Supplementary Materials:

S1. The calculation of the elapsed time to reach the boundary of a hexahedron in trajectory tracking

Since the coordinate system of a given meteorological field is expressed in terms of latitude and longitude in the horizontal direction and m on the ground in the vertical direction, the edges of the hexahedron extending in the east-west and north-south directions are not parallel to the x- and y-axes of the UTM coordinate system considered here. As such, it takes some effort to calculate $\Delta t_{\mathrm{x}}, \Delta t_{\mathrm{y}}$. As the planes of the hexahedron, $z = z_0$, and $z = z_1$, are perpendicular to the other planes, $\Delta t_{\mathrm{x}}, \Delta t_{\mathrm{y}}$ is defined as the time for a particle with velocity (u, v) moving in a quadrangle whose corners are points $\mathrm{A}(x_{10}, y_{10}), \mathrm{B}(x_{11}, y_{11}), \mathrm{C}(x_{00}, y_{00}), \mathrm{D}(x_{01}, y_{01})$ to reach the edge of the quadrangle. For example, in calculating Δt_{x}, which the particle moves toward the edge AB or CD is not sure, time to reach the edge AB, Δt_{x1}, and time to reach the edge CD, Δt_{x2}, are calculated. If the sign of calculated time is positive, it means that the particle moves toward the edge, the time is adopted. As shown in Figure S.1, Using the tangential component and parallel component of a wind vector $\overrightarrow{\mathrm{v_H}}$ to the edge AB, $\overrightarrow{v_{H\perp}}, \overrightarrow{v_{H\parallel}}$ and the tangential component and the parallel component of the vector from the particle coordinate O to the corner A, $\overrightarrow{OA}$ to the edge AB, $\overrightarrow{OA_{\perp}}, \overrightarrow{OA_{\parallel}}$, the following equation is obtained:

$$\overrightarrow{OA_{\parallel}} = \frac{\overrightarrow{OA} \cdot \overrightarrow{AB}}{\left|\overrightarrow{AB}\right|^2} \overrightarrow{AB}$$

then the following equation is obtained:

$$\begin{aligned} \Delta t_{x1} &= \frac{\left|\overrightarrow{OA_{\perp}}\right|}{\left|\overrightarrow{v_{H\perp}}\right|} \\ &= \frac{\left|\overrightarrow{OA} - \overrightarrow{OA_{\parallel}}\right|}{\left|\overrightarrow{v_H} - \overrightarrow{v_{H\parallel}}\right|} \end{aligned}$$

In the actual calculation, it is recommended to calculate using both the x and y component of $\overrightarrow{\mathrm{v_H}}, \overrightarrow{\mathrm{OA}}$ and the equation using the x component is obtained as follows:

$$\Delta \mathrm{t_{x1x}} = \frac{(\mathrm{x_{10}} - \mathrm{x_n}) - \dfrac{(\mathrm{x_{10}} - x_n)(x_{11} - x_{10}) + (\mathrm{y_{10}} - y_n)(y_{11} - y_{10})}{(x_{11} - x_{10})^2 + (y_{11} - y_{10})^2} \times (x_{11} - x_{10})}{u - \dfrac{u(x_{11} - x_{10}) + v(y_{11} - y_{10})}{(x_{11} - x_{10})^2 + (y_{11} - y_{10})^2} \times (x_{11} - x_{10})} \quad \ldots (\mathrm{S.4})$$

Δt_{x1x} and Δt_{x1y} are obtained using this equation and when $\Delta t_{x1x}=\Delta t_{x1y}$, Δt_{x1} is determined to be Δt_{x1x}. If $\Delta t_{x1} \leqq 0$, Δt_{x2}, time to reach the edge CD. If the denominator of Δt_{x1x} is zero, it means that the velocity vector of the particle and the edge AB or CD is parallel, so $\Delta t_{\mathrm{x}}=\infty$. Additionally, the combination of the sign of Δt_{x1} and Δt_{x2} is 0 and negative, the particles immediately returns to the domain before the move. In this case, if the particle return to the domain before the move, the combination of the sign of Δt_{x1} and Δt_{x2} is 0 and negative again, and then the particle doesn't move at the border of the domains, so the calculation is terminated. Δt_{y} is calculated with the same method.

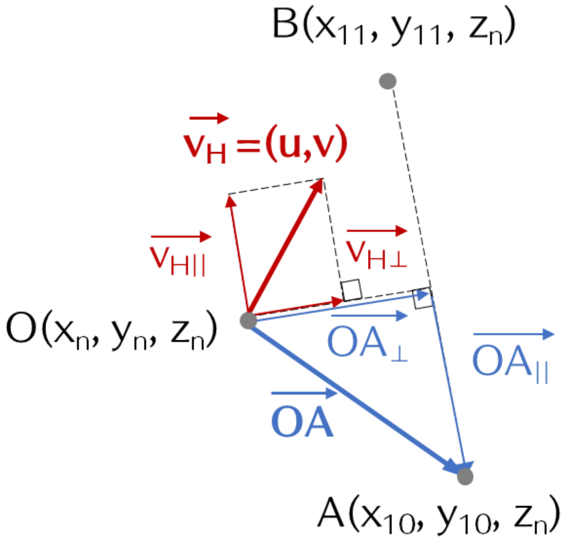

Figure S1. Schematic figure for calculating arrival time.

S2. Vertical travel distance from segregation to the achievement of the terminal velocity

In Tephra2 and Tephra4D, trajectories are calculated assuming that the initial settling velocity of the particle is equal to the terminal velocity. In this section, we consider whether the actual vertical travel distance from the start of the fall to the achievement of the terminal velocity should be taken into account. The distance x can be obtained by numerically solving the following second-order differential equation:

$$\frac{d^2x}{dt^2} = g - \frac{3C_D\rho_a}{4d\rho_p}\left(\frac{dx}{dt}\right)^2 \dots (S1)$$

where g is gravitational acceleration, C_{D} is drag coefficient, ρ_{a} is the fluid density, d is particle diameter, ρ_{p} is the particle density. In Tephra4D, the drag coefficient is obtained by Suzuki (1983):

$$C_{\mathrm{D}} = \frac{24}{R_{\mathrm{a}}}F^{-0.32} + 2\sqrt{1.07 - F} \dots (\mathrm{S2})$$

$$R_{\mathrm{a}} = \frac{\rho_{\mathrm{a}}d}{\eta_{\mathrm{a}}}\frac{dx}{dt}$$

where R_{a} is Raynolds number, F is shape factor. Taking the difference every 0.01 seconds calculating x, x at the point where the acceleration became less than 0.05m/s^2 and almost reached the terminal velocity were 0.3m, 19.1m, 174m for particles with diameters of 0.1mm, 1mm, 10mm respectively in Tephra2, and 0.15m, 6.8m, 91.2m for particles with the same diameter set in Tephra4D. These values are shorter than the travel distance from the segregation to the deposit enough to neglect.

S3. Wind field in the cases of current study

Each wind field in the cases of current study is shown in **Error! Reference source not found.**. Concerning below 2 km asl, wind was strong at the onset of eruption L ans S and weak at the onset of eruption M1 and M2.

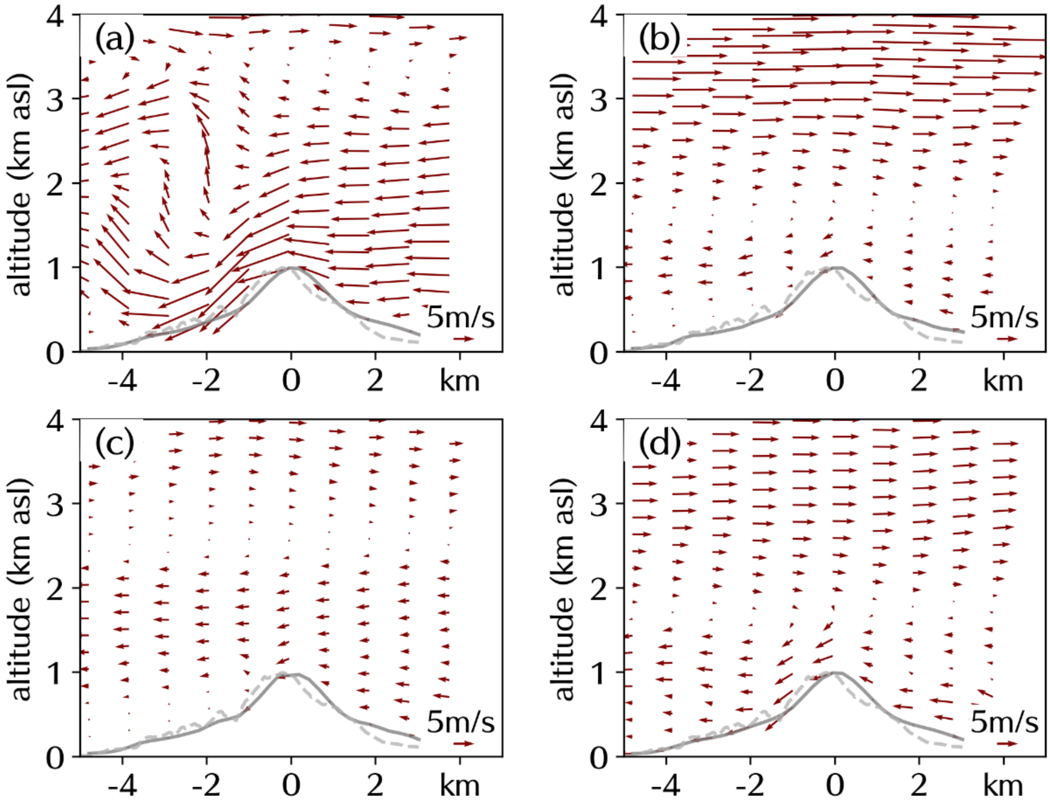

Figure S2. wind velocity field in east-west section through the Minamidake crater at the onset of eruption: (**a**) L, (**b**) M1, (**c**) M2, (**d**) S. Solid gray lines are topographic features used for trajectory calculations and dashed gray lines are topographic features used for calculating the tephra deposit load distribution. The vertical direction is emphasized twice as much as the horizontal direction in both elevation and wind.