

Article

Equivalent Modes of Reimbursement in Augmented Contests

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Abstract: This article presents an equivalence theorem in the context of Tullock's augmented lottery contest with external or internal cost reimbursement. Three alternative modes of reimbursement are studied. The equivalence implies that, even though the augmented contest is vulnerable to framing biases, it is strategically neutral.

Keywords: Tullock; contest; Equivalent Modes; reimbursement

1. Introduction

In neoclassical economics, by the rationality principle, an individual who faces a set of alternatives chooses the most preferred alternative. When two alternatives are presented to Individual 1, he/she may prefer alternative x to y because of bias due to a framing effect and not because of the paradigmatic preference maximization assumption¹. Individual 2 may reject alternative x because it is illegal and prefer alternative y . Individual 3 may reject both x because it is illegal and y because it transgresses some moral principle or is not politically feasible. In such circumstances, if a third alternative z is available, he/she may prefer it. The problematic situations described above do not apply if the three alternatives are equivalent, equally advancing the individual's objective.

In a macroeconomic context, elected officials may face different methods of raising money from the public. During an election period, they may not support tax increases and seek an alternative option. In the context of economic policy, it is possible that two government programs are associated with framing-effect biases that determine the citizens' attitude regarding the desirable program. This means that a contest designer who takes into account the political feasibility of the alternative programs must also take into account these framing biases. It is also possible that a program is disqualified on the grounds of legal problems. Political or legal feasibility may, therefore, pave the way to the choice of a third legal alternative, the political feasibility of which may be related to the ability to simply explain and justify it to the public².

The purpose of this article is to present an equivalence result in the context of Tullock's [3] augmented all-pay rent-seeking lottery contests in which the following three alternative different reimbursement options are considered: internal funding by the loser, uniform internal reimbursement and external uniform funding, and internal and non-uniform external funding³.

Reimbursement mechanisms are thoroughly studied in the contest literature. Forms of reimbursement of expenses include internal funding, where the loser reimburses the expenses of the winner. This form is relevant in rent-seeking and litigation, where the loser pays his own litigation costs, as well as part of the winner's costs [10,11]⁴. Hehenkamp et al. [13] showed that contestants behave as if they aim to maximize relative payoff, a goal that is not only furthered by increasing one's own payoff, but also by lowering the payoff of others. Their results can explain why litigants spend too much compared to what is at stake in litigation cases.



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Cohen et al. [14] studied the following four funding modes of expenses reimbursement: internal/external funding for the winner/loser, without a combination of them. A comparison is made of the performance of the four models in terms of the planner's profit, total efforts, the ratio of winning probabilities (preserving the contests' fairness/affirmative action/discrimination), and the optimal return rate to maximize the planner's goal. This study identifies the best financing method according to the alternative performance criteria and the alternative repayment methods. Their study, nevertheless, did not show any equivalence between the modes. A mechanism for the reimbursement of expenses may result in the fulfillment of the sufficient conditions for the existence of multiple equilibria in Tullock contests. In this situation, asymmetric equilibria arise even under symmetric prizes and cost structures [15].

The equivalence of the three reimbursement options is understood in terms of the contestants' payoffs and winning probabilities and the contest designer's objective, which is usually represented by the sum of the contestants' equilibrium efforts. In the contest literature, since the agents' efforts are transferred to the designer or enhance competition that is deemed desirable, the designer is often assumed to maximize the total efforts of the contestants, Fu and Wu [6], Szymanski [16]. However, sometimes, since the agents' efforts are costly to society, the designer may want to reduce the agents' total effort, as in the work of Sela [9]. The equivalence implies that, even though the augmented contest is vulnerable to framing biases, it is strategically neutral. The equivalence could serve as an important benchmark case (e.g., in a laboratory experiment) for identifying the potential framing biases associated with the different reimbursement schemes of the three models.

2. The Three Types of Funding

Reimbursement may be internal (from the loser to the winner or vice versa), external (a third party reimburses the winner/loser) or mixed (internal and external). In addition, the external subsidization can be uniform (the designer reimburses part of the expenses of both players) or non-uniform (the designer grants a subsidy to just one contestant). Our study compares the designer's and the players' performance under three different representative variants of funding based on internal funding by the loser, mixed funding with uniform subsidization and mixed funding with non-uniform subsidization.

2.1. Model A—Internal Funding by the Loser

One can consider a Tullock contest with two players $i \in \{1, 2\}$ who compete for a prize that they value at V_i . We can denote the effort of player i by x_i , and the discriminatory power or accuracy parameter of the CSF by r . With no loss of generality, $V_1 \geq V_2$. According to the first mechanism, the losing player reimburses the winner for α percent of his/her expenses. An important very natural common legal application of this mechanism is the English rule according to which in a litigation contest, the losing party covers part of the winning party's expenses [10].

The payoff functions in this case are as follows:

$$\pi_i = (V_i + \alpha x_i) \left[\frac{x_i^r}{x_i^r + x_j^r} \right] - (\alpha x_j) \left[1 - \frac{x_i^r}{x_i^r + x_j^r} \right] - x_i, \quad i, j \in \{1, 2\}, \quad i \neq j. \quad (1)$$

2.2. Model B—Uniform Internal and External Funding

Sometimes, in addition to the internal reimbursement by the loser, the designer attempts to stimulate the total efforts by participating in the financing of the expenses of both players. If the designer feels committed to neutrality, possibly because of the uncertainty regarding the identity of the more likely winner or because of a sense of commitment to fairness (unbiasedness), he/she may grant a uniform subsidy of β percent for each unit

of effort and, in addition, the losing player reimburses the winner at α percent of his/her actual net cost. The contestants' payoffs for $i, j \in \{1, 2\}$, $i \neq j$ are as follows:

$$\pi_i = (V_i + \alpha(1 - \beta)x_i) \left[\frac{x_i^r}{x_i^r + x_j^r} \right] - (\alpha(1 - \beta)x_j) \left[1 - \frac{x_i^r}{x_i^r + x_j^r} \right] - x_i(1 - \beta). \quad (2)$$

2.3. Model C—Internal and Non-Uniform External Funding

Here, in contrast to Model B, the designer makes a stronger effort to incentivize winning by granting the winner of the contest a non-uniform higher subsidy of β percent of the total effort. The losing player still reimburses the winner for part of his/her actual net cost. Such a non-uniform external intervention might be due to the designer's desire to reveal the truth; he/she applies a stronger means to incentivize winning expecting larger efforts that, in the context of litigation, clearly increase the quality of the contest, viz., the evidence on the basis of which the court makes a decision. The contestants' payoffs in this case for $i, j \in \{1, 2\}$, $i \neq j$ become the following:

$$\pi_i = (V_i + \alpha(1 - \beta)x_i + \beta(x_i + x_j)) \left[\frac{x_i^r}{x_i^r + x_j^r} \right] - (\alpha(1 - \beta)x_j) \left[1 - \frac{x_i^r}{x_i^r + x_j^r} \right] - x_i. \quad (3)$$

We will focus on the special case of a lottery contest (the Tullock CSF in which $r = 1$)—an assumption that is quite common in the contest literature. Under this assumption, although the three funding programs are essentially different in the funding mechanism, it is shown below that the three situations are equivalent.

3. The Equivalence Theorem

Theorem 1. *In lottery contests (where $r = 1$), the three models, A, B, and C, are equivalent in terms of the designer's profit and the players' payoffs and odds of winning.*

Proof. The first-order equilibrium conditions in Models B and C are given by

$$\frac{\partial \pi_i^{B,C}}{\partial x_i} = \frac{V_i x_j}{(x_i + x_j)^2} - (1 - \alpha)(1 - \beta) = 0. \quad (4)$$

The corresponding equilibrium efforts are

$$x_i^{*B,C} = \frac{V_i^2 V_j}{(1 - \alpha)(1 - \beta)(V_i + V_j)^2}. \quad (5)$$

Equations (4) and (5) are also valid for Model A when $\beta = 0$.

Below are the equivalent second-order terms in all three models.

$$\frac{\partial^2 \pi_i^{A,B,C}}{\partial^2 x_i} = -\frac{2 V_i x_j}{(x_i + x_j)^3} < 0.$$

Accordingly, the contestants' probabilities of winning are the same in all three models.

$$p_i^{*A,B,C} = \frac{x_i^*}{x_i^* + x_j^*} = \frac{V_i}{V_i + V_j}. \quad (6)$$

The expected payoffs Equation (7) are identical in all three models and are obtained by substituting Equation (5) into Equation (1), Equations (2) or (3) as follows:

$$\pi_i^{*A,B,C} = \frac{V_i \left(V_i^2 - \alpha (V_i^2 + V_j^2) \right)}{(1 - \alpha)(V_i + V_j)^2}. \quad (7)$$

With Equation (7), to keep the payoffs and efforts non-negative in all three models, A, B, and C, it must be assured that

$$0 \leq \alpha \leq \frac{V_2^2}{V_1^2 + V_2^2}.$$

Finally, we prove that the planner's profit in all three models, A, B, and C, is the same. The planner's profit in Model A is equal to the total effort X^* , which attains its maximum value at the maximal α . The planner's profit in models B and C is equal to the total efforts minus the subsidy $X^*\beta$, i.e., $(1 - \beta)X^*$ and it also reaches its peak at the maximum α . Hence in all three cases,

$$\pi_d^* = \frac{V_1 V_2}{(1 - \alpha)(V_1 + V_2)}.$$

The maximum profit is obtained when α is maximal and equal to

$$\pi_{dmax}^* = \frac{V_2 (V_1^2 + V_2^2)}{V_1 (V_1 + V_2)}.$$

□

Even though there is no external funding in A, in contrast to B and C, the external funding rate β is symmetrical for both players and; therefore, only the efforts change. Since the rivals' efforts are reduced by the same proportion, $\frac{1}{(1-\alpha)}$ in A and $\frac{1}{(1-\alpha)(1-\beta)}$ in B and C, they leave the contestants' payoffs and winning probabilities unchanged and this is also the case for the designer's profit. By simplifying the payoff functions of Player 1 and Player 2 (see (2) and (3)) in Models B and C, we obtain the following identical payoffs:

$$\pi_i = V_i \frac{x_i}{x_i + x_j} - (1 - \beta) x_i + \alpha(1 - \beta)(x_i - x_j), \quad i, j \in \{1, 2\}, \quad i \neq j.$$

The first term is identical to the term that appears in a standard Tullock's contest. The second term implies that each player receives a subsidy of β of his/her effort. The third term means that the stronger Player 1, with the higher prize valuation, receives proportion α of the net difference (taking subsidy rate β into account) between his/her and his/her weaker rival's efforts, and the weaker Rival 2 pays 1 this difference. To conclude, the strategic equivalence of the augmented contest in the three cases, namely equality in the performance of the players and of the designer, is explained in the following two ways: the identical payoff functions in Models B and C and the proportional reduction in the contestants' efforts.

Finally, we explain why the planner's profit is the same in all three models. The profit of the planner in A is equal to the total efforts X^{*A} . In models B and C, each player has an incentive to increase his/her effort, relative to model A, in a proportion equal to the subsidy given by the planner. Algebraically, in B and C, it is the net total efforts that take into account the subsidy awarded, $(1 - \beta)X^{*B,C}$. Since $X^{*B,C} = \frac{X^{*A}}{(1-\beta)}$, we can obtain the following equation: $X^{*A} = (1 - \beta)X^{*B,C}$.

Note that for the general case of r , under symmetry in the winning prizes ($V_1 = V_2 = V$) and whenever a unique Nash equilibrium in pure strategies exists, $x_i^{*A} = \frac{rV}{2(2-(1+r)\alpha)}$,

$x_i^{*B} = \frac{rV}{2(1-\beta)(2-(1+r)\alpha)}$ and $x_i^{*C} = \frac{rV}{2(2-(1+r)(\alpha-\alpha\beta+\beta))}$. Thus, models A and B remain equivalent, but the equivalence between A and C and between B and C no longer holds. That is, the equivalence theorem is valid only for lottery contests.

4. Conclusions

Contest planners often hesitate about the mechanism they will use, when wishing to achieve the main goal for which the contest is held. However, it is important for them to ensure that the mechanism is practical, applicable by the regulator and acceptable to the players. In addition, as is well known, the way economic alternatives are presented may also bias the decision makers (framing effect).

In this paper, we presented three different modes of reimbursement of the contestants' expenses that are strategically neutral in terms of the planner's profit, the players' payoffs, and the winning probabilities. The equivalence allows a planner to offer the involved agents (the regulator and the contestants) a number of alternatives to choose from, realizing that in a lottery contest, they are equivalent. The most agreed upon mode of reimbursement will be preferred by the contest designer.

The main limitation of our study is the assumption of a lottery contest ($r = 1$), which certainly restricts the applicability of the equivalence result. The reason is that it cannot be taken for granted that the underlying technology is accurately described by the assumed lottery contest success function; it may rather entail more or less random components, implying that $r \neq 1$. Notice, however, that the lottery contest assumption is prevalent in the contest literature, and the equivalence result could serve as an important benchmark case, especially in laboratory experiments, for identifying the potential framing biases associated with the different reimbursement schemes.

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Notes

- ¹ As shown by Kahneman and Tversky [1]. In their example, 600 people are exposed to a dangerous disease. Plan A has a chance to save 1/3 of the patients, while Plan B has a chance that 2/3 of the patients will die. A significant majority of the respondents preferred Plan A, even though Plan A and B are equivalent.
- ² Aiche et al. [2] present an example of equivalence. In Tullock contests in which the common value of the prize is uncertain, there is a formal equivalence between the contest and a Cournot oligopoly with asymmetric information.
- ³ The most widely studied form of intervention in the contest literature is discrimination or favoritism; see the work of Chowdhury et al. [4], Fu and Wu [5,6] and Mealem and Nitzan [7]. Cohen and Sela [8] studied a complete reimbursement of the winner's costs by a third party. Recently, Sela [9] suggested a new form of discrimination that purports to limit the socially harmful efforts of the contestants; penalizing (rewarding) the player who makes the larger (smaller) effort.
- ⁴ Riley [12] studied a two-player all-pay-auction with a mechanism for reimbursement of refundable-only expenses to the winner. He showed that in equilibrium, under asymmetry, no agent will submit a strictly positive bid amount with positive probability. His results also explain why expenditures in rent-seeking contests fall short of the value of the prize.
- ⁵ Chowdhury and Sheremeta [17] showed that structurally different contests can be strategically equivalent in terms of equilibrium efforts rather than the designer's objective function, namely net efforts, as in our setting. Their condition for equivalent contests that yield identical equilibrium payoffs can be used to derive the set of all funding systems that are equivalent to our three types of funding.

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