Article

Analysis of interval data envelopment efficiency model considering different distribution characteristics — Based on environmental performance evaluation of manufacturing industry

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0. supplementary materials

0.1. Theorem

Theorem 1. *Assume that $\xi_1, \xi_2, \cdots$, and ξ_n are independent uncertain variables obeying regular uncertainty distributions $h_1, h_2, \cdots$, and h_n, respectively, and $h_1(x), h_2(x), \cdots$, and $h_n(x)$, and $h_0(x)$ are real-valued functions.*

$$M\{\sum_{i=1}^{n} h_i(x)\xi_i \le h_0(x)\} \ge \alpha$$

holds if and only if

$$\sum_{i=1}^{n} h_i^{+}(x)\phi_i^{-1}(\alpha) - \sum_{i=1}^{n} h_i^{-}(x)\phi_i^{-1}(1-\alpha) \le h_0(x)$$

where

$$h_i^{+}(x) = \begin{cases} h_i(x) & h_i(x) > 0 \\ 0 & h_i(x) \le 0 \end{cases}$$

$$h_i^{-}(x) = \begin{cases} -h_i(x) & h_i(x) < 0 \\ 0 & h_i(x) \ge 0 \end{cases}$$

Theorem 2. *The equivalent deterministic form of chance Constraint (12-1) in Model (12) is as follows:*

$$\sum_{j=1}^{n} -\lambda_j(\alpha_{rj}y_{rjU} + (1-\alpha_{rj})y_{rjL}) \le (-S_r^{+}) - ((1-\alpha_{rj})y_{rj_0U} + \alpha_{rj}y_{rj_0L})$$

Theorem 3. *The certain deterministic form of the Constraint (16-2) in the model is as follows:*

$$\sum_{j=1}^{n} \lambda_j(\overline{e}_{ij} - (\sqrt{3}\overline{\sigma}_{ij})/\pi ln((\overline{\alpha}_{ij})/(1-\overline{\alpha}_{ij}))) + S_i^{-} \le \theta * ((\overline{e}_{x_{ij_0}} - (\sqrt{3}\overline{\sigma}_{x_{ij_0}})/\pi ln((\overline{\alpha}_{ij})/(1-\overline{\alpha}_{ij}))))$$

0.2. Proof

Constraint

$$\sum_{j=1}^{n} \lambda_j\widetilde{y}_{rj} \ge \widetilde{y}_{rj_0} + S_r^{+}$$

Multiplied by (-1) *on the left and right sides, the inequation is subjected to a linear distribution, which is converted to the following:*

$$-\sum_{j=1}^{n}\lambda_j\widetilde{y}_{rj} \leq (-S_r^+) - \widetilde{y}_{rj_0}$$

Furthermore, let

$$\widetilde{x}_{ij} \sim \ell\,[x_{ijL}, x_{ijU}], \widetilde{y}_{rj} \sim \ell\,[y_{rjL}, y_{rjU}], \widetilde{z}_{rj} = -\widetilde{y}_{rj} \sim \ell\,[-y_{rjU}, -y_{rjL}]$$

So Constraint (12-1) could be converted as follows:

$$M\{\sum_{j=1}^{n}\lambda_j\widetilde{z}_{rj} \leq (-S_r^+) + \widetilde{z}_{rj_0}\} \geq 1 - \alpha_{rj}$$

Through Definition 4, the corresponding linear conversion could be obtained as follows:

$$\sum_{j=1}^{n}\lambda_j(\alpha_{rj}z_{rjL} + (1-\alpha_{rj})z_{rjU}) \leq (-S_r^+) + ((1-\alpha_{rj})z_{rj_0U} + \alpha_{rj}z_{rj_0L})$$

We subsequently substitute and reorganize Constraint (12-1). The deterministic form of chance Constraint (12-2) of Model (12) is as follows:

$$\sum_{j=1}^{n}\lambda_j(\overline{\alpha}_{ij}x_{ijL} + (1-\overline{\alpha}_{ij})x_{ijU}) + (S_i^-) \leq \theta * ((1-\overline{\alpha}_{ij})x_{ij_0L} + \overline{\alpha}_{ij}x_{ij_0U})$$

0.3. Proof

$$\sum_{j=1}^{n}\lambda_j\widetilde{y}_{rj} \geq \widetilde{y}_{rj_0} + S_r^+$$

This can be multiplied by (-1) *on the left and right sides and then the inequation is subject to normal distribution, which is converted to the following:*

$$-\sum_{j=1}^{n}\lambda_j\widetilde{y}_{rj} \leq (-S_r^+) - \widetilde{y}_{rj_0}$$

Let

$$\widetilde{x}_{ij} \sim Normal\ (\overline{e}_{ij}, \overline{\sigma}_{ij}), \widetilde{y}_{rj} \sim Normal\ (e_{rj}, \sigma_{rj})$$

Then

$$\widetilde{z}_{rj} = -\widetilde{y}_{rj} \sim Normal\ [-y_{rjU}, -y_{rjL}]$$

So the Constraint (16-1) can be converted as follows:

$$M\{\sum_{j=1}^{n}\lambda_j\widetilde{z}_{rj} \leq (-S_r^+) + \widetilde{z}_{rj_0}\} \geq 1 - \alpha_{rj}$$

Through Definition 9, we could get the following normal conversion:

$$\sum_{j=1}^{n}\lambda_j(e_{z_{rj}} - (\sqrt{3}\sigma_{z_{rj}})/\pi ln((\alpha_{rj})/(1-\alpha_{rj}))) \leq (-S_r^+) + (e_{z_{rj_0}} - (\sqrt{3}\sigma_{z_{rj_0}})/\pi ln((\alpha_{rj})/(1-\alpha_{rj})))$$

We subsequently substitute and reorganize Constraint (16-1).