

The original Stefan formula is as follows:

$$\text{MSFD} = \sqrt{2k_f \frac{n_f \cdot \text{FI}}{p_b \text{WL}}} \quad (\text{S1})$$

where MSFD is the annual maximum seasonally frozen depth (m), FI is the air-freezing index ($^{\circ}\text{C}$), k_f represents the thermal conductivity of frozen ground ($\text{W m}^{-1} \text{ }^{\circ}\text{C}^{-1}$), n_f is the n factor during the freezing season, which is equal to the ratio between the surface-freezing index and air-freezing index, p_b is the soil bulk density (kg m^{-3}), W is the soil water content by weight (dimensionless), and L is the latent heat of fusion (J kg^{-1}) [9, 31].

In fact, Equation (S1) is usually simplified as:

$$\text{MSFD} = E\sqrt{\text{FI}} \quad (\text{S2})$$

where E is the edaphic factor, and can be defined as [43]:

$$E = \sqrt{\frac{2k_f \cdot n_f}{p_b \text{WL}}} \quad (\text{S3})$$

Equation (S2) is the simplified Stefan formula. To estimate the MSFD, the air-freezing index (i.e., FI) at each meteorological station was calculated firstly. In this study, FI is the sum of all daily temperatures below $0 \text{ }^{\circ}\text{C}$ from September to August. Secondly, we estimated the constant E value for each station using the observational MSFD, FI, and Equation (S2) based on all the observation data from 1981 to 2010. Based on the FI and constant E value for each station, we can obtain the estimated MSFD from 1975 to 1984 and 2011 to 2014, and then the estimation accuracy of the MSFD is verified compared to the observed MSFD.