

Article

Navigating Uncertainty: A Framework for Optimising Public Transport Networks' Performance

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Abstract: Public transport (PT) networks face significant challenges in achieving optimal outcomes due to the presence of risk and uncertainty. Despite the importance of optimising PT networks' performance, limited research has applied risk management tools to tackle this issue. In response, this study presents a three-stage framework to optimise PT networks' performance in uncertain conditions. First, we establish a PT criteria matrix using an analytic hierarchy process to develop a criteria model and calculate the criteria weightings. Second, we propose a multi-aspiration-level goal programming approach to optimise a PT network's performance based on the weighted results. To manage uncertainty, we use Monte Carlo simulation to analyse the probability of the optimal solution. Finally, to validate our approach, we apply the three-stage framework to three case study areas in Australia. The results of this research offer significant insights into identifying the likelihood of criteria optimisation scenarios, thereby assisting decision makers in allocating resources for optimising the delivery of PT network performance solutions in accordance with government requirements.

Keywords: uncertainty; public transport network optimisation; three-stage model; sampling; multiple-criteria decision making



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1. Introduction

1.1. Public Transport Networks

Public transportation is essential for the daily operation of society and is also considered a viable way to address the environmental issues that are caused by the increasing number of private vehicles. Due to its significance for sustainable development, public transport (PT) is being advocated by many countries, regions, and organisations, such as the UN-Habitat [1]. A PT network is a network formed by various types of PT, such as buses and trains, and an optimised PT network can not only provide residents easy access to PT but also help better address environmental issues and contribute to the sustainability of society. However, optimising a PT network is associated with uncertainty and risk, which can have great impacts on optimising outcomes.

The amount of recent research on PT decision making under uncertain conditions is increasing, with a focus on identifying the level of uncertainty that is associated with system input variables [2,3]. Additionally, in the PT multicriteria optimisation decision making problem, current research only considers one or two processes in terms of evaluation, optimisation, and uncertainty [3–6]. Studies about combining these three processes are limited. Therefore, this study's integration of evaluation, optimisation, and uncertainty processes in PT performance within a novel framework provides improved performance.

1.2. Uncertainty and Risks in Public Transport Networks

The delivery of optimised results is, in practice, impacted by the uncertainty of events. Uncertainty and variation are problematic when trying to optimise PT networks' performance. According to Altieri et al. (2017), PT is a complex system, whose quality analysis is challenging because it must consider the risks and uncertainties that are associated with human reasoning [7]. Additionally, there are numerous risks and uncertainties associated with user demand, operations, and traffic conditions that must be considered when PT performance improvement is being considered [8]. Therefore, whenever a PT optimisation model is developed to replicate a complex system, its output will always be uncertain.

Uncertainty is usually related to risk, which is defined as the influence of uncertainty on objectives or criteria [9–11]. Appropriate identification of major sources of risk can eliminate or at least reduce the probability of discovering new sources of uncertainty during the modelling process [4]. Thus, the uncertainty or risk identification process of the criteria is required to deliver the project results. Risk management is the tool that provides methods for mitigating project risk.

Risk management employs both qualitative and quantitative techniques. Dalmau (2022) used risk management to forecast the likelihood of airspace user rerouting, which aids the flow manager in air traffic flow management [12]. Similarly, Budzynski et al. (2021) examined PT's response to hazards using a qualitative method, risk registers [13]. To model the likelihood of project objectives, this study employs a quantitative risk management tool.

In the PT sector, risk management models have already been used to model input and uncertainty [14,15], and recent optimisation under uncertainty problems in PT frequently employs quantitative risk management methods, which assist DMs in determining the probability of the optimal solution [16,17].

1.3. Monte Carlo Simulation for Managing Uncertainty

Uncertainty cannot be fully investigated due to limited knowledge or the randomness of some model components. Monte Carlo simulation (MCS) is a quantitative risk analysis method based on a probabilistic model that employs probability distributions to model uncertainty [18,19]. The results assist DMs in managing risk and uncertainty to complete the project.

MCS is a risk management tool that is widely used in many fields, including medicine and project management. For example, MCS is used in medicine to assess the likelihood of viral transmission [20]. Yang et al. (2020) employed MCS to model uncertainty in a project to assess the health of land ecosystems [21]. Kannan et al. (2021) used MCS to analyse the sensitivity of VIKOR and grey relational analysis in a sustainable location of a solar site selection project [22]. MCS is also used to improve the reliability of assessment results in a lake eutrophication level evaluation project [23]. In most cases, MCS is used to assess the likelihood of project outcomes.

MCS has attracted the interest of PT researchers in recent years. For instance, Manzo et al. (2015) used MCS to analyse uncertainty in a four-stage transport model [4]. This study focused on investigating how the uncertainty of model parameters and inputs influences the model outputs. Conway et al. (2018) utilised MCS to account for variation and uncertainty in accessibility metrics when planning PT sketches [24]. Furthermore, Pencheva et al. (2021) applied MCS to determine the waiting time of passenger vehicles in PT areas [25]. Research shows that despite the increased optimisation and uncertainty analysis of PT, the existing studies focus more on single aspects of PT. Consequently, an effective framework for optimising a PT network's performance under uncertain conditions in multiple aspects is increasingly necessary to propose optimal plans and strategies while considering uncertainty.

1.4. Research Contribution

Probabilistic analysis is a commonly used technique for addressing evaluation-based issues in project management. MCS is also used in mitigating uncertainty that is related

to model inputs and outputs in various application areas. Despite its effectiveness in addressing project management issues, little research has used MCS to address the problem of optimising PT network performance. This study only examines one aspect of physical performance. In PT network performance optimisation problems, the probability of a scenario (scenario analysis) is thus required.

Multiple-criteria decision making (MCDM) and goal programming (GP) methods provide a variety of frameworks, and a few MCDM and GP methods have been used to optimise PT performance to meet the goals and requirements of DMs [5,26,27]. To solve multicriteria optimisation problems, the analytic hierarchy process (AHP) is an MCDM method that is frequently combined with the GP approach [5,27].

Previous research proposed the public transport criteria matrix (PTCM)-AHP-Multi-Aspiration-Level Goal Programming (MALGP) model for optimising PT networks' performance [27,28]. The model considers the basic PT infrastructure level, sustainable development level, PT service level, and economic benefit level for optimisation. In some cases, due to uncertainty occurring during the optimisation process, it is difficult for DMs to deliver an optimal solution. Previous research lacks an analysis of uncertainty that is related to criteria uncertainty.

Despite the current literature, a multicriteria optimisation method that combines these three processes in PT optimisation under uncertain conditions is still lacking. In the pursuit of creating a comprehensive tool to optimise PT performance under ambiguous circumstances at various levels of aspiration, several important aspects of enhancing PT performance have been overlooked [5,16]. To bridge this gap, this study proposes a three-stage approach for optimising PT networks' performance under uncertain conditions. The models optimise four levels of criteria with uncertainty to achieve the DMs' PT network optimisation goals. The primary goal of this study is to determine the level of criteria uncertainty, and a sensitivity analysis is performed to guide the optimisation process. MCS results can be used to assist DMs in making PT network optimisation decisions, as well as to precisely indicate the probability of the uncertainty rate when delivering criteria outcomes.

Compared with the current research, this study introduces the following novel contributions: 1. The three-stage model framework considers multiple aspects of PT network criteria. 2. The three-stage model framework is developed to evaluate and optimise PT networks' performance under uncertain conditions. 3. The validity of optimal solutions is examined in the case study areas.

The remainder of this paper is organised as follows: Section 2 explains the framework of the proposed three-stage PT network performance optimisation under uncertain conditions. Section 3 presents the input data of the three case study areas. Section 4 discusses the analysis results of the three case study areas, and the conclusions and future directions of the research are presented in Section 5.

2. Materials and Methods

In this study, we combine the PTCM-AHP, MALGP, and MCS models into a three-stage framework to optimise PT networks' performance under uncertain conditions. PT networks face significant challenges in achieving optimal outcomes due to the presence of risk and uncertainty. Despite the importance of optimising PT networks' performance, there has been limited research that applies risk management tools to tackle this issue. In response, this research presents a three-stage framework to optimise PT networks' performance under uncertain conditions. First, we use the established PT network criteria matrix. Second, we propose a MALGP approach to optimise PT networks' performance based on the weighted results. To manage uncertainty, we use MCS to analyse the probability of the optimal solution. The results of this research offer significant insights into identifying the likelihood of criteria optimisation scenarios, thereby assisting DMs in allocating resources for optimising the delivery of PT network performance solutions in accordance with government requirements.

Figure 1 depicts the three-stage approach for optimising the uncertain PT network performance, which includes an AHP process, a MALGP process, and an MCS process. The following sections review the specifics of each stage.

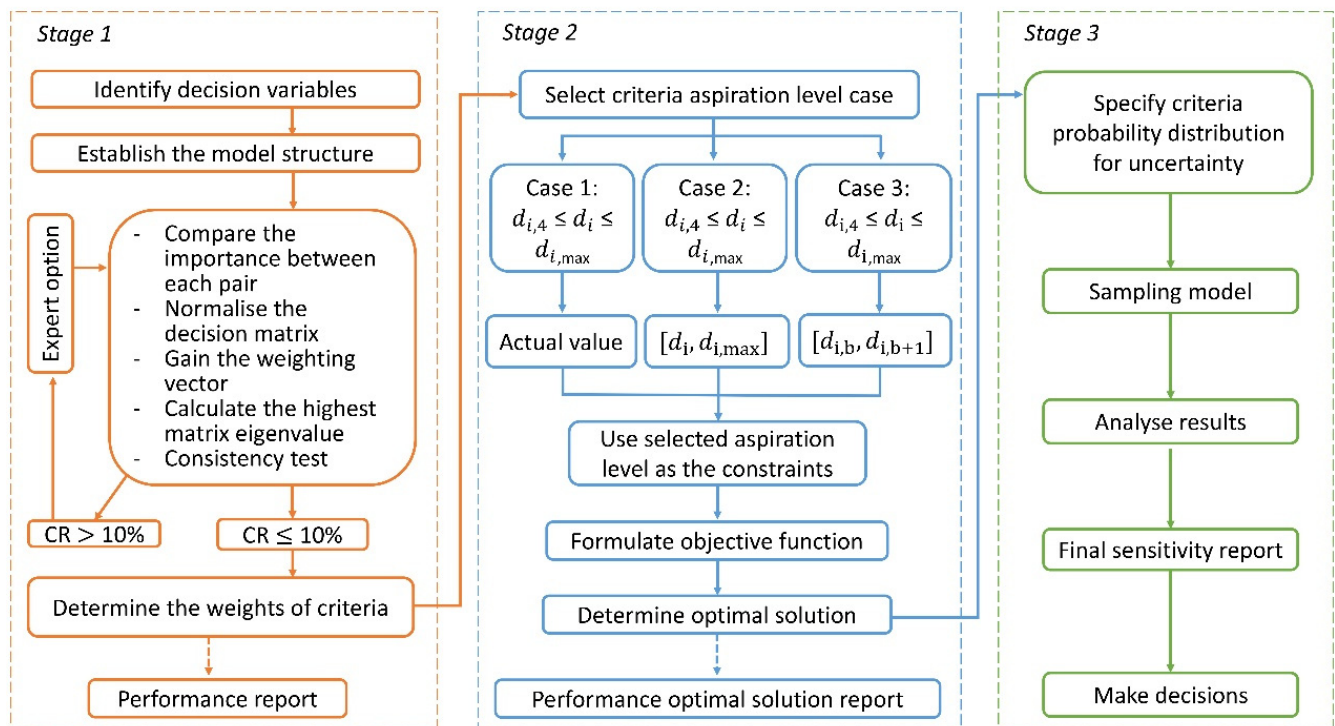


Figure 1. The proposed three-stage process for optimising public transport networks' performance under an uncertain process.

2.1. AHP Process

AHP is a structured model for analysing and solving complex decision issues [29,30]. To implement AHP to solve problems, there are three steps: criteria priority weight calculation, issue decomposition, and criteria comparison analysis [31]. In this study, first, the model decomposes the PT network performance evaluation problem into numerous levels. Second, to obtain the weight of each criterion, the model uses pairwise comparisons that assign the relative importance between two criteria [29,30,32]. Based on the AHP process, the PTCM-AHP model was proposed to evaluate a PT network's performance [28]. The following subsections review the specifics of the AHP process.

2.1.1. PTCM-AHP Model Structure

The decision variables of the AHP model have been described by Lin et al. (2021) [28]. Additional details of the PT network performance criteria can be found in Lin et al. (2021) [28]. The criteria were selected from existing PT evaluation assessments and indices [33–37]. These criteria are used to determine the PTCM-AHP model structure.

The PTCM-AHP model is based on four levels: the basic PT infrastructure level, the PT service level, the economic benefit level, and the sustainable development level [28]. Figure 2 presents the hierarchy of the PT network performance criteria of the PTCM-AHP model. The model includes 4 levels of criteria and 15 subcriteria.

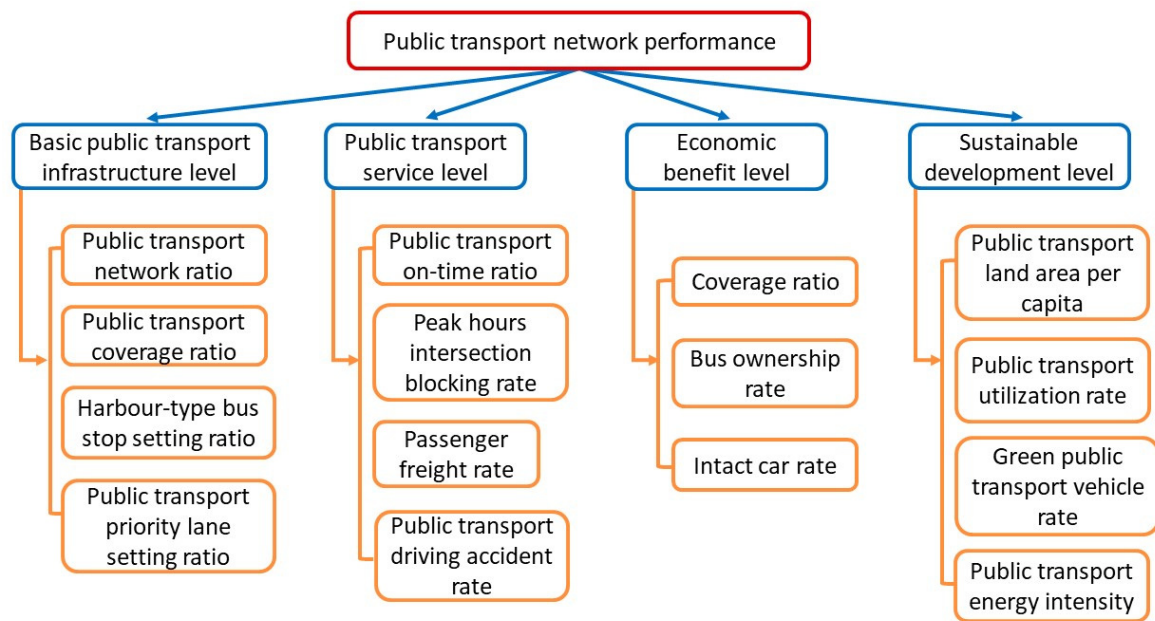


Figure 2. Hierarchy structure of public transport network performance criteria.

- The PT infrastructure level includes the harbour-type bus stop setting ratio, PT coverage ratio, PT priority lane setting ratio, and PT network ratio.
- The PT service level contains four subcriteria: passenger freight rate, PT on-time ratio, PT driving accident rate, and peak hours intersection blocking rate.
- The economic benefit level contains the intact car rate, coverage ratio, and bus ownership rate.
- The level of sustainable development considers the PT utilisation rate, PT energy intensity, PT land area per capita, and green PT vehicle rate.

Once the PTCM-AHP model structure was established, the process of determining criteria weights was undertaken to test and calculate the results of the weightings. The details of the weighting process are shown in the following section.

2.1.2. Criteria Weight Determination

The major steps for determining the weights of criteria are described below [28].

- (1) Construct the problem in a hierarchical structure and determine the criteria and subcriteria.
- (2) Create the decision matrix $C = (C_{ig})$ and perform pairwise comparison between criteria and subcriteria. C_{ig} indicates the importance values for criteria (i) and (g), which are between 1 and 9, provided by experts.
- (3) Normalise the decision matrix C to be matrix $D = (d_{ig})$:

$$d_{ig} = \frac{c_{ig}}{\sum_{i=1}^n C_{ig}}$$

- (4) Calculate the arithmetic mean of matrix D rows to obtain the prioritisation vector (w):

$$w = \frac{\sum_{g=1}^n d_{ig}}{n}$$

- (5) Fulfil the calculation result of the highest matrix eigenvalue $T_{\max}$:

$$Cw = T_{\max}w \text{ and } T_{\max} \approx T = \frac{\sum_{i=1}^n T_i}{n}.$$

- (6) Verify the consistency of the results. Hence, the consistency ratio (CR) must be calculated. RI is the random index. The formulations of the consistency index (CI) and CR for each matrix C are shown below:

$$CI = \frac{T_{\max} - n}{n - 1}$$

$$CR = \frac{CI}{RI}$$

- (7) Repeat steps 2–6 until $CR \leq 10\%$. When $CR \leq 10\%$, the model result is deemed internally coherent.

Hence, we can eventually identify the weight of the PTCM-AHP model criteria and subcriteria, which are used as coefficient values in the MALGP process. The case study area's performance report is also created to identify the city's PT network's performance score and show each criterion's performance score, which are calculated based on the case study areas' criteria actual value. The results of the city performance report will be used to determine the criteria aspiration level used in the calculation of criteria goal values in the MALGP process. The criteria weights and performance results of the case study areas' PTCM-AHP model results can be found in Lin et al. (2021) [28].

2.2. Multi-Aspiration-Level Goal Programming (MALGP) Process

GP is often combined with AHP to assist DMs, which can address MCDM problems and identify optimal solutions [38,39]. The outputs of the AHP process are used to define the objective function criteria priority of GP [5]. The model minimises the objective function by selecting the criteria aspiration level from numerous criterion input values [5]. Based on GP, MCGP further develops a model that allows DMs to address multiple goals or aspiration levels per criterion [40–43]. However, MCGP does not consider the selection of a criterion goal level among various aspiration-level cases. Hence, the establishment of MALGP helps DMs choose different aspiration levels to solve the PT network performance optimisation problem [27]. The model takes the selection of the criteria aspiration level into consideration to help DMs in performance optimisation. The MALGP process is shown below.

2.2.1. Criteria Aspiration Level Case Selection

The MALGP model includes the criterion case selection process. The aspiration level criterion is selected based on the actual value of the criteria. The details of the criteria-level grades can be found in Lin et al. (2021) [28]. According to Figure 3, the process contains three cases [27].

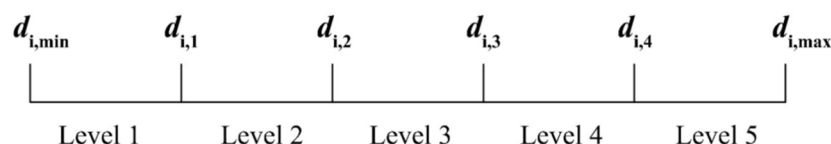


Figure 3. Grade level scale for subcriteria [27].

Case 1: The actual value is the aspiration value for the i th criterion when the i th criterion's actual value is greater than $d_{i,max}$.

Case 2: The aspiration value of the i th criterion is less than $d_{i,max}$ but greater than the actual value when the actual value for the i th criterion is less than $d_{i,max}$ but greater than $d_{i,4}$.

Case 3: The aspiration value of the i th criterion is the $(i + 1)$ th aspiration level when the actual value level for the i th criterion is level 1, 2, 3, or 4.

Then, the criteria aspiration-level case of the model can be identified. In the calculation step, the conditions of the objective function formulation are based on the criteria aspiration-level selection results.

2.2.2. Objective Function Formulation

After the case selection process for the criteria aspiration level, we establish the objective function formulation for the PT network performance optimisation process. The MALGP model uses criteria weights as coefficients in the model's objective function [5]. The notations and formulation for the MALGP objective function are shown as follows [27]:

Notations:

s : criteria number, $s = 1, 2, \dots, e$;

i : goal number, $i = 1, 2, \dots, n$;

R_i : weight assigned for i th priority;

x_s : s th decision variable;

b_{is} : coefficient of the s th criteria for the i th goal;

p_i : positive deviation;

q_i : negative deviation;

d_i : aspiration grade level for goal i , $i = 1, 2, \dots, 5$.

$$\text{Min} \sum_{i=1}^n R_i(p_i + q_i)$$

subject to

$$\sum_{s=1}^e b_{is}x_s - p_i + q_i = d_i, \\ p_i, q_i, x_s \geq 0,$$

Case 1: If the constraint of d_i is the actual value of the criterion,

$$d_i \geq d_{i,\max}$$

Case 2: If the constraint of d_i is chosen between the criterion's actual value and $d_{i,\max}$,

$$d_{i,A} \leq d_i \leq d_{i,\max}$$

Case 3: If the criterion's actual value is less than $d_{i,A}$ and the criterion's goal value is less than $d_{i,\max}$,

$$d_{i,\min} \leq d_i \leq d_{i,\max}$$

During the MALGP model process, the constraint functions are based on the selected grade for the criteria aspiration level and considering the relation of the criteria. The details of the case study areas' objective functions and constraints can be found in Lin et al. (2022) [27].

2.3. Monte Carlo Simulation (MCS) Process

In this process, we used MCS to model the probability of optimal scenario delivery. The proposed method was used to calculate the possibility of an optimal solution. MCS performs calculations, allowing for multiple simulations of a project. The process was used to quantitatively analyse project risk and identify the probability of the best solution by randomly selecting criteria values [44,45]. MCS analyses risk and uncertainty using a probability distribution. This study assumed that the DMs must control each criterion's performance and that the criteria probability was within a range of $-5\%/+10\%$. The model outcomes were analysed to identify the probability of and confidence level for achieving the goals. The results are obtained using @risk software Version 8.3. The details of the MCS process are shown below.

2.3.1. Criteria Probability Distribution Identification

Before we begin simulating the optimisation results, we must first determine the probability of the criteria. The types of criteria probability distributions must be chosen during the identification process. According to Figure 4, the criteria sampling process uses a triangular probability distribution because the minimum, most likely, and maximum values can be estimated. The MALGP process outputs are used as the most likely value of criteria in the MCS process. Table 1 shows the criteria ratings for the uncertainty level, which can be used to calculate the minimum and maximum values of the criteria. The level of uncertainty is divided into five categories: very high, high, medium, low, and very low.

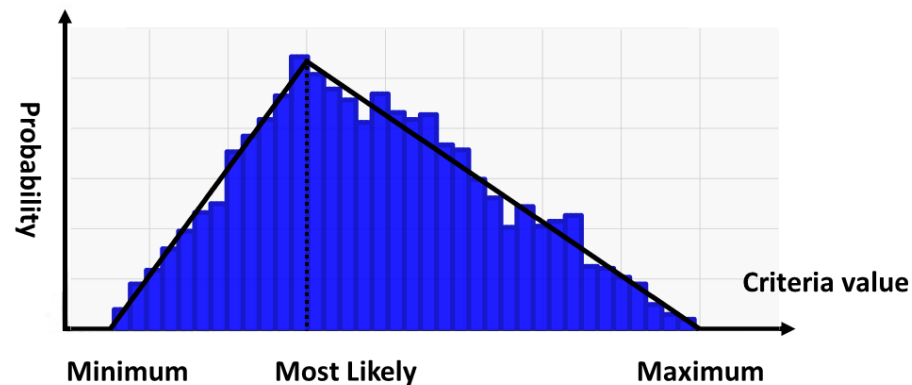


Figure 4. Triangular distribution of criteria for public transport network performance optimisation.

Table 1. Uncertainty level [46].

Uncertainty Level	Min	Most Likely	Max
Very high	50%	100%	200%
High	75%	100%	150%
Medium	85%	100%	125%
Low	90%	100%	115%
Very low	95%	100%	110%

Thus, the criteria's risk and uncertainty levels need to be identified. To determine the input of the criteria, the uncertainty and risk level of a criterion are selected based on the risk rating recommendation and existing risk ratings for the criteria. The current PT risk assessment shows that the risk level of PT driving accident rates is high [47]. Based on existing risk ratings, the uncertainty level of the intersection blocking rate during peak hours, coverage rate, PT land area per capita, and PT utilisation rate are medium [48]. Other criteria's uncertainty levels are very low, since the optimisation process can be controlled under the government implementation plan. After the criteria uncertainty levels have been identified, the results are utilised in the sampling process. During the criteria sampling, the sampling model needs to be selected. The details of the sampling model selection are shown in the following subsection.

2.3.2. Sample Selection

MCS uses a random sampling process. Monte Carlo (MC) sampling can recreate the full input distribution by making random selections across the entire probability distribution with large iterations [49]. With high iteration, the model results are closer to the actual situation. Hence, this study used MCS performed by means of MC sampling. The details of the model input for the criteria are described in Section 3.

3. Case Study

The analysis was implemented in three study areas in Australia, including the City of Bayswater, the City of Cockburn, and the City of Stonnington. Stonnington and Bayswater

are suburbs close to Melbourne and Perth Central Business District, respectively. Cockburn is a suburb in the south of Perth. In these cities, trains and buses are the major means of public transport, and the main land use type is residential. The details of the case studies can be found in Lin et al. (2021) [28]. The locations and areas of these three cities are shown in Figure 5.

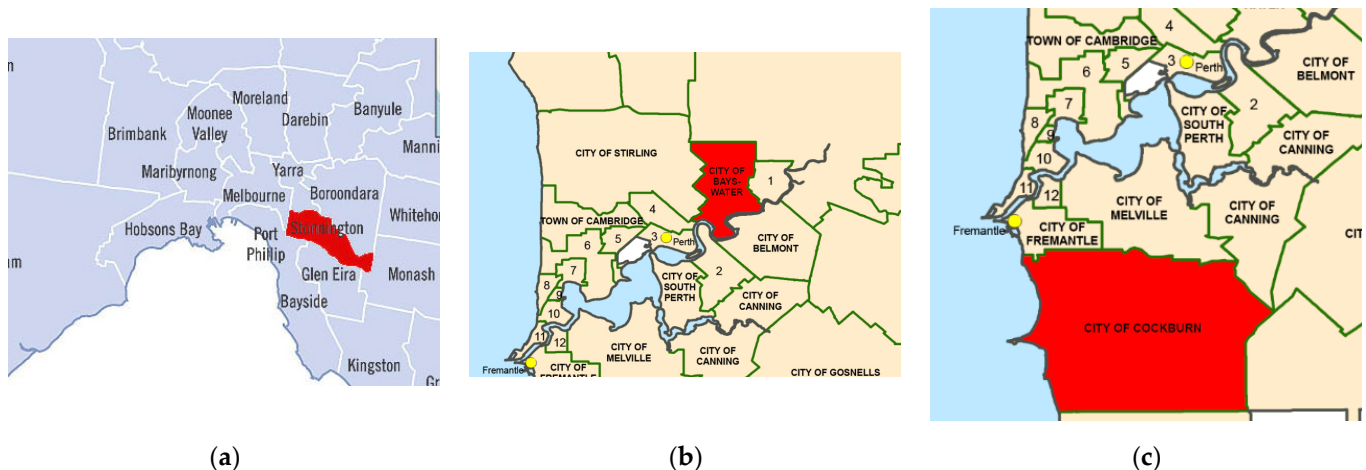


Figure 5. (a) City boundary of Stonnington; (b) city boundary of Bayswater; (c) city boundary of Cockburn [28].

The MCS was conducted to analyse the likelihood of achieving PT network performance optimisation goals. The input data for the three case study areas were derived from AHP and MALGP outputs. As demonstrated in Section 2.1, the PTCM-AHP model calculates the criteria weights that are later utilised in MALGP for the optimisation process [27,28]. The criteria weights are presented in Lin et al. (2021) [27]. The mean value of the criteria for MCS was extracted from the MALGP criteria optimising results, and the details can be found in Lin et al. (2022) [28].

The sources of uncertainty for the optimisation process of public transport networks' performance have not been fully investigated. Thus, the degree of uncertainty for each criterion is defined based on the existing risk rating, which is discussed in Section 2.3.1. This analysis focuses on the uncertainty of the implementation criteria of the optimisation results. The risk level of criteria is defined based on an existing risk assessment of the uncertainty level.

According to the risk rating description, the uncertainty level is medium for the intersection blocking rate during peak hours, coverage rate, PT land area per capita, and PT utilisation rate. Based on existing PT risk assessments, the uncertainty level of the PT driving accident rate is high. The criteria's mean values are each criterion's optimal value. The remaining criteria have very low uncertainty levels. Thus, the minimum and maximum values for the criteria were calculated.

The type of probability distribution for all criteria sampling was assumed to be triangularly distributed, since the minimum, most likely, and maximum values can be estimated. The model input list of the three cities for MCS is shown in Tables A1–A3. The sampling result is more likely to display the distribution accurately with a high number of draws. Thus, the criteria used 5000 draws by applying MC sampling.

Sensitivity analyses on the three case study areas were implemented. Each PT criteria performance was calculated on 5000 model runs. To explore the criteria model outputs' uncertainty, criteria uncertainty was investigated via the criteria coefficient value, criteria optimising value's impacts on the model output, and the criteria's probability of reaching the DMs' optimisation goals. The details of the model's results and sensitivity analyses are shown in the next section.

4. Results and Discussion

Sensitivity analyses were implemented in the three case study areas. The most likely values for the criteria during the optimisation process were also determined. Finally, the results reveal the critical sensitive criteria that governments must take into account to manage uncertainty for future optimisation plans and strategies for the case study areas. Section 4.1 identifies the most sensitive criteria and the criteria's most likely values during the optimisation process. Section 4.2 shows the most important criteria of the MCS model's output for the study areas. Section 4.3 determines the probability of sensitive criteria to achieve the government requirements.

4.1. Sensitivity Analysis

According to Tables A4–A6, all cities' outputs are influenced by the on-time rate. Based on the output of the probability distribution for the case study areas, three cities have a 50th percentile chance of achieving the performance optimisation goals for each criterion. Except for the on-time rate, other criteria have at least a 60% likelihood of achieving the optimal solution.

Figures 6–8 show the coefficient values of the criteria for the three case study areas. The y-axis displays the names of the criteria from top to bottom in an order of sensitive influence to the criteria. The x-axis indicates the coefficient values of the associated criteria.

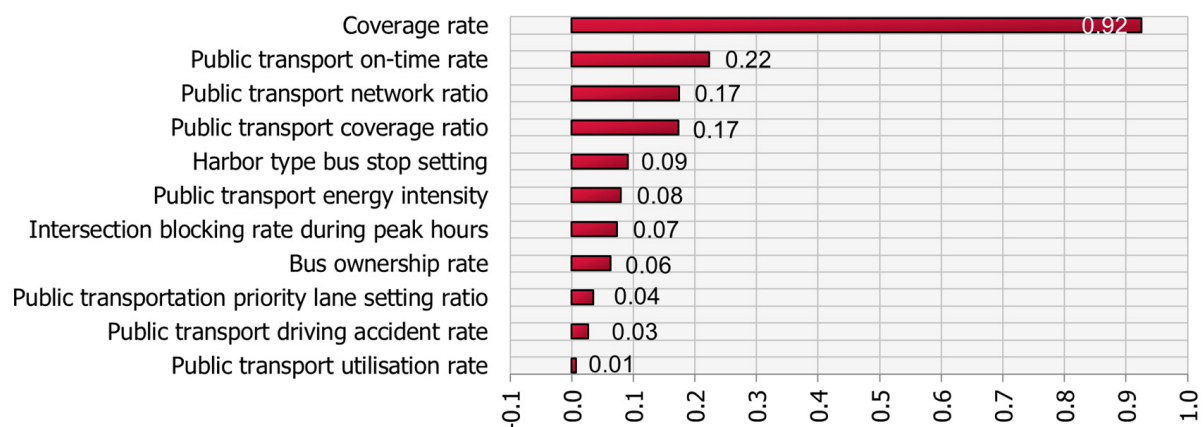


Figure 6. Bayswater PT network's performance criteria's coefficient values.

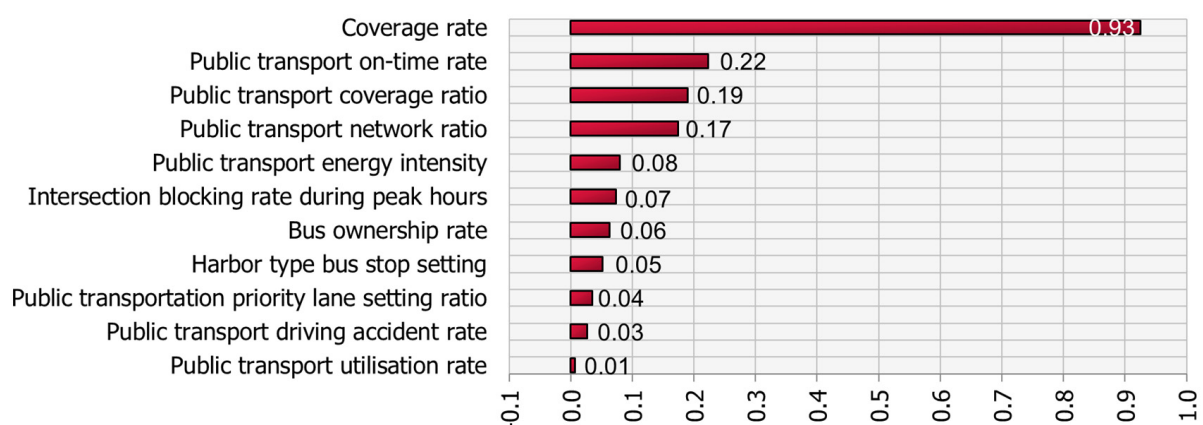


Figure 7. Cockburn PT network's performance criteria's coefficient values.

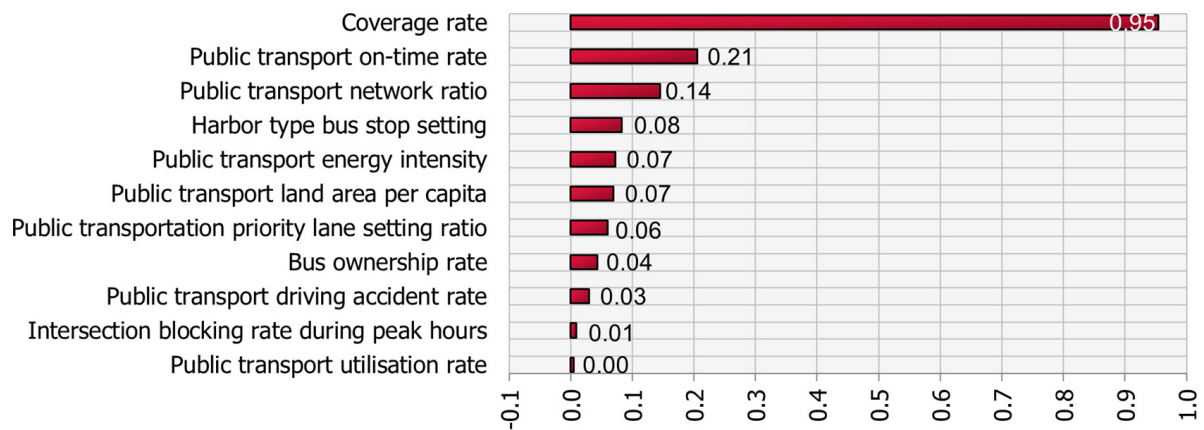


Figure 8. Stonnington PT performance criteria's coefficient values.

According to the results, the most sensitive criterion for all cities is the coverage rate. This criterion's coefficient value is over 0.9 for the three cities. According to Table A7, Bayswater and Cockburn's most likely values are both 103.33%. The two cities' minimum and maximum values are 85.08% and 124.5%, respectively. Table A7 suggests that the most likely value for Stonnington is 155%. The Stonnington minimum and maximum values are 127.76% and 186.52%, respectively. To control and minimise the uncertainty of this most sensitive criterion's optimisation process, the DMs should consider improving the PT service's commercial revenue and reducing the operating expenses for all cities' optimisation scenarios.

Figures 6–8 effectively offer an overall interpretation of the model based on each criterion. However, the relative importance of the criteria on the model output has not been discovered. For this reason, Figures 9–11 show the criteria for optimising the impacts of the input on MCS output.

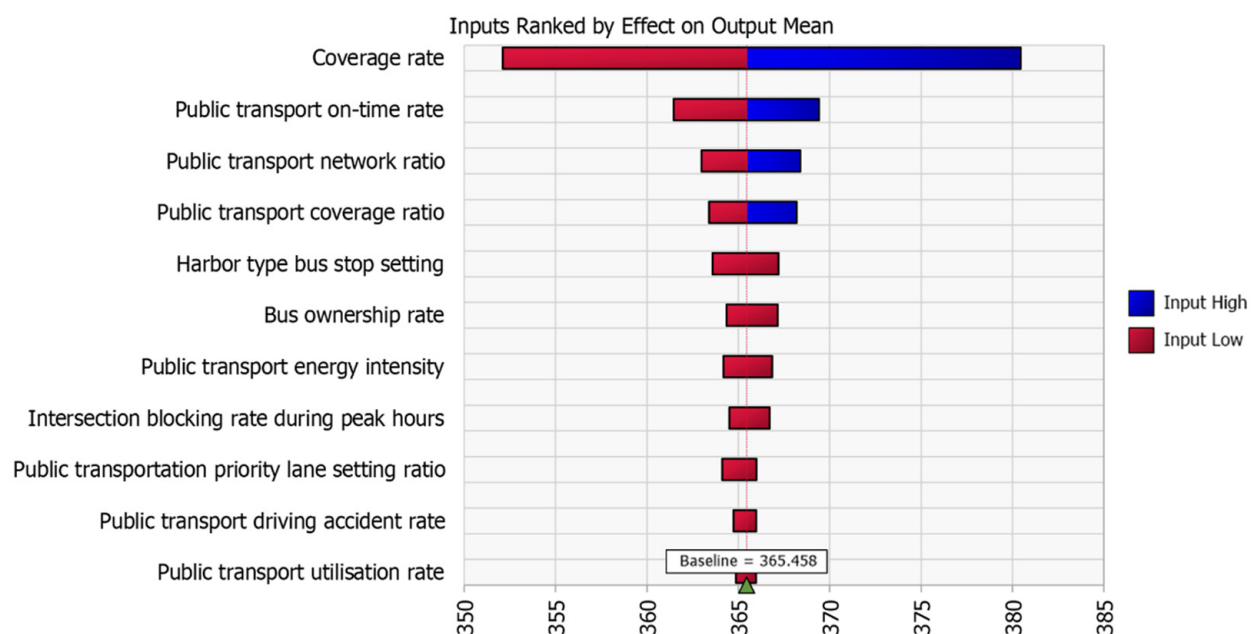


Figure 9. Bayswater: criteria's optimising value's impact on model output.

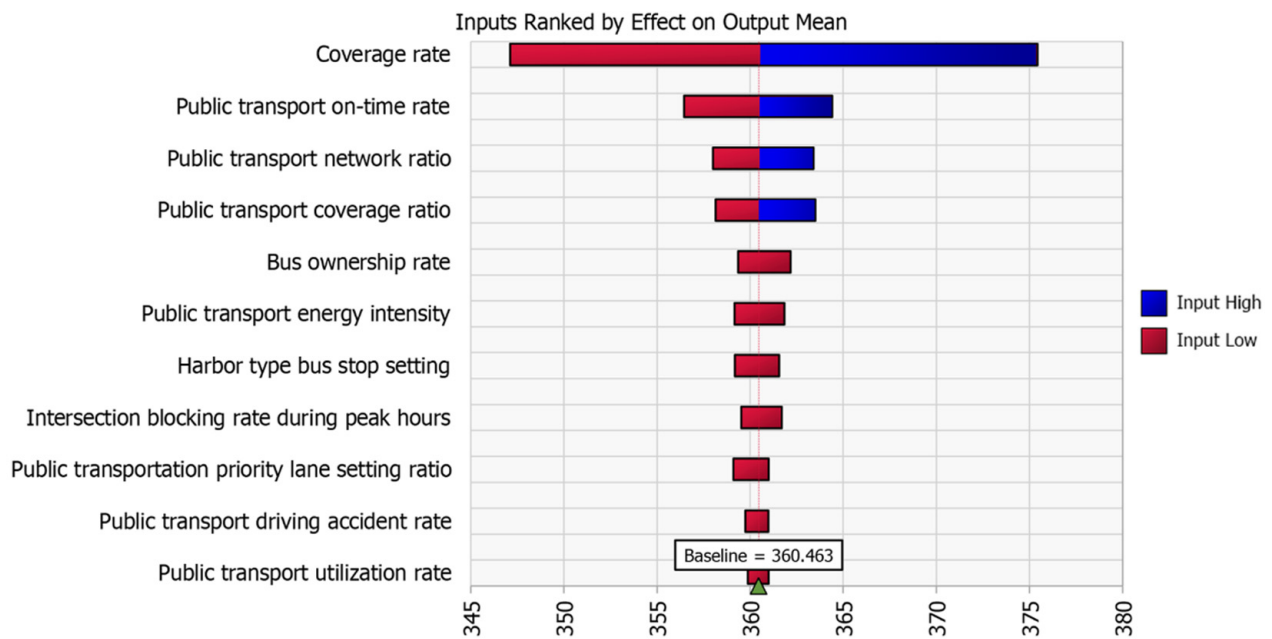


Figure 10. Cockburn: criteria's optimising value's impact on model output.

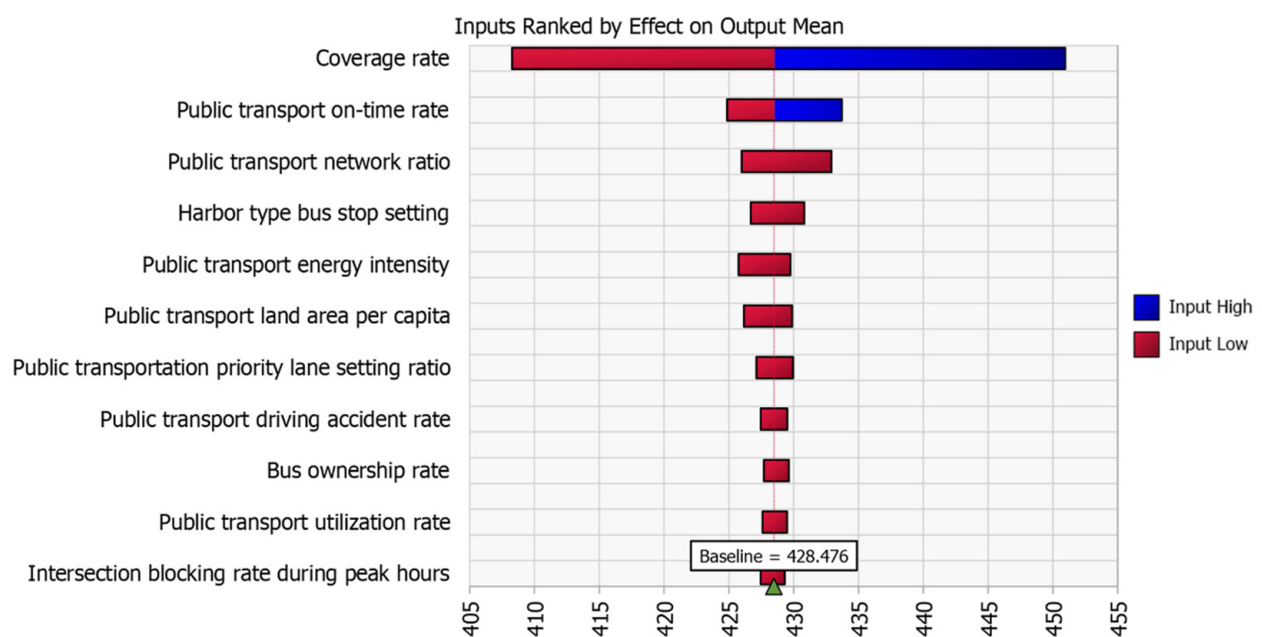


Figure 11. Stonnington: criteria's optimising value's impact on model output.

4.2. Permutation Feature Importance

Figures 9–11 show the impacts of the three cities' criteria for optimising the value on the model results. The y-axis demonstrates the name of the criteria, based on importance magnitude, from top to bottom. The x-axis indicates the criteria's impact on the model output. The line colour shows the impact of the criteria on the model output, which supports the DMs in analysing the criteria's impact on the city optimisation solution.

The figures show that the coverage rate has the highest impact on the model output of the three case study areas. Furthermore, the higher the coverage rate value is, the greater the influence on the model output is. However, this criterion suggests a baseline result when the coverage rate input is low.

For Bayswater, other criteria, such as the PT on-time rate, the PT network ratio, and the PT coverage ratio, also have a high impact on the output (as shown in Figure 9).

Similarly, Figure 10 demonstrates that these three criteria have a high influence on the model output for Cockburn. The results of the coverage rate also apply to these three criteria.

Finally, Figure 11 suggests that the higher the PT on-time rate requirement is, the higher the impact on the model optimisation results for Stonnington is. Except for the criteria mentioned above, a higher other criteria requirement has a low influence on the model optimisation output for the three cities. The figure results also validate the criteria weighting results of the PTCM-AHP model. The PT network ratio and PT coverage ratio, PT on-time rate, and coverage rate are the most important variables for the basic PT infrastructure level, PT service level, and economic benefit level, respectively [28].

Figures 9–11 provide a method to analyse the effect of each criterion on the model outputs. However, DMs are often subject to government requirements to control the optimisation process. Therefore, it is necessary to identify the probability of criteria that meet the government requirements.

4.3. Test Accuracy

Finally, we determined the criteria's probability distribution in the PT network performance optimisation process. The following section identifies the probability of the criterion that meets the DMs' requirements. DMs require the criteria probability to be within a range of $-5\%/+10\%$. Figures 12 and 13 show the probability of the criteria reaching the requirements for the three cities. The y-axis displays the probability of achieving the criterion-optimising values. The x-axis indicates the input values of the associated criteria.

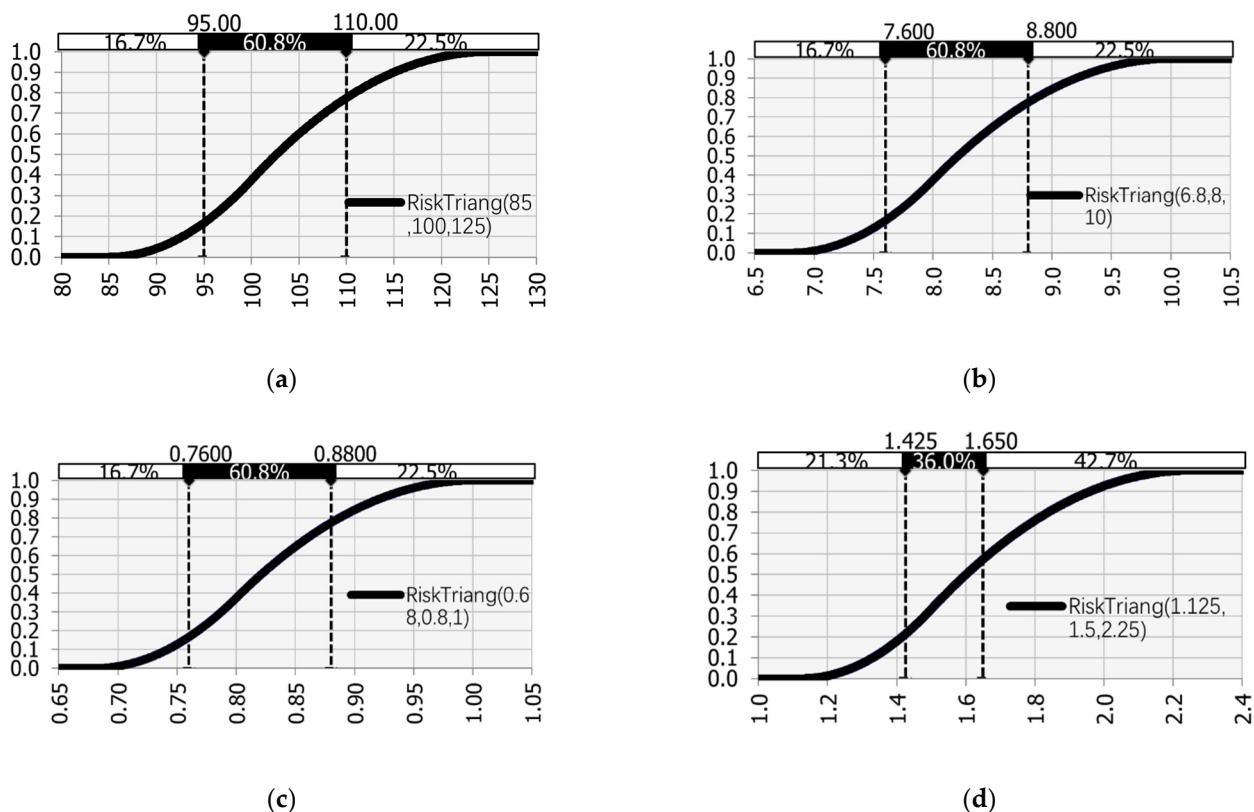
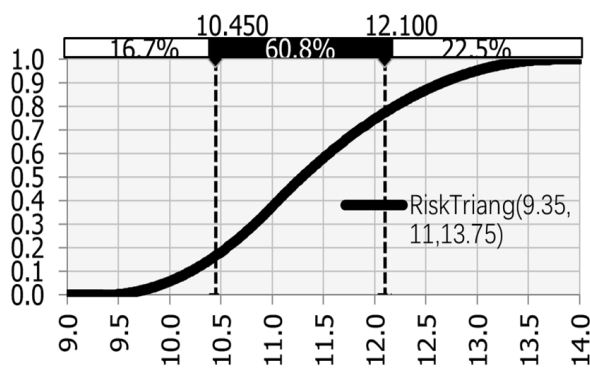
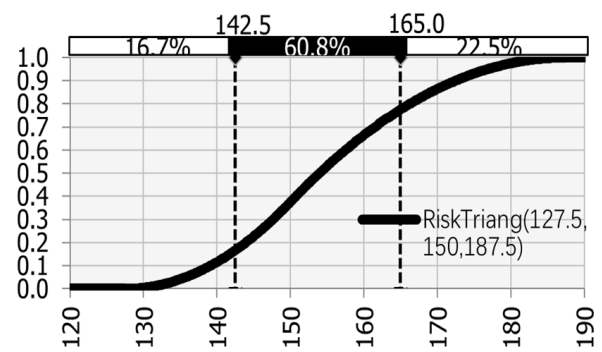


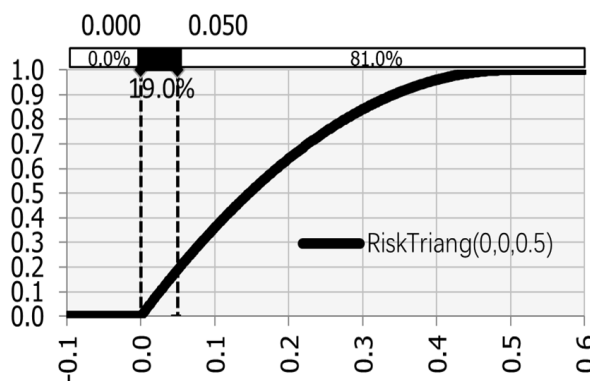
Figure 12. Bayswater and Cockburn criterion probability distribution for reaching DMs' optimising goals. (a) Coverage rate probability distribution. (b) Intersection blocking rate during peak hours probability distribution. (c) PT utilisation rate probability distribution. (d) PT driving accident rate probability distribution.



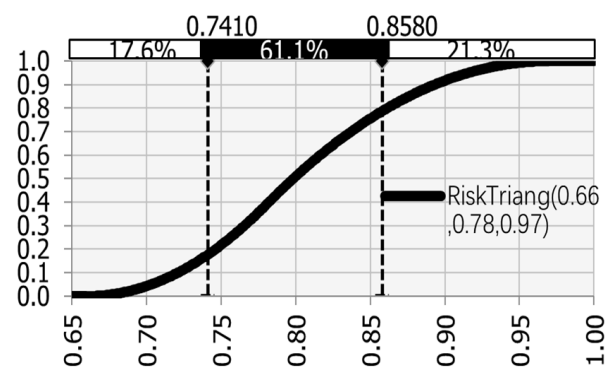
(a)



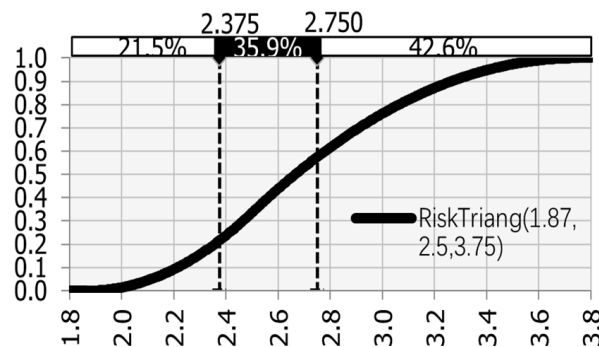
(b)



(c)



(d)



(e)

Figure 13. Stonnington: criteria's probability distribution for reaching the DMs' goals. (a) PT land area per capita probability distribution. (b) Coverage rate probability distribution. (c) Intersection blocking rate during peak hours probability distribution. (d) PT utilisation rate probability distribution. (e) PT driving accident rate probability distribution.

Since the uncertainty levels of most criteria are very low, most criteria have a 100% probability of meeting the government requirements. For Bayswater and Cockburn, there are four criteria uncertainty levels that are higher than very low, including the coverage rate, intersection blocking rate during peak hours, PT utilisation rate, and PT driving accident rate. The details of the criteria's probability distribution for Bayswater and Cockburn are shown in Figure 12. Five criteria for Stonnington have an uncertainty level that is higher

than the ‘very low’ level. Figure 13 shows the probability distribution of these five criteria, namely, the PT land area per capita, coverage rate, intersection blocking rate during peak hours, PT utilisation rate, and PT driving accident rate.

For Bayswater and Cockburn, Figure 12a shows that a coverage rate of 60.8% reaches the government goal. According to Lin et al. (2021), this criterion has the highest weight in the economic benefit level [28]. Hence, when cities implement the optimisation scenario for economic benefit level, DMs are advised to plan ahead, which requires the implementation of a management plan during the optimisation process to mitigate the uncertainty.

Figure 12b,c demonstrate that both cities have a probability of 60.8% of achieving the requirement for intersection blocking rate during peak hours and PT utilisation rate. Lin et al. (2021) show that these two criteria have low priority to achieve the optimal goal [28]. Hence, DMs just need to effectively monitor and control the process to deliver optimisation scenarios.

Figure 12d shows that the PT driving accident rate has a low probability, i.e., 36%, of achieving the government requirement. According to Table A1, although the priority of this criterion is low, the government still requires a management plan for the optimisation scenario. Since this criterion has a high uncertainty level, the delivery of the optimal solution will be influenced.

For Stonnington, Figure 13a,b show that both criteria have a probability of 60.8% of fulfilling the government’s requirements. The criterion of PT land area per capita is not of the highest importance at the sustainable development level, but its weight is higher than that of the coverage rate. The coverage rate is the most important criterion in the economic benefit level, for which the government needs to apply management plans to optimise PT network performance. Hence, DMs are also advised to implement management action to achieve an optimal solution.

According to Figure 13c, although Stonnington has only a probability of 19% of achieving the DMs’ requirements for the criterion of the intersection blocking rate during peak hours, the evaluation results show that the actual value achieved the highest level, which is level A in Lin et al. (2021) [28]. Since it is difficult to further improve and achieve optimising results in the criterion performance, the DMs can instead focus on maintaining the current performance while controlling and optimising the criterion performance.

Figure 13d shows that there is a probability of 61.1% of achieving the government requirement for the PT utilisation rate. Since the weight of this criterion is low, the DMs are advised to implement monitoring and control during the optimisation process.

According to Figure 13e, the probability of Stonnington’s PT driving accident rate is similar to the other two case study areas, which is 35.9%. The criterion uncertainty level is high. Thus, Stonnington also suggests implementing actions to mitigate the risks during the optimisation process.

Figures 12 and 13 are useful for analysing the probability distribution of each criterion to fulfil the governments’ requirements. This approach helps governments allocate resources for delivering case study area optimisation solutions.

This research establishes a solid framework for optimising PT networks’ performance in the face of uncertainty. The combination of the PTCM-AHP model, the MALGP model, and MCS enables DMs to make informed decisions based on criteria weights while optimising the PT network and accounting for uncertainty. The findings of this study help advance PT network optimisation methodologies and provide practical advice for improving urban transportation systems. DMs gain insights into the relative importance of criteria, propose optimal solutions, and assess the probability of criteria optimisation in uncertain environments by integrating the PTCM-AHP model, the MALGP model, and MCS.

5. Conclusions

To mitigate the criteria uncertainty involved in the process of optimising PT networks’ performance, this paper proposes a three-stage optimisation model for optimising public

transport networks' performance under uncertain conditions. First, the PTCM-AHP model was used to identify the weights of the model criteria and evaluate the case study areas' PT networks' performance. The obtained weights were then used by the second model, MALGP, to propose the three cases' PT network performance optimisation solutions. Finally, MCS was implemented to analyse the sensitive criteria, discover the optimal solution under criteria uncertainty, and identify the likelihood of criteria optimisation based on DMs' requirements for the three case study areas. The research results indicate that the coverage rate is the most sensitive criterion for these three cities. Furthermore, a higher coverage rate and PT on-time rate requirement will lead to a higher impact on the model optimising result for all cities. Last, although the PT driving accident rate has a low priority and probability of achieving the DMs' requirements, this criterion has a high level of risk. Governments still need to implement management plans to achieve optimised solutions.

The model proposed in this paper can be used in the following areas: First, the DMs can use the model to evaluate the performance of a PT network. The model also provides the weights of criteria for the optimisation process. Second, the model is based on criteria weights and the governments' goal for the criteria performance to propose an optimisation solution for the case study areas. Third, the model results identify the sensitive criteria and the criteria's optimising value's impact on the delivery of a PT network's performance optimisation solution. Fourth, the outcome of this research can be used to identify the likelihood of a criteria optimisation scenario. Based on government requirements, the MCS results were combined with weighted results, which provide a reference for DMs to allocate resources for optimising the delivery of PT network performance solutions.

Despite innovations in the three-stage optimisation framework design, the models and theories used in this study still have scope for improvement. Future research should consider overcoming the relevant limitations. In terms of calculating city performance scores and the optimisation processes, the processes of conducting results necessitate collaboration with statistical programming software to enhance efficiency.

This model, however, did not consider the actual risk events and their corresponding risk treatments. Hence, the framework can provide qualitative risk management methods for the proposed associated risk treatments. Further work should go beyond the risk analysis to achieve performance optimisation. Moreover, future research should consider risk information that is received from other subject sources.

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Appendix A

Table A1. Bayswater model inputs.

Variable	Risk Level	Min	Mean Value	Max	Shape
PT network ratio	Very low	47.5	50	55	Triangular
PT coverage ratio	Very low	47.5	50	55	Triangular
Harbour-type bus stop setting ratio	Very low	23.75	25	27.5	Triangular
Public transportation priority lane setting ratio	Very low	9.5	10	11	Triangular
PT on-time rate	Very low	90.25	95	100	Triangular
Intersection blocking rate during peak hours	Medium	6.8	8	10	Triangular
Passenger freight rate			1.75		
PT driving accident rate	High	1.125	1.5	2.25	Triangular
Coverage rate	Medium	85	100	125	Triangular
Bus ownership rate	Very low	17.1	18	19.8	Triangular
Intact car rate			100		
PT land area per capita			20.47		
PT utilisation rate	Medium	0.68	0.8	1	Triangular
Green public transport vehicle rate			100		
PT energy intensity	Very low	0	0	3	Triangular

Table A2. Cockburn model inputs.

Variable	Risk Level	Min	Mean Value	Max	Shape
PT network ratio	Very low	47.5	50	55	Triangular
PT coverage ratio	Very low	47.5	55	55	Triangular
Harbour-type bus stop setting ratio	Very low	14.25	15	16.5	Triangular
Public transportation priority lane setting ratio	Very low	9.5	10	11	Triangular
PT on-time rate	Very low	90.25	95	100	Triangular
Intersection blocking rate during peak hours	Medium	6.8	8	10	Triangular
Passenger freight rate			1.75		
PT driving accident rate	High	1.125	1.5	2.25	Triangular
Coverage rate	Medium	85	100	125	Triangular
Bus ownership rate	Very low	17.1	18	19.8	Triangular
Intact car rate			100		
PT land area per capita			26.23		
PT utilisation rate	Medium	0.68	0.8	1	Triangular
Green public transport vehicle rate			100		
PT energy intensity	Very low	0	0	3	Triangular

Table A3. Stonnington model inputs.

Variable	Risk Level	Min	Mean Value	Max	Shape
PT network ratio	Very low	57.74	60.78	66.86	Triangular
PT coverage ratio			83.72		
Harbour-type bus stop setting ratio	Very low	33.25	35	38.5	Triangular
Public transportation priority lane setting ratio	Very low	24.11	25.38	27.92	Triangular
PT on-time rate	Very low	80.75	85	93.5	Triangular
Intersection blocking rate during peak hours	Medium	0	0	0.5	Triangular
Passenger freight rate			2.33		
PT driving accident rate	High	1.87	2.5	3.75	Triangular
Coverage rate	Medium	127.5	150	187.5	Triangular
Bus ownership rate	Very low	17.1	18	19.8	Triangular
Intact car rate			100		
PT land area per capita	Medium	9.35	11	13.75	Triangular
PT utilisation rate	Medium	0.66	0.78	0.97	Triangular
Green public transport vehicle rate			100		
PT energy intensity	Very low	28.5	30	33	Triangular

Table A4. Bayswater summary statistics in total.

Name Description Cell Function	Public Transport Network Ratio Input B1	Public Transport Coverage Ratio Input B2	Harbour- Type Bus Stop Setting Input B3	Public Transporta- tion Priority Lane Setting Ratio Input B4	Public Transport on-Time Rate Input B5	Intersection Blocking Rate during Peak Hours Input B6	Public Transport Driving Accident Rate Input B7	Coverage Rate Input B8	Bus Ownership Rate Input B9	Public Transport Utilisation Rate Input B10	Public Transport Energy Intensity Input B11
Percentiles											
1%	47.9423	52.6870	14.3888	9.5849	90.8506	6.9989	1.18211	87.248	17.2467	0.70310	0.0160
10%	48.8639	53.7908	14.6576	9.7764	92.3835	7.4063	1.32963	92.678	17.5913	0.74373	0.1488
20%	49.3899	54.3977	14.8250	9.8893	93.2820	7.6744	1.41561	95.884	17.7949	0.76918	0.3182
25%	49.6484	54.6520	14.8904	9.9343	93.6105	7.7776	1.44542	97.148	17.8787	0.77872	0.4051
30%	49.8488	54.8737	14.9565	9.9764	93.8986	7.8663	1.47927	98.335	17.9534	0.78834	0.4959
35%	50.0314	55.0881	15.0156	10.0128	94.2359	7.9576	1.50881	99.338	18.0208	0.79696	0.5847
40%	50.2108	55.3207	15.0798	10.0493	94.5160	8.0470	1.54068	100.347	18.0893	0.80540	0.6768
45%	50.4319	55.5341	15.1414	10.0921	94.7778	8.1328	1.57329	101.406	18.1637	0.81324	0.7787
50%	50.6512	55.7906	15.2022	10.1350	95.0402	8.2244	1.60420	102.528	18.2412	0.82232	0.8829
55%	50.8657	56.0201	15.2677	10.1778	95.3043	8.3087	1.63717	103.779	18.3266	0.83136	0.9869
60%	51.0791	56.2985	15.3355	10.2232	95.5614	8.4048	1.67112	104.954	18.3989	0.84100	1.0974
65%	51.3197	56.5476	15.4093	10.2753	95.8875	8.5046	1.70788	106.238	18.4950	0.85092	1.2179
70%	51.5816	56.8067	15.4895	10.3302	96.1834	8.6281	1.75044	107.614	18.5947	0.86190	1.3509
75%	51.9094	57.1131	15.5744	10.3941	96.5048	8.7509	1.79612	109.069	18.6995	0.87317	1.4920
80%	52.2371	57.4395	15.6634	10.4586	96.8769	8.8751	1.83853	110.589	18.8310	0.88616	1.6728
90%	53.0088	58.3971	15.9010	10.6244	97.7727	9.1921	1.96308	114.884	19.1074	0.91765	2.0782
99%	54.4025	59.7433	16.3227	10.8925	99.2561	9.7271	2.14651	121.810	19.5711	0.97468	2.6878

Table A5. Cockburn summary statistics in total.

Name Description Cell Function	Public Transport Network Ratio Input B1	Public Transport Coverage Ratio Input B2	Harbour- Type Bus Stop Setting Input B3	Public Transporta- tion Priority Lane Setting Ratio Input B4	Public Transport on-Time Rate Input B5	Intersection Blocking Rate during Peak Hours Input B6	Public Transport Driving Accident Rate Input B7	Coverage Rate Input B8	Bus Ownership Rate Input B9	Public Transport Utilisation Rate Input B10	Public Transport Energy Intensity Input B11
Percentiles											
1%	47.9423	47.8973	23.7617	9.5849	90.8506	6.9989	1.18211	87.248	17.2467	0.70310	0.0160
10%	48.8639	48.9007	24.2686	9.7764	92.3835	7.4063	1.32963	92.678	17.5913	0.74373	0.1488
20%	49.3899	49.4525	24.5842	9.8893	93.2820	7.6744	1.41561	95.884	17.7949	0.76918	0.3182
25%	49.6484	49.6836	24.7076	9.9343	93.6105	7.7776	1.44542	97.148	17.8787	0.77872	0.4051
30%	49.8488	49.8851	24.8322	9.9764	93.8986	7.8663	1.47927	98.335	17.9534	0.78834	0.4959
35%	50.0314	50.0801	24.9431	10.0128	94.2359	7.9576	1.50881	99.338	18.0208	0.79696	0.5847
40%	50.2108	50.2916	25.0554	10.0493	94.5160	8.0470	1.54068	100.347	18.0893	0.80540	0.6768
45%	50.4319	50.4856	25.1615	10.0921	94.7778	8.1328	1.57329	101.406	18.1637	0.81324	0.7787
50%	50.6512	50.7187	25.2660	10.1350	95.0402	8.2244	1.60420	102.528	18.2412	0.82232	0.8829
55%	50.8657	50.9273	25.3788	10.1778	95.3043	8.3087	1.63717	103.779	18.3266	0.83136	0.9869
60%	51.0791	51.1805	25.4955	10.2232	95.5614	8.4048	1.67112	104.954	18.3989	0.84100	1.0974
65%	51.3197	51.4069	25.6226	10.2753	95.8875	8.5046	1.70788	106.238	18.4950	0.85092	1.2179
70%	51.5816	51.6424	25.7606	10.3302	96.1834	8.6281	1.75044	107.614	18.5947	0.86190	1.3509
75%	51.9094	51.9210	25.9067	10.3941	96.5048	8.7509	1.79612	109.069	18.6995	0.87317	1.4920
80%	52.2371	52.2177	26.0600	10.4586	96.8769	8.8751	1.83853	110.589	18.8310	0.88616	1.6728
90%	53.0088	53.0883	26.4690	10.6244	97.7727	9.1921	1.96308	114.884	19.1074	0.91765	2.0782
99%	54.4025	54.3121	27.1947	10.8925	99.2561	9.7271	2.14651	121.810	19.5711	0.97468	2.6878

Table A6. Stonnington summary statistics in total.

Name Description Cell Function	Public Transport Network Ratio Input B1	Harbour- Type Bus Stop Setting Input B2	Public Transporta- tion Priority Lane Setting Ratio Input B3	Public Transport on-Time Rate Input B4	Intersection Blocking Rate during Peak Hours Input B5	Public Transport Driving Accident Rate Input B6	Coverage Rate Input B7	Bus Ownership Rate Input B8	Public Transport Land Area per Capita Input B9	Public Transport Utilisation Rate Input B10	Public Transport Energy Intensity Input B11
Percentiles											
1%	58.2778	33.5281	24.3450	81.471	0.00195	1.9805	130.731	17.2430	9.6035	0.68273	28.7683
10%	59.3986	34.2305	24.8002	83.100	0.02521	2.2067	139.076	17.5887	10.1992	0.72272	29.3080
20%	60.0381	34.6167	25.0836	84.059	0.05237	2.3556	143.939	17.7927	10.5512	0.74777	29.6645
25%	60.3525	34.7785	25.1945	84.441	0.06522	2.4130	145.626	17.8731	10.6959	0.75717	29.8038
30%	60.5962	34.9196	25.3064	84.800	0.07794	2.4622	147.540	17.9486	10.8251	0.76664	29.9308
35%	60.8182	35.0561	25.4064	85.108	0.09474	2.5131	149.203	18.0126	10.9410	0.77512	30.0418
40%	61.0363	35.2041	25.5152	85.419	0.11044	2.5666	150.871	18.0813	11.0507	0.78331	30.1546
45%	61.3052	35.3399	25.6195	85.783	0.12673	2.6185	152.555	18.1552	11.1681	0.79084	30.2794
50%	61.5718	35.5031	25.7224	86.148	0.14482	2.6740	154.151	18.2334	11.2904	0.79954	30.4072
55%	61.8327	35.6491	25.8333	86.511	0.16374	2.7251	155.854	18.3206	11.4251	0.80822	30.5345
60%	62.0922	35.8263	25.9481	86.898	0.18214	2.7834	157.607	18.4025	11.5393	0.81747	30.6699
65%	62.3847	35.9849	26.0731	87.340	0.20550	2.8438	159.505	18.4920	11.6908	0.82698	30.8174
70%	62.7032	36.1497	26.2089	87.807	0.22669	2.9187	161.703	18.5880	11.8482	0.83752	30.9803
75%	63.1019	36.3447	26.3526	88.350	0.24970	2.9931	164.062	18.6894	12.0135	0.84833	31.1531
80%	63.5003	36.5524	26.5034	88.898	0.27635	3.0683	166.252	18.7954	12.2210	0.86079	31.3745
90%	64.4387	37.1618	26.9058	90.308	0.34050	3.2604	172.684	19.0948	12.6572	0.89100	31.8711
99%	66.1334	38.0184	27.6197	92.586	0.44673	3.5847	182.156	19.5776	13.3888	0.94571	32.6177

Table A7. Case study areas' MCS model results.

Criteria	Most Likely Value		
	Bayswater	Cockburn	Stonnington
PT network ratio	50.83	50.83	61.79
PT coverage ratio	50.83	55.91	-
Harbour-type bus stop setting ratio	25.33	25.33	35.58
Public transportation priority lane setting ratio	10.16	10.16	25.8
PT on-time rate	95.08	95.08	86.41
Intersection blocking rate during peak hours	8.26	8.26	0.16
PT driving accident rate	1.62	1.62	2.7
Coverage rate	103.33	103.33	155
Bus ownership rate	18.3	18.3	18.3
PT land area per capita	-	-	11.36
PT utilisation rate	0.83	0.83	0.8
PT energy intensity	1	1	30.5

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