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Improving GNSS/INS Tightly Coupled Positioning by Using BDS-3 Four-Frequency Observations in Urban Environments

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Abstract: Vehicular positioning in urban environments has become a research hotspot in recent years, and global navigation satellite system/inertial navigation system (GNSS/INS) tightly coupled positioning is a commonly used method. With the broadcast of BDS-3 and Galileo four-frequency signals, the multi-frequency and multi-system tightly coupled positioning method provides more possibilities for vehicular positioning in urban environments. At present, most of the studies on multi-frequency and multi-system mainly focus on static or baseline solutions, and there are few studies on the urban dynamic environment. In this paper, based on the triple-frequency GPS/BDS-2/INS tightly coupled positioning model, the BDS-3 four-frequency observation is introduced to conduct a preliminary study on the performance of GPS/BDS-2/BDS-3/INS tightly coupled positioning. Taking into account the positioning accuracy and calculation efficiency of the tightly coupled positioning, single epoch extra-wide-lane/wide-lane (EWL/WL) observation is used to participate in the modeling of tightly coupled positioning. The EWL/WL ambiguity is solved by the geometry-free (GF) method, in which triple-frequency carrier ambiguity resolution (TCAR) and four-frequency carrier ambiguity resolution (FCAR) are used for triple-frequency and four-frequency observations, respectively. Finally, the positioning performance of the tightly coupled method in this paper is evaluated through vehicular experiment. The experimental results show that in the urban dynamic environment, due to the higher quality of the linear combination of BDS-3 four-frequency, the positioning accuracy of the BDS-3/INS tightly coupled was slightly better than that of the triple-frequency BDS-2/INS. Compared with GPS/BDS-2/INS, the GPS/BDS-2/BDS-3/INS tightly coupled positioning accuracy increased by 29.1% and 58.7% in horizontal and vertical directions, respectively, which can realize decimeter positioning accuracy in urban environments.

Keywords: GNSS/INS; tightly coupled positioning; BDS-3 four-frequency; EWL/WL; urban environment



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1. Introduction

With the demands of applications such as mobile measurement, autonomous driving, and intelligent transportation increasing, vehicular positioning in urban environments has attracted much attention in recent years [1–3]. As the main application scenarios of smart carriers, the characteristics of the building structure and physical environment in cities are complex and diverse. Especially in complex environments such as tree occlusion, urban canyon, viaduct, and tunnel, it will cause global navigation satellite system (GNSS) signal attenuation, occlusion, and even interruption, seriously interfere with the quality of observation signal, and have a very adverse impact on positioning accuracy and reliability. Therefore, it is difficult to obtain reliable high-precision positioning results in complex environments such as urban canyons only by relying on GNSS [4,5].

To overcome this limitation, GNSS is often integrated and used with other sensors, such as inertial navigation system (INS), camera, lidar, and odometer [6–9]. Among them, the combination of GNSS and INS is the most common and has a wide range of applications, mainly including the loosely coupled and the tightly coupled integration. Compared with the loosely coupled architecture, the tightly coupled solution has distinctive advantages in positioning accuracy and reliability [10]. Therefore, considering the cost and the stability of the technical solution, the tightly coupled integration is more suitable for vehicular positioning in urban environments.

In the past few years, the research on GNSS/INS tightly coupled integration positioning has evolved from single-system to multi-system. The positioning methods are from single point positioning (SPP) to precision positioning, such as real-time kinematics (RTK) and precise point positioning (PPP). The observation model is from pseudorange observation to carrier phase observation. The current main research contents focus on two aspects: model construction and ambiguity resolution. In [11], a differential global positioning system/BeiDou navigation satellite system (GPS/BDS)/INS tightly coupled positioning model based on carrier phase was proposed. Subsequently, Ref. [12] studied the feasibility of a single-frequency GNSS RTK/INS tightly coupled positioning model to achieve high-precision positioning in urban environments. With the development of PPP, its integration with INS was gradually being explored by researchers. A PPP/INS tightly coupled positioning model was proposed, which has obvious improvements in positioning accuracy and initial convergence time [13]. To further achieve high-precision and reliable urban positioning and reduce the PPP convergence time, a GNSS PPP/INS tightly coupled integrated positioning model with atmospheric augmentation was proposed [14]. In addition, the constraint information (zero speed, non-holonomic, attitude, etc.) of the vehicle itself was introduced into the tightly coupled measurement model, which can further enhance the ability of vehicle high-precision positioning in the complex environment [15].

The correct resolution of ambiguity is a key issue to achieve high-precision dynamic positioning. In tightly coupled positioning, there are generally two kinds of methods to estimate the ambiguity parameters [16]. One is to take the INS position information as an additional constraint observation to assist the independent resolution of ambiguity [17]. The other is to estimate the ambiguity parameter together with other parameters to be estimated in the tightly coupled state equation [18]. However, the correct fixation of the ambiguity in the above two methods inevitably depend on the accuracy of pseudorange measurement. When the pseudorange measurement has a relatively large error due to multipath or noise, the positioning accuracy of the tightly coupled system will be decreased [19]. To avoid the difficulties caused by ambiguity resolution, a model using GPS time differential carrier phase (TDCP) to assist INS was proposed [20].

The aforementioned tightly coupled positioning models basically use single-frequency/dual-frequency carrier phase and pseudorange observations to participate in measurement update, and its development has been relatively mature. However, there are still problems in the process of ambiguity resolution. For example, once the ambiguity is fixed incorrectly or re-fixed, the stability of the tightly coupled positioning system is insufficient. In addition, if the least-squares ambiguity decorrelation adjustment (LAMBDA) method is used to fix the ambiguity, it is necessary to search for the correct integer solution through the float ambiguity solution and its covariance matrix, which increases the computational burden.

At present, all major satellite navigation systems support broadcast data with three or more frequencies for navigation and location services. With the development of multi-frequency signals, more redundant observations can be provided, which helps to resolve ambiguity and improve positioning performance [21]. The research on triple-frequency positioning was started in the 1990s. Forsell and Hatch respectively proposed a triple-frequency carrier ambiguity resolution (TCAR) method and a cascaded integer decomposition (CIR) method based on the geometry-free (GF) model. They mainly adopted the idea of stepwise rounding and fixing to achieve the solution of the triple-frequency ambigu-

ity [22,23]. Subsequently, Feng and Li applied the TCAR method to the geometry-based (GB) model and the geometry-free ionospheric (GIF) model to further improve the strength of the ambiguity model [24,25]. No matter which model is used to resolve the ambiguity, the main idea is to use the advantages of the extra-wide-lane/wide-lane (EWL/WL) combination, such as a longer wavelength, a small ionospheric delay scale factor, and a small observation noise scale factor [26]. Since the combined observation of EWL/WL can realize the rapid positioning of RTK in a single epoch, this is very meaningful for application scenarios such as autonomous driving [27,28]. Xiao combined the triple-frequency ambiguity resolution method with tightly coupled to build a triple-frequency differential GNSS/INS tightly coupled model [29]. Compared with the traditional tightly coupled model, it can obtain better real-time performance and considerable accuracy. The key is to avoid the increase of filtering order and operation burden. However, for the narrow lane (NL) ambiguity, when the observation environment is poor or the baseline is long, it is difficult to solve the ambiguity with the GF-based TCAR method, and its reliability cannot be guaranteed [30]. To ensure the system reliability of vehicle positioning in urban environments, a tightly coupled positioning model based on the differential inter-system bias (DISB) triple-frequency WL observation and INS was proposed to alleviate the impact of a lesser number of visible satellites and difficult to fix ambiguity in urban environments [31].

There are few studies on multi-frequency GNSS/INS tightly coupled positioning, mainly focusing on GPS and BDS-2 systems. With the full completion of the BDS-3 system, it can support the public broadcast of four-frequency signals (B1I, B1C, B3I, and B2a) data. Compared with the BDS-2 system, the BDS-3 system has obvious advantages in terms of system coverage, spatial signal accuracy, availability, and continuity [32]. The BDS-3 four-frequency signal provides more EWL/WL signals, and the linear combination quality is better, which can improve the efficiency of ambiguity resolution and positioning performance [33,34]. However, the research of BDS-3 four-frequency is still based on the baseline solution, and there is no research on positioning in the dynamic environment. Therefore, based on the triple-frequency GNSS/INS tightly coupled positioning model, this paper introduces the BDS-3 four-frequency observations, uses the GF model to fix the EWL/WL combined observations, and initially evaluates the influence of the BDS-3 four-frequency on GNSS/INS tightly coupled positioning. By introducing four-frequency signal observation information, more high-quality redundant observations are obtained, which further improves the positioning accuracy and reliability of the tightly coupled system.

The rest of this paper is arranged as follows: the observation equation of the BDS-3 four-frequency linear combination is first introduced in Section 2. On this basis, the appropriate EWL/WL linear combination is selected and the ambiguity gets resolved, and finally the tightly coupled positioning measurement model is constructed by using the EWL/WL observation values with fixed ambiguity. In Section 3, the single epoch ambiguity resolution and positioning results of the GNSS/INS tightly coupled using BDS-3 four-frequency in urban dynamic environments are given through vehicular experiment. The experimental results are analyzed and discussed in Section 4, and some conclusions are given in Section 5.

2. Methods

2.1. The Linear Combinations of Four-Frequency BDS-3 Observations

According to official information, BDS-3 four-frequency signals generally include B1C, B1I, B3I, and B2a; and their frequencies and wavelengths are shown in Table 1 [35]. The BDS-3 four-frequency double-difference (DD) carrier phase and pseudorange linear combination observations can be expressed as follows:

$$\nabla\Delta\Phi_{(i,j,k,m)} = \frac{i \cdot f_1 \cdot \nabla\Delta\Phi_1 + j \cdot f_2 \cdot \nabla\Delta\Phi_2 + k \cdot f_3 \cdot \nabla\Delta\Phi_3 + m \cdot f_4 \cdot \nabla\Delta\Phi_4}{i \cdot f_1 + j \cdot f_2 + k \cdot f_3 + m \cdot f_4} \quad (1)$$

$$\nabla\Delta P_{(i,j,k,m)} = \frac{i \cdot f_1 \cdot \nabla\Delta P_1 + j \cdot f_2 \cdot \nabla\Delta P_2 + k \cdot f_3 \cdot \nabla\Delta P_3 + m \cdot f_4 \cdot \nabla\Delta P_4}{i \cdot f_1 + j \cdot f_2 + k \cdot f_3 + m \cdot f_4} \quad (2)$$

where i, j, k, m are the combination coefficients of four-frequency signals; f_1, f_2, f_3 , and f_4 corresponded to B1C, B1I, B3I, and B2a, respectively; $\nabla\Delta\Phi_i$ and $\nabla\Delta P_i$ represent the DD carrier phase and pseudorange observations corresponding to the frequency (the unit is meter). The corresponding linear combination frequency, wavelength, and integer ambiguity are respectively expressed as follows:

$$f_{(i,j,k,m)} = i \cdot f_1 + j \cdot f_2 + k \cdot f_3 + m \cdot f_4 \quad (3)$$

$$\lambda_{(i,j,k,m)} = \frac{c}{i \cdot f_1 + j \cdot f_2 + k \cdot f_3 + m \cdot f_4} \quad (4)$$

$$\nabla\Delta N_{(i,j,k,m)} = i \cdot \nabla\Delta N_1 + j \cdot \nabla\Delta N_2 + k \cdot \nabla\Delta N_3 + m \cdot \nabla\Delta N_4 \quad (5)$$

where c is the speed of light; $\nabla\Delta N_i$ is the DD integer ambiguity of the corresponding frequency. Under a short baseline, the ionospheric and tropospheric delay error, satellite clock error, and receiver clock error can be eliminated through DD. The observation equation corresponding to the above DD linear combination observations can be written as follows:

$$\nabla\Delta\Phi_{(i,j,k,m)} = \nabla\Delta\rho + \lambda_{(i,j,k,m)} \nabla\Delta N_{(i,j,k,m)} + \varepsilon_{\nabla\Delta\Phi_{(i,j,k,m)}} \quad (6)$$

$$\nabla\Delta P_{(i,j,k,m)} = \nabla\Delta\rho + \varepsilon_{\nabla\Delta P_{(i,j,k,m)}} \quad (7)$$

where $\nabla\Delta\rho$ is the DD geometric distance from the satellite to the receiver; $\varepsilon_{\nabla\Delta\Phi_{(i,j,k,m)}}$ and $\varepsilon_{\nabla\Delta P_{(i,j,k,m)}}$ respectively represent the corresponding linear combination observation noise.

Generally, it is assumed that the phase observation noise of each frequency is equal and independent, and the noise accuracy of the corresponding DD carrier phase linear combination observation can be written as follows:

$$\begin{aligned} \sigma_{\varepsilon_{\nabla\Delta\Phi_{(i,j,k,m)}}}^2 &= \frac{(i \cdot f_1)^2 + (j \cdot f_2)^2 + (k \cdot f_3)^2 + (m \cdot f_4)^2}{(i \cdot f_1 + j \cdot f_2 + k \cdot f_3 + m \cdot f_4)^2} \sigma_{\varepsilon_{\nabla\Delta\Phi}}^2 \\ &= \mu_{(i,j,k,m)}^2 \sigma_{\varepsilon_{\nabla\Delta\Phi}}^2 \end{aligned} \quad (8)$$

where $\sigma_{\varepsilon_{\nabla\Delta\Phi}}$ is the equivalent DD carrier phase observation noise accuracy; $\mu_{(i,j,k,m)}$ is the phase noise factor (PNF) of linear combination observation. In addition, $\beta_{(i,j,k,m)}$ is the scale factor (ISF) corresponding to the first-order ionospheric delay of linear combination, it can be expressed as follow:

$$\beta_{(i,j,k,m)} = \frac{f_1^2(i/f_1 + j/f_2 + k/f_3 + m/f_4)}{i \cdot f_1 + j \cdot f_2 + k \cdot f_3 + m \cdot f_4} \quad (9)$$

Table 1. The frequency and wavelength information of BDS-3 four-frequency signals.

Number	Signals	Frequency (MHz)	Wavelength (m)	Code Chipping Rate (Mcps)
1	B1C	1575.420	0.1903	1.023
2	B1I	1561.098	0.1920	2.046
3	B3I	1268.520	0.2363	10.23
4	B2a	1176.450	0.2548	10.23

2.2. The Linear Combination Selection of BDS-3 Four-Frequency Observations

The linear combination observation model is given in Section 2.1. Considering the arbitrariness of the combination coefficient, an infinite number of linear combination observations can be constructed theoretically. To improve the single epoch EWL/WL ambiguity resolution, it is necessary to construct high-quality linear combined observations. For a high-quality linear combination, the following conditions need to be met, such as a long signal wavelength, a small ionospheric delay scale factor, and a small phase noise factor [36].

According to the above conditions, Table 2 shows some typical EWL/WL linear combinations of BDS-3 four-frequency signals. Among them, the wavelengths greater than 2.93 m are regarded as EWL, which are $\nabla\Delta\Phi_{(1,-1,0,0)}$, $\nabla\Delta\Phi_{(-4,5,-3,2)}$, $\nabla\Delta\Phi_{(0,0,1,-1)}$, and $\nabla\Delta\Phi_{(-1,2,-3,2)}$ respectively; and the wavelength from 0.75 m to 2.93 m is WL, which is $\nabla\Delta\Phi_{(1,0,-1,0)}$ and $\nabla\Delta\Phi_{(0,1,0,-1)}$ respectively. In EWL, the ISF of $\nabla\Delta\Phi_{(-4,5,-3,2)}$ is the smallest, but the PNF is the largest. Compared with the other three, $\nabla\Delta\Phi_{(0,0,1,-1)}$ is the most qualified, $\nabla\Delta\Phi_{(0,0,1,-1)}$ is chosen to fix the EWL ambiguity. In WL, the wavelength and PNF of the two combinations $\nabla\Delta\Phi_{(1,0,-1,0)}$ and $\nabla\Delta\Phi_{(0,1,0,-1)}$ are not much different, where the PNF of $\nabla\Delta\Phi_{(0,1,0,-1)}$ is 5.082. From the PNF point of view, we choose $\nabla\Delta\Phi_{(0,1,0,-1)}$ to fix the WL ambiguity.

Table 2. High-quality signals and the corresponding information of BDS-3 four-frequency.

Number	(i,j,k,m)	$\lambda_{(i,j,k,m)}$	$\beta_{(i,j,k,m)}$	$\mu_{(i,j,k,m)}$
1	(1,−1,0,0)	20.932	−1.009	154.858
2	(−4,5,−3,2)	5.861	−0.052	214.747
3	(0,0,1,−1)	3.256	−1.663	18.791
4	(−1,2,−3,2)	3.185	−0.489	60.338
5	(1,0,−1,0)	0.977	−1.242	6.591
6	(0,1,0,−1)	0.779	−1.351	5.082

It should be pointed out that when the four-frequency signal is used to fix the ambiguity, it is generally better to choose three EWL/WL linear combinations. For example, an EWL linear combination $\nabla\Delta\Phi_{(1,-1,0,0)}$ is added on the basis of $\nabla\Delta\Phi_{(0,0,1,-1)}$ and $\nabla\Delta\Phi_{(0,1,0,-1)}$. It is proven in [37] that the accuracy of the two is equivalent. It can be found from the literature that adding an EWL combination $\nabla\Delta\Phi_{(1,-1,0,0)}$ has little effect on the positioning result, which is basically the same. However, since the wavelength of the EWL combination $\nabla\Delta\Phi_{(1,-1,0,0)}$ is 20.932 m, its ambiguity is easily fixed and can be used to assist the ambiguity fixing of other combinations. In addition, in order to use four frequencies of BDS-3, the use of combination $\nabla\Delta\Phi_{(1,-1,0,0)}$ is added in the paper. Therefore, $\nabla\Delta\Phi_{(1,-1,0,0)}$, $\nabla\Delta\Phi_{(0,0,1,-1)}$, and $\nabla\Delta\Phi_{(0,1,0,-1)}$ are finally selected for single epoch EWL/WL ambiguity resolution.

The optimal pseudorange linear combination can be determined by a similar method. The optimal pseudorange linear combination is mainly used to generate virtual signals with higher accuracy, to assist in the fixed ambiguity. Under the short baseline, the optimal pseudorange linear combination selection of BDS-3 four-frequency has higher accuracy than the original pseudorange observation signal. Since the pseudorange observation accuracy of B1C, B1I, B3I, and B2a is different, the equal weight model is not suitable. To efficiently and reliably estimate the accuracy of these different pseudorange observations, the code chipping rate is used. According to the code chipping rate information, the pseudorange linear combination of BDS-3 four-frequency is selected as $\nabla\Delta P_{(0,1,1,0)}$, which has higher accuracy than the original pseudorange observation signal [36].

2.3. BDS-3 Four-Frequency EWL/WL Single Epoch Ambiguity Resolution Method

For four-frequency carrier phase observations, the ambiguity resolution method is similar to that of triple-frequency. In [36], it is recommended to use the FCAR method based on the GF model. The main reason is that the method is simple and can more intuitively study the single epoch fixation effect of ambiguity. In addition, considering the balance of positioning accuracy and calculation efficiency, the GF-FCAR method is used to solve the ambiguity of four-frequency observations in this paper. In the case of a short baseline, the GF model is used to resolve the EWL ambiguity by rounding to an integer, and then the

WL ambiguity is fixed using four-frequency EWL observations with a fixed ambiguity. The ambiguity resolution process of the EWL/WL based on the GF-FCAR model is as follows:

$$\nabla\Delta\hat{N}_{(i_e,j_e,k_e,m_e)} = \frac{\nabla\Delta P - \nabla\Delta\Phi_{(i_e,j_e,k_e,m_e)}}{\lambda_{(i_e,j_e,k_e,m_e)}} + \frac{\varepsilon\nabla\Delta P - \varepsilon\nabla\Delta\Phi_{(i_e,j_e,k_e,m_e)}}{\lambda_{(i_e,j_e,k_e,m_e)}} \quad (10)$$

$$\nabla\Delta\tilde{N}_{(i_e,j_e,k_e,m_e)} = \text{round}[\nabla\Delta\hat{N}_{(i_e,j_e,k_e,m_e)}] \quad (11)$$

$$\nabla\Delta\tilde{\Phi}_{(i_e,j_e,k_e,m_e)} = \nabla\Delta\Phi_{(i_e,j_e,k_e,m_e)} - \lambda_{(i_e,j_e,k_e,m_e)} \cdot \nabla\Delta\tilde{N}_{(i_e,j_e,k_e,m_e)} \quad (12)$$

where $\nabla\Delta\hat{N}_{(i_e,j_e,k_e,m_e)}$ and $\nabla\Delta\tilde{N}_{(i_e,j_e,k_e,m_e)}$ are the EWL float ambiguity and integer ambiguity, respectively; $\text{round}[\cdot]$ represents the rounding calculation.

$$\nabla\Delta\hat{N}_{(i_w,j_w,k_w,m_w)} = \frac{\nabla\Delta\tilde{\Phi}_{(i_e,j_e,k_e,m_e)} - \nabla\Delta\Phi_{(i_w,j_w,k_w,m_w)}}{\lambda_{(i_w,j_w,k_w,m_w)}} + \frac{\varepsilon\nabla\Delta\Phi_{(i_e,j_e,k_e,m_e)} - \varepsilon\nabla\Delta\Phi_{(i_w,j_w,k_w,m_w)}}{\lambda_{(i_w,j_w,k_w,m_w)}} \quad (13)$$

$$\nabla\Delta\tilde{N}_{(i_w,j_w,k_w,m_w)} = \text{round}[\nabla\Delta\hat{N}_{(i_w,j_w,k_w,m_w)}] \quad (14)$$

$$\nabla\Delta\tilde{\Phi}_{(i_w,j_w,k_w,m_w)} = \nabla\Delta\Phi_{(i_w,j_w,k_w,m_w)} - \lambda_{(i_w,j_w,k_w,m_w)} \cdot \nabla\Delta\tilde{N}_{(i_w,j_w,k_w,m_w)} \quad (15)$$

It should be noted that in short baseline positioning, the CF-FCAR method can generally be fixed to the NL ambiguity. However, the wavelength of NL observation is less than 0.19 m, which is smaller than the original carrier observation. Compared with the EWL/WL observation, it is more difficult to fix the ambiguity. In the urban dynamic environment, due to the influence of the surrounding physical environment, the noise and multipath error of the observation signal will be further amplified after DD and linear combination, the NL ambiguity is difficult to be fixed by rounding. Therefore, in order to ensure that the reliability of the positioning system is not affected by the above situation, the ambiguity of the four-frequency observation is only fixed at the WL level.

2.4. Multi-Frequency GNSS/INS Tightly Coupled Integration

2.4.1. System State Equation

The dynamic model of the GNSS/INS integrated system is mainly described by the INS error equation. The psi-angle error equation model in the navigation frame (n -frame, the three axes point to ENU, respectively) is used, it is expressed as follows [38]:

$$\begin{bmatrix} \delta\dot{\mathbf{r}}^n \\ \delta\dot{\mathbf{v}}^n \\ \dot{\boldsymbol{\psi}}^n \end{bmatrix} = \begin{bmatrix} -\boldsymbol{\omega}_{en}^n \times \delta\mathbf{r}^n + \delta\mathbf{v}^n \\ -(2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \delta\mathbf{v}^n - \boldsymbol{\psi}^n \times \mathbf{f}^n + \delta\mathbf{g}^n + \mathbf{C}_b^n \delta\mathbf{f}_{ib}^b \\ -(\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \boldsymbol{\psi}^n - \mathbf{C}_b^n \delta\boldsymbol{\omega}_{ib}^b \end{bmatrix} \quad (16)$$

where the superscript (n), the subscript (i) and (e) represent the navigation frame, the inertial (i -frame), and the earth-center-earth-fixed frame (e -frame), respectively; $\delta\mathbf{r}^n$, $\delta\mathbf{v}^n$ and $\boldsymbol{\psi}^n$ are the position, velocity, and attitude error vector, respectively; $\mathbf{C}_b^n$ is the rotation matrix from the body frame (b -frame) to the n -frame; $\boldsymbol{\omega}_{ie}^n$ is the angular velocity of earth rotation; $\boldsymbol{\omega}_{en}^n$ is the rotation vector from the e -frame to the n -frame; $\mathbf{f}^n$ is the specific force vector; $\delta\mathbf{g}^n$ is the gravity error vector in the n -frame; $\delta\mathbf{f}_{ib}^b$ and $\delta\boldsymbol{\omega}_{ib}^b$ are the accelerometer and gyro error vector in the b -frame, respectively.

In addition, inertial device errors are the most influential error sources in inertial navigation, including gyro errors and accelerometer errors. The error equation of the inertial sensor is as follows:

$$\delta\boldsymbol{\omega}_{ib}^b = \mathbf{s}_g \cdot \boldsymbol{\omega}_{ib}^b + \mathbf{b}_g + \mathbf{w}_g \quad (17)$$

$$\delta\mathbf{f}^b = \mathbf{s}_a \cdot \mathbf{f}^b + \mathbf{b}_a + \mathbf{w}_a \quad (18)$$

where s_g , s_a , b_g and b_a are the scale factor and bias vector of gyro and accelerometer, respectively; w_g and w_a are white noise.

For the device error of INS, the bias error is mainly considered. It can generally be represented by error modeling, such as first-order Markov process, random walk, and random constant. Here the bias error is modeled as a random walk, which can be expressed as follows:

$$\begin{cases} \dot{b}_a = w_{ba} \\ \dot{b}_g = w_{bg} \end{cases} \quad (19)$$

where w_{ba} and w_{bg} represent their random white noise.

The system state equation is determined by the INS dynamic model, and its system continuous state equation is defined as follows:

$$\dot{X} = F \cdot X + G \cdot W \quad (20)$$

where F represents the state transition matrix of the system; W is the noise vector of the system; G is the dynamic noise matrix of the system; X represents the system state vector, which can be written as follows:

$$X = [\delta r, \delta v, \psi, b_g, b_a] \quad (21)$$

According to the above error equation, F , G , and W can be derived, which are expressed as follows:

$$F = \begin{bmatrix} F_{rr} & F_{rv} & 0 & 0 & 0 \\ F_{vr} & F_{vv} & f^n \times & 0 & C_b^n \\ F_{\psi r} & F_{\psi v} & -(\omega_{ie}^n + \omega_{en}^n) \times & -C_b^n & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (22)$$

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_b^n & 0 & 0 \\ -C_b^n & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \quad W = \begin{bmatrix} w_g \\ w_a \\ w_{bg} \\ w_{ba} \end{bmatrix} \quad (23)$$

where F_{rr} and F_{rv} indicate state coefficients related to position; F_{vr} and F_{vv} indicate coefficients related to velocity; $F_{\psi r}$ and $F_{\psi v}$ indicate coefficients related to attitude, see [38] for specific derivation.

The continuous state Equation (18) is discretized and can be written as follows:

$$X_k = \Phi_{k,k-1} X_{k-1} + w_{k-1} \quad (24)$$

where $\Phi_{k,k-1}$ is the discrete state transition matrix; w_{k-1} is the discrete system noise vector, and its covariance matrix is Q_k . They can be obtained by Equations (25) and (26), which are specifically expressed as follows [39]:

$$\Phi_{k,k-1} \approx I + F \cdot \Delta t \quad (25)$$

$$Q_k \approx \frac{1}{2} (\Phi_{k,k-1} G_k Q_k G_k^T + G_k Q_k G_k^T \Phi_{k,k-1}^T) \cdot \Delta t \quad (26)$$

where Δt represents the Kalman filtering time interval.

2.4.2. Measurement Equation

The measurement equation represents the relationship between the state parameter to be estimated and the measured value in the same epoch. The general discrete measurement equation is expressed as follows:

$$\mathbf{Z}_k = \mathbf{H}_k \mathbf{X}_k + \mathbf{V}_k \quad (27)$$

where $\mathbf{Z}_k$ represents the measurement vector at the epoch k ; $\mathbf{H}_k$ is the measurement design matrix, which reflects the relationship between the system state and the measurement vector; and $\mathbf{V}_k$ is the measurement noise vector.

In the GNSS/INS tightly coupled system, the measurement vector at the epoch k is generally expressed as:

$$\mathbf{Z}_k = \nabla \Delta \Phi_{G,k} - \nabla \Delta \rho_{I,k} \quad (28)$$

where $\nabla \Delta \Phi_{G,k}$ represent the GNSS DD carrier observation at the epoch k ; and $\nabla \Delta \rho_{I,k}$ is the DD geometric distance calculated by INS.

Aiming at the multi-frequency GNSS/INS tightly coupled positioning model, this paper uses the single epoch DD WL observation obtained by the multi-frequency EWL/WL linear combination to replace the traditional DD carrier phase observation and pseudorange observation. The forms of four-frequency and triple-frequency are basically the same. Therefore, the measurement vector of the multi-frequency GNSS/INS tightly coupled system can be expressed as:

$$\begin{aligned} \nabla \Delta \tilde{\Phi}_{WL} - \nabla \Delta \rho_I &= (\nabla \Delta l_{WL} - \nabla \Delta l_I) \delta x + (\nabla \Delta m_{WL} - \nabla \Delta m_I) \delta y + \\ &= (\nabla \Delta n_{WL} - \nabla \Delta n_I) \delta z + (\nabla \Delta \varepsilon_{WL} - \nabla \Delta \varepsilon_I) \end{aligned} \quad (29)$$

where $\nabla \Delta \tilde{\Phi}_{WL}$ represents the multi-frequency WL observation after the ambiguity is fixed. $\nabla \Delta l$, $\nabla \Delta m$, and $\nabla \Delta n$ are the DD direction cosine; $ffix$, $ffiy$, and $ffiz$ represent the three components of the position error in the three directions; $\nabla \Delta \varepsilon$ is the DD measurement noise.

Let,

$$\nabla \Delta \mathbf{e} = \begin{bmatrix} \nabla \Delta l_{WL} - \nabla \Delta l_I & \nabla \Delta m_{WL} - \nabla \Delta m_I & \nabla \Delta n_{WL} - \nabla \Delta n_I \end{bmatrix} \quad (30)$$

$$\nabla \Delta \boldsymbol{\varepsilon} = \nabla \Delta \varepsilon_{WL} - \nabla \Delta \varepsilon_I \quad (31)$$

Considering other state vectors, $\mathbf{H}_k$ and $\mathbf{V}_k$ can be written as follows:

$$\mathbf{H}_k = \begin{bmatrix} \nabla \Delta \mathbf{e}_{n \times 3} & \mathbf{0}_{n \times 12} \end{bmatrix} \quad (32)$$

$$\mathbf{V}_k = \begin{bmatrix} \nabla \Delta \boldsymbol{\varepsilon}^1 & \nabla \Delta \boldsymbol{\varepsilon}^2 & \dots & \nabla \Delta \boldsymbol{\varepsilon}^n \end{bmatrix}^T \quad (33)$$

where n represents the number of visible satellites. Therefore, the measurement equation is written in matrix form in combination with Formulas (29) to (33):

$$\begin{bmatrix} \nabla \Delta \tilde{\Phi}_{WL}^1 - \nabla \Delta \rho_I^1 \\ \nabla \Delta \tilde{\Phi}_{WL}^2 - \nabla \Delta \rho_I^2 \\ \vdots \\ \nabla \Delta \tilde{\Phi}_{WL}^n - \nabla \Delta \rho_I^n \end{bmatrix} = \begin{bmatrix} \nabla \Delta \mathbf{e}^1 & \mathbf{0}_{1 \times 12} \\ \nabla \Delta \mathbf{e}^2 & \mathbf{0}_{1 \times 12} \\ \vdots & \vdots \\ \nabla \Delta \mathbf{e}^n & \mathbf{0}_{1 \times 12} \end{bmatrix} \cdot \mathbf{X} + \begin{bmatrix} \nabla \Delta \boldsymbol{\varepsilon}^1 \\ \nabla \Delta \boldsymbol{\varepsilon}^2 \\ \vdots \\ \nabla \Delta \boldsymbol{\varepsilon}^n \end{bmatrix} \quad (34)$$

2.4.3. Integration Scheme

Figure 1 is a flow chart of the multi-frequency GNSS/INS tightly coupled integrated positioning method proposed in this paper. Based on the GPS/BDS-2 triple-frequency WL observation, the BDS-3 four-frequency carrier observation is added to participate in the tightly coupled positioning. Firstly, the BDS-3 four-frequency DD linear combination observation model is constructed; and then the BDS-3 four-frequency optimal EWL/WL

linear combination observation is selected according to certain criteria. Then, the EWL/WL ambiguity is solved by the GF model, in which the TCAR method is used for GPS/BDS-2 triple-frequency observations, and the FCAR method is used for BDS-3 four-frequency observations. Finally, the difference between the WL observation with the fixed ambiguity and the DD geometric distance calculated by the INS is taken as the measurement vector into the extended Kalman filter (EKF). The estimated positioning error is used to correct the approximate coordinates calculated by the INS, and the accurate positioning result after the current epoch correction is output, to realize the multi-frequency tightly coupled GNSS/INS integrated positioning.

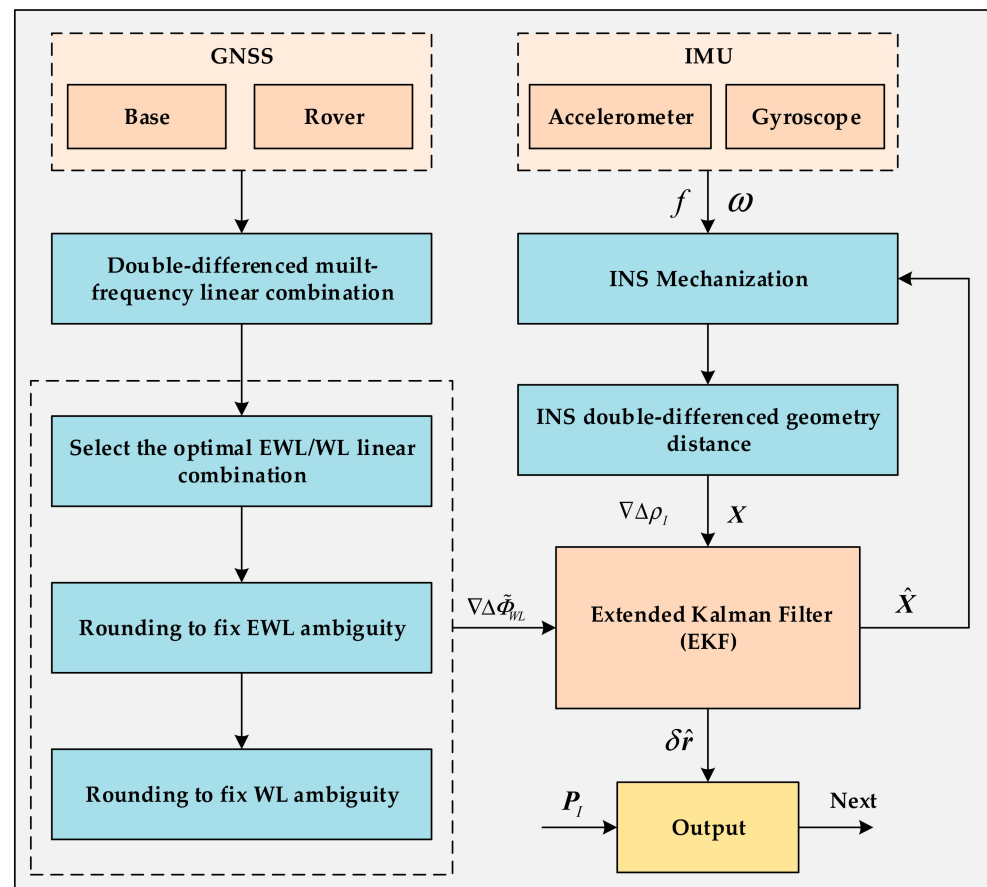


Figure 1. The flow chart of the multi-frequency GNSS/INS tightly coupled positioning method.

3. Results

3.1. Field Test Description

To evaluate the positioning performance of the multi-frequency GNSS/INS tightly coupled after using the BDS-3 four-frequency observations in urban environments, a land vehicular test of the integrated navigation system was carried out in Nanjing, China on 22 October 2020. The experimental test hardware platform and equipment were shown in Figure 2. The inertial data acquisition was from a MEMS-IMU (ADIS16488A). The IMU was composed of three MEMS gyros and three quartz accelerometers. Its output frequency was 200 Hz. The main performance indicators can be obtained from Table 3. The GNSS data were collected by two receivers composed of Sinan K708 board cards, both of which support receiving BDS-3 data, one of which was located on the roof of a college of Southeast University as a base station. For the sampling rate of the GNSS receiver, both the base station and the rover were set to 1 Hz. In addition, the lever arm offset between the GNSS antenna phase center of the rover station and the IMU measurement center has been accurately measured before the test.

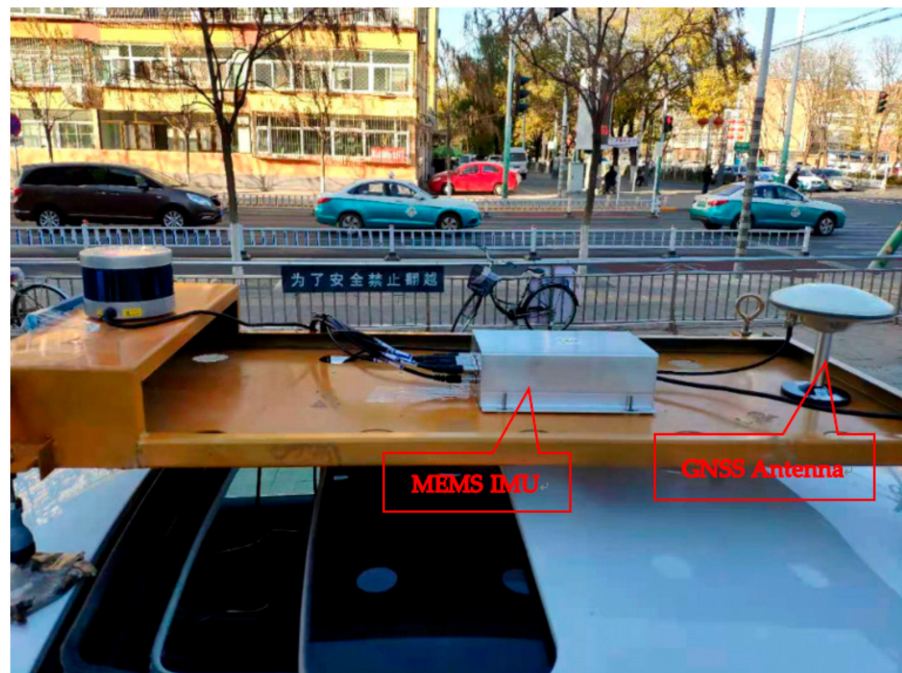


Figure 2. The hardware platform and equipment of the vehicular experiment.

Table 3. The specific performance indicator of the MEMS-IMU.

IMU	Bias Instability (1 σ)	Random Walk (1 σ)	Bias Repeatability (1 σ)
Gyro.	5.1°/h	0.26°/ $\sqrt{h}$	$\pm 0.2^\circ/s$
Accel.	0.07 mg	0.029 m/s/ $\sqrt{h}$	± 16 mg

The experimental data collection time lasted for about 40 min, in which the test route included scenes such as open sky conditions and trees occluded. This paper took about 15 min of data for processing, including the above environment time lasting as long as possible. The vehicular trajectory was shown in Figure 3, and the maximum baseline length did not exceed 5 km. In the experiment, the post-processing tightly coupled differential GNSS/INS integrated positioning and smoothing result solved by the commercial software Waypoint Inertia Explorer was used as a reference value. The initial navigation parameter information of the INS can be obtained through the reference value. The vehicular speed and attitude changes during the experiment were shown in Figure 4. The driving speed was within 20 m/s.

The experiment collected satellite observation data of the GPS, BDS-2, and BDS-3 systems. Among them, GPS and BDS-2 have triple-frequency observation data, namely L1, L2, and L5 and B1, B2, and B3 respectively. BDS-3 has four-frequency observation data, namely B1C, B1I, B3I, and B2a. In the process of data processing, triple-frequency EWL/WL linear combination was constructed for triple-frequency observations, and four-frequency EWL/WL linear combination was constructed for four-frequency observations. Among them, the EWL and WL combinations of triple-frequency GPS/BDS-2 are (0,−1,1) and (1,−1,0), respectively, and the EWL combination of pseudorange is (0,1,1). The selection of the above EWL/WL combination can be referred to [40]. To research the influence of BDS-3 four-frequency observations on GNSS/INS tightly coupled positioning, firstly, the signal quality of GPS/BDS-2 triple-frequency observations and BDS-3 four-frequency observations in this experimental environment were given, including the number of visible satellites and corresponding position dilution of precision (PDOP), as well as the signal-to-noise ratio (SNR) of satellite observations. Secondly, the single epoch EWL/WL ambiguity resolution

was analyzed, due to the different observation quality of triple-frequency data and four-frequency data, their ambiguity resolution will also be different. Finally, GNSS/INS tightly coupled positioning results with or without BDS-3 four-frequency observations were given.



Figure 3. The vehicular trajectory of the experiment (Green: reference result; red: our result).

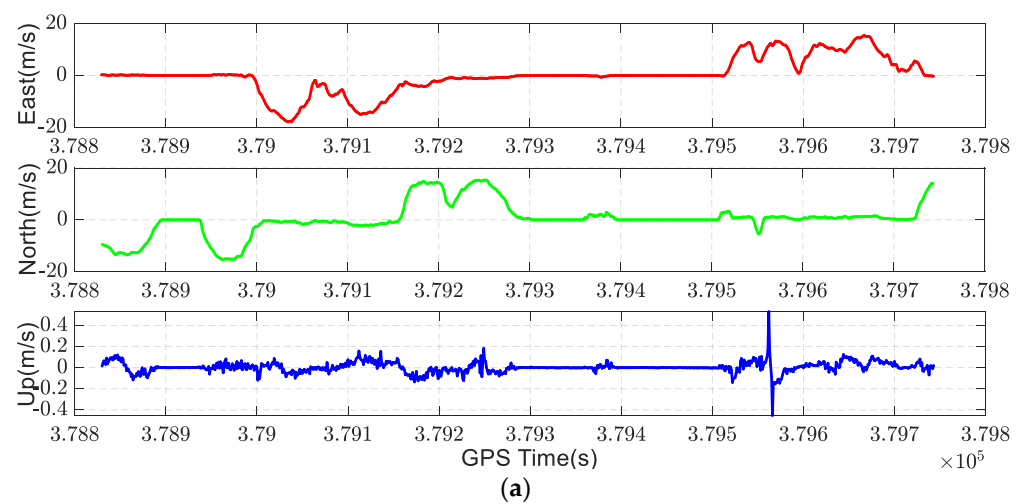


Figure 4. Cont.

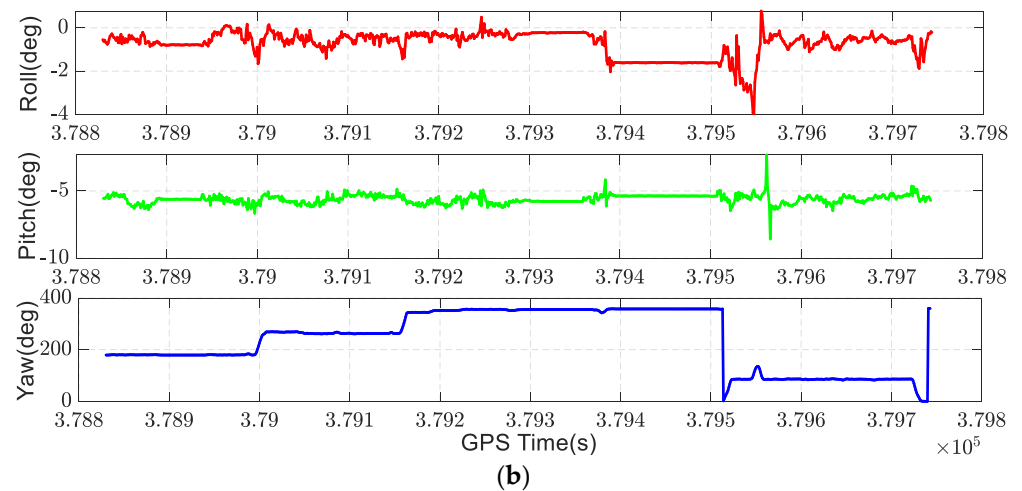


Figure 4. The vehicular speed and attitude in the field test: (a) the vehicular speed; and (b) the vehicular attitude.

3.2. Satellite Availability

In this section, the signal quality of satellite observations during the experiment was evaluated in terms of the number of visible satellites, PDOP, and SNR. The number of visible satellites and the PDOP value of the rover station during the experiment are shown in Figure 5, which is mainly divided into three types: GPS/BDS-2, BDS-3, and GPS/BDS-2/BDS-3. Among them, GPS and BDS-2 are combined here. One reason is that compared with BDS-2, the number of visible satellites of GPS is basically at 4, and there are only 3 satellites in a few time periods, so its PDOP value cannot be calculated. The other is to study the influence of BDS-3 on GNSS/INS tightly coupled positioning based on GPS/BDS-2, and the experimental result will be more intuitive.

According to Figure 5a, it can be found that there were about 5 visible satellites of BDS-3, which can be used for positioning. GPS/BDS-2 had a relatively large number of visible satellites, about 11 on average. However, during some periods of the experiment, due to the occlusion of trees, frequent satellite changes occurred. Especially in the last stage, the number of visible satellites decreased significantly. The PDOP value in Figure 5b was also further explained. When the number of visible satellites increased, its PDOP value also significantly improved.

Figure 6 shows the SNR of the triple-frequency and four-frequency observations during the experiment. The triple-frequency signal shows the observation quality through the BDS-2 C07 satellite SNR, and the four-frequency signal shows the observation quality through the BDS-3 C36 satellite SNR. As can be seen from Figure 6 that the SNR of the three frequencies (B1, B2, B3) of the C07 satellite was above 30 dB-Hz, and most of the time period was 35–40 dB-Hz. The SNR of the four frequencies (B1C, B1I, B2a, B3I) of the C36 satellite was also above 30 dB-Hz. The SNR of B2a was between 35–40 dB-Hz, and the other three frequencies were between 40–50 dB-Hz. Compared with the three frequencies of BDS-2, the SNR of each frequency of BDS-3 was overall higher, and the signal observation quality was better.

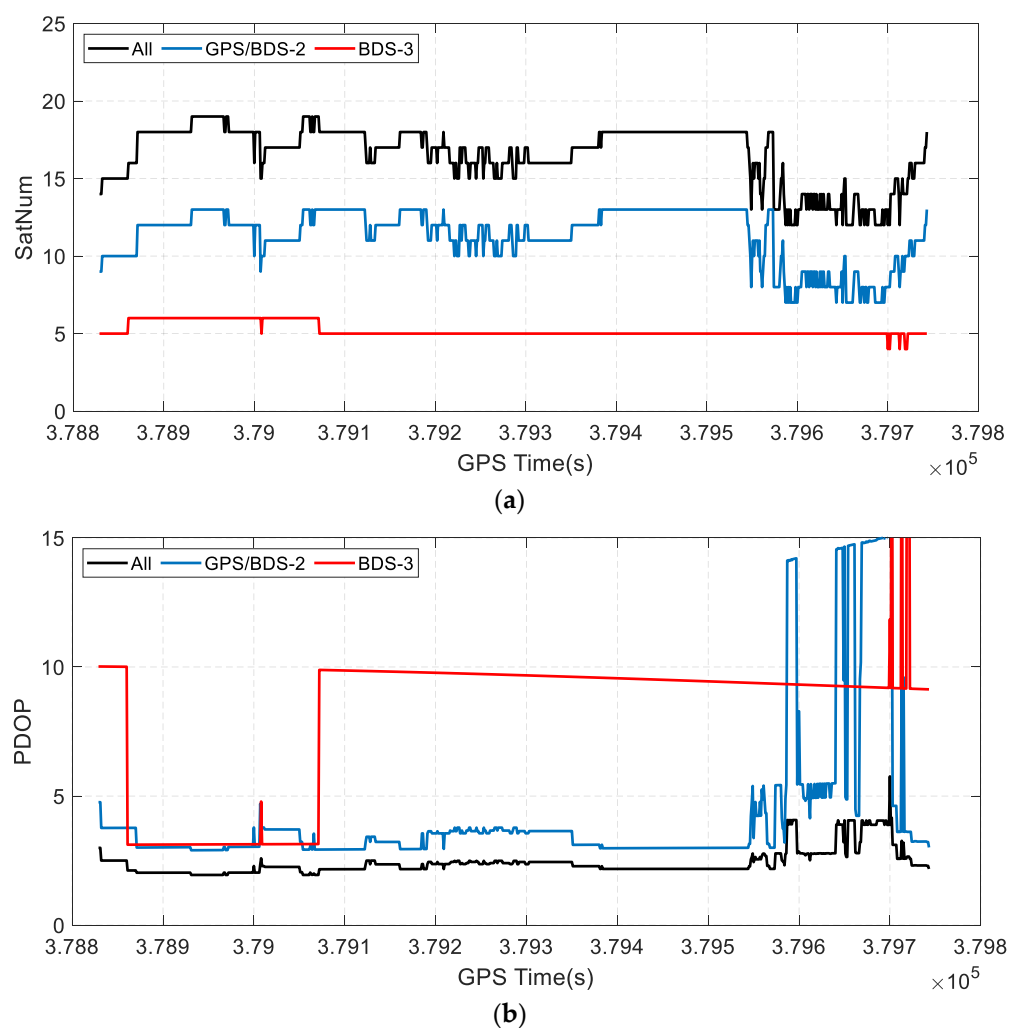


Figure 5. The number of visible satellites and PDOP of GPS/BDS-2, BDS-3, and GPS/BDS-2/BDS-3 (all) in the rover station: (a) number of satellites; (b) PDOP.

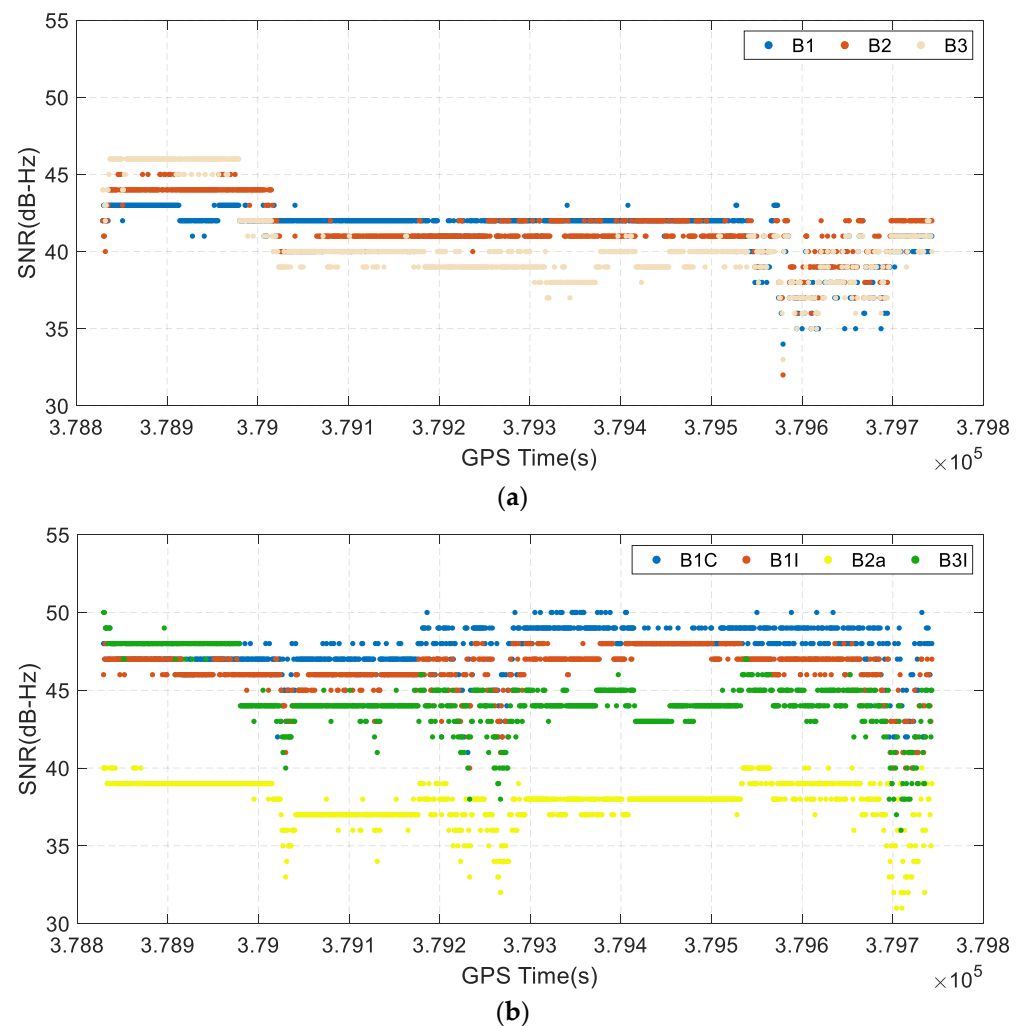


Figure 6. The signal-to-noise ratio of triple-frequency and four-frequency observations in the experiment: (a) BDS-2 C07; (b) BDS-3 C36.

3.3. Single Epoch EWL/WL Ambiguity Resolution

For single epoch EWL/WL ambiguity resolution, there is no need to detect and identify cycle slips since the ambiguity is processed epoch-by-epoch. To evaluate the performance of each linear combination, the ambiguity resolution success rate used is widely adopted. It is defined as follows:

$$P = \frac{N_{suc}}{N_{all}} \times 100\% \quad (35)$$

where P is the ambiguity resolution success rate; N_{suc} and N_{all} are the epoch numbers of successful ambiguity resolution and total epochs, respectively. However, in dynamic positioning, there is no prior reference solution information, so it is impossible to directly judge whether the ambiguity is fixed successfully. Therefore, it was considered to use the ambiguity fractions to evaluate the fixation of ambiguity. The ambiguity fractions represent the difference between the single epoch float ambiguity and the true ambiguity [33]. When the probability distribution of fractions is within a certain range, it can be considered that within this range, its ambiguity is successfully fixed.

The performance of the single epoch EWL AR was first studied. The EWL ambiguity fractions of the GPS/BDS-2 and BDS-3 are given in Figure 7, different colors represent different satellite pairs. Among them, BDS-3 has two EWL ambiguity fractions, namely EWL1 and EWL2. EWL1 can be used to assist the ambiguity fix of EWL2. As can be seen that the trend of the EWL ambiguity fractions of GPS/BDS-2 and BDS-3 was basically the

same, within 0.2 cycles. A few scattered points were affected by the noise and multipath of pseudorange observations in the EWL AR calculation. Table 4 shows mean and STD values for EWL ambiguity fractions of GPS/BDS-2, BDS-3 EWL1, and BDS-3 EWL2. It can be found from the table that most EWL ambiguity fractions were within the cycle, and the average value were almost zero. For GPS/BDS-2, the STD of the EWL ambiguity fractions was 0.068 cycles, and for BDS-3, it was 0.058 cycles and 0.069 cycles, respectively. In the EWL AR, the ambiguity fixed rates of GPS/BDS-2 and BDS-3 were 98.9%, 99.0%, and 98.8%, respectively, within 0.2 cycles, which can be solved reliably by using the single epoch method.

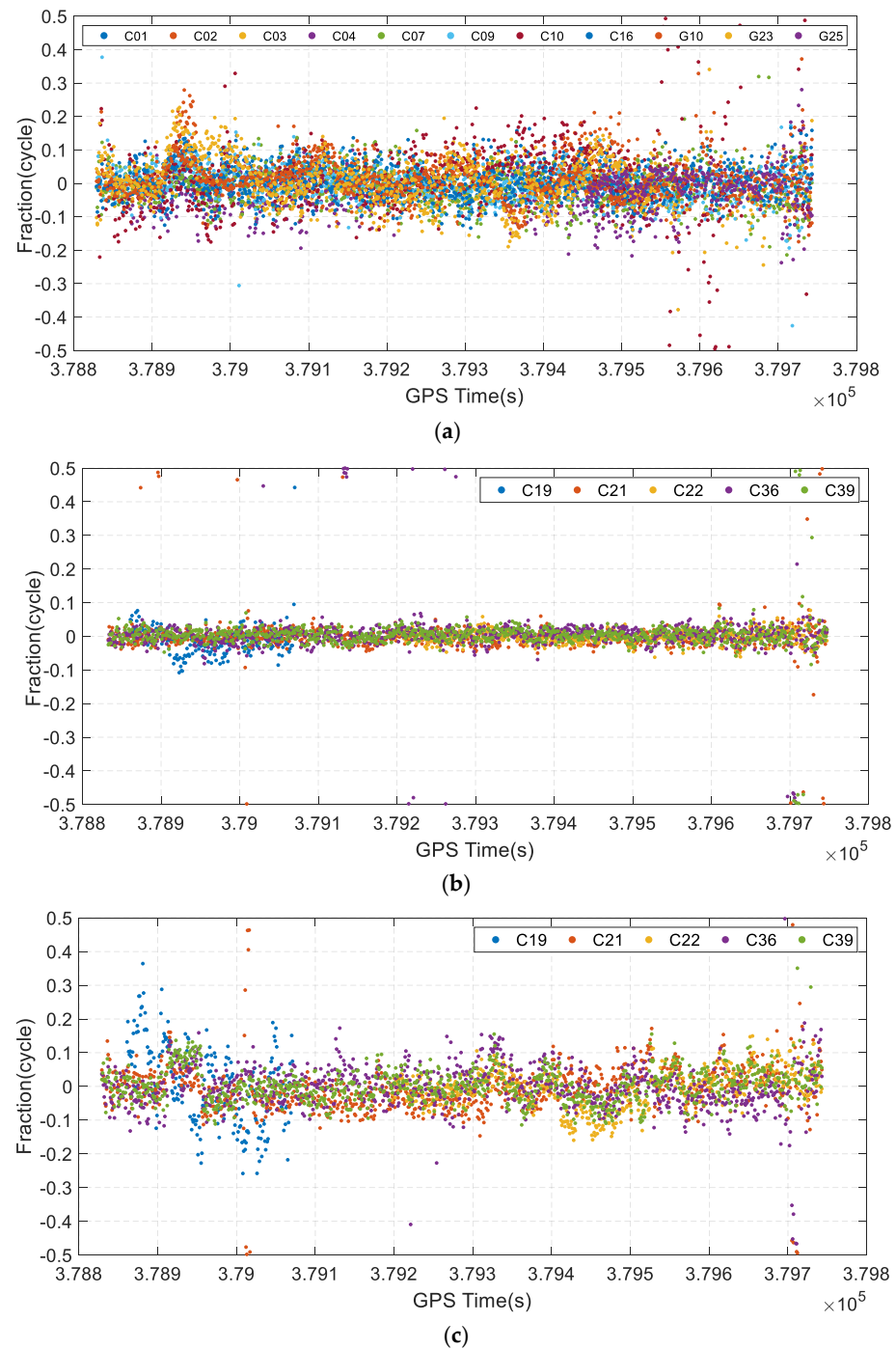
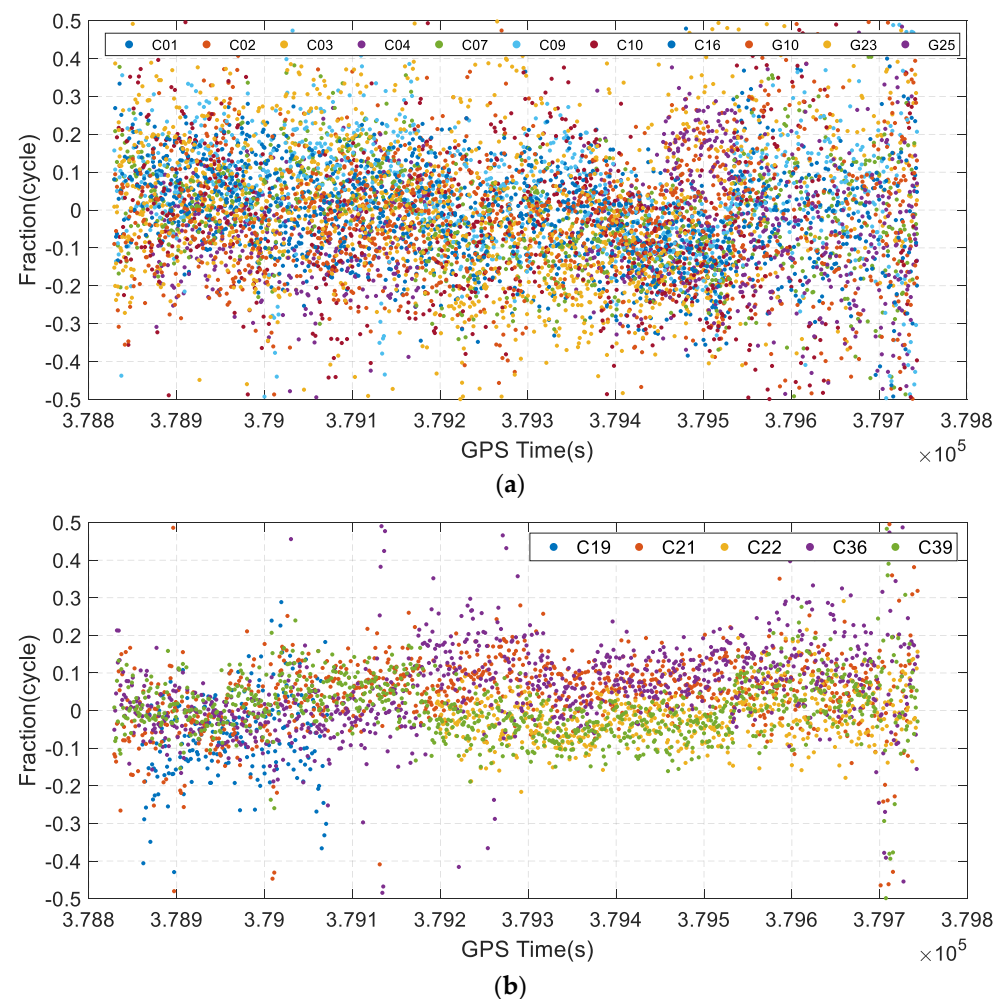


Figure 7. The EWL ambiguity fractions of the GPS/BDS-2 and BDS-3: (a) GPS/BDS-2; (b) BDS-3 EWL1; (c) BDS-3 EWL2.

Table 4. Mean and STD values for EWL ambiguity fractions of GPS/BDS-2, BDS-3.

	GPS/BDS-2	BDS-3 EWL1	BDS-3 EWL2
Mean/cycle	−0.002	0.001	−0.002
STD/cycle	0.068	0.058	0.069

Similarly, the WL ambiguity was still rounded and fixed using the GF model. Figure 8 shows the WL ambiguity fractions of GPS/BDS-2 and BDS-3. As can be seen, unlike EWL, the WL ambiguity fractions of GPS/BDS-2 and BDS-3 had a significant gap. Among them, the WL ambiguity fractions of BDS-3 was basically within 0.3 cycles, which indicated the feasibility of single epoch WL AR. While the WL ambiguity fractions of GPS/BDS-2 was more scattered, which had something to do with the quality of observation data in the dynamic environment. Table 5 shows mean and STD values for WL ambiguity fractions. It can be seen that the mean and STD value of WL ambiguity fractions of BDS-3 were better than GPS/BDS-2, and their values were less than 0.03 and 0.2 cycles, respectively. The WL ambiguity fractions of GPS/BDS-2/BDS-3 were also given here.

**Figure 8.** The WL ambiguity fractions of the GPS/BDS-2 and BDS-3: (a) GPS/BDS-2; (b) BDS-3.**Table 5.** Mean and STD values for WL ambiguity fractions of GPS/BDS-2, BDS-3, and all.

	GPS/BDS-2	BDS-3	GPS/BDS-2/BDS-3
Mean/cycle	−0.112	0.022	−0.001
STD/cycle	0.164	0.104	0.147

To further ensure the reliability of WL AR, the probability distribution of GPS/BDS-2 and BDS-3 WL ambiguity fractions under different thresholds $|\delta|$ are given in Table 6, i.e., ambiguity fixed rate. It can be seen that the smaller the threshold, the smaller the probability of their ambiguity fractions. Among them, the probability of ambiguity fractions of BDS-3 was still 94.7% within 0.2 cycles, while GPS/BDS-2 was significantly worse. For the static baseline solution, to ensure the reliability of AR, the threshold was generally set to 0.2 cycles. However, for the positioning in the urban dynamic environment, the observation was significantly affected by noise and multipath and was not stable. Hence, the threshold was set to within 0.4 cycles. At this time, the ambiguity fractions probability of GPS/BDS-2 and BDS-3 was 97.7% and 99.1%, respectively, which can guarantee the reliability of the positioning results most of the time.

Table 6. The probability distribution of WL ambiguity fractions of GPS/BDS-2, BDS-3, and GPS/BDS-2/BDS-3 under different thresholds $|\delta|$.

$ \delta /\text{Cycle}$	GPS/BDS-2	BDS-3	GPS/BDS-2/BDS-3
<0.45	0.989	0.995	0.990
<0.4	0.977	0.991	0.982
<0.3	0.927	0.983	0.945
<0.2	0.789	0.947	0.842

3.4. Tightly Coupled GNSS/INS Positioning Performance

After the above analysis, the positioning performance of the GNSS/INS tightly coupled with or without BDS-3 four-frequency observations in urban environments was evaluated. First, the tightly coupled positioning error of the BDS-2 triple-frequency and BDS-3 four-frequency with INS was given, as shown in Figure 9. Through comparison, it can be found that the positioning error of the BDS-3/INS tightly coupled was smaller than the BDS-2/INS tightly coupled in the E, N, and U directions. In the vertical direction, the tightly coupled positioning errors of the two were obviously different. In the last period of time, there were fewer outliers in the BDS-3/INS tightly coupled positioning result, and the positioning error became smaller.

The statistical results of the tightly coupled positioning error of BDS-2/INS and BDS-3/INS are given in Table 7. By comparison with the BDS-2/INS, the BDS-3/INS tightly coupled positioning accuracy was increased by 31.1% and 57.3% in the horizontal and vertical directions, respectively, which further verified the above situation. Furthermore, it was explained that the combination of BDS-3 four-frequency WL observation and INS can meet the decimeter/meter-level positioning requirements in the urban environment, and its tightly coupled positioning has better positioning accuracy and reliability than BDS-2/INS.

Table 7. Positioning error statistics of the BDS-2/INS and BDS-3/INS tightly coupled.

	RMS/m				
	E	N	U	2D	3D
BDS-2/INS	0.510	0.350	1.951	0.512	2.047
BDS-3/INS	0.239	0.260	0.834	0.353	0.906
Improvement	53.1%	25.7%	57.3%	31.1%	55.7%

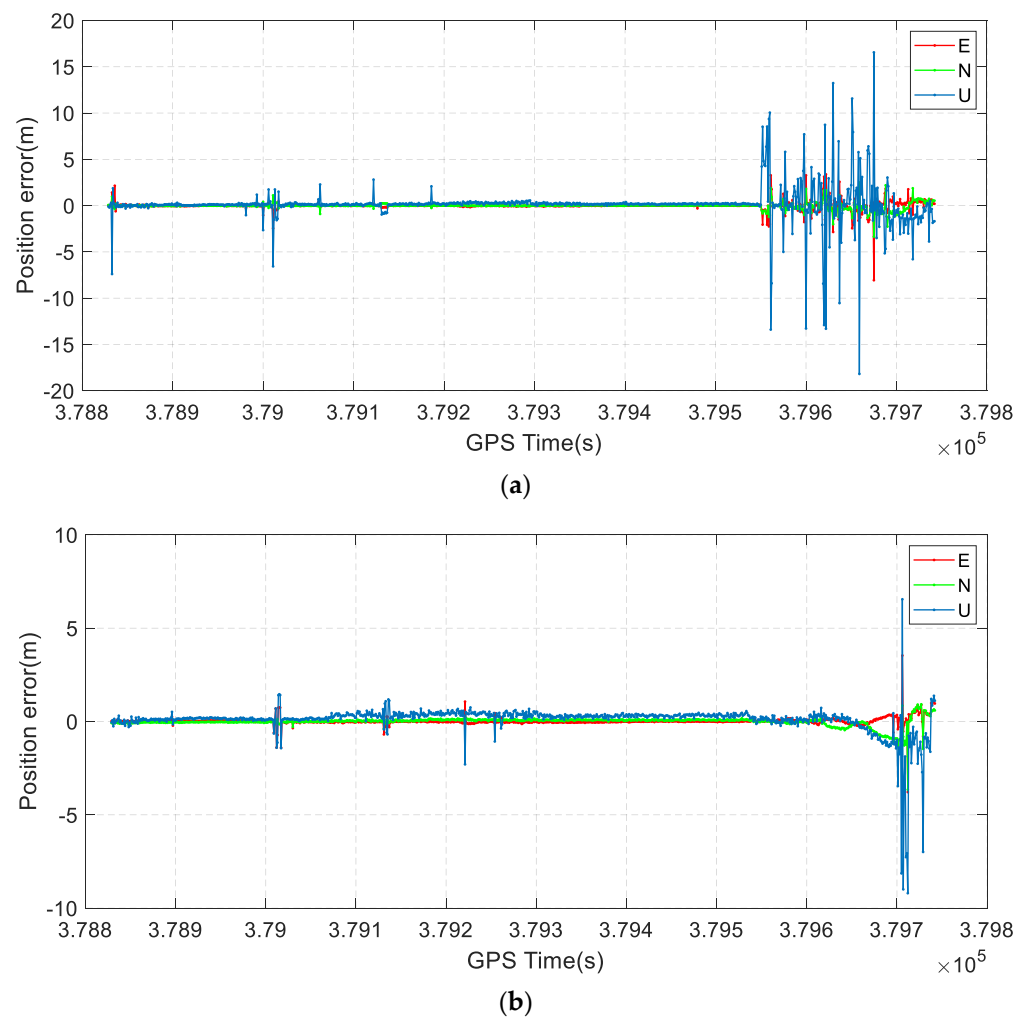


Figure 9. The tightly coupled positioning error of the BDS-2 triple-frequency and BDS-3 four-frequency with INS: (a) BDS-2/INS; (b) BDS-3/INS.

After analyzing the positioning performance of BDS-3 four-frequency WL observation and its tightly coupled positioning with INS, the positioning error of GPS/BDS-2/INS tightly coupled positioning with or without BDS-3 four-frequency observations was finally given, as shown in Figure 10. As can be clearly found, the positioning accuracy of the tightly coupled with the participation of the BDS-3 four-frequency has been greatly improved, and the positioning error was smaller in the place where the observation environment was poor in the last segment.

Through further quantitative analysis, the statistical results of the GNSS/INS tightly coupled positioning error with or without BDS-3 four-frequency WL observation were given in Table 8. Compared with GPS/BDS-2/INS, the GPS/BDS-2/BDS-3/INS tightly coupled positioning accuracy was improved by 36.4%, 23.8%, and 58.7% in the E, N, and U directions, respectively. In particular, the vertical was the highest. It was believed that the satellite structure distribution of BDS-3 was better, which was a very important reason. On the whole, the GPS/BDS-2/BDS-3/INS tightly coupled positioning can achieve decimeter-level positioning accuracy in the horizontal and vertical directions, respectively, which is in line with theoretical expectations.

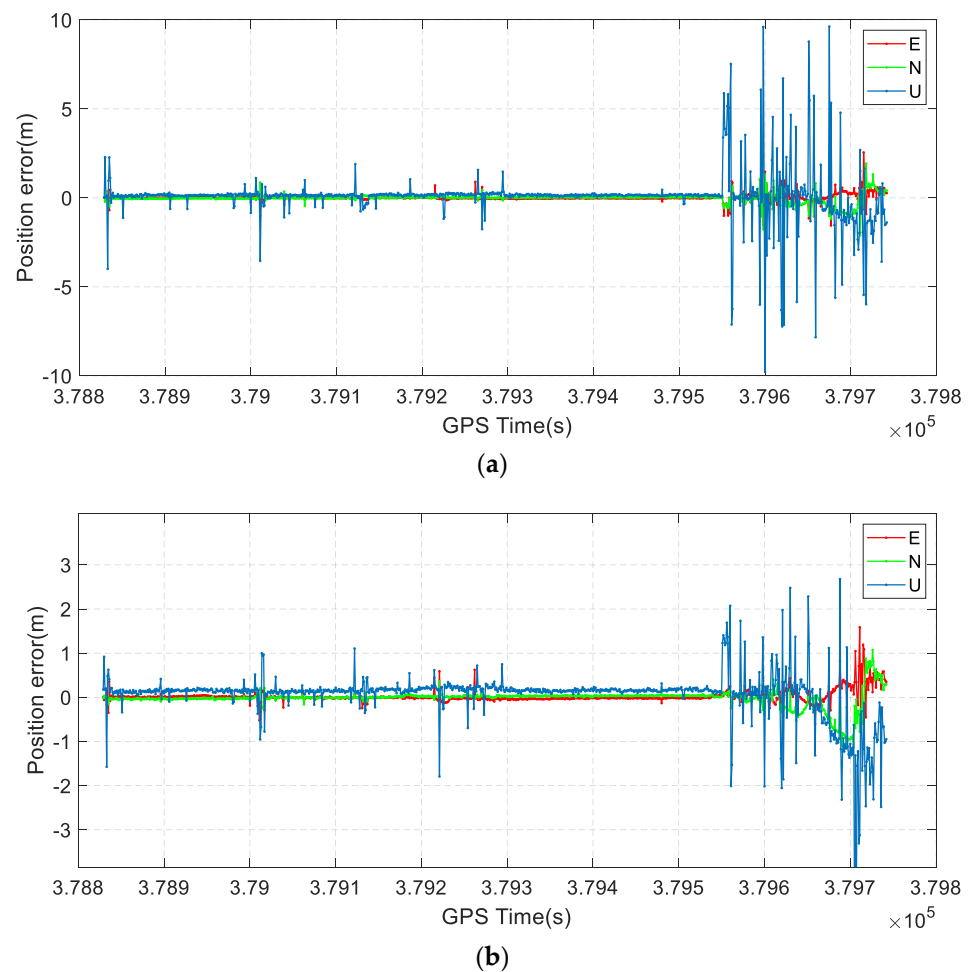


Figure 10. The GNSS/INS tightly coupled positioning results of different satellite systems and INS: (a) GPS/BDS-2/INS; (b) GPS/BDS-2/BDS-3/INS.

Table 8. Positioning error statistics of the GNSS/INS tightly coupled with or without BDS-3.

	RMS/m				
	E	N	U	2D	3D
GPS/BDS-2/INS	0.236	0.269	1.345	0.358	1.392
GPS/BDS-2/BDS-3/INS	0.150	0.205	0.558	0.254	0.613
Improvement	36.4%	23.8%	58.7%	29.1%	56.0%

4. Discussion

In Section 3, through a set of vehicular experiments, the GNSS/INS tightly coupled positioning performance with or without BDS-3 four-frequency observations in urban environments was evaluated. First of all, from the observation data, the number of visible satellites of BDS-3 was obviously less than that of GPS/BDS-2, but it was basically about 5, which can complete positioning independently. However, in the last part of the experiment, the observation environment became significantly worse, and the PDOP value also reflected the distribution of its satellite structure. At this time, the PDOP of GPS/BDS-2 was worse than that of BDS-3. This was also one of the reasons why the positioning result of BDS-3 was better than that of GPS/BDS-2 in the vertical direction. In addition, the SNR was also one of the indicators reflecting the quality of the observation data. Compared with the BDS-2 triple-frequency signal, the BDS-3 four-frequency signal had a higher SNR as a whole. B2a was the lowest, but it was still above 30 dB-Hz. It can be found from Figure 6

that in the last part of the experiment, the SNR of the three frequencies of BDS-2 has all been reduced, which further shows that the quality of the BDS-2 observation data is poor.

Secondly, the single epoch EWL AR of GPS/BDS-2 and BDS-3 can basically be reliably fixed from the perspective of the single epoch EWL/WL AR. Among them, the influence of pseudorange observation noise and multipath was not obvious, mainly because the wavelength of EWL was longer and AR was easier. However, in a dynamic environment, there were still a few points that were relatively scattered, which can be seen from the ambiguity fractions. In WL AR, the ambiguity fractions of GPS/BDS-2 was more scattered than that of BDS-3. There were two reasons for this phenomenon. For one thing, GPS/BDS-2 had large pseudorange noise and multipath in the dynamic environment; in another respect, its EWL wavelength and noise were relatively large and the calculation of WL was affected by them. The mean and STD value of the WL ambiguity fractions in Table 5 also verified the above phenomenon. In the dynamic environment, the reliability of AR can only be further explained by the probability distribution of ambiguity fractions, because there was no reference integer solution. Compared with GPS/BDS-2, BDS-3 had a higher fixed rate of ambiguity at different thresholds, within 94%. To be more rigorous, the threshold can be set to within 0.3. At this time, GPS/BDS-2 and BDS-3 still have fixed rates of 92.7% and 98.3%. Therefore, it can be believed that WL AR was relatively reliable for most of the time.

Finally, it was believed that the BDS-3/INS tightly coupled positioning result was better by comparing with the BDS-2/INS tightly coupled positioning, which was mainly reflected in the vertical direction. This situation was analyzed in detail above, and it had something to do with the quality of the observation. In addition, the experimental results also show that the BDS-3 four-frequency WL observation can achieve decimeter/meter-level positioning accuracy in urban environments. On this basis, the GNSS/INS tightly coupled positioning results with or without the BDS-3 four-frequency were compared with each other. In the horizontal direction and the vertical direction, the positioning accuracy of GPS/BDS-2/BDS-3 tightly coupled was significantly improved, reaching 29.1% and 58.7%, respectively. This clearly reflects the advantages of multi-frequency and multi-system positioning in urban environments. In general, it can be believed that the participation of BDS-3 four-frequency observations can improve the accuracy and reliability of GNSS/INS tightly coupled positioning in urban environments. This is not only to add one more satellite system, but more importantly, to build a higher-quality four-frequency linear combination, which helps rapid fixing of EWL/WL ambiguity and improvement of positioning accuracy.

The influence of BDS-3 four-frequency WL observations on GNSS/INS tightly coupled positioning is focused on in this paper. However, the quality control of tightly coupled positioning in urban environments is expected to be further studied. In addition, the collection of multi-frequency data in complex urban environments is limited by equipment and other factors. We have also made preparations for this and will further study it in the future.

5. Conclusions

The BDS-3 four-frequency WL observation is introduced to improve the positioning accuracy and reliability of the GNSS/INS tightly coupled system in urban environments based on the triple-frequency GPS/BDS-2/INS tightly coupled positioning model in this paper. The positioning performance of GNSS/INS tightly coupled with or without BDS-3 four-frequency is evaluated through vehicular experiment. The experimental results show that the positioning accuracy of GPS/BDS-2/BDS-3/INS tightly coupled is 0.254 m and 0.558 m in the horizontal and vertical direction, respectively. By comparison with GPS/BDS-2/INS, the results have increased by 29.1% and 58.7% in the horizontal and vertical direction, respectively. In consequence, the multi-frequency and multi-system tightly coupled positioning with the participation of BDS-3 four-frequency WL observation can achieve fast and reliable decimeter-level positioning accuracy in urban environments, which is in line with theoretical expectations. In addition, the BDS-3 four-frequency has a

higher quality linear combination, which has a slightly higher positioning accuracy than BDS-2/INS; and the BDS-3/INS tightly coupled can also achieve decimeter/meter-level positioning accuracy.

The next step is to study the quality control of tightly coupled positioning in urban environments, especially complex environments, which is expected to achieve a more stable urban vehicular positioning system.

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