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Study on Suppression of Transverse Mechanical Coupling of Rectangular Double-Laminated Bending Vibration Elements

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Abstract: Rectangular double-laminated piezoelectric elements are widely used in low frequency transducers. At present, most of these piezoelectric elements use a piezoelectric ceramic laminated structure. In this work, the bending vibration properties of double-laminated piezoelectric ceramic elements with different transverse dimensions were analyzed by the finite element method. It was found that there was a strong transverse mechanical coupling, and the transverse dimensions had a great effect on the bending vibration. To suppress the transverse mechanical coupling effect, a double-laminated piezoelectric element based on a 2-2 piezoelectric composite was designed. The simulation results show that the bending vibration performance of the double-laminated composite element is not affected by the transverse size. The 2-2 piezoelectric composite double-laminated and ceramic elements were prepared. In addition, simulation and experimental results indicate that the transverse mechanical coupling of the bending vibration of the piezoelectric composite double-laminated element is effectively suppressed, the vibration frequency is reduced by about 100 Hz, and the vibration displacement is increased by 2.2 times, in comparison with the piezoelectric ceramic double-laminated element.

Keywords: piezoelectric composite; double lamination; bending vibration; transverse coupling; finite element simulation

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1. Introduction

Due to its simple structure and it being lightweight, the double-laminated bending vibration element can obtain much lower resonance frequencies than the longitudinal, thickness, and radial vibration elements of a smaller size [1]. It is widely used in low-frequency transducers [2–4]. The structure of the double-laminated element is formed by gluing together two pieces of piezoelectric ceramics with the same or opposite polarization direction [5]. When the polarization direction of the two piezoelectric plates is opposite, the excitation power is connected in series. When the polarization direction of the two piezoelectric plates is the same, the excitation power is connected in parallel. Under the electric field excitation, when one piece of the piezoelectric element is elongated at a certain time, the other piece is shortened, and the whole piezoelectric element produces bending vibration [6]. The bending vibration elements of double laminations are mostly disc-shaped and rectangular, and are mainly made of piezoelectric ceramics and piezoelectric composite materials. Much research has been undertaken in recent years to study these elements. Aronov gave the resonance frequency and electromechanical coupling coefficients of non-uniform double-laminated elements by combining theory and experiment [7]. Ding et al. calculated and analyzed the bending vibration element of double-laminated elements composed of metal and piezoelectric ceramic elements under fixed boundaries [8]. Zhang et al. used the Rayleigh method to study the bending vibration element of double-laminated elements [9].

Compared with piezoelectric ceramics, piezoelectric composites can improve electromechanical coupling coefficients, remove radial or transverse modes, reduce the frequency, and change the impedance of the transducer. Smith et al. compared the performance differences between 1-3 piezoelectric composites and conventional piezoelectric ceramics. The advantages of 1-3 piezoelectric composites were pointed out and applied to ultrasound medical imaging [10,11]. Shui et al. studied a transducer fabricated with 2-2 piezoelectric composites and analyzed the effect of multiple modes on the transducer's performance. The thickness electromechanical coupling coefficient of 2-2 piezoelectric composites was calculated for the first time by theoretical analysis and combined with experiments [12]. Liu et al. made a comparison between a 2-2 piezoelectric composite double-laminated element, a 1-3 piezoelectric composite double-laminated element, and a piezoelectric ceramic double-laminated element. It was concluded that the vibration displacement of 2-2 piezoelectric composites was greater and the resonant frequency was lower [13]. Lv et al. used 2-2 piezoelectric composites to fabricate double-laminated and triple-laminated elements, which were analyzed by filling them with different polymers and under different boundary conditions. The 2-2 piezoelectric composites made of silicone rubber were obtained to have lower resonant frequencies, higher electromechanical coupling coefficients, and larger bending vibration displacements. The performance enhancement is more obvious under fixed boundary conditions [14,15]. Chen et al. analyzed the 2-2 piezoelectric composite bimorph by the analytical method. Compared with the piezoelectric bimorph, the piezoelectric composite bimorph had a larger bending vibration displacement amplitude and better acoustic radiation capability [16]. Relevant studies have shown that changes in transverse dimensions can lead to significant changes in the frequency, displacement, and other parameters of bending vibration of the double-laminated element [17,18], which is generally detrimental to the design and use of double lamination.

Therefore, this work uses the finite element method to analyze the law of bending vibration characteristics of double-laminated element changing with the transverse dimensions, and to determine the deep reasons for the changes of bending vibration frequency and vibration displacement. We propose an improved structure, which uses piezoelectric composite to replace piezoelectric ceramic sheets. The 2-2 piezoelectric composite is bonded by piezoelectric ceramics and silicone rubber. Silicone rubber has low Young's modulus and strong decoupling ability, which can reduce the influence of transverse dimensions on the performance of double-laminated bending vibration elements. Compared with the piezoelectric ceramic laminated element, the transverse mechanical coupling effect of the rectangular double-laminated bending vibration element made of 2-2 piezoelectric composite is effectively suppressed. Its vibration displacement is larger and the working frequency is lower, which provides a good way to prepare high-performance low-frequency transducers.

2. Bending Vibration Analysis of Rectangular Piezoelectric Ceramic Double Laminated

Figure 1 shows the geometry of a rectangular piezoelectric ceramic double-laminated element in series. The length of the double lamination is L, the width is W, and the thickness is T, thus the thickness of each piezoelectric ceramic is $T_0 = T/2$.

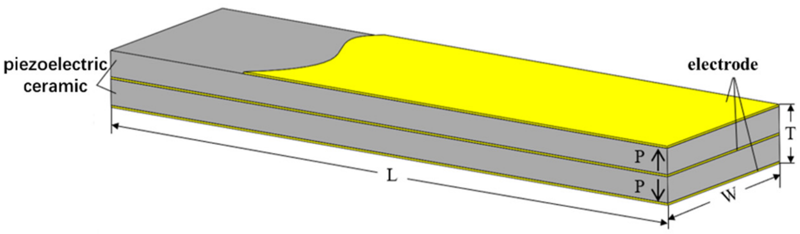


Figure 1. Rectangular double-laminated geometry.

In order to analyze the influence of transverse dimensions on the bending vibration performance of the double-laminated element, the finite element software ANSYS (15.0, ANSYS, Inc., Pittsburgh, PA, USA) is used to simulate the double-laminated element with different widths. The length and thickness of the selected piezoelectric ceramics are

L = 50 mm and T = 4 mm. The type of piezoelectric ceramic is PZT-5A and solid5 is selected as the unit type. The material parameters are shown in Table 1 [19–22]. When dividing the mesh, the side length of each cell mesh is set as 1 mm, and the function of "SWEEP" is used to sweep slightly. The influence of the adhesive layer is ignored. The alternating voltage loads of 1 V and 0 V are respectively applied on the upper and lower surfaces of the double-laminated element, and the boundary is set as the free boundary. The harmonic response analysis is adopted: the frequency range is set as 3000–7000 Hz, the number of steps is 200, the step length of each step is 20 Hz, and the damping coefficient is 0.02. Bending vibration modes with different widths are obtained, as shown in Figure 2. By extracting the resonant frequency and maximum displacement of the bending vibration under different widths, the curves of resonant frequency and maximum vibration displacement with width are obtained, as shown in Figure 3.

Table 1. Material Parameters.

Parameter	PZT-5A	Silicone Rubber
ρ (kg/m^3)	7750	1050
E (10^{10} N/m^2)	/	2.55×10^{-4}
σ	/	0.48
$c^E{}_{11}$ (10^{10} N/m^2)	12.1	/
$c^E{}_{12}$ (10^{10} N/m^2)	7.54	/
$c^E{}_{13}$ (10^{10} N/m^2)	7.52	/
$c^E{}_{33}$ (10^{10} N/m^2)	11.1	/
$c^E{}_{44}$ (10^{10} N/m^2)	2.11	/
$c^E{}_{66}$ (10^{10} N/m^2)	2.26	/
e_{31} (C/m^2)	-5.4	/
e_{33} (C/m^2)	15.8	/
e_{15} (C/m^2)	12.3	/
$\varepsilon^S{}_{11}/\varepsilon_0$	916	3.3
$\varepsilon^S{}_{33}/\varepsilon_0$	830	3.3

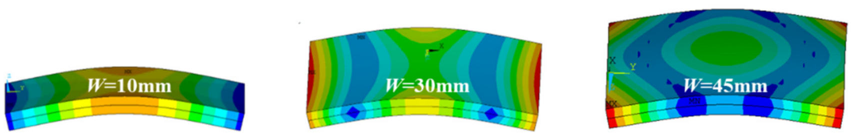


Figure 2. Bending vibration mode of piezoelectric ceramic double lamination.

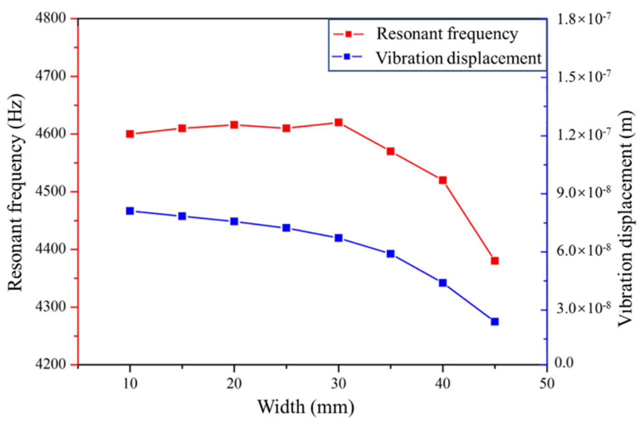


Figure 3. The curve of the resonance frequency and the maximum vibration displacement with width.

Figure 2 shows that the bending vibration mode of piezoelectric ceramic double-laminated elements changes significantly when the width is changed. In other words, when the width is small, the double-laminated element has a good bending vibration mode. With the increase of the width, the influence of the transverse bending vibration or the transverse contraction vibration on the length bending vibration is gradually enhanced, and it is difficult for the double-laminated element to maintain a good bending vibration mode. Figure 3 shows that the resonant frequency and displacement of the bending vibration of the double-laminated element also fluctuates greatly with the width change, which is mainly caused by the influence of strong transverse mechanical vibration on the bending vibration mode when the width changes.

3. Transverse Decoupling Design Based on Piezoelectric Composites

To solve the problem that piezoelectric ceramic double-laminated element is strongly affected by transverse mechanical vibration, this work proposes a double-laminated structure based on 2-2 piezoelectric composites, as shown in Figure 4. In Figure 4, the whole piezoelectric ceramic plate is replaced by a 2-2 piezoelectric composite. The reason for choosing the 2-2 piezoelectric composite is that the ceramic strips of the piezoelectric composite are filled with a polymer that can inhibit mechanical vibration coupling, and the slender state of the ceramic strip can provide a good structural foundation for the bending vibration of the double-laminated element. The polymers of piezoelectric composites mainly include epoxy resin, silicone rubber, and other materials. In general, the lower the Young's modulus of the polymer, the stronger the decoupling effect. To achieve a better decoupling effect, silicone rubber with low Young's modulus is selected as the polymer material in this work.

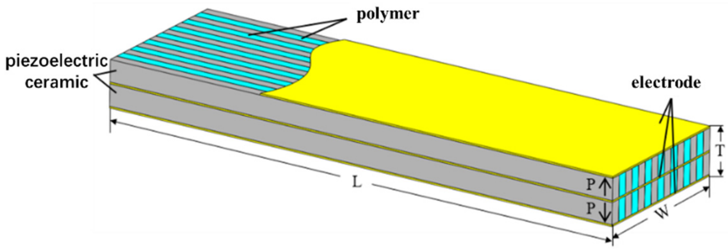


Figure 4. Double-laminated 2-2 piezoelectric composites.

In order to design a piezoelectric composite double-laminated element with more resistance to bending vibration, the influence of piezoelectric phase volume fraction on the element performance is investigated in this paper. The length L = 50 mm, width W = 9.68 mm, and thickness T = 4 mm is selected for the 2-2 piezoelectric composite double-laminated element. The width of the polymer phase remains unchanged at 0.56 mm, and the width of the piezoelectric phase is changed to 4.56 mm, 2.86 mm, 2 mm, 1.488 mm, 1.147 mm, and 0.56 mm, corresponding to piezoelectric phase volume fraction of 89.06%, 83.57%, 78.12%, 72.66%, 67.17%, and 50%, respectively. The boundary conditions and other conditions are the same as those of the piezoelectric ceramic in the earlier simulation. PZT-5A piezoelectric ceramics are still selected as piezoelectric ceramics, and the unit type is solid5. The polymer is 704 silicone rubber, and the unit type is solid45 (see Table 1 for material parameters). The curves of resonant frequency and maximum vibration displacement with volume fraction are shown in Figure 5.

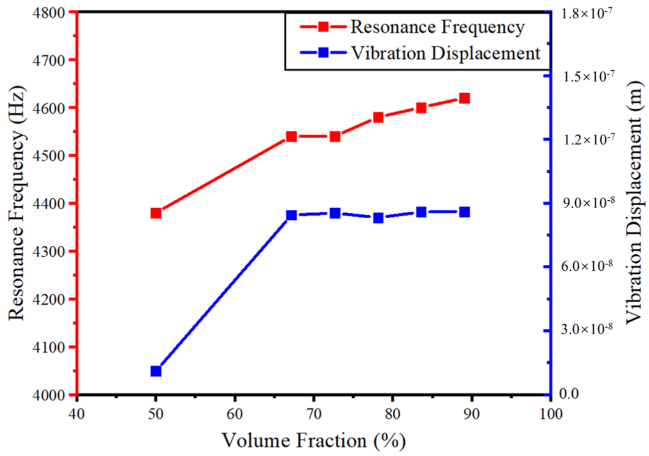


Figure 5. The curve of the resonance frequency and maximum vibration displacement with volume fraction of piezoelectric phase.

It can be seen from Figure 5 that the volume fraction of piezoelectric ceramics increases from 50% to about 70%, and the resonant frequency of the laminated element has a jump. When the volume fraction exceeds 70%, the resonant frequency rises slowly. The vibration displacement of the laminated element first increases rapidly with the increase of the volume fraction of piezoelectric ceramics, and then gradually flattens around 67% of the volume fraction and remains basically constant. In summary, the piezoelectric phase volume fraction of the laminated element is suitable between 65–90%. Combined with the actual operation process, the laminated element with a volume fraction of 78.13% is selected for analysis in this paper.

The piezoelectric composite double-laminated elements of different widths are simulated. The piezoelectric phase volume fraction of the laminated element is 78.12%, the piezoelectric phase width is 2 mm, and the polymer phase width is 0.56 mm. All other conditions are kept constant during the simulation. The vibration mode of the piezoelectric composite double-laminated elements obtained by simulation is shown in Figure 6. The variation curves of the resonant frequency and the maximum vibration displacement with the width are shown in Figure 7. In order to compare with piezoelectric ceramic double-laminated elements, the simulation results of piezoelectric ceramic double-laminated elements are also listed in Figure 7.

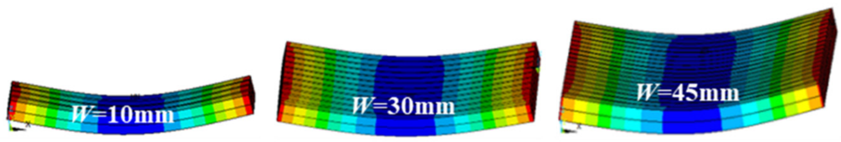


Figure 6. Vibration mode of piezoelectric composite double-laminated.

Figure 6 shows that the piezoelectric composite double-laminated element can always maintain a good bending vibration mode when the width changes and its bending vibration mode is not affected by the transverse size. Figure 7 shows that the bending vibration resonance frequency f_s and the maximum vibration displacement d of the piezoelectric composite double-laminated element do not change with the width W. The resonance frequency is always kept at around 4500 Hz, and the vibration displacement is around 8.5×10^{-8} m. The resonant frequency and vibration displacement of piezoelectric ceramic double-laminated elements are significantly affected by the width, especially since the vibration displacement decreases rapidly with the increase of the width. The reason is that the transverse mechanical coupling caused by the increase of the width inhibits the bending vibration of the piezoelectric ceramic double-laminated element. For piezoelectric composite double-laminated elements, silicone rubber has a good decoupling effect, which can maintain a good bending vibration mode for the double-laminated element, so that the resonant frequency and vibration displacement are not affected by the width size.

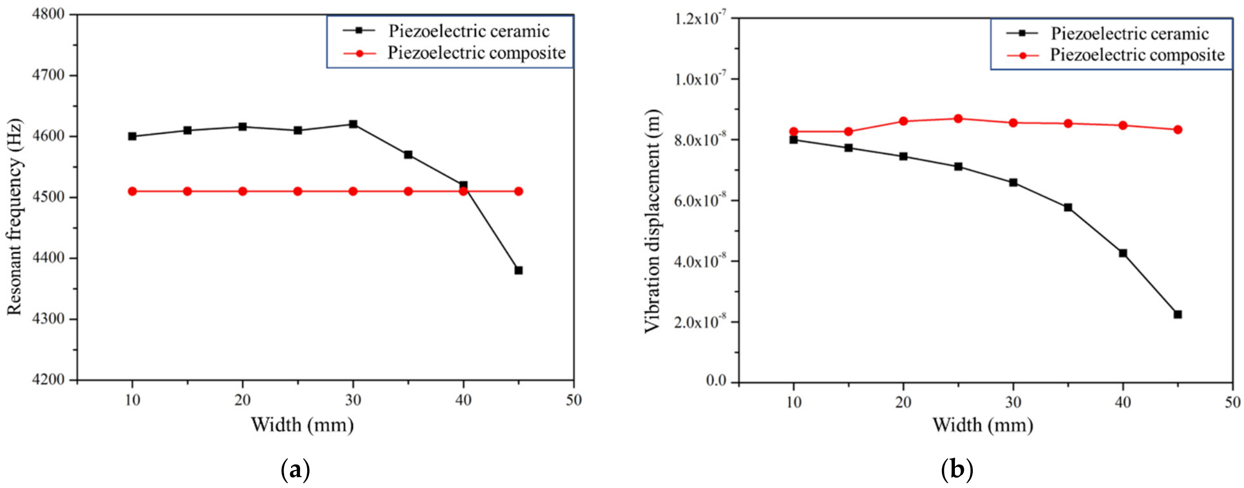


Figure 7. The curve of the resonance frequency and maximum vibration displacement of piezoelectric composite with width. (**a**) Resonant frequency; (**b**) Maximum vibration displacement.

In addition, it can be seen from Figure 7a that the resonant frequency of a composite double-laminated element is lower than that of a ceramic double-laminated element. This is because the stiffness of the piezoelectric composite double-laminated element is significantly reduced due to the addition of the polymer, which then reduces the resonant frequency of the bending vibration. In other words, under the same size, piezoelectric composite double-laminated element will be able to obtain a lower resonant frequency. Figure 7b also shows that the vibration displacement of the piezoelectric composite double-laminated element is significantly improved compared with the piezoelectric ceramic double-laminated element.

4. Fabrication and Performance Test of the Double Lamination

In order to verify whether the transverse decoupling proposed can be realized, based on the simulation results, the ceramic double-laminated elements of the same shape and size and the 2-2 piezoelectric composite double-laminated elements are prepared to compare the differences in their properties. The structural parameters of the fabricated double-laminated elements are as follows: L = 50 mm, W = 30 mm, T = 4 mm.

The preparation steps of a ceramic double-laminated element are shown in Figure 8. Firstly, a 2.5 mm PZT-5A piezoelectric ceramic is selected for grinding. A thickness of 0.5 mm is removed to make its thickness 2 mm, but the positive electrode is kept. Secondly, the positive electrodes of two polished piezoelectric ceramic plates are bonded together with epoxy resin. Finally, the upper and lower surfaces of the double-laminated element were coated with silver with a thickness of about 200 nm using a magnetron sputtering instrument. Tape is used as a mask during sputtering to prevent short circuiting between two electrodes.

As shown in Figure 9, the 2-2 piezoelectric composite double-laminated element is prepared by the cutting-filling method. Firstly, 5 mm thick PZT-5A piezoelectric ceramic is selected for cutting (the number of slices is 12), and the substrate is retained. Secondly, the slit of the piezoelectric ceramic is infilled with silicone rubber. Thirdly, the previously retained substrate is polished and removed, but the original positive surface is retained. Fourthly, the two polished ceramic positive surfaces are bonded with epoxy resin. Finally, the upper and lower surfaces were sputtered with a magnetron sputtering instrument. The sputtered material was silver with a thickness of about 200 nm. Tape is used as a mask during sputtering to prevent short circuiting between two electrodes.

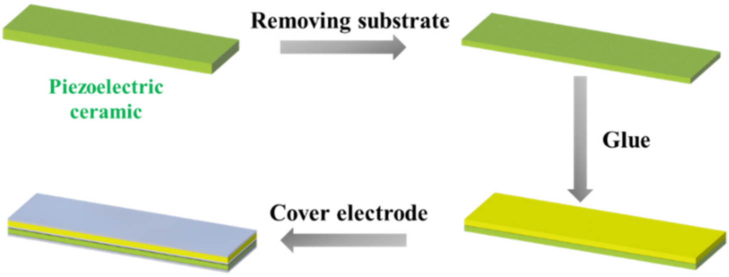


Figure 8. Flow chart of ceramic double-laminated element preparation.

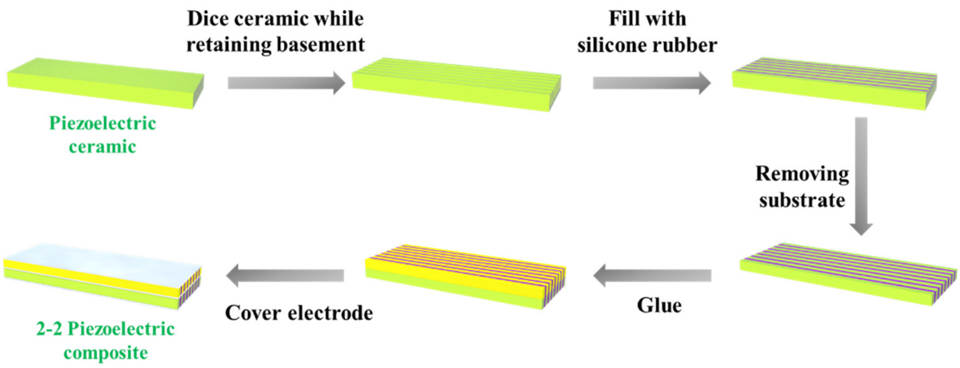
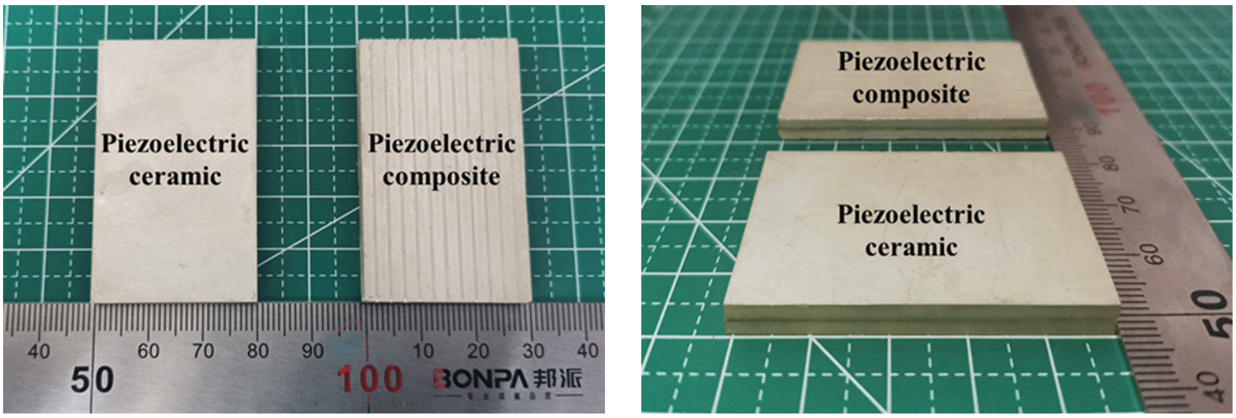


Figure 9. Flow chart of 2-2 piezoelectric composite double-laminated element preparation.

The samples of rectangular ceramic double-laminated element and rectangular piezoelectric composite double-laminated element prepared according to the above method are shown in Figure 10. Figure 10a shows a ceramic double-laminated element on the left and a piezoelectric composite double-laminated element on the right. Figure 10b shows a piezoelectric composite double-laminated element on the upper side and a ceramic double-laminated element on the lower side. Impedance analyzer and laser Doppler vibrometer are used for testing. The impedance analyzer is used to test the resonant frequency of elements. The laser Doppler vibrometer is used to observe the actual vibration of elements, and the vibration mode diagram is shown in Figure 11. The test results are compared with the simulation results, and the results are shown in Table 2.



(**a**) (**b**)

Figure 10. The physical picture of double-laminated elements. (**a**) vertical view; (**b**) side view.

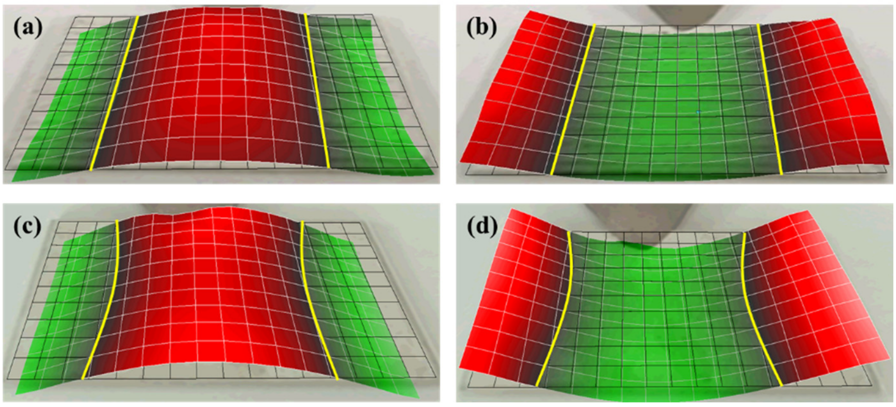


Figure 11. Vibration mode diagram of double-laminated elements. (**a**) 1/2 vibration period of the ceramic double-laminated element; (**b**) 2/2 vibration period of the ceramic double-laminated element; (**c**) 1/2 vibration period of the piezoelectric composite double-laminated element; (**d**) 2/2 vibration period of the piezoelectric composite double-laminated element.

Table 2. Comparison between experimental test performance and simulation performance.

	Type	Resonant Frequency (Hz)	Maximum Vibration Displacement ($^{-8}$ m)
Simulation	ceramic	4623	6.5
	2-2	4510	8.5
Experiment	ceramic	4700	3.5
	2-2	4580	7.9

It can be seen from Table 2 that compared with ceramic double-laminated elements, piezoelectric composite double-laminated elements have a lower resonant frequency and a larger vibration displacement. The finite element simulation is basically consistent with the actual measured resonant frequency, and the numerical error is not more than 1.5%. However, the vibration displacement of the double-laminated element obtained by simulation is far greater than the actual measured value, which may be caused by the following reasons: (1) the piezoelectric ceramic material parameters selected during actual fabrication may not conform to the material parameters given by simulation; and (2) the environment during simulation is too ideal, while the material is in the external environment during actual measurement, which may cause other defects in the elements.

The vibration mode diagram is shown in Figure 11. Figure 11a,b is a vibration period of the ceramic double-laminated element, and Figure 11c,d is a vibration period of the piezoelectric composite double-laminated element. It can be seen from Figure 11 that bending vibration occurred on both double-laminated elements. At the same time and position, the vibration amplitude of piezoelectric composite double-laminated element is obviously greater than that of ceramic double-laminated element. This is due to the decoupling effect of piezoelectric composites, which reduces the influence of transverse size on bending vibration and effectively suppresses transverse mechanical coupling.

5. Discussion

In this work, 2-2 piezoelectric composite is proposed to replace the piezoelectric ceramic sheet to suppress the transverse mechanical coupling of laminated elements. The simulation and experimental results show that: (1) under the same size, piezoelectric composites can reduce the influence of transverse size on bending vibration, which is due to the decoupling effect of piezoelectric composites; (2) compared with piezoelectric ceramics, 2-2 piezoelectric composites can produce greater vibration displacement because the Young's modulus of silicone rubber in piezoelectric composites is smaller and has reduced stiffness, which can produce greater deformation; and (3) compared with piezoelectric ceramics, 2-2 piezoelectric composites can obtain lower resonant frequency because the 2-2 piezoelectric composite has a low longitudinal wave velocity, and the material with low longitudinal wave velocity can achieve low vibration frequency.

6. Conclusions

In this work, the transverse mechanical coupling problem of double-laminated elements is studied using finite element simulation and experiment. The specific results are as follows:

(1) The bending vibration characteristics of rectangular piezoelectric ceramic double lamination are simulated by using the finite element method. The simulation results show that with the increase of transverse dimensions, the longitudinal bending vibration and transverse bending vibration of the double-laminated element have resonance coupling, and the transverse mechanical vibration coupling is gradually enhanced.

(2) In order to suppress the transverse coupling, a double-laminated structure based on 2-2 piezoelectric composites is proposed in this work. The influence of the volume fraction of piezoelectric phase on its performance is discussed, and the simulation analysis and physical preparation of the structure are carried out. The experimental results are basically consistent with the simulation results. The experimental results show that the 2-2 piezoelectric composite double-laminated element reduces the influence of transverse size on bending vibration and effectively suppresses transverse coupling. Compared with piezoelectric ceramics, the resonant frequency of 2-2 piezoelectric composite is reduced by 100 Hz and the vibration displacement is increased by 2.2 times.

Therefore, the piezoelectric composite double-laminated bending vibration element produced in this work effectively suppresses the coupling of longitudinal bending vibration and transverse bending vibration, and reduces the influence of transverse size on bending vibration. This paper provides convenient advice for the design of double lamination and lays a foundation for the manufacture of low frequency transducers, and has high practical value in the fields of underwater and air sound.

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