3}\right.$, $\left.R^{2}=0.953 ; P_{s}=1.0 P_{d}, P_{s}=1.910 U_{s}{ }^{3}, R^{2}=0.954\right)$.

Table 2. Drag coefficients at various water velocities.

| Drag <br> Coefficients | $\mathbf{0 . 1 5}$ <br> $\mathbf{m} / \mathbf{s}$ | $\mathbf{0 . 3 0}$ <br> $\mathbf{m} / \mathbf{s}$ | $\mathbf{0 . 4 5}$ <br> $\mathbf{m} / \mathbf{s}$ | $\mathbf{0 . 6 0}$ <br> $\mathbf{m} / \mathbf{s}$ | $\mathbf{0 . 7 5}$ <br> $\mathbf{m} / \mathbf{s}$ | $\mathbf{0 . 9 0}$ <br> $\mathbf{m} / \mathbf{s}$ | $\mathbf{1 . 0 5}$ <br> $\mathbf{m} / \mathbf{s}$ | $\mathbf{1 . 2 0}$ <br> $\mathbf{m} / \mathbf{s}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $C_{d}$ | 0.0149 | 0.0130 | 0.0120 | 0.0113 | 0.0108 | 0.0104 | 0.0101 | 0.0098 |
| Calibration |  |  |  | $C_{d}=0.126-0.140$ |  |  |  |  |

### 4.4. Estimating the Velocity at which Anaerobic Metabolism Begins

After correcting the drag coefficient, the drag power of the crucian carp was compared with the aerobic power (Figure 7). It was found that the drag power, $P_{d}$, of the crucian carp began to exceed their aerobic power, $P_{s}$, at a water velocity of $0.52 \mathrm{~m} / \mathrm{s}$. Since drag power exceeding aerobic power likely indicates the initiation of anaerobic metabolism, the swimming speed corresponding to the point of divergence between $P_{d}$ and $P_{s}$ can be viewed as the velocity at which anaerobic metabolism is initiated, and the excess of $P_{d}$ over $P_{s}$ as an estimate of non-aerobic cost. As water velocity increased to $0.90 \mathrm{~m} / \mathrm{s}$, the power provided by aerobic metabolism reached a maximum from which it scarcely increased further. However, as water velocity increased, more power was needed, and the non-aerobic cost continued to increase. This non-aerobic cost is unfavorable to the upstream migration of fish, so, during the design of fishways, it is very important to obtain well-founded estimates of the water velocity at which anaerobic metabolism begins [53], despite the difficulty of this task [29].

In this study, by comparing the drag power and aerobic power of the crucian carp, a method of estimating the velocity at which anaerobic metabolism begins was proposed. An approximate estimate of the carp initial velocity under $20 \pm 1^{\circ} \mathrm{C}$ water temperature conditions for anaerobic metabolism of $0.52 \mathrm{~m} / \mathrm{s}$ was obtained, a finding consistent with the observation that excess post-exercise oxygen costs begin to be observed in the range $60-80 \% U_{\text {crit }}$ [28]. This value may be different for different temperatures, species and environments. In this study, for crucian carp swimming at $60 \% U_{\text {crit }}(0.51 \mathrm{~m} / \mathrm{s})$, drag power, $P_{d}$, was almost equal to the capacity of aerobic respiration to generate energy for swimming $\left(P_{d}=90 \% P_{s}\right)$; for crucian carp swimming at $80 \% U_{c r i t}, P_{d}$ was almost two times $P_{s}$; for carp swimming at $U_{c r i t}, P_{d}$ was almost 2.5 times $P_{s}$. However, as swimming speed surpassed $U_{c r i t}$, sample sizes were significantly reduced, and the quality of these estimates inevitably deteriorated. Anaerobic costs become necessary when fish swim at high speeds, and when
these exceed a certain limit, may result in irreversible damage to their health [29]. Further research is necessary to determine the maximum speeds at which fish can swim without incurring irreversible health damage.

## 5. Conclusions

Measurement results for the metabolic rates of crucian carp swimming at various speeds were obtained. A method for estimating the velocity at which anaerobic metabolism begins was proposed. A flaw of this method lies in the requirement for an assumption regarding the energy efficiency of the test fish (i.e., that it lies in the range 0.90-1.0) for calibration of the drag coefficient $C_{d}$, since the excess post-exercise oxygen consumption at high speed is not measured. A pressing need for accurate measurements of the energy efficiency and excess post-exercise oxygen consumption of a wide range of fish species remains.

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## Appendix A



Figure A1. Velocity-modulator frequency relationship. The flow velocity in the test area had a linear relationship with the frequency displayed by the motor, fitting the equation $y=0.03788 x$ (where x is the modulator frequency and $y$ is the water velocity).

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