sustainability

MDPI

Article

A Measurement of Social Cohesion in Poland's NUTS2 Regions in the Period 2010–2019 by Applying Dynamic Relative Taxonomy to Interval-Valued Data

Marek Walesiak [1] and Grażyna Dehnel [2,*]

[1] Department of Econometrics and Computer Science, Wroclaw University of Economics and Business, 53-345 Wrocław, Poland

[2] Department of Statistics, Poznań University of Economics and Business, 61-875 Poznań, Poland

* Correspondence: grazyna.dehnel@ue.poznan.pl

Abstract: Composite indicators play an important role in the analysis of socio-economic phenomena. A number of different approaches to constructing composite indicators have been proposed in the literature. Depending on the degree of compensation, they can be divided into compensatory, partially compensatory, and non-compensatory. The following article focuses on the method of relative taxonomy and its dynamic modification. While this method is typically applied to metric data, the authors propose using the dynamic approach for interval-valued data, which describes objects of interest more precisely. Metric data are of an atomic nature; i.e., an observation of each variable is expressed as one real number. In contrast, each observation of an interval-valued variable is expressed as an interval. By making use of interval-valued data, it is possible to assess objects not only at the regional level but also at a lower level of territorial aggregation, taking into account spatial variation across districts that make up each region. The study described in the article was conducted by applying relative taxonomy in its dynamic approach to interval-valued data in order to measure the level of social cohesion in Poland's NUTS2 regions during the period 2010–2019. The target dataset was obtained by aggregating numeric data about social cohesion in districts (LAU1) at the level of regions. The lower and upper limit of the interval for each region was based on district-level data and corresponded to the 2nd and 8th decile, respectively (60% of observations), which helped to mitigate the effect of outliers. By applying dynamic relative taxonomy to interval-valued data, it was possible to graphically represent changes in the level of social cohesion that took place across 17 Poland's NUTS2 regions between 2010 and 2019. It was found that during the reference period, the level of social cohesion in the regions systematically improved. Despite the observed variation, the distance between the regions consistently decreased over time. The level of social cohesion was found to be higher in regions that had received more EU funding to support regional development.

Keywords: social cohesion; relative taxonomy; composite indicators; dynamic approach; interval-valued data

Citation: Walesiak, M.; Dehnel, G. A Measurement of Social Cohesion in Poland's NUTS2 Regions in the Period 2010–2019 by Applying Dynamic Relative Taxonomy to Interval-Valued Data. *Sustainability* **2023**, *15*, 3752. https://doi.org/10.3390/su15043752

Academic Editors: Tomasz Kijek, Aleksandra Kowalska, Arkadiusz Kijek and Anna Matras-Bolibok

Received: 18 December 2022
Revised: 12 February 2023
Accepted: 15 February 2023
Published: 17 February 2023

1. Introduction

Social cohesion is a multifaceted phenomenon, which means that there are various possible ways of identifying and measuring it [1]. The pursuit of social cohesion involves efforts to eliminate regional inequalities, which result from a differential allocation of goods and services and limited access to them. Social cohesion can be analyzed and measured for more or less complex territorial units on the regional level, such as NUTS2.

Systematic monitoring and measurement of social cohesion require the use of adequate methods and reliable statistical data. Social cohesion can be assessed on the basis of primary data (cf., e.g., [2–10]) or secondary data (cf., e.g., [11–16]). Primary data about social cohesion come from sample surveys, while secondary data are provided mainly by official statistics.

It is worth noting that studies aimed at assessing the level of social cohesion are usually based on primary data collected by means of questionnaires, which are designed to elicit non-standard information that is particularly relevant to the analysis of social cohesion. However, the use of primary data is associated with certain limitations. Indicators derived from questionnaire data represent weak measurement scales and are subjective since what they actually reflect are respondents' attitudes, opinions, and preferences. Given the high costs of such targeted surveys, it is not feasible to use them for a systematic measurement of social cohesion to obtain a longer time series (e.g., for the period 2010–2019). Moreover, statistical data collected in surveys are usually atomic, which means that an observation of each variable is expressed as a single category or real number. Hence, the research problem of our study is how to assess changes in the level of social cohesion without using primary data.

Secondary data sources can be a good alternative with respect to some of these limitations. For one thing, they tend to provide systematic information about social cohesion for longer time series. Another advantage is that relevant data are often available at lower levels of territorial aggregation (e.g., LAU1 districts). Because secondary sources usually contain interval or ratio data and are objective (independent of respondents), they can provide not only atomic but also interval-valued information. It has to be admitted, however, that secondary data are also not free from problems, which come down to two main limitations: the scope of available data and the time when they are released. As regards the first issue, variables measured in social surveys conducted by official statistics may not cover all aspects of social cohesion. The other problem is that secondary data are released with some delay. Despite these limitations, the empirical study described in this article was based on secondary data from official statistical sources, which were used to track changes in social cohesion that took place over the period of 10 years.

The purpose of the study was to measure changes in the level of social cohesion that took place across Poland's NUTS2 regions between 2010 and 2019. The study was conducted using composite indicators derived from interval-valued data [17]. Interval-valued variables characterize objects of interest more accurately than metric data. Classic data are of an atomic nature; i.e., an observation of each variable is expressed as a single real number. In contrast, an observation of each interval-valued variable is expressed as an interval. Studies by Gioia and Lauro [18], Brito, Noirhomme-Fraiture, and Arroyo [19] provide different examples of interval data.

Composite indicators play an important role in the analysis of socio-economic phenomena. A number of different approaches to constructing composite indicators have been proposed in the literature. Depending on the degree of compensation, they can be divided into compensatory, partially compensatory, and non-compensatory ([20], p. 3). Studies concerning multi-criteria decision making (MCDM) make use of non-compensatory methods (see [21–23]). An overview of compensatory and partially compensatory aggregate measures applied to different types of data can be found in Walesiak and Dehnel [24]; Walesiak, Dehnel, and Dudek [25]. Many different concepts and applications for ranking sets of objects based on aggregate measures have been developed, including classic data matrix (an aggregate measure that accounts for the pattern of development [26]; aggregate measures accounting for the pattern and the anti-pattern of development [27]; the TOPSIS measure [28]); ordinal data (an aggregate measure in the form of GDM2 distance measure from the pattern of development [29]); fuzzy numbers (for example, fuzzy TOPSIS [30]; an interval-valued intuitionistic fuzzy synthetic measure (I-VIFSM) based on Hellwig's approach [31]); the conditions of spatial dependence [32]; combining multidimensional scaling with ranking sets of objects for classical data [33]; aggregate measures with a penalty function [34,35]; aggregate measures with the adjustment to the surroundings of a given object [36]; an iterative approach to ranking sets of objects, whereby in each iteration the highest ranked object receives the next position in the ranking and is eliminated from the set of objects [37]; and an aggregate measure covering special cases of a generalized mean (or power mean of order r): minimum, harmonic mean, geometric mean, arithmetic mean, quadratic mean, cubic mean, and maximum [38]. A guide for constructing and using composite indicators for policy makers, academics, the media, and other interested parties can be found in publications [39,40].

Aggregate measures are used to rank sets of objects in terms of innovativeness, competitiveness, well-being, social cohesion, sustainable development, poverty and social exclusion, social inclusion, customer satisfaction, quality of life, and quality of health services.

Composite indices are widely used as synthetic measures to assess social cohesion. Duhaime et al. [2] assess the level of social cohesion in the Canadian Artic using six sets of indices. Bernard and Chan's definition of social cohesion was used as the basis for the VALCOS (VALeurs et COhésion Sociale) index developed for European countries ([3,41,42]). Two social cohesion indices: a national average SCI and a Social Cohesion Index Variance-Adjusted (SCIVA) to assess the national level of social cohesion for African countries were developed by [4]. Balcerzak [12] analyzed social cohesion in EU countries using a synthetic measure of development put forward by [26]. There are also many other proposals and studies contributing to the measurement of social cohesion using composite indicators ([5,13], among others).

Aggregate measures based on interval-valued data are also becoming increasingly popular. The following measures have been proposed: an aggregated measure of the development of interval data [43]; interval-valued TOPSIS [44]; an approach combining multidimensional scaling with rankings of sets of objects based on interval-valued data [45]; interval-based composite indicators [46,47]; and composite indices based on the performance interval approach [38]. Composite indicators based on interval-valued data were used to study well-being in Italian regions [38], poverty in Italian regions [46], territorial variation in poverty across Polish provinces [43], energy efficiency and green entrepreneurship [47], health systems of selected countries of the world [44], and economic efficiency of medium-sized manufacturing enterprises in districts of Wielkopolska province [45].

This article focuses on the method of relative taxonomy proposed by Wydymus [48] and its dynamic modification [24]. This approach was extended by Walesiak, Dehnel, and Dudek [25] to include robust measures of central tendency. Both approaches were developed with respect to classic, metric data. The article proposes to extend the use of the dynamic relative taxonomy method to interval-valued data in the assessment of social cohesion. The main advantage of the proposed approach over those relying on metric, atomic data is that it can be used to assess regions (NUTS2 units) not only based on region-level data but, thanks to the use of interval-valued data, also by taking into account within-region variation at lower levels of spatial aggregation, i.e., at district level (LAU1), which is lost once such data are aggregated to average region-level values. In the assessment of cohesion, the point is not only to ensure that the average situation is good, but also that there are no significant spatial differences in the region. The research procedure proposed in Section 4 offers the choice of five measures of central tendency: mean, trimmed mean, median, winsorized mean, and biweight mean. The mean is the optimal estimator of Gaussian data for location measures. The presence of outliers can have considerably distorted classical statistical methods that are valid under the assumption of normality. Outliers cause the distributions of the analyzed variables to be skewed. To deal with this problem, several robust alternatives to the mean, which are less sensitive to outliers, have been proposed. In the context of relative taxonomy, two kinds of robust location estimation methods are particularly useful [49,50]: L-estimators (median, trimmed mean, and winsorized mean) and M-estimators (biweight mean). The median is defined as the value separating the higher half from the lower half of the data. The trimmed mean is calculated after trimming a fraction of observations (e.g., 20%) from each end. In the winsorized mean, rather than just dropping the top and bottom trim percent, these extreme values are replaced with values at the trim and one—trim quantiles. The concept of an iterative reweighed measure of central tendency, called the biweight, was proposed by Beaton and Tukey [51]. The biweight (bisquare weight) estimator represents a class of robust M-estimators. In M-estimation, each observation is weighted according to a function (e.g., Tukey's) selected for its special properties [52]. Results of the analysis of the degree of compatibility between rankings of objects based on robust measures of central tendency are better than those obtained after employing the arithmetic mean. Of the robust measures of central tendency tested in the study, the best results were obtained for the trimmed mean, which helped to mitigate the effect of outliers on the variables used for measuring the level of social cohesion in the regions.

The article starts with a review of some of the theoretical and empirical work on social cohesion. Variables and data availability are characterized in Section 3. Section 4 describes each step of our methodological approach to assessing social cohesion across Poland's regions. Section 5 presents the model specification in detail, including the selection of the measure of central tendency in dynamic relative taxonomy. Section 6 contains a summary of the results and their interpretation. The article ends with Discussion and Conclusions.

2. An Overview of the Social Cohesion Concept

Since social cohesion has a positive effect on economic development and its stability, more and more studies and analyses are being undertaken to investigate it. Nowadays, social cohesion is perceived as a prerequisite of political stability and security. It contributes to prosperity and economic growth, while its absence or insufficient levels require increased public spending [53,54]. Social cohesion derives from social policy. One of the first and most prominent frameworks in this regard was developed by Canadian Policy Research [55]. Modern understanding of social cohesion is the result of numerous theoretical and methodological approaches that can be found in the literature ([1,4,8,41,42,55–61]). They represent two main discourses: the academic discourse focuses on sociology and psychology and on the conceptual and analytic understanding of social cohesion [43], with emphasis on integration and social stability ([6,9,10,57,62,63]). In contrast, in the policy discourse, social cohesion is viewed as a prerequisite of economic well-being ([1,7,8,41,42,55,56]). This discourse is described as problem-driven because it refers to the numerous economic and social problems resulting from unequal income distribution, employment, poverty and social exclusion, housing issues, limited access to health care and education, and participation in political and public life ([7,8,42]). Both types of discourses differ with regard to certain aspects of social cohesion, which is a reflection of differences between theoretical perspectives offered by social sciences and concerns expressed by representatives of particular policy fields. Despite these disparities, there is a strong conceptual overlap between them. Both discourses capture six core dimensions: social relations, sense of belonging, orientation towards the common good, (in)equality, quality of life, and shared values. The last three are sometimes treated as constitutive components and, in other cases, as antecedents or consequences of social cohesion [64]. Changes in the way social cohesion has been conceptualized over the years reflect the increasing role of socio-cultural and political indicators and the declining importance of the economic sphere.

Contemporary approaches to social cohesion are more strongly connected with its operationalization and utility for policy makers, which is determined by two main objectives: (1) minimizing inequalities and social exclusion; and (2) strengthening social relations, interactions, and ties ([53,65]). This approach is present in cohesion policies pursued by the EU, OECD, and the World Bank [65]. The Council of Europe, for example, defines social cohesion as "the capacity of a society to ensure the well-being of all its members, minimising disparities and avoiding marginalisation" ([66], p. 3). So, generally speaking, social cohesion is identified with changes aimed at improving the living conditions of society, and hence with social progress. The level and development of social cohesion over time and across regions and societies can only be monitored on the basis of measurable indicators. This is where composite indicators come into play. However, the measurement of social cohesion is always associated with certain difficulties as there is no single indicator that covers all aspects of social cohesion. In each case, it is necessary to thoroughly quantify the magnitude of social problems, depending on which concept of social cohesion has been adopted. One of the proposed indicators enabling a more comprehensive measurement of social development is the Social Progress Index—SPI (EU-SPI for EU countries). This synthetic indicator [67] describes the level of social progress on the basis of 50 social and environmental variables. It takes no account of economic indicators. A more detailed overview of the social cohesion concept and its measurement, including the strategic priorities of cohesion policy in the European Union, can be found in Walesiak and Dehnel [15].

3. Variables and Data Availability

In our study, changes in social cohesion were assessed following the concept of the EU-SPI [67,68], where three dimensions of social progress are distinguished: basic human needs, foundations of well-being, and opportunities. Twenty-two metric variables were selected to assess the level of social cohesion in Poland (see Table 1). Changes that took place over such a long period (2010–2019) could only be tracked using secondary data, which, unfortunately, did not contain variables included in the EU-SPI, such as "Trust in the legal system", "Trust in the police", "Making friends", "Volunteering", or "Gender employment gap". After a thorough analysis of secondary data sources, a set of variables was selected that was as close as possible to those included in the EU-SPI [67].

Table 1. Variables applied in the assessment of social cohesion.

Dimensions	Variables
Basic human needs	y1—death rate among persons below the age of 60
	y2—users of water treatment services (% of total population)
	y3—percentage of all dwellings equipped with central heating
	y4—mean useful floor area of a dwelling per inhabitant in m^2
	y5—average number of persons per room
	y6—length of the sewerage network in relation to the length of the water supply network in %
	y7—number of doctors and dentists per 10,000 population
	y8—crimes reported (criminal offenses, against life and health, against property) per 10,000 population
Foundations of well-being	y9—mean gross monthly wage in PLN
	y10—children enrolled in nursery schools per 1000 children aged 3–5
	y11—pupils taking obligatory classes of English in primary and intermediate schools (% of all pupils)
	y12—people participating in cultural events (organized by cultural centers and clubs) per 1000 population
	y13—area of public greenspace (parks, residential greenspace) per 10,000 population (in ha)
	y14—length of municipal and district improved hard surface roads per 10,000 population (in km)
	y15—road accidents per 100,000 population
Opportunities	y16—dependency ratio (ratio of the dependent and elderly population per 100 working-age population)
	y17—total unemployment rate in %
	y18—percentage share of women in the labor force
	y19—percentage share of young adults (up to the age of 25) among registered unemployed
	y20—percentage share of long-term unemployed (over 12 months) in the population of registered unemployed in %
	y21—beneficiaries of social assistance at the place of residence (below the means test threshold) per 1000 population
	y22—places in stationary social welfare facilities per 10,000 population

Source: authors' compilation.

Variables y2–y4, y6, y7, y9–y14, and y22 are examples of stimulants (where higher values are preferred), variables y1, y5, y8, y15–y17, and y19–y21 are destimulants (where lower values are preferred), and y18 is a nominant (with the nominal value of 50%). Statistical data on social cohesion including 22 variables about 380 districts of Poland in the period 2010–2019 come from the Local Data Bank (BDL) maintained by Statistics Poland. Poland has a three-tier system of administrative division, consisting of 16 provinces (Pol. *województwo*), 380 districts (Pol. *powiat*), and 2477 communes (Pol. *gmina*). The empirical study was conducted at the level of NUTS2 regions, which were established in 2018 [69]. Poland is divided into 17 regions, 15 of which correspond to 15 provinces, while the remaining two were created by splitting Mazowieckie into two parts: one covering the metropolitan area of Warsaw (Warszawski stołeczny, consisting of 10 districts) and the second, encompassing the rest of the province (Mazowiecki regionalny, with 32 districts). The number of districts in the remaining regions ranges from 12 (Opolskie) to 36 (Śląskie).

District-level data for the following variables are missing (NA):

- variable y7 for 2012;
- variables y8 and y12 for 2010–2012;
- variable y11 for 2019;
- variable y15 for 2010;
- no data for Wałbrzych, a town with district status, for 2010–2012.

The variables in Table 1 are treated as equally important in the assessment of social cohesion. For this reason, the same weights are used in the procedure of constructing aggregate measures (8)–(10), which is described in Section 4. Equal weighting is the most common scheme appearing in the development of compensatory and partially compensatory composite indicators ([20,39,40,43]). To weigh a variable means to give it greater or lesser importance than other variables in determining the composite index. The introduction of differentiated weights usually causes changes in the ranking of objects. Therefore, the use of such weights requires substantive justification.

4. Social Cohesion across Poland's Regions—Research Methodology

In order to create a ranking of regions in terms of social cohesion, the authors propose applying a dynamic version of relative taxonomy to interval-valued data. The procedure consists of the following steps:

1. A set of interval-valued data was created in two steps:
   a. Classic metric data about social cohesion (22 variables) in the period 2010–2019 were collected for 380 districts. The observations of m variables for n' objects and T periods were combined into one data matrix:

$$\left[y_{ijt}\right]_{n'\cdot T\times m}=\begin{bmatrix} y_{111} & y_{121} & \cdots & y_{1m1} \\ \vdots & \vdots & \cdots & \vdots \\ y_{n'11} & y_{n'21} & \cdots & y_{n'm1} \\ \cdots & \cdots & \cdots & \cdots \\ y_{11T} & y_{12T} & \cdots & y_{1mT} \\ \vdots & \vdots & \cdots & \vdots \\ y_{n'1T} & y_{n'2T} & \cdots & y_{n'mT} \end{bmatrix} \quad (1)$$

   where $i=1,\ldots,n'$—object's number ($n'=380$: data for 380 Polish districts), $j=1,\ldots,m$—variable number ($m=22$: variables describing social cohesion—see Table 1), $t=1,\ldots,T$—period number (years from 2010 to 2019).

   b. Stimulants, destimulants, and nominants were identified in the set of variables. The first two terms were introduced by Hellwig [26], while the third one, 'nominant' was proposed by Borys [70]. Their definitions can be found in Walesiak [71]. Instead of describing variables as stimulants and destimulants, Mazziotta and Pareto [34,35] use the terms 'positive polarity' (increasing values of the index correspond to the phenomenon improvement) and 'negative polarity' (increasing values of the index correspond to the phenomenon worsening). Hwang and Yoon ([28], p. 130) use the concepts of 'benefit' (larger values of a variable are more preferred) and 'cost' (larger values of a variable are less preferred). Destimulants D and nominants N were converted into stimulants using the ratio transformation (cf. e.g., [72], p. 18):

$$x_{ijt}=\left(y_{ijt}^{D}\right)^{-1} \quad (2)$$

$$x_{ijt} = \frac{\min\left\{nom_j; y_{ijt}^N\right\}}{\max\left\{nom_j; y_{ijt}^N\right\}} \tag{3}$$

where nom_j—nominal level of the j-th variable.

c. Metric (atomic) data were aggregated for each of the 17 regions to obtain interval-valued data. The lower and upper limits of the interval for each region were based on district-level data and corresponded to the 2nd and 8th decile, respectively (60% of observations). The 2nd and 8th decile were chosen as the lower and upper limits of the interval to mitigate the influence of outliers, which would not be limited by the minimum and maximum values selected. Given the type of data used in the analysis (as noted in Section 3, the number of districts in NUTS2 regions ranges from 12 to 36), the choice of the 5th and 95th percentile, or the 1st and 9th decile would not be enough to mitigate the influence of outliers. As a result, a single data table was created containing m interval-valued variables describing n objects in T periods:

$$\begin{bmatrix} \left[x_{111}^l, x_{111}^u\right] & \left[x_{121}^l, x_{121}^u\right] & \cdots & \left[x_{1m1}^l, x_{1m1}^u\right] \\ \vdots & \vdots & \cdots & \vdots \\ \left[x_{n11}^l, x_{n11}^u\right] & \left[x_{n21}^l, x_{n21}^u\right] & \cdots & \left[x_{nm1}^l, x_{nm1}^u\right] \\ \cdots & \cdots & \cdots & \cdots \\ \left[x_{11T}^l, x_{11T}^u\right] & \left[x_{12T}^l, x_{12T}^u\right] & \cdots & \left[x_{1mT}^l, x_{1mT}^u\right] \\ \vdots & \vdots & \cdots & \vdots \\ \left[x_{n1T}^l, x_{n1T}^u\right] & \left[x_{n2T}^l, x_{n2T}^u\right] & \cdots & \left[x_{nmT}^l, x_{nmT}^u\right] \end{bmatrix} \tag{4}$$

where $\left[x_{ijt}^l, x_{ijt}^u\right]$—the observation of the j-th interval-valued variable for the i-th object in t-th period ($x_{ijt}^l \leq x_{ijt}^u$);

x_{ijt}^l and x_{ijt}^u—the lower and the upper bound (limit) of the interval;
$i = 1, \ldots, n$—object number ($n = 17$: 17 regions);
$j = 1, \ldots, m$—variable number ($m = 22$: variables describing social cohesion—see Table 1);
$t = 1, \ldots, T$—period number (years from 2010 to 2019).

2. Values of each j-th variable were relativized. Relativization of interval-valued data requires special treatment. The lower and upper limits of the interval of the j-th variable for n objects in T periods are combined into one vector containing $2n \cdot T$ observations:

$$\left[x_{ijt}\right] = \left[x_{1j1}^l, \ldots, x_{nj1}^l, \ldots, x_{1jT}^l, \ldots,\ x_{njT}^l, x_{1j1}^u, , \ldots, x_{nj1}^u, \ldots, x_{1jT}^u, \ldots,\ x_{njT}^u\right]'. \tag{5}$$

Values of each j-th variable were relativized according to the following $2n \cdot T \times 2n \cdot T$ matrix:

$$\begin{bmatrix} 1 & \cdots & x^u_{njT}/x^l_{1j1} \\ \vdots & \vdots & \vdots \\ x^l_{1j1}/x^l_{nj1} & \cdots & x^u_{njT}/x^l_{nj1} \\ \cdots & \cdots & \cdots \\ x^l_{1j1}/x^l_{1jT} & \cdots & x^u_{njT}/x^l_{1jT} \\ \vdots & \vdots & \vdots \\ x^l_{1j1}/x^l_{njT} & \cdots & x^u_{njT}/x^l_{njT} \\ x^l_{1j1}/x^u_{1j1} & \cdots & x^u_{njT}/x^u_{1j1} \\ \vdots & \vdots & \vdots \\ x^l_{1j1}/x^u_{nj1} & \cdots & x^u_{njT}/x^u_{nj1} \\ \cdots & \cdots & \cdots \\ x^l_{1j1}/x^u_{1jT} & \cdots & x^u_{njT}/x^u_{1jT} \\ \vdots & \vdots & \vdots \\ x^l_{1j1}/x^u_{njT} & \cdots & 1 \end{bmatrix} \quad (6)$$

As a result of relativization, variable values are dimensionless. When the numerator is not greater than the denominator, the relativization formula produces values included in the interval $(0; 1]$; otherwise, values are included in the interval $(1; \infty)$.

3. The average similarity of a given relativized observation with respect to other relativized observations of the j-th variable for each column of matrix (6) was calculated using the arithmetic mean or one of the robust measures of central tendency: median, trimmed mean, winsorized mean, and biweight mean. The corresponding formulas can be found in Section 3.1 of the article by Walesiak, Dehnel, and Dudek [25]. The same formulas can also be found in [51,52,73]. After this operation, observations for each variable in period t from 1 to n are the lower limits of intervals while observations from $n + 1$ to $2n$ are the upper limits:

$$\begin{bmatrix} \left[z^l_{111}, z^u_{111}\right] & \left[z^l_{121}, z^u_{121}\right] & \cdots & \left[z^l_{1m1}, z^u_{1m1}\right] \\ \vdots & \vdots & \cdots & \vdots \\ \left[z^l_{n11}, z^u_{n11}\right] & \left[x^l_{n21}, z^u_{n21}\right] & \cdots & \left[z^l_{nm1}, x^u_{nm1}\right] \\ \cdots & \cdots & \cdots & \cdots \\ \left[z^l_{11T}, z^u_{11T}\right] & \left[z^l_{12T}, z^u_{12T}\right] & \cdots & \left[z^l_{1mT}, z^u_{1mT}\right] \\ \vdots & \vdots & \cdots & \vdots \\ \left[z^l_{n1T}, z^u_{n1T}\right] & \left[z^l_{n2T}, z^u_{n2T}\right] & \cdots & \left[z^l_{nmT}, z^u_{nmT}\right] \end{bmatrix} \quad (7)$$

The data table (7) is equivalent to a normalized data table in multivariate statistical analysis.

4. The outputs of step 3 were used to calculate values of the composite indicator SM_{it} for the lower limit (l) and the upper limit (u) of the interval using the following formulas:

$$SM^l_{it} = f\left(\frac{1}{z^l_{ijt}}\right) \quad (8)$$

$$SM^l_{it} = f\left(\frac{1}{z^l_{ijt}}\right) \quad (9)$$

where f—mean, median, trimmed mean, winsorized mean, or biweight mean.

The question of selecting the optimal measure of central tendency in dynamic relative taxonomy is addressed in Section 5.

Values of the composite indicator SM_{it} for the center (c) of the interval $\left[SM_{it}^{l}; SM_{it}^{u}\right]$ were calculated using the formula:

$$SM_{it}^{c} = \left(SM_{it}^{l} + SM_{it}^{u}\right)/2 \tag{10}$$

Values of SM_{it} given by (8)–(10) can be greater or smaller than one. The smaller the value of SM_{it}, the better the position of object i relative to other objects in a time interval from $t = 1$ to $t = T$.

5. Results of the ordering of objects obtained by applying dynamic relative taxonomy to interval-valued data were presented graphically.

Unlike the static approach, the dynamic approach shows not only relations between the objects in specific periods, but also changes in the phenomenon of interest that took place between objects over the entire reference period. Formulas (8)–(10) can be extended by including different weights for variables, which express their individual contribution to the aggregate phenomenon. There are three ways of determining variable weights (see, e.g., [24]). They can be determined either based on expert judgment, or using algorithms based on information included in primary (raw) data, or by combining both these methods. More information about how to select variable weights can be found in Becker et al. [74] and Greco et al. [75]. The problem of determining variable weights has not yet been satisfactorily solved. For this reason, authors of empirical studies often assume that the variables are equally important from the perspective of the research problem (see e.g., [75]). Moreover, the research methodology proposed in point 4 of the study allows conducting the analysis taking into account the missing data (NA). This type of data (NA) is not included in the calculation process of SM_{it} aggregate measure.

Compared to other methods of ordering a set of objects, relative taxonomy for interval-valued data has certain limitations:

- The interval between the lower and upper limit $\left[x_{ijt}^{l}, x_{ijt}^{u}\right]$ contains only positive real numbers. This is not a serious problem, given that analyses of economic phenomena usually involve positive numbers;
- Composite indicators SM_{it} do not have an upper limit, which does not disqualify them.

5. Social Cohesion across Poland's Regions—Selection of the Measure of Central Tendency in Dynamic Relative Taxonomy

In order to select one of the five measures of central tendency in the procedure described in Section 4, the similarity of rankings of objects with respect to individual variables was compared with the ranking based on the composite indicator. This comparison of the rankings can be justified by the fact that the overall ranking is a product of the rankings obtained for the individual variables. The following procedure was used:

1. The lower limit (l), the upper limit (u) (see data table (7)), and the center (c) of the interval $\left[SM_{it}^{l}; SM_{it}^{u}\right]$ were used to calculate values of composite indicators SM_{it}^{l}, SM_{it}^{u}, and SM_{it}^{c}. In each case, five measures of central tendency were employed, which resulted in five different rankings for each composite indicator.
2. The rankings obtained in step 1 were compared to individual rankings created on the basis of individual variables ($j = 1, \ldots, m$) for the lower limit, the upper limit, and the center of the interval (see data table (4)) using Kendall's tau correlation coefficient.
3. Based on m results obtained in step 2 (separately for five measures of central tendency and the lower, upper bound, and the center of the interval), the mean value is calculated (see Table 2). Higher mean values represent a greater degree of similarity

between the rankings of objects with respect to individual variables and the ranking based on the composite indicator.

Table 2. The degree of compatibility between the rankings of objects based on individual variables and the ranking based on the aggregate measure SM_{it}.

No.	Measure of Central Tendency	Lower Bound of Interval		Upper Bound of Interval		Center of Interval	
		Kendall's Tau	Rank	Kendall's Tau	Rank	Kendall's Tau	Rank
1	Mean	0.2256	5	0.2590	3	0.2453	5
2	Trimmed mean (10%)	0.2470	2	0.2596	1	0.2593	1
3	Median	0.2387	4	0.2499	5	0.2528	4
4	Winsorized mean (10%)	0.2400	3	0.2595	2	0.2568	3
5	Biweight mean	0.2496	1	0.2555	4	0.2577	2

Source: authors' calculations using R program [76].

On the basis of the comparison from Table 2, it should be stated that the results in the assessment of the degree of compliance of the ordering of objects obtained with the use of robust measures of the central tendency are better than the results obtained with the use of the arithmetic mean. The robust measures help to mitigate the impact of outliers in the variables considered in the study on the rankings of regions in terms of social cohesion. Of the four robust measures of central tendency, the best results were obtained by employing the trimmed mean.

6. Results Measuring Social Cohesion in the Regions by Means of Relative Taxonomy for Interval-Valued Data

The method of dynamic relative taxonomy (described in Section 4) was used to measure changes in the level of social cohesion that took place between 2010 and 2019 across 17 regions of Poland. The main benefit of the dynamic approach is that it shows not only relations between the objects in specific periods, but also changes in the phenomenon of interest that took place between objects over the entire reference period. The assessment is based on values of the composite indicator SM_{it}, which is used for interval-valued data. This means that social cohesion in each region is assessed at a lower level of spatial aggregation, i.e., on the basis of district-level data. This approach accounts for internal variation within the regions. According to the results obtained in Section 5, the trimmed mean was used to calculate the composite indicators in the procedure of dynamic relative taxonomy (described in Section 4).

Values of the composite indicators were calculated for the lower and upper limit and as well as the center of the interval (using formulas (8)–(10)): SM^l_{it}, SM^u_{it}, and SM^c_{it} (see Table 3). Smaller values of SM_{it} indicate that the position of region i in terms of social cohesion is better in relation to other regions in a given year and over the entire reference period 2010–2019.

Figure 1 contains choropleth maps showing the level of social cohesion in the regions in 2010 and 2019 based on the composite indicator SM^c_{it} for the center of the interval. The class intervals are ordered from the lowest [0.8–0.84] (the highest level of social cohesion) to the highest [1.24–1.28] (the lowest level of social cohesion). The direction of changes in all regions is the same: the level of social cohesion increased considerably over the period of 10 years. There were, however, differences in the rate of changes, which are discussed in more detail below.

Table 3. The values of the composite indicator for center, lower, and upper bound of interval showing changes in the assessment of the level of social cohesion of Polish regions from 2010 to 2019 (sorted by center of interval 2019 values).

i	Region	Interval Bound	SM_{it} Aggregate Measure Values										
			2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	Δ
1	Śląskie	center	0.9828	0.9666	0.9794	0.9607	0.9356	0.8915	0.8704	0.8432	0.8405	0.8125	−0.1702
		lower	1.1764	1.1415	1.1542	1.1410	1.1070	1.0511	1.0221	1.0044	0.9979	0.9798	−0.1966
		upper	0.7891	0.7918	0.8046	0.7803	0.7642	0.7319	0.7188	0.6819	0.6831	0.6453	−0.1439
2	Warszawski stołeczny	center	0.9633	0.9493	0.9491	0.9417	0.9306	0.8904	0.8657	0.8460	0.8258	0.8129	−0.1504
		lower	1.1196	1.0991	1.1036	1.1089	1.0788	1.0312	1.0085	0.9798	0.9583	0.9534	−0.1662
		upper	0.8070	0.7996	0.7946	0.7746	0.7824	0.7495	0.7230	0.7121	0.6933	0.6724	−0.1345
3	Opolskie	center	1.0164	0.9849	0.9520	0.9796	0.9525	0.9058	0.8942	0.8543	0.8503	0.8186	−0.1978
		lower	1.1706	1.1249	1.0824	1.1377	1.1019	1.0506	1.0363	0.9777	0.9851	0.9519	−0.2188
		upper	0.8622	0.8449	0.8216	0.8215	0.8031	0.7611	0.7520	0.7309	0.7155	0.6854	−0.1769
4	Zachodniopomorskie	center	1.0780	1.0260	1.0274	1.0236	0.9952	0.9521	0.9466	0.8908	0.8827	0.8539	−0.2242
		lower	1.2519	1.1773	1.1953	1.2067	1.1837	1.1220	1.1198	1.0627	1.0496	1.0179	−0.2340
		upper	0.9042	0.8748	0.8595	0.8405	0.8066	0.7823	0.7735	0.7188	0.7157	0.6898	−0.2144
5	Lubuskie	center	1.1178	1.0727	1.0744	1.0811	1.0162	0.9683	0.9435	0.9019	0.8886	0.8543	−0.2635
		lower	1.3176	1.2453	1.2329	1.2728	1.1908	1.1359	1.0973	1.0505	1.0338	1.0068	−0.3108
		upper	0.9180	0.9001	0.9158	0.8894	0.8416	0.8007	0.7897	0.7533	0.7434	0.7018	−0.2162
6	Dolnośląskie	center	1.0819	1.0301	1.0415	1.0665	1.0000	0.9627	0.9425	0.9106	0.8901	0.8685	−0.2135
		lower	1.2826	1.2109	1.2298	1.2862	1.2008	1.1543	1.1336	1.0876	1.0638	1.0512	−0.2315
		upper	0.8812	0.8492	0.8531	0.8467	0.7991	0.7711	0.7514	0.7336	0.7164	0.6858	−0.1955
7	Pomorskie	center	1.0986	1.0758	1.0835	1.0948	1.0385	0.9896	0.9431	0.9036	0.9055	0.8728	−0.2258
		lower	1.3228	1.2751	1.2823	1.3250	1.2366	1.1892	1.1181	1.0691	1.0901	1.0496	−0.2732
		upper	0.8744	0.8766	0.8847	0.8645	0.8403	0.7900	0.7681	0.7382	0.7209	0.6959	−0.1785
8	Wielkopolskie	center	1.1072	1.0765	1.0388	1.0552	0.9960	0.9605	0.9293	0.9035	0.8838	0.8773	−0.2299
		lower	1.3221	1.2683	1.2225	1.2609	1.1836	1.1418	1.1038	1.0703	1.0498	1.0529	−0.2692
		upper	0.8923	0.8847	0.8551	0.8495	0.8084	0.7791	0.7549	0.7366	0.7177	0.7016	−0.1906
9	Podkarpackie	center	1.1626	1.1317	1.1264	1.0933	1.0436	1.0160	0.9798	0.9464	0.9244	0.8918	−0.2708
		lower	1.3479	1.3233	1.3087	1.2950	1.2341	1.2144	1.1684	1.1339	1.1130	1.0820	−0.2658
		upper	0.9774	0.9401	0.9440	0.8916	0.8532	0.8176	0.7911	0.7589	0.7358	0.7016	−0.2758
10	Małopolskie	center	1.0935	1.0837	1.1030	1.0841	1.0441	0.9939	0.9676	0.9438	0.9316	0.8949	−0.1985
		lower	1.2911	1.2589	1.3046	1.2931	1.2510	1.1870	1.1494	1.1353	1.1354	1.0872	−0.2039
		upper	0.8958	0.9085	0.9014	0.8751	0.8373	0.8008	0.7858	0.7523	0.7279	0.7027	−0.1931
11	Świętokrzyskie	center	1.1834	1.1434	1.1389	1.1071	1.0579	1.0283	0.9875	0.9511	0.9355	0.9134	−0.2700
		lower	1.4011	1.3345	1.3397	1.3083	1.2503	1.2274	1.1761	1.1324	1.1191	1.1000	−0.3012
		upper	0.9657	0.9522	0.9382	0.9058	0.8655	0.8292	0.7989	0.7698	0.7519	0.7268	−0.2388
12	Lubelskie	center	1.2201	1.1490	1.1809	1.1445	1.0767	1.0617	1.0100	0.9823	0.9532	0.9275	−0.2926
		lower	1.5304	1.3930	1.4376	1.4151	1.3151	1.3174	1.2402	1.2116	1.1718	1.1481	−0.3824
		upper	0.9097	0.9051	0.9242	0.8740	0.8382	0.8061	0.7799	0.7530	0.7346	0.7070	−0.2027
13	Kujawsko-pomorskie	center	1.2298	1.1748	1.1721	1.1459	1.1014	1.0547	1.0346	0.9852	0.9566	0.9297	−0.3001
		lower	1.4770	1.3949	1.3715	1.3997	1.3248	1.2613	1.2335	1.1697	1.1336	1.1131	−0.3639
		upper	0.9826	0.9548	0.9727	0.8921	0.8780	0.8482	0.8356	0.8007	0.7796	0.7464	−0.2362
14	Warmińsko-mazurskie	center	1.2030	1.1692	1.1821	1.1120	1.0918	1.0633	1.0402	0.9968	0.9787	0.9319	−0.2711
		lower	1.3832	1.3486	1.3640	1.2813	1.2580	1.2261	1.2088	1.1522	1.1383	1.0787	−0.3045
		upper	1.0227	0.9898	1.0002	0.9427	0.9256	0.9005	0.8716	0.8414	0.8192	0.7851	−0.2377
15	Łódzkie	center	1.1360	1.1287	1.1464	1.1122	1.0715	1.0443	1.0012	0.9690	0.9520	0.9320	−0.2040
		lower	1.3579	1.3401	1.3612	1.3256	1.2729	1.2602	1.1964	1.1591	1.1459	1.1267	−0.2311
		upper	0.9141	0.9174	0.9316	0.8989	0.8701	0.8284	0.8060	0.7789	0.7580	0.7372	−0.1768
16	Podlaskie	center	1.2320	1.1885	1.2215	1.1182	1.0833	1.0511	1.0145	1.0301	1.0137	0.9488	−0.2831
		lower	1.5244	1.4581	1.5281	1.3531	1.3058	1.2822	1.2354	1.2932	1.2873	1.1815	−0.3429
		upper	0.9395	0.9189	0.9150	0.8834	0.8607	0.8201	0.7936	0.7670	0.7401	0.7162	−0.2233
17	Mazowiecki regionalny	center	1.2692	1.2381	1.2556	1.2392	1.1723	1.1284	1.1134	1.0443	1.0127	0.9945	−0.2747
		lower	1.5457	1.4948	1.5245	1.5276	1.4416	1.3720	1.3609	1.2735	1.2184	1.2143	−0.3314
		upper	0.9928	0.9814	0.9867	0.9508	0.9030	0.8848	0.8659	0.8150	0.8070	0.7748	−0.2180
	Mean	center	1.1280	1.0935	1.0984	1.0800	1.0357	0.9978	0.9697	0.9355	0.9192	0.8903	−0.2377
		lower	1.3425	1.2876	1.2966	1.2905	1.2316	1.1896	1.1534	1.1155	1.0995	1.0703	−0.2722
		upper	0.9135	0.8994	0.9002	0.8695	0.8398	0.8059	0.7859	0.7554	0.7388	0.7103	−0.2031
	Standard deviation	center	0.0866	0.0800	0.0889	0.0711	0.0618	0.0650	0.0621	0.0592	0.0540	0.0497	−0.0369
		lower	0.1227	0.1109	0.1259	0.1021	0.0879	0.0928	0.0874	0.0890	0.0824	0.0725	−0.0502
		upper	0.0611	0.0551	0.0598	0.0466	0.0414	0.0433	0.0414	0.0378	0.0352	0.0342	−0.0269
	Range	center	0.3060	0.2888	0.3065	0.2974	0.2417	0.2380	0.2477	0.2011	0.1879	0.1820	−0.1214
		lower	0.4261	0.3958	0.4457	0.4187	0.3628	0.3407	0.3524	0.3154	0.3290	0.2624	−0.1637
		upper	0.2336	0.1980	0.2056	0.1762	0.1615	0.1686	0.1528	0.1594	0.1361	0.1398	−0.0938

$\Delta = SM_{i2019} - SM_{i2010}$. Source: authors' calculations using R program [76].

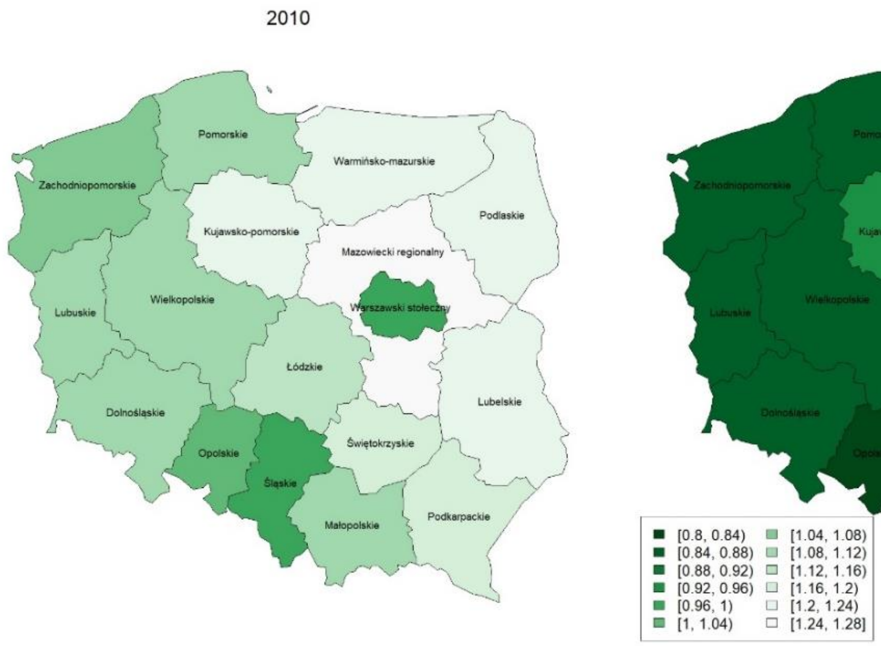

Figure 1. A comparison of social cohesion in the regions in 2010 and 2019 based on values of the composite indicator SM_{it}^{c} for the center of interval. Source: calculation and plot produced using the R program [76].

The biggest improvements in the value of the composite indicator SM_{it}^{c} in the period between 2010 and 2019 can be observed in the regions, which, in 2010, were ranked between 11th and 17th (see Table 3, Figure 2). This group includes the regions of Eastern Poland (Podkarpackie, Lubelskie, Podlaskie, and Warmińsko-mazurskie) as well as Świętokrzyskie, Kujawsko-pomorskie, and Mazowiecki regionalny. The change in the value of the composite indicator $\Delta = SM_{i2019} - SM_{i2010}$ for these regions is in the range $(-0.265; -0.305]$. As a result, Kujawsko-pomorskie, Lubelskie, and Podkarpackie gained two places in the ranking. A considerable improvement can be observed in the case of Lubuskie ($\Delta = -0.2635$), which moved from the 9th place in 2010 up to the 5th in 2019. Following a relatively weak improvement over the period of 10 years, the regions of Łódzkie ($\Delta = -0.204$) and Małopolskie ($\Delta = -0.1985$) dropped in the ranking: Łódzkie by five places (from 10th to 15th place) and Małopolskie by four places (from 6th to 10th place).

The considerable improvement in the level of social cohesion observed in the study was mainly due to funds provided by the EU as part of various programs aimed at supporting regional development.

The Pearson linear correlation coefficient was calculated between the absolute values of the increment of the aggregate measure for the center of the interval for each region $|\Delta| = \left|SM_{i2019}^{c} - SM_{i2010}^{c}\right|$ (see Table 3) and the average amount of EU funding per its inhabitant in the analyzed period (see Table 4). Due to the lack of data, two regions were not included in the calculations: Warszawski stołeczny and Mazowiecki regionalny. Until 2018, these two regions constituted one unit and were assessed jointly as the province of Mazowieckie. As the results of the analysis show, the improvement in the level of social cohesion is strongly positively correlated with the amount of EU subsidies per inhabitant of the region (the correlation coefficient $r = 0.64$, $p\text{-}value = 0.0103$) (the analysis was based on data from the following reports: [77–79]). According to the assumptions of the EU social cohesion policy for 2004–2020, financial assistance was first of all provided to regions characterized by lower levels of development, as measured by GDP per capita. The biggest amount of EU funding per inhabitant in the period 2010–2020 (The calculation was made using population data for Poland for the middle year of the reference period, i.e., 2014 [80]) was given to the following regions of Eastern Poland, which are among the poorest regions in the EU: Warmińsko-mazurskie (EUR 1600), Świętokrzyskie (EUR 1405), Lubelskie (EUR 1343), Podlaskie (EUR 1321), and Podkarpackie (EUR 1297). The smallest amount of EU funding per capita was given to two regions, which, until 2018, constituted one administrative unit—the province of Mazowieckie (EUR 588)—and to Wielkopolskie and Śląskie (both received less than EUR 1000 per capita). Taking into account the ratio of the funding received by each region in the period 2010–2020 to its regional GDP in 2020 expressed as a percentage, the regions of Eastern Poland were the leaders again: Warmińsko-mazurskie (17.66%), Lubelskie (14.76%), Świętokrzyskie (14.70%), Podkarpackie (13.75%), and Podlaskie (13.36%), while Mazowieckie (2.61%), Wielkopolskie (6.08%), or Dolnośląskie (6.71%) were at the bottom of the ranking. These results confirm the positive influence of EU funding on the implementation of cohesion policy objectives: the level of social cohesion improved a lot more in the regions that received more funding. The policy of fund allocation also contributed to decreasing the regional variation in social cohesion, which is analyzed in more detail in this section.

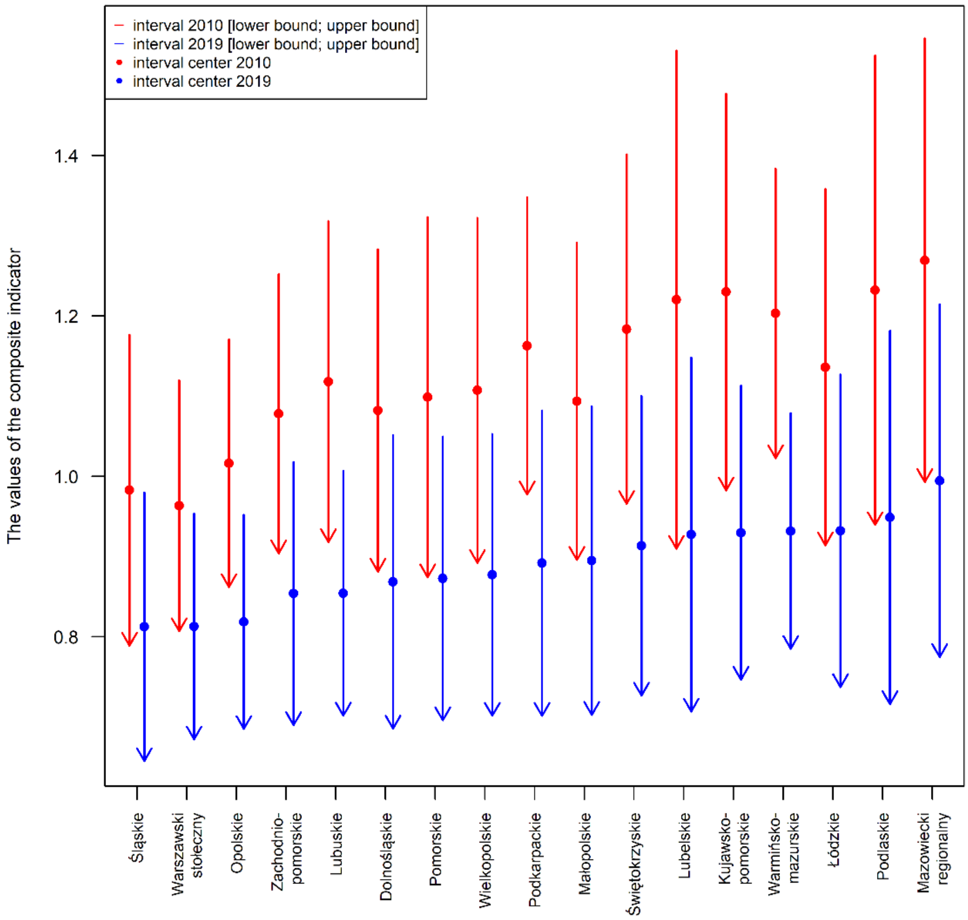

Figure 2. The values of the composite indicator for interval center and intervals [lower bound; upper bound] for Polish regions in 2010 and 2019 (sorted by center of interval 2019 values). Source: charts created using R program [76].

Figure 2 contains three choropleth maps showing the level of social cohesion in 17 regions on the basis of interval-valued data for 2019 in comparison with 2010 (composite indicators SM_{it}^{l}, SM_{it}^{u}, SM_{it}^{c}).

Compared to 2010, the range of intervals for the composite indicator SM_{it} in 2019 decreased for 16 regions, which indicates that the district-level variation in social cohesion in these regions declined. The increase in the value of the composite indicator $\Delta = SM_{i2019} - SM_{i2010}$ (see Table 3, Figure 3, Figure 4b,c) indicates that much of the decrease in the district-level variation in the regions was due to the improved level of social cohesion in districts found near the lower limit of the interval. The only exception is

Podkarpackie, where the range of the interval of the composite indicator SM_{it} slighly increased.

Table 4. Absolute values of the increment of the aggregate measure for the center of the interval for each region $|\Delta| = |SM^{c}_{i2019} - SM^{c}_{i2010}|$ and the average amount of EU funding per capita received by Polish regions from 2010 to 2019 (by decreasing values of $|\Delta|$).

Region	The Increment of the Aggregate Measure for the Center of the Interval $\lvert\Delta\rvert$	The Average Amount of EU Funding per Capita
Kujawsko-pomorskie	0.3001	1169.5
Lubelskie	0.2926	1342.7
Podlaskie	0.2831	1320.6
Warmińsko-mazurskie	0.2711	1604.1
Podkarpackie	0.2708	1297.4
Świętokrzyskie	0.2700	1405.1
Lubuskie	0.2635	1133.5
Wielkopolskie	0.2299	915.2
Pomorskie	0.2258	1030.5
Zachodniopomorskie	0.2242	1209.1
Dolnośląskie	0.2135	1012.1
Łódzkie	0.2040	1127.9
Małopolskie	0.1985	1074.0
Opolskie	0.1978	1185.2
Śląskie	0.1702	969.4

$|\Delta| = |SM^{c}_{i2019} - SM^{c}_{i2010}|$. Source: Table 3 and reports [77–79].

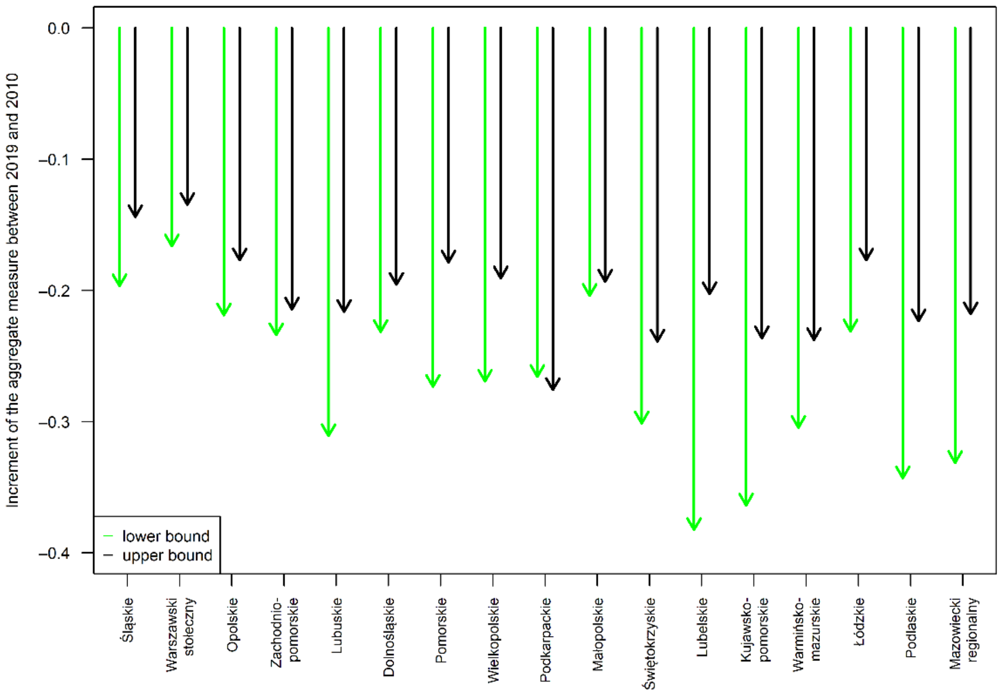

Figure 3. Increment of the values of the aggregate measure for Polish regions between 2019 and 2010 for lower and upper bound of interval. Source: charts created using R program [76].

Line charts in Figures 5 and 6 illustrate changes in the level of social cohesion in 17 regions in the period from 2010 to 2019.

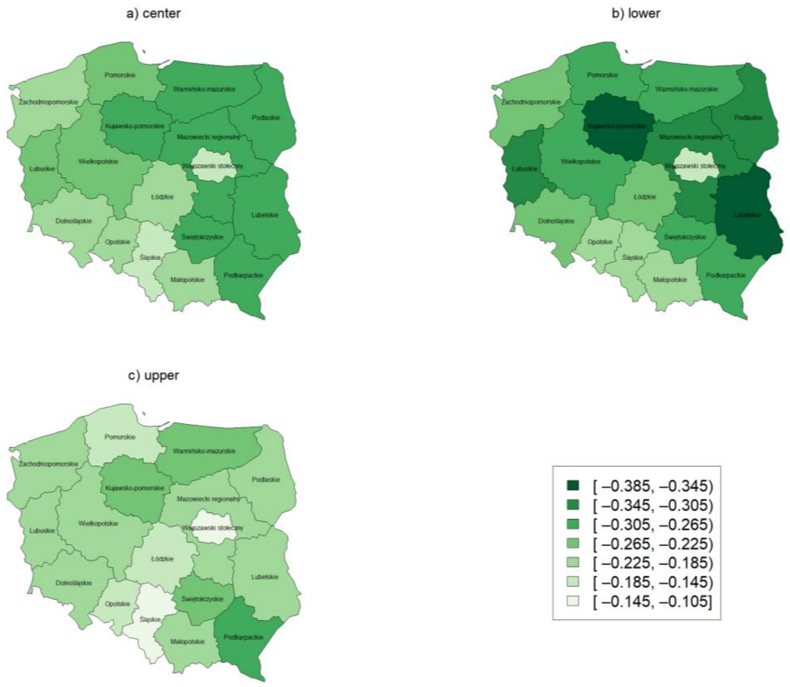

Figure 4. Increase in the values of the composite indicator ($\Delta = SM_{i2019} - SM_{i2010}$) for the regions between 2019 and 2010 for center (**a**), lower (**b**), and upper (**c**) limits of the interval. Source: charts created using R program [76].

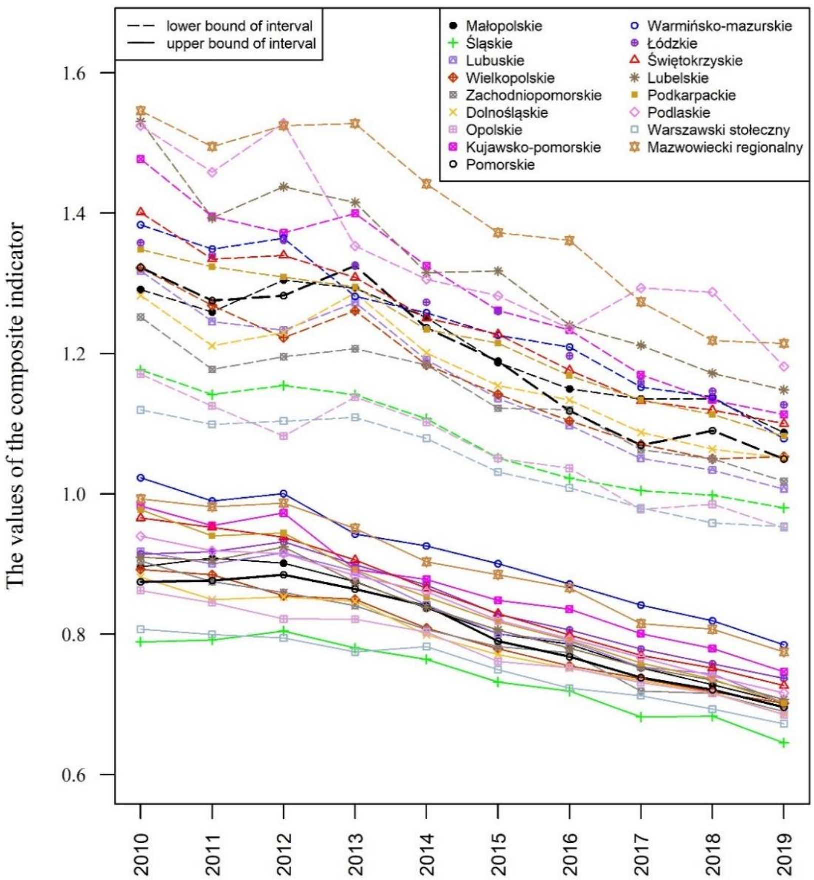

Figure 5. Values of the composite indicators SM_{it}^{l} and SM_{it}^{u} for the lower and upper limit of the interval for the regions in 2010–2019. Source: charts created using R program [76].

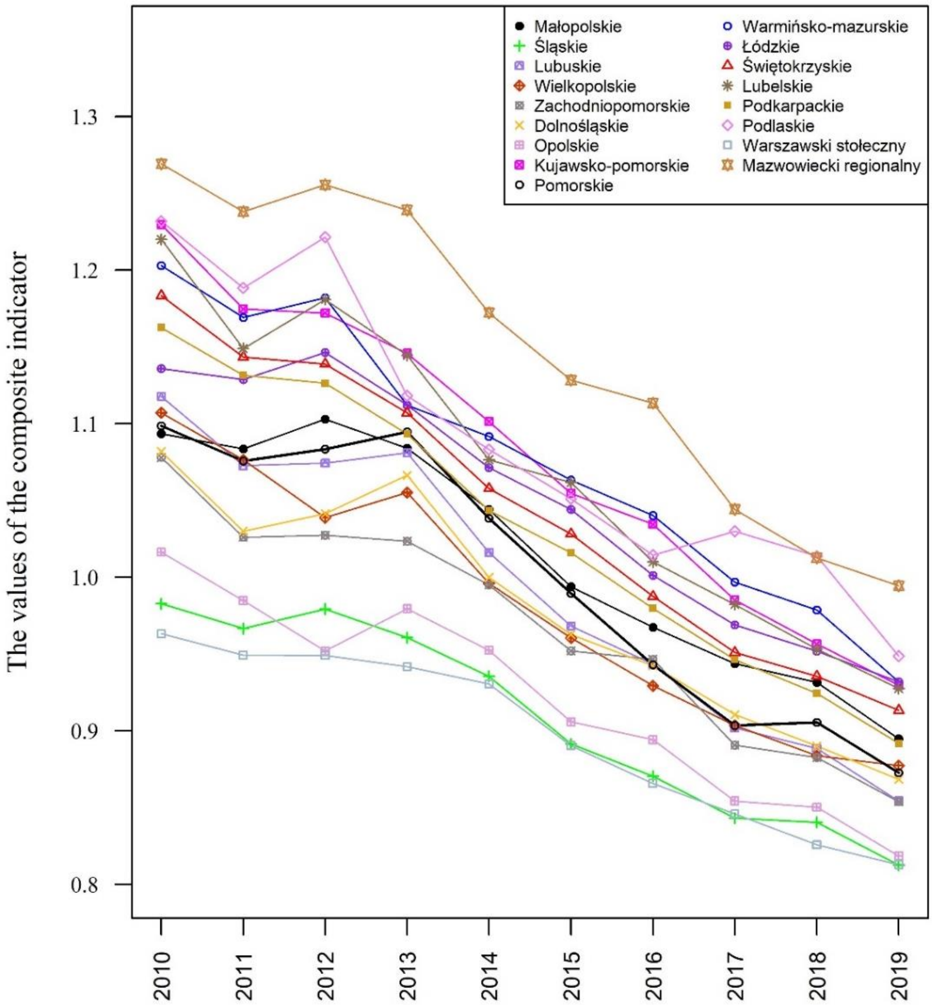

Figure 6. Values of the composite indicator SM_{it}^{c} for the center of the interval for the regions in 2010–2019. Source: charts created using R program [76].

Over the entire reference period, one can observe a systematic decline in the value of, and variation in, the composite indicator (see Table 3, Figures 5 and 6). This means, first of all, that the level of social cohesion kept improving. Secondly, the regional variation kept declining, as evidenced by the range and standard deviation of the composite indicator (see Table 3). The range of the composite indicator for the center of the interval fell from $R_{2010}^{SM^c} = 0.3060$ in 2010 to $R_{2019}^{SM^c} = 0.1820$ in 2019. The same trend can be observed for the lower and upper limits of the interval. However, the decline in the variability of the composite indicator is mainly due to the decline in the lower limit of the interval (from $R_{2010}^{SM^l} = 0.4261$ in 2010 to $R_{2010}^{SM^l} = 0.2624$ in 2019) (see Table 3), which is also confirmed by Figure 5.

7. Discussion and Conclusions

Social cohesion policies implemented by EU countries are largely assessed at the regional level since their main objective is to support measures that contribute to equalizing economic and social conditions in all regions of the EU. A wide range of policy measures and their effects reflects the multidimensional character of social cohesion, which requires a complex process of measurement and assessment. The proposed method of relative taxonomy in its dynamic version [25] can be used to assess the level of social cohesion in a given object (region) and in a given year relative to the level observed in all objects (regions) over the entire reference period, i.e., between 2010 and 2019. By making use of interval-valued data, it is possible to assess objects not only at the regional level but also at a lower level of territorial aggregation, taking into account spatial variation across districts that make up each region. The proposed procedure employs robust measures of central tendency, which were tested in another study by Walesiak, Dehnel, and Dudek [26]. Their inclusion helps to mitigate the impact of outliers occurring in the variables used to construct the composite indicator, which was employed to create the ranking of regions.

The study spanning the period of 10 years could only be conducted by making use of secondary data sources. The scope of information available in these sources determined what variables could be selected for the study. The main limitations of the secondary sources were due to the fact that certain variables were released with a delay (resulting from the official schedule of statistical surveys) and the unavailability of some variables included in the EU-SPI that were associated with certain components of the Opportunity dimension, such as "Personal Rights" or "Tolerance and Inclusion". The final set of 22 variables was selected to ensure that they were as close as possible to those included in the EU-SPI. The practical benefit of the proposed approach, which is based on secondary data sources, is that it then enables systematic assessment of the level of social cohesion for longer time series without the need to conduct costly surveys to obtain primary data. Moreover, statistical data collected in surveys are usually atomic, which means that an observation of each variable is expressed as a single category or real number.

The results of the study clearly indicate that the level of social cohesion in all regions kept improving throughout the entire reference period, though the rate of this improvement varied. More dynamic changes were observed in regions where the level of social cohesion was lower at the start of the reference period. Consequently, as time went on, initial disparities between the regions gradually decreased. The fastest improvement in social cohesion could be observed in eastern regions of Poland and in Kujawsko-pomorskie. The smallest increase in social cohesion was found in Warszawski stołeczny and in south-western regions. Values of the composite indicator used to measure social cohesion and create rankings were found to be closely correlated with the amount of EU funding received by each region. According to the guidelines for the allocation of EU funding to support regional development, financial assistance was first to be provided to regions characterized by lower levels of development, as measured by GDP per capita. The results confirm the positive influence of EU funding on the implementation of cohesion policy objectives: improvements in the level of social cohesion were much more considerable in the regions that received more funding for regional programs. This is also confirmed by changes in the values of the composite indicator: the gradually decreasing disparities in social cohesion between the regions are largely due to the amount of EU funding received in the reference period.

The problem of assessing social cohesion in NUTS2 regions is not limited to Poland but is relevant for all EU countries as well as similar regions outside the EU. Hence, the usefulness of the proposed approach should also be verified using data from other countries, e.g., OECD countries.

Author Contributions: Conceptualization, M.W.; methodology, M.W. and G.D.; software, M.W., validation, M.W. and G.D.; formal analysis, M.W. and G.D.; investigation, M.W. and G.D.; resources, M.W. and G.D.; data curation, M.W.; writing—original draft preparation, M.W. and G.D.; writing—review and editing, M.W. and G.D.; visualization, M.W.; supervision, M.W. and G.D. All authors have read and agreed to the published version of the manuscript.

Funding: The project is financed by the Ministry of Education and Science in Poland under the program "Regional Initiative of Excellence" 2019–2023 project number 015/RID/2018/19 total funding amount: 10 721 040.00 PLN.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The dataset and an R script, which implements the procedure described in Section 4, (steps 2–4 that support the findings of this study) are available from the corresponding author upon request.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Jenson, J. Defining and Measuring Social Cohesion, Commonwealth Secretariat, London. 2010. Available online: https://www.files.ethz.ch/isn/151856/Jenson%20ebook.pdf (accessed on 5 February 2023).
2. Duhaime, G.; Searles, E.; Usher, P.J.; Myers, H.; Frechette, P. Social cohesion and living conditions in the Canadian Arctic: From theory to measurement. *Soc. Indic. Res.* **2004**, *66*, 295–318. [CrossRef]
3. Acket, S.; Borsenberger, M.; Dickes, P.; Sarracino, F. Measuring and validating social cohesion: A bottom-up approach. In Proceedings of the International Conference on Social Cohesion and Development, Paris, France, 20–21 January 2011; pp. 20–21. Available online: https://search.oecd.org/development/pgd/46839973.pdf (accessed on 5 February 2023).
4. Langer, A.; Stewart, F.; Smedts, K.; Demarest, L. Conceptualising and Measuring Social Cohesion in Africa: Towards a Perceptions-Based Index. *Soc. Indic. Res.* **2017**, *131*, 321–343. [CrossRef]
5. Schaeffer, M. Can competing diversity indices inform us about why ethnic diversity erodes social cohesion? A test of five diversity indices in Germany. *Soc. Sci. Res.* **2013**, *42*, 755–774. [CrossRef] [PubMed]
6. Janmaat, J.G. Social Cohesion as a Real-life Phenomenon: Assessing the Explanatory Power of the Universalist and Particularist Perspectives. *Soc. Indic. Res.* **2011**, *100*, 61–83. [CrossRef]
7. Bottoni, G. A Multilevel Measurement Model of Social Cohesion. *Soc. Indic. Res.* **2018**, *136*, 835–857. [CrossRef]
8. Dickes, P.; Valentova, M.; Borsenberger, M. Construct Validation and Application of a Common Measure of Social Cohesion in 33 European Countries. *Soc. Indic. Res.* **2010**, *98*, 451–473. [CrossRef]
9. Dickes, P.; Valentova, M. Construction, Validation and Application of the Measurement of Social Cohesion in 47 European Countries and Regions. *Soc. Indic. Res.* **2013**, *113*, 827–846. [CrossRef]
10. Rajulton, F.; Ravanera, Z.R.; Beaujot, R. Measuring Social Cohesion: An Experiment using the Canadian National Survey of Giving, Volunteering, and Participating. *Soc. Indic. Res.* **2007**, *80*, 461–492. [CrossRef]
11. Botterman, S.; Hooghe, M.; Reeskens, T. 'One Size Fits All'? An Empirical Study into the Multidimensionality of Social Cohesion Indicators in Belgian Local Communities. *Urban Stud.* **2012**, *49*, 185–202. [CrossRef]
12. Balcerzak, A.P. Wielowymiarowa analiza spójności społecznej w krajach Unii Europejskiej w kontekście strategii Europa 2020 [Multidimensional analysis of social cohesion in the European Union countries in the context of the Europe 2020 strategy]. In *Aktualne Trendy w Zarządzaniu Środowiskiem [Current Trends in Environmental Management]*; Bartniczak, B., Trzeciak, K., Eds.; Wydawnictwo AD REM: Jelenia Góra, Poland, 2015; pp. 343–352.
13. Holgado, M.M.; Salinas, J.A.; Rodríguez, J.A. A synthetic indicator to measure the economic and social cohesion of the regions of Spain and Portugal. *Rev. De Econ. Mund.* **2015**, *39*, 223–240. Available online: https://digibug.ugr.es/handle/10481/36997 (accessed on 5 February 2023).
14. Rodríguez, J.A.; Martín, J.M.; Salinas, J.A.; Zermeño, K.A.; Añaños, K.G. A Spatial Analysis of the Achievements, in Terms of Regional Development, Accomplished by the Initial EU-Member Cohesion Fund Beneficiaries Using a Synthetic Indicator. *Sustainability* **2019**, *11*, 2343. [CrossRef]
15. Walesiak, M.; Dehnel, G. The Measurement of Social Cohesion at Province Level in Poland Using Metric and Interval-Valued Data. *Sustainability* **2020**, *12*, 7664. [CrossRef]
16. Sung-Geun, K. Measuring Regional Social Cohesion by Objective Indices: The Case of Korea. *Int. J. Community Well-Being* **2022**, *5*, 587–615. [CrossRef]
17. Billard, L.; Diday, E. *Symbolic Data Analysis: Conceptual Statistics and Data Mining*; John Wiley: Chichester, UK, 2006. [CrossRef]
18. Gioia, F.; Lauro, C.N. Principal component analysis on interval data. *Comput. Stat.* **2006**, *21*, 343–363. [CrossRef]
19. Brito, P.; Noirhomme-Fraiture, M.; Arroyo, J. Editorial for Special Issue on Symbolic Data Analysis. *Adv. Data Anal. Classif.* **2015**, *9*, 1–4. [CrossRef]
20. El Gibari, S.; Gómez, T.; Ruiz, F. Building composite indicators using multicriteria methods: A review. *J. Bus. Econ.* **2019**, *89*, 1–24. [CrossRef]
21. Banihabib, M.E.; Hashemi-Madani, F.-S.; Forghani, A. Comparison of Compensatory and non-Compensatory Multi Criteria Decision Making Models in Water Resources Strategic Management. *Water Resour. Manag.* **2017**, *31*, 3745–3759. [CrossRef]
22. Munda, G. *Social Multi-Criteria Evaluation for a Sustainable Economy*; Springer: Berlin/Heidelberg, Germany, 2008. [CrossRef]
23. Munda, G.; Nardo, M. Noncompensatory/nonlinear composite indicators for ranking countries: A defensible setting. *Appl. Econ.* **2009**, *41*, 1513–1523. [CrossRef]
24. Walesiak, M.; Dehnel, G. A dynamic approach to relative taxonomy in the assessment of changes in the social cohesion of polish provinces in 2010–2018. *Argum. Oeconomica* **2022**, *2022*, 37–65. [CrossRef]
25. Walesiak, M.; Dehnel, G.; Dudek, A. A dynamic approach to relative taxonomy and robust measures of central tendency. *Commun. Stat.-Simul. Comput.* **2022**, 1–17. [CrossRef]

26. Hellwig, Z. Procedure of Evaluating High-Level Manpower Data and Typology of Countries by Means of the Taxonomic Method. In *Towards a system of Human Resources Indicators for Less Developed Countries, Papers Prepared for UNESCO Research Project, Ossolineum*; Gostkowski, Z., Ed.; The Polish Academy of Sciences Press: Wrocław, Poland, 1972; pp. 115–134.
27. Hellwig, Z. Wielowymiarowa analiza porównawcza i jej zastosowanie w badaniach wielocechowych obiektów gospodarczych [Multivariate comparative analysis and its application in multifeature economic objects]. In *Metody i Modele Ekonomiczno-matematyczne w Doskonaleniu Zarządzania Gospodarką Socjalistyczną [Economic and Mathematical Methods and Models in the Improvement of Socialist Economy Management]*; Welfe, W., Ed.; PWE (Polish Economic Publishers): Warszawa, Poland, 1981; pp. 46–68.
28. Hwang, C.-L.; Yoon, K. *Multiple Attribute Decision Making—Methods and Applications. A State-of-the-Art Survey*; Springer-Verlag: New York, NY, USA, 1981; p. 186. [CrossRef]
29. Walesiak, M. Distance measure for ordinal data. *Argum. Oeconomica* **1999**, *2*, 167–173. Available online: https://www.dbc.wroc.pl/dlibra/publication/52780/edition/41291 (accessed on 31 January 2023).
30. Chen, C.-T. Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets Syst.* **2000**, *114*, 1–9. [CrossRef]
31. Roszkowska, E.; Jefmański, B. Interval-Valued Intuitionistic Fuzzy Synthetic Measure (I-VIFSM) Based on Hellwig's Approach in the Analysis of Survey Data. *Mathematics* **2021**, *9*, 201. [CrossRef]
32. Pietrzak, M.B. Taksonomiczny miernik rozwoju (TMR) z uwzględnieniem zależności przestrzennych [Taxonomic measure of development (TMD) with the inclusion of spatial dependence]. *Przegląd Stat. (Stat. Rev.)* **2014**, *61*, 181–201. Available online: https://ps.stat.gov.pl/Article/2014/2/181-202 (accessed on 31 January 2023).
33. Walesiak, M. Visualization of linear ordering results for metric data with the application of multidimensional scaling. *Ekonometria* **2016**, *2*, 9–21. [CrossRef]
34. Mazziotta, M.; Pareto, A. On a Generalized Non-compensatory Composite Index for Measuring Socio-economic Phenomena. *Soc. Indic. Res.* **2016**, *127*, 983–1003. [CrossRef]
35. Mazziotta, M.; Pareto, A. Measuring Well-Being Over Time: The Adjusted Mazziotta–Pareto Index Versus Other Non-compensatory Indices. *Soc. Indic. Res.* **2018**, *136*, 967–976. [CrossRef]
36. Łysoń, P.; Szymkowiak, M.; Wawrowski, Ł. Badania porównawcze atrakcyjności turystycznej powiatów z uwzględnieniem ich otoczenia [Comparative studies of the tourist attractiveness of Polish poviats with regard to their surroundings], Wiadomości Statystyczne. *Pol. Stat.* **2016**, *61*, 45–57. [CrossRef]
37. Sokołowski, A.; Markowska, M. Iteracyjna metoda liniowego porządkowania obiektów wielocechowych [Iterative method for linear ordering of multidimensional objects]. *Przegląd Stat. (Stat. Rev.)* **2017**, *64*, 153–162. [CrossRef]
38. Mazziotta, M.; Pareto, A. Composite Indices Construction: The Performance Interval Approach. *Soc. Indic. Res.* **2022**, *161*, 541–551. [CrossRef]
39. Nardo, M.; Saisana, M.; Saltelli, A.; Tarantola, S.; Hoffman, A.; Giovannini, E. *Handbook on Constructing Composite Indicators*; OECD Publishing: Paris, France, 2005. [CrossRef]
40. OECD. *Handbook on Constructing Composite Indicators: Methodology and User Guide*; OECD: Paris, France, 2008; ISBN 978-92-64-04345-9.
41. Bernard, P. La cohésion sociale: Critique dialectique d'un quasi-concept. *Lien Soc. Polit.* **1999**, *41*, 47–59. [CrossRef]
42. Chan, J.; To, H.-P.; Chan, E. Reconsidering Social Cohesion: Developing a Definition and Analytical Framework for Empirical Research. *Soc. Indic. Res.* **2006**, *75*, 273–302. [CrossRef]
43. Młodak, A. On the construction of an aggregated measure of the development of interval data. *Comput. Stat.* **2014**, *29*, 895–929. [CrossRef]
44. Fu, Y.; Tian, X.; Rui, K.; Luo, H.; Yu, L. Correction to: Constructing Composite Indicators with Collective Choice and Interval-Valued TOPSIS: The Case of Value Measure. *Soc. Indic. Res.* **2020**, *152*, 1213. [CrossRef]
45. Dehnel, G.; Walesiak, M. A comparative analysis of economic efficiency of medium-sized manufacturing enterprises in districts of Wielkopolska province using the hybrid approach with metric and interval-valued data. *Stat. Transit. New Ser.* **2019**, *20*, 49–67. [CrossRef]
46. Drago, C. The Analysis and the Measurement of Poverty: An Interval-Based Composite Indicator Approach. *Economies* **2021**, *9*, 145. [CrossRef]
47. Drago, C.; Gatto, A. An interval-valued composite indicator for energy efficiency and green entrepreneurship. *Bus. Strat. Environ.* **2022**, *31*, 2107–2126. [CrossRef]
48. Wydymus, S. Rozwój gospodarczy a poziom wynagrodzeń w krajach Unii Europejskiej—Analiza taksonomiczna [Economic development and income level in UE countries—Taxonomic analysis]. *Zesz. Nauk. Uniw. Szczecińskiego* **2013**, *756*, 631–645. Available online: https://wneiz.pl/nauka_wneiz/frfu/57-2013/FRFU-57-631.pdf (accessed on 5 February 2023).
49. Goodwyn, F. The Utility of Robust Means in Statistics, Texas A&M University. Paper presented at the annual meeting of the Southwest Educational Research Association, New Orleans, 1 February 2012. Available online: https://files.eric.ed.gov/fulltext/ED529101.pdf (accessed on 5 February 2023).
50. Wilcox, R.R. *Fundamentals of Modern Statistical Methods. Substantially Improving Power and Accuracy*; Springer: New York, NY, USA; Dordrecht, The Netherland; Berlin/Heidelberg, Germany; London, UK, 2010. [CrossRef]
51. Beaton, A.E.; Tukey, J.W. The Fitting of Power Series, Meaning Polynomials, Illustrated on Band-Spectroscopic Data. *Technometrics* **1974**, *16*, 147. [CrossRef]

52. Rousseeuw, P. Robust Statistics. Part 1: Introduction and Univariate Data, LARS-IASC School, May 2019. Available online: https://cseweb.ucsd.edu/~slovett/workshops/robust-statistics-2019/slides/donoho-univariate.pdf (accessed on 5 February 2023).
53. Berger-Schmitt, R. Considering Social Cohesion in Quality of Life Assessments: Concept and Measurement. *Soc. Indic. Res.* **2002**, *58*, 403–428. [CrossRef]
54. Cuellar, R. Social cohesion and democracy. International Institute for Democracy and Electoral Assistance. International IDEA. 2009. Available online: https://www.idea.int/sites/default/files/publications/chapters/the-role-of-the-european-union-in-democracy-building/eu-democracy-building-discussion-paper-27.pdf (accessed on 5 February 2023).
55. Jenson, J. *Mapping Social Cohesion: The State of Canadian Research*; Canadian Policy Research Networks: Ottawa, ON, Canada, 1998; pp. 109–128. Available online: http://www.cccg.umontreal.ca/pdf/CPRN/CPRN_F03.pdf (accessed on 5 February 2023).
56. Beauvais, C.; Jenson, J. Social Cohesion: Updating the State of the Research, Canadian Policy Research Networks Ottawa CPRN Discussion Paper No. F|22. 2002. Available online: http://oaresource.library.carleton.ca/cprn/12949_en.pdf (accessed on 5 February 2023).
57. Hulse, K.; Stone, W. Social cohesion, social capital and social exclusion. *Policy Stud.* **2007**, *28*, 109–128. [CrossRef]
58. Novy, A.; Swiatek, D.C.; Moulaert, F. Social Cohesion: A Conceptual and Political Elucidation. *Urban Stud.* **2012**, *49*, 1873–1889. [CrossRef]
59. Klein, C. Social Capital or Social Cohesion: What Matters For Subjective Well-Being? *Soc. Indic. Res.* **2013**, *110*, 891–911. [CrossRef]
60. Ariely, G. Does Diversity Erode Social Cohesion? Conceptual and Methodological Issues. *Political Stud.* **2014**, *62*, 573–595. [CrossRef]
61. Fonseca, X.; Lukosch, S.; Brazier, F. Social cohesion revisited: A new definition and how to characterize it. *Innov. Eur. J. Soc. Sci. Res.* **2019**, *32*, 231–253. [CrossRef]
62. Berger, P.L. *The Limits of Social Cohesion: Conflict and Mediation in Pluralist Societies*; Routledge: New York, NY, USA, 2018. [CrossRef]
63. Friedkin, N.E. Social Cohesion. *Annu. Rev. Sociol.* **2004**, *30*, 409–425. [CrossRef]
64. Schiefer, D.; van der Noll, J. The Essentials of Social Cohesion: A Literature Review. *Soc. Indic. Res.* **2017**, *132*, 579–603. [CrossRef]
65. Jenson, J. Intersections of Pluralism and Social Cohesion. *Global Centre for Pluralism*. 2019, pp. 1–25. Available online: https://www.pluralism.ca/wp-content/uploads/2019/01/Jane-Jenson-Social-Cohesion.pdf (accessed on 5 February 2023).
66. New Strategy and Council of Europe Action Plan for Social Cohesion Council of Europe. 2010. Available online: https://www.coe.int/t/dg3/socialpolicies/socialcohesiondev/source/2010Strategy_ActionPlan_SocialCohesion.pdf (accessed on 5 February 2023).
67. Annoni, P.; Bolsi, P. The Regional Dimension of Social Progress in Europe: Presenting the New EU Social Progress Index, Luxembourg, Publications Office of the EU. 2020. Available online: https://ec.europa.eu/regional_policy/sources/work/202006_spi_en.pdf (accessed on 17 January 2023).
68. Annoni, P.; Dijkstra, L. *The EU Regional Social Progress Index: Methodological Note*; European Commission: Brussels, Belgium, 2016. Available online: https://ec.europa.eu/regional_policy/information-sources/maps/social-progress/2016_en (accessed on 5 February 2023).
69. Commission Regulation (EU) 2016/2066 of 21 November 2016. Amending the Annexes to Regulation (EC) No 1059/2003 of the European Parliament and of the Council on the Establishment of a Common Classification of Territorial Units for Statistics (NUTS), The European Commission, Brussels. 2016. Available online: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32016R2066&from=EN (accessed on 5 February 2023).
70. Borys, T. Metody normowania cech statystycznych w badaniach porównawczych [Methods of characteristics normalization in statistical comparative studies]. *Przegląd Stat. (Stat. Rev.)* **1978**, *25*, 227–239.
71. Walesiak, M. The choice of normalization method and rankings of the set of objects based on composite indicator values. *Stat. Transit. New Ser.* **2018**, *19*, 693–710. [CrossRef]
72. Walesiak, M. *Uogólniona Miara Odległości w Statystycznej Analizie Wielowymiarowej [The Generalized Distance Measure in Multivariate Statistical Analysis]*; Wydawnictwo Akademii Ekonomicznej we Wrocławiu: Wrocław, Poland, 2002; Available online: http://keii.ue.wroc.pl/pracownicy/mw/2002_Walesiak_Uogolniona_miara_odleglosci_wyd_1.pdf (accessed on 5 February 2023).
73. Rousseeuw, P.J.; Leroy, A.M. *Robust Regression and Outlier Detection*; John Wiley & Sons: New York, NY, USA, 1987.
74. Becker, W.; Saisana, M.; Paruolo, P.; Vandecasteele, I. Weights and importance in composite indicators: Closing the gap. *Ecol. Indic.* **2017**, *80*, 12–22. [CrossRef]
75. Greco, S.; Ishizaka, A.; Tasiou, M.; Torrisi, G. On the methodological framework of composite indices: A review of the issues of weighting, aggregation, and robustness. *Soc. Indic. Res.* **2019**, *141*, 61–94. [CrossRef]
76. R Core Team. *R: A Language and Environment for Statistical Computing*; R Foundation for Statistical Computing: Vienna, Austria, 2022; Available online: https://www.R-project.org/ (accessed on 13 October 2022).
77. The Impact of the Cohesion Policy on the Social and Economic Development of Poland and of its Regions in the Period 2004–2019 Ministry of Development Funds and Regional Policy, Warsaw. 2020. Available online: https://www.ewaluacja.gov.pl/media/99612/BROSZURA_ENG_web.pdf (accessed on 5 February 2023).

78. Fundusze Europejskie w Polsce. Programy regionalne 2014–2020 [European Funds in Poland. Regional Programs 2014–2020] Biuletyn Ministerstwa Infrastruktury i Rozwoju [Bulletin of the Ministry of Infrastructure and Development], 36, ISSN 1899-783X. 2014. Available online: https://www.funduszeeuropejskie.gov.pl/media/1065/Fundusze_Europejskie_w_Polsce_biuletyn_nr_36.pdf (accessed on 5 February 2023).
79. Poland National Strategic Reference Framework 2007–2013 in Support of Growth and Jobs. National Cohesion Strategy (2007), Document adopted by the European Commission on 7th May 2007, Warsaw, Ministry of Regional Development. Available online: https://www.funduszeeuropejskie.gov.pl/media/92053/NSRO_an_20_07.pdf (accessed on 5 February 2023).
80. GUS. Stan i Struktura Ludności Oraz Ruch Naturalny w Przekroju Terytorialnym w 2014 r. [Size and Structure of Population and Vital Statistics in Poland by Territorial Division in 2014], Informacje i Opracowania Statystyczne, Warszawa. 2015. Available online: https://stat.gov.pl/obszary-tematyczne/ludnosc/ludnosc/ludnosc-stan-i-struktura-ludnosci-oraz-ruch-naturalny-w-przekroju-terytorialnym-stan-w-dniu-31-xii-2014-r-,6,17.html (accessed on 5 February 2023).