



Article Improvement of Wind-Induced Responses of Twin Towers Using Modal Substructure Method with Link Bridges

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Abstract: A new wind-resistant design optimization method for twin towers by utilizing link bridges, named as the Modal Substructure (MSS) method, is proposed. The MSS method combines the benefits of engineering design approaches and theoretical analysis methods to achieve efficient wind-induced vibration control tailored to specific twin tower projects. The method involves three main steps: (1) establishing control objectives based on the wind-induced response characteristics of the twin towers, (2) determining control strategies by analyzing the modal acceleration characteristics of the twin towers, and (3) performing parameter optimization of the link bridge, including assessing the damping ratio, mass ratio, and frequency ratio of the bridge. By applying the MSS method, optimal configurations for the link bridge can be identified, leading to effective vibration reduction effects. The wind-induced responses of the twin towers exhibit three distinct types: predominance of out-of-phase response, predominance of in-phase response, and equal importance of in-phase and out-of-phase responses. Each response type necessitates the implementation of specific control strategies. We propose a two-section link bridge design approach: the upper section functions as a tuned mass damper to effectively control the in-phase response, while the lower section is designed as a "stiffness + damping component" to reduce the out-of-phase response.

Keywords: twin towers; link bridge; wind-induced response; control of dynamic response

1. Introduction

Super-tall twin towers have gained worldwide recognition and appreciation from property owners and architects due to their exceptional aesthetic design and outstanding architectural functionalities. However, the dynamic behavior of these towers under wind loads becomes a primary concern, often surpassing the seismic considerations which are more commonly emphasized for conventional buildings. Consequently, there arises a need to delve into the optimization of wind-resistant design strategies for super-tall twin towers.

Compared to single towers, the aerodynamic characteristics of the twin towers are often more complex [1–3]. Lim [4,5] conducted synchronized tests on twin towers with spacing ratios ranging from 0.25 to 3, using dual high-frequency force balance experiments. The investigation involved examining the correlation coefficients and coherence functions to explore the aerodynamic correlation between the twin towers. Substantial differences were observed between the overall aerodynamic forces of the twin towers and those of single towers. Song [6] conducted rigid model pressure tests on twin towers with spacing ratios from 0.25 to 2, investigating the correlation between aerodynamic forces of the twin towers.

Furthermore, based on previous research [7], it is found that the existence of a structural link significantly influences the wind-induced responses of the linked twin towers. The presence of a structural link leads to the redistribution of wind loads between the twin



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Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). towers. In comparison to the detached twin towers, the wind-induced responses of the tower with originally larger response values will decrease after being linked, while the tower with initially smaller response values will experience an increase. Therefore, we propose a hypothesis that adjusting the structural parameters of the link (such as damping ratio, mass, and natural frequency) could potentially improve the wind-induced responses of the twin towers.

Currently, the wind-resistant design methods for the linked twin towers can be categorized into two types:

- (1) Engineering design methods. These methods are employed for specific twin tower projects, using commercial finite element software to establish detailed building models and conduct modal analyses. By combining the generalized aerodynamic forces obtained from wind tunnel tests with the structural dynamic characteristics of the linked twin towers, the wind-induced responses and wind loads are calculated [8]. The advantage of this approach lies in obtaining highly reliable wind-induced responses that can directly guide engineering projects. However, its drawback is that the results obtained are only applicable to the specific project and are not efficient when used for analyzing different link conditions. Consequently, this method is not conducive to guiding wind-resistant design optimization for the linked twin towers.
- (2) Theoretical analysis methods. Common simplified calculation models for theoretical analysis include lumped mass models [9], rigid panel models [10,11], and cantilever beam models [12–14], among others. Using modal analyses of these simplified models, the natural frequencies and mode shapes of the twin towers are obtained. Utilizing hypothesized modal shapes, Lim et al. [4] introduced a 3D simplified theoretical model to assess the dynamic attributes of the linked twin towers. This model delved into the influence of link stiffness on the frequencies of these structures. While Lim's model offers a concise equation for computing wind-induced responses of the linked twin towers, its prognostications for generalized forces and estimations of wind-induced responses entail certain ambiguities stemming from the overly simplified modal correction coefficients [15]. Addressing the limitations of Lim's model, Song [10] proposed a more comprehensive 3D analysis model for the linked twin towers that does not require assumed modal shapes. Based on the rigid panel models, Song derived the mass and stiffness matrices of the linked twin towers and obtained their modal shapes by solving the characteristic equation. Huang [16] simplified the linked twin towers into a lumped-mass model and found that the variations in the link stiffness have minimal impact on the in-phase mode of the twin towers, but significantly affects the out-of-phase mode.

The advantage of the theoretical analysis approach is its high computational efficiency, facilitating the calculation of wind-induced responses for different link conditions. Nevertheless, its limitation lies in the significant disparity between the overly simplified theoretical models and the actual structures, leading to insufficient accuracy in the wind response results and making it difficult to directly guide engineering projects.

Due to the limitations of the engineering design methods and theoretical analysis methods, this paper introduces a new method, named as the Modal Substructure (MSS) method, for wind-resistant design optimization of twin towers by using link bridges. The MSS method is an approximate calculation method exhibiting sufficient accuracy, which avoids the drawbacks of repetitive modeling in engineering design methods and the diminished computational accuracy in theoretical analysis methods. Firstly, a wind tunnel test of square twin-towers with various building spacing is conducted to obtain the overall aerodynamic forces on each tower. Secondly, the primary steps to improve wind-induced responses of the twin towers via the MSS method are proposed. Thirdly, the wind-resistant design optimization process for twin towers by use of link bridges are given under various building spacing. Finally, valuable insights and conclusions drawn from the research findings are presented.

2. Wind Tunnel Experimental Setups

2.1. Testing Model Configurations

To investigate the wind-induced responses of the twin towers, two identical square buildings were selected for wind tunnel experiments. The wind tunnel model was fabricated at a geometric scale of 1:300, which represents twin towers of 300 m in height and 45 m in width and was tested in the Boundary Layer Wind Tunnel Laboratory of Zhejiang University. To maintain a consistent spacing between the two towers throughout their height, the two models were affixed using a small bar at the top of the two models, as depicted in Figure 1.



Figure 1. Pressure model of the square twin towers.

Each building model was instrumented with 240 pressure taps on the facades. Using a synchronous pressure acquisition system, the wind pressures on the model surfaces were measured at a sampling rate of 312.5 Hz for 90 s per wind direction during the wind tunnel testing. Based on the model similarity principles [17], this corresponds to about 2.5 h recording at a 3 Hz sampling rate in full scale.

To emphasize the vortex-induced across-wind responses, a typical open wind exposure was simulated, which represents a power-law index of 0.15 for mean velocity profile and a turbulence intensity of 9% at the building height. Figure 2 defines the coordinates and wind directions.



Figure 2. Definition of coordinates and wind directions.

Based on the previous research [6,18], it has been identified that the most critical wind directions for symmetric twin towers are 0° and 90°. Therefore, the twin towers were tested for the building spacing s/B = 0.5,1 and 2.0 at wind directions of 0° and 90°.

2.2. Data Processing

With synchronous pressure integrations, the overall generalized aerodynamic forces on each tower were determined as follows.

Generalized force :
$$GF_j(t) = \int_A p(t)\Phi_j(z)dA \quad j = x, y$$
 (1)

where p(t) is the time series of wind pressures on the model surface; dA indicates the calculated area of the measurement point; and $\Phi_i(z)$ denotes the *j*-th modal shape function.

The equations of motion are expressed in terms of generalized coordinates, and the wind-induced response of the structure can be obtained by solving the following equations of motion:

$$\left\{\ddot{\eta}_{j}(t)\right\} + 2\zeta\omega_{j}\left\{\dot{\eta}_{j}(t)\right\} + \omega_{j}^{2}\left\{\eta_{j}(t)\right\} = \frac{GF_{j}(t)}{M_{j}} \quad j = x, y$$
⁽²⁾

where $\eta_j(t)$ is the generalized displacement; ζ is the structural damping ratio; ω_j is the natural angular frequency ($\omega_j = 2\pi f_j$); and M_j is the generalized mass.

The frequency domain method is utilized for calculation, which provides an intuitive representation of the structural dynamic characteristics and facilitates the determination of statistical values for the wind-induced response. When employing the frequency domain method, the root mean square value of the acceleration for the *j*-th mode can be obtained from the following equation:

$$\sigma_{aj} = \sqrt{\int \frac{S_{GF_j}(f)}{M_j^2 \left[\left(1 - \left(f/f_j \right)^2 \right)^2 + \left(2\zeta(f/f_j) \right)^2 \right]} \cdot df} \approx \frac{1}{M_j} \sqrt{\frac{\pi f_j S_{GF_j}(f_j)}{4\zeta}}$$
(3)

where $S_{GF_j}(f)$ is the spectrum of the generalized force, and f_j is the natural frequency of the *j*-th mode.

3. Description of MSS Method

The MSS method is an approximate approach for calculating the wind-induced responses of the linked twin towers. The methodology incorporates the concept of substructure [19] to perform an energy analysis on both the main towers and the link bridge. By means of theoretical deduction, the motion equations governing the twin tower structures and the link bridge were derived to evaluate their wind-induced responses [20]. The principle of the MSS method is to first obtain the structural dynamic characteristics of the detached twin towers using commercial structural analysis software. Based on this, a parameterized theoretical analysis model for the linked twin towers is established. Subsequently, parameter analysis and optimization are conducted on the link bridge of the twin towers, aiming to achieve wind-induced vibration control. The MSS method allows for wind-resistant design optimization tailored to specific twin tower projects, thereby avoiding the problems associated with repetitive modeling in engineering design methods and reduced computing accuracy in theoretical analysis methods.

In order to improve the wind-induced responses of the linked twin towers, the Modal Sub-Structure (MSS) method was utilized to optimize the link bridge design with the objective of lessening the wind-induced accelerations. The wind-resistant design optimization process for twin towers encompasses three primary steps as follows:

Step 1: Establishment of control objectives.

During this step, an assessment is made to determine the necessity of improving the wind-induced response, considering the specific engineering project's design conditions, including the design wind speed, structural load capacity limits, and wind-induced acceleration limits.

Step 2: Formulation of control strategies.

A comprehensive analysis of the data obtained from the wind tunnel experiments reveals key factors contributing to heightened wind-induced responses. Building upon this analysis, well-defined control strategies are devised. Notably, the MSS method suggests adopting a "stiffness + viscous damper" link bridge configuration to effectively control the wind-induced response of out-of-phase modes. In addition, link bridges serve as tuned mass dampers (TMD) to mitigate the wind-induced responses of in-phase modes. By strategically applying out-of-phase forces on the twin towers via the link bridges, significant reduction in the wind-induced response of in-phase modes can be achieved.

Step 3: Parameter optimization of the link bridge.

In this step, various link bridge parameters, including the damping ratio, mass, and frequency, are investigated, and their corresponding wind-induced responses are analyzed using the MSS method. This enables the identification of effective and feasible combinations of bridge parameters, providing valuable guidance for further detailed structural design optimization of the link bridge.

4. Application of MSS Method in Wind Resistance Design of Linked Twin Towers *4.1. Structural Design Parameters*

In a typical twin-tower configuration characterized by symmetrical geometrical shapes and structural attributes, the most important modes of vibrations to wind-induced effects can be classified into two categories [18]: the in-phase modes and the out-of-phase modes, as illustrated in Figure 3.



Figure 3. Illustration of fundamental structural modes for the detached twin towers. (**a**) *y*-in mode; (**b**) *x*-in mode; (**c**) *y*-out mode; and (**d**) *x*-out mode.

The structural design parameters of the twin towers and the link bridge are given in Table 1.

Table 1. Design parameters of the twin towers and the link bridge.

Design Parameter	Value
the tower height	300 m
the tower width	45 m
the floor height	4 m
the floor mass	2500 ton/floor
natural frequency of each tower	0.125 Hz (in <i>x</i> & <i>y</i> direction)
damping ratio of each tower	2%
the location of the link bridge	on the top floor
the mass of the link bridge	1600 ton(about 1.5% generalized mass of the tower)
other design parameters of the link bridge	to be determined

The other design parameters of the link bridge used in the MSS method are as follows:

where m_B is the mass of link bridge, ω_B is the natural frequency of link bridge, c_B is the damping of link bridge, M_1 and M_2 is the mass of tower1 and tower2, and ω_j is the natural frequency of the tower.

4.2. Improving the Wind-Induced Response in 0° Wind Direction

According to previous research findings [18], the wind-resistant design of twin towers is primarily influenced by wind-induced responses under wind directions of 0° (or 180°) and 90° (or 270°). The methods for controlling wind-induced responses in the wind direction of 0° by utilizing link bridges are illustrated as follows.

4.2.1. s/B = 2

Firstly, the wind-resistant design optimization steps for twin towers with building spacing s/B = 2 are outlined using the MSS method:

Step 1: Control objectives are established based on the wind-induced responses of the detached twin towers, representing the most unfavorable scenario. The dominance of the across-wind response is evident, as depicted in Figure 4, where the root mean square accelerations for the detached twin towers are computed using Equation (3) with respect to the reference wind speed. Thus, the primary control objective for the twin towers with s/B = 2 is to lessen the across-wind response.



Figure 4. Roof top accelerations for the detached twin towers.

Step 2: Control strategies are determined by analyzing the wind-induced accelerations of the first four modes (*x*-in mode, *x*-out mode, *y*-in mode, and *y*-out mode) of the detached twin towers, as depicted in Figure 5. It is observed that wind speeds below 60 m/s are characterized by the out-of-phase response (a_{x-out}) dominating the across-wind response. Additionally, the out-of-phase response exhibits a distinctive vortex-induced resonance behavior, while the in-phase across-wind response (a_{x-in}) resembles a galloping response.



Figure 5. Modal accelerations of the first four modes for the detached twin towers.

Based on this analysis, the wind-induced vibration control strategy for the twin towers with s/B = 2 primarily targets the out-of-phase mode in the across-wind response. To this end, a novel mechanical model, presented in Figure 6, has been introduced to effectively mitigate the out-of-phase across-wind response. The strategy involves enhancing the link bridge's stiffness and incorporating viscous dampers for effective control.



Figure 6. Schematic diagram for the control of the out-of-phase modal response by increasing the stiffness and setting viscous dampers.

Step 3: The optimization of the link bridge parameters is carried out using the MSS method, analyzing the wind-induced accelerations of the linked twin towers. Three distinct link bridge connection cases are considered in this analysis:

Case 1: In this scenario, only the support stiffness is employed to constrain the relative displacement of the twin towers, with no use of viscous dampers to control their relative vibration. The frequency ratio of the link bridge is set to 3 ($\mu_{\omega} = 3$), representing a typical elastic constraint.

Case 2: Similar to Case 1, only the support stiffness is utilized to restrict the relative displacement of the twin towers, without incorporating viscous dampers. With the natural frequency of the single tower being 0.125 Hz, a frequency ratio of 8 indicates that the

natural frequency of the link bridge in the *x*-direction is 1 Hz, representing a typical rigid constraint between the link bridge and the twin towers.

Case 3: In this configuration, both the viscous dampers and support stiffness are implemented to constrain the relative vibration and displacement of the twin towers, respectively. The frequency ratio of the link bridge remains set at 3 ($\mu_{\omega} = 3$), and the damping ratio of the viscous dampers is 10% ($\zeta_B = 10\%$).

The wind-induced accelerations of the linked twin towers are calculated by using the MSS method for each case. The transfer function of the linked twin towers for the wind loads can be obtained via the MSS method [20] as

$$|H_T(i\omega)|^2 = \frac{1}{\omega_j^4 (D_R^2 + D_I^2)} \left[\left(1 - \left(\frac{\omega}{\omega_B}\right)^2 \right)^2 + 4\zeta_B^2 \left(\frac{\omega}{\omega_B}\right)^2 \right]$$
(4)

$$D_{R} = 1 + \frac{1}{4} \mu_{Mj} \gamma_{Dj}^{2} \left(\frac{\omega_{B}}{\omega_{j}}\right)^{2} - \left(1 + \mu_{Mj} \left(\gamma_{Aj}^{2} + \left(\frac{1}{4} + \zeta_{B}^{2}\right)\gamma_{Dj}^{2}\right)\right) \left(\frac{\omega}{\omega_{j}}\right)^{2} - \left(\frac{\omega}{\omega_{B}}\right)^{2} - 4\zeta_{j}\zeta_{B} \left(\frac{\omega}{\omega_{j}}\right) \left(\frac{\omega}{\omega_{B}}\right) + \left(\frac{\omega}{\omega_{j}}\right)^{2} \left(\frac{\omega}{\omega_{B}}\right)^{2}$$
(5)

$$D_{I} = 2\zeta_{j}\left(\frac{\omega}{\omega_{j}}\right) - 2\zeta_{j}\left(\frac{\omega}{\omega_{j}}\right)\left(\frac{\omega}{\omega_{B}}\right)^{2} + 2\zeta_{B}\left(\frac{\omega}{\omega_{B}}\right) + \zeta_{B}\mu_{Mj}\gamma_{Dj}^{2}\left(\frac{\omega_{B}}{\omega_{j}}\right)\left(\frac{\omega}{\omega_{j}}\right) - 2\zeta_{B}\left(1 + \mu_{Mj}\left(\gamma_{Aj}^{2} + \frac{1}{4}\gamma_{Dj}^{2}\right)\right)\left(\frac{\omega}{\omega_{j}}\right)^{2}\left(\frac{\omega}{\omega_{B}}\right)$$

$$(6)$$

where ω_j is the natural frequency of each tower, ζ_j is the damping ratio of each tower, ω_B is the natural frequency of the link bridge, ζ_B is the damping ratio of the link bridge, γ_{Aji} is the average displacement of the two towers, and γ_{Dji} is the relative displacement of the two towers.

The variance of accelerations under wind excitation can thus be obtained as:

$$\sigma_a^2 = \int_0^\infty \omega^4 |H_T(i\omega)|^2 S_F(\omega) d\omega \tag{7}$$

where $S_F(\omega)$ is the spectrum of the generalized force.

Figure 7 shows the root mean square wind-induced building accelerations as a function of the reference wind speed. In this context, the accelerations of the detached twin towers are determined using Equation (3), while the accelerations of the linked twin towers are calculated using Equation (7).

In Case 1, the relative displacement of the twin towers is constrained by setting the support stiffness without the installation of viscous dampers to control their relative vibration. Consequently, the out-of-phase mode's natural frequency increases, leading to a higher critical wind speed for vortex-induced oscillations. Case 1 proves effective when the design wind speed remains below the critical wind speed.

Comparing the outcomes of Case 1 with Case 2, it is evident that the critical wind speed for vortex-induced resonance increases with higher support stiffness. Nevertheless, utilizing either Case 1 or Case 2 could have adverse consequences if the design wind speed significantly surpasses the vortex-induced resonance wind speed of the detached twin towers. This is because the link bridge's stiffness induces vortex-induced resonance at higher wind speeds, resulting in higher response values compared to the resonance response at lower wind speeds, especially in Case 2.

The most effective approach to mitigate wind-induced vibration is represented by Case 3, wherein a viscous damper is incorporated to limit the relative vibration between the twin towers, alongside support stiffness to restrict their relative displacement. This combined approach not only elevates the critical wind speed but also reduces the amplitude of the vortex-induced response. Importantly, it is noteworthy that the mentioned three connection cases have no negative impact on the along-wind response.



Figure 7. Wind-induced building accelerations (*s*/*B* = 2): (a) Along-wind, and (b) across-wind.

4.2.2. s/B = 1

Prior investigations [18] have revealed that alterations in the spacing between the twin towers exert a significant influence on their aerodynamic characteristics and consequent wind-induced responses. Therefore, it is imperative to separately address the wind-induced responses and wind-resistance design optimization for twin towers with different building spacings. In this section, particular attention is devoted to the scenario where the relative spacing between the twin towers is 1 (s/B = 1).

Figure 8 presents the standard deviations of tower accelerations and modal accelerations of the detached twin towers, with the displayed values indicating the higher acceleration between the two towers. As depicted in Figure 8a, when s/B = 1, the wind-induced response remains predominantly in the across-wind direction. However, it is noteworthy that the out-of-phase component dominates the across-wind response only when the reference wind speed is relatively small (less than 55 m/s). Conversely, at higher reference wind speeds, both the out-of-phase and in-phase components assume equal importance, as illustrated in Figure 8b.



Figure 8. Wind-induced accelerations of the detached twin towers (s/B = 1): (**a**) Towers' accelerations, and (**b**) modal accelerations.

Hence, in the case of twin towers with s/B = 1, when the design wind speed is relatively low (below 55 m/s), the "stiffness + viscous damper" approach mentioned earlier proves effective in controlling the across-wind response. Conversely, if the design wind speed exceeds 55 m/s, concurrent control of both out-of-phase and in-phase components becomes imperative for effectively mitigating the across-wind response. This entails utilizing a structurally robust link to govern the out-of-phase response while employing the link bridge as a tuned mass damper (TMD) to regulate the in-phase response. The corresponding schematic mechanical model is presented in Figure 9.

To optimize the parameters of the link bridge, a comprehensive analysis is performed on wind-induced accelerations of the twin towers, considering three distinct link bridge connection cases:

Case 1: In this instance, solely the support stiffness is employed to restrict the relative displacement of the twin towers, devoid of any accompanying viscous dampers to control their relative vibration. The frequency ratio of the link bridge is set to 3 ($\mu_{\omega} = 3$), indicative of a typical elastic constraint.

Case 2: Similarly, in Case 2, only the support stiffness is utilized to constrain the relative displacement of the twin towers, while no accompanying viscous dampers are incorporated to control their relative vibration. The frequency ratio of the link bridge is adjusted to 8 ($\mu_{\omega} = 8$), representing a typical rigid constraint.

Case 3: Following the design methodology of the tuned mass dampers (TMD), horizontal constraints and damping elements are implemented, setting the frequency ratio of the link bridge to 1 ($\mu_{\omega} = 1$) and the damping ratio to 5%. Simultaneously, a rigid link is established between the twin towers to restrict their relative displacement. The values of the link bridge's parameters used in the MSS method can be found in Table 2.



Figure 9. Mechanical model schematic diagram for simultaneous control of in-phase and out-of-phase modal responses in the *x* direction.

Table 2. The values of the link bridge's parameters.

The Parameters of in-Phase Mode	The Parameters of Out-of-Phase Mode
mass ratio $\mu_M = 0.015$	mass ratio $\mu_M = 0.015$
frequency ratio $\mu_\omega = 1$	frequency ratio $\mu_\omega = 8$
damping ratio $\zeta_B = 5\%$	damping ratio $\zeta_B = 0\%$

Utilizing the MSS method to evaluate the wind-induced responses of the twin towers, we obtained the standard deviation of wind-induced accelerations for various cases as a function of the reference wind speed, considering a relative spacing of 1 (s/B = 1), as depicted in Figure 10.

As depicted in Figure 10, when the relative spacing is 1 (s/B = 1), the impact of reducing wind-induced response using Case 1 is not pronounced, primarily due to relatively large in-phase components. Conversely, Case 2, involving the rigid connection of the link bridge between the twin towers, effectively controls the out-of-phase response, thereby resulting in a modest reduction in the total wind-induced response. Nevertheless, this approach proves less effective when the reference wind speed surpasses 60 m/s because the in-phase response is not controlled. In contrast, the measures implemented in Case 3 are capable of simultaneously controlling both in-phase and out-of-phase responses, leading to substantial vibration reduction across all wind speeds.



Figure 10. Wind-induced building accelerations (*s*/*B* = 1): (a) Along-wind, and (b) across-wind.

4.2.3. s/B = 0.5

Figure 11 displays the standard deviations of building accelerations and modal accelerations concerning the detached twin towers with a relative spacing of 0.5 (*s*/*B* = 0.5). The accelerations depicted in Figure 11a correspond to the higher acceleration value among the two towers.

The observations from Figure 11 indicate that for a spacing ratio of s/B = 0.5, the wind-induced response of the detached twin towers remains predominantly governed by the across-wind direction. Furthermore, vortex-induced vibration predominantly occurs in the in-phase mode. Notably, around the critical wind speed, the across-wind response is primarily composed of in-phase modal response. This is consistent with the findings in reference [18], where the in-phase aerodynamic force of twin towers becomes dominate at s/B = 0.5.

Consequently, it becomes imperative to concurrently control both the out-of-phase and in-phase modal responses for the twin towers with such a spacing ratio to effectively mitigate the across-wind response of the twin towers.



Figure 11. Wind-induced accelerations of the detached twin towers (s/B = 0.5): (**a**) Towers' accelerations, and (**b**) modal accelerations.

To conduct the parameter optimization of the link bridge, an extensive investigation of wind-induced accelerations of the twin towers is undertaken, considering three typical connection cases of link bridges:

Case 1: In this scenario, a small support stiffness is applied to restrict the relative displacement of the twin towers, without any accompanying installation of viscous dampers to control their relative vibration. The link bridge frequency ratio is set to 3 ($\mu_{\omega} = 3$), representing a typical elastic constraint.

Case 2: On the other hand, a large support stiffness is employed to constrain the relative displacement of the twin towers, while no viscous dampers are installed to control their relative vibration. The link bridge frequency ratio is adjusted to 8 ($\mu_{\omega} = 8$), indicative of a typical rigid constraint.

Case 3: For this particular instance, the link bridge design adheres to the principle of the tuned mass damper (TMD), incorporating the appropriate horizontal constraints and damping elements. This ensures that the bridge frequency ratio equals 1 ($\mu_{\omega} = 1$), while the damping ratio of the damping elements is set at 5%. Furthermore, a structural link with robust stiffness is integrated between the two towers, directly constraining their relative displacement.

By using the MSS method to calculate the wind-induced response of the twin towers, the wind-induced accelerations of the twin towers under different cases as a function of reference wind speed can be obtained for a spacing ratio of 0.5 (s/B = 0.5), as illustrated in Figure 12.



Figure 12. Wind-induced building accelerations (*s*/*B* = 0.5): (a) Along-wind, and (b) across-wind.

A comparative analysis of the accelerations in the three cases was conducted to assess the vibration reduction efficacy of each method. The findings revealed that when s/B = 0.5, the across-wind response was predominantly governed by the in-phase mode response. Consequently, concurrent control of responses in both in-phase and out-of-phase modes became imperative. As a result, Case 3 exhibited a superior vibration reduction effect compared to Case 1 and Case 2.

In conclusion, the three spacing ratios examined (s/B = 2, 1, and 0.5) represent distinct typical scenarios encompassing wind-induced vibration control in twin towers. These scenarios encompass predominance of out-of-phase modal response, equal significance of out-of-phase and in-phase responses, and predominance of in-phase response. Consequently, through judiciously designing the connection mode between the link bridge and the twin towers in line with the actual aerodynamic conditions, the wind-induced response of the main towers can be effectively mitigated.

In practical applications, achieving control of out-of-phase modal response via rigid connection is generally more straightforward compared to designing the link bridge as a tuned mass damper (TMD) for controlling the in-phase modal response, which entails certain technical challenges during implementation. In TMD design, it is essential to consider the displacement stroke of the link bridge relative to the main tower during wind-induced vibrations. The calculation of the link bridge displacement can be readily accomplished using the MSS method, which applies the transfer function of the bridge for wind loads [20]. Subsequently, bridge displacement is obtained using the integration of the transfer function multiplied by the generalized aerodynamic power spectrum. As illustrated in Figure 13 for the case of a spacing ratio s/B = 0.5, employing the control method of Case 3, this relative displacement between the link bridge and the main tower is referred to as "stroke" in TMD design, and controlling the stroke of the bridge becomes crucial to meet construction requirements. Consequently, in practical engineering, the utilization of stiffened nonlinear damping elements, such as the V2-type damping element (where the damping force is proportional to the square of velocity) commonly used in TMD systems for most super high-rise buildings, is recommended.



Figure 13. Horizontal displacement stroke of the link bridge.

4.3. Improving the Wind-Induced Response in 90° Wind Direction

Subsequently, an investigation into the wind-induced responses of the twin towers in the wind direction of 90° is conducted, with a specific focus on exploring the approaches to controlling wind-induced vibrations via the utilization of link bridges. Analogous to the preceding discussion, the case of a spacing ratio of 2 (s/B = 2) is first discussed.

4.3.1. s/B = 2

Figure 14a depicts the root mean square accelerations as a function of the reference wind speed, with the displayed values representing the higher acceleration among the two towers. Meanwhile, Figure 14b provides an overview of the modal accelerations of the structurally detached twin towers.

Based on Figure 14, when the spacing ratio is s/B = 2.0, the wind-induced response continues to be predominantly governed by the across-wind component. However, unlike in the wind direction of 0°, the in-phase and out-of-phase components of the across-wind response exhibit more comparable magnitudes in the wind direction of 90°. This finding is in line with the observations made in reference [18], where it is demonstrated that the power spectra of in-phase and out-of-phase aerodynamic forces are nearly identical at the peak points for a spacing ratio of s/B = 2.0. Consequently, it becomes imperative to simultaneously control both the out-of-phase and in-phase components to effectively control the across-wind response.



Figure 14. Wind-induced accelerations of the detached twin towers (s/B = 2): (**a**) Towers' accelerations, and (**b**) modal accelerations.

In the wind direction of 0° , the out-of-phase component, which primarily governs the across-wind response, can be mitigated using the utilization of axial stiffness along the *x*-direction of the link bridge. This approach provides a substantial constraint force, thereby achieving a nearly rigid connection. However, in the wind direction of 90° , controlling the out-of-phase component of the across-wind response necessitates the utilization of bending and shearing stiffness along the *y*-direction of the link bridge. Yet, it is worth noting that the attainable constraint stiffness is relatively limited, especially for a long-span link bridge. Analyses of various engineering projects reveal that employing fixed supports at both ends of a link bridge, restraining the out-of-phase vibrations in the *y*-direction, can achieve a bridge frequency ratio of approximately 3 to 4. Hence, this aspect should be thoughtfully considered when defining the relevant design parameters in the MSS method.

Subsequently, the parameter optimization of the control method was carried out by analyzing wind-induced accelerations of the linked twin towers under three typical link bridge connection cases:

Case 1: The link bridge is rigidly connected to the twin towers, resulting in a frequency ratio of 8 for the in-phase mode and a frequency ratio of 4 for the out-of-phase mode. However, no constraints are imposed on the utilization of viscous dampers to control the relative vibrations between the two towers.

Case 2: Similar to Case 1, the link bridge is rigidly connected to the twin towers, with the same frequency ratios for the in-phase and out-of-phase modes. In addition, to effectively controlling the vibrations of the out-of-phase mode, viscous dampers are introduced, with the damping ratio (ζ_B) set at 10%.

Case 3: In this case, the link bridge is specifically designed with horizontal constraint stiffness and damping of the bearings, following the principles of tuned mass dampers. As a result, the frequency ratio of the in-phase mode is set to 1 (i.e., $\mu_{\omega} = 1$). Furthermore, viscous dampers are installed at the bearing locations with a damping ratio of 5% (i.e., $\zeta_B = 10\%$). Simultaneously, a "stiffness + damping component" is introduced between the two towers to effectively control the vibrations of the out-of-phase mode, efficiently constraining the relative displacement of the twin towers. The schematic diagram of the mechanical model, capable of simultaneously controlling the in-phase and out-of-phase modal responses in the *y*-direction, is illustrated in Figure 15.



Figure 15. Schematic of the mechanical model for simultaneously controlling the in-phase and out-of-phase modal responses in the *y*-direction.

Using the MSS method, we conducted an analysis of the wind-induced responses of the twin towers. The wind-induced accelerations of the twin towers, focusing on a spacing ratio of 2 (s/B = 2), were carefully examined as a function of the reference wind speed, and the results are presented in Figure 16.

A comparative study of the wind-induced accelerations among different cases revealed a notable similarity in the proportion of in-phase and out-of-phase components of the acrosswind response in the wind direction of 90°. Consequently, Case 3, which effectively controls both the in-phase and out-of-phase responses, demonstrates superior vibration reduction effectiveness. Worth mentioning, in Case 3, the link bridge is ingeniously designed as a tuned mass damper, and the horizontal displacement of the link bridge is thoughtfully illustrated in Figure 17.



Figure 16. Wind-induced building accelerations in the wind direction of $90^{\circ}(s/B = 2)$: (a) Along-wind, and (b) across-wind.



Figure 17. Horizontal displacement stroke of the link bridge in the wind direction of 90° .

4.3.2. s/B = 1 and 0.5

Figure 18 presents the building accelerations and modal accelerations for the detached twin towers (s/B = 1 and 0.5). The accelerations shown in Figure 18 represent the higher acceleration among the two towers.



Figure 18. Wind-induced accelerations of the detached twin towers: (a) Towers' accelerations (s/B = 1), (b) towers' accelerations (s/B = 0.5), (c) modal accelerations (s/B = 1), and (d) modal accelerations (s/B = 0.5).

Upon examination of Figure 18, it is evident that when s/B = 2.0, the wind-induced response of the twin towers is primarily dominated by the out-of-phase across-wind component. Subsequently, we delve into the analysis of three distinct cases:

Case 1: The link bridge is rigidly connected to the twin towers, resulting in a frequency ratio of 8 for the in-phase mode and a frequency ratio of 2 for the out-of-phase mode, due to the relatively low horizontal bending stiffness of the bridge. Meanwhile, no viscous dampers are installed to restrain the relative vibrations between the two towers.

Case 2: Similarly, the link bridge is rigidly connected to the twin towers, but now yielding a frequency ratio of 8 for the in-phase mode and a frequency ratio of 4 for the outof-phase mode. Again, no viscous dampers are present to restrain the relative vibrations between the twin towers.

Case 3: In this scenario, the link bridge is rigidly connected to the twin towers, resulting in an in-phase mode frequency ratio of 8 and an out-of-phase mode frequency ratio of 4. Notably, viscous dampers are installed to effectively control the out-of-phase vibrations, with a damping ratio (ζ_B) set at 10%. The wind-induced accelerations of the twin towers, as a function of the reference wind speed in different cases, are shown in Figure 19 for the spacing ratios of 1 and 0.5 (*s*/*B* = 1 and 0.5).



Figure 19. Wind-induced building accelerations (s/B = 2): (**a**) Along-wind (s/B = 1), (**b**) along-wind (s/B = 0.5), (**c**) across-wind (s/B = 1), and (**d**) across-wind (s/B = 0.5).

As evident from Figure 19, in the wind direction of 90° , the across-wind response for both *s*/*B* ratios of 1 and 0.5 is predominantly governed by the out-of-phase component. Consequently, Case 3, which incorporates rigid connections between the bridge and towers while simultaneously implementing viscous dampers, demonstrates the most effective reduction in vibration.

5. Discussion

In summary, the wind-induced response of the twin towers exhibits three distinct scenarios: predominance of out-of-phase response, predominance of in-phase response, and equal importance of in-phase and out-of-phase responses.

To address the improvement of wind-induced responses of the twin towers via the utilization of link bridges, this research proposes a comprehensive mechanical model, depicted in Figure 20, which effectively controls both the in-phase and out-of-phase modal responses in both *x* and *y* directions by using the link bridges. To evaluate the vibration reduction efficiency of various link bridge configurations, the Modal Sub-Structure (MSS) method is employed, facilitating the determination of optimal connecting methods and design parameters for link bridges. This offers essential technical guidance for the wind-resistant design of the linked twin towers.

In Scenario 1, where out-of-phase modal response dominates, the link bridge of strong stiffness is rigidly connected with the core tube of the towers, alongside the inclusion of viscous dampers to enhance energy dissipation, as shown in Figure 20c.

In Scenario 2, characterized by the predominance of in-phase modal response, the link bridge should be designed as a tuned mass damper (TMD), utilizing the mass of the bridge to exert a force in the opposite direction. The spring-damping constraint system for the bridge is designed based on the principle of TMD, as illustrated in Figure 20b.



Figure 20. Schematic diagram of the general mechanical model which can control the in-phase and out-of-phase modal responses in both x and y directions: (**a**) the whole link bridge, (**b**) the upper section of the link bridge, and (**c**) the lower section of the link bridge.

In Scenario 3, where in-phase and out-of-phase modal responses hold equal importance, a two-section bridge design is recommended, as illustrated in the mechanical model schematic diagram of Figure 20a. The upper section serves the purpose of controlling the in-phase response as a TMD, while the lower section is a strong link rigidly connected with the towers.

The wind-resistant optimization strategies for super-tall twin towers can be classified into two categories: aerodynamic optimization and structural optimization. Aerodynamic optimization measures yield significant benefits, yet they often entail alterations to the building's appearance, potentially creating conflicts with architectural designs. Traditional structural optimization methods primarily focus on enhancing structural stiffness and load-bearing capacity, typically leading to escalated material expenses.

In contrast, the utilization of link bridges for wind-resistant optimization in connected twin towers circumvents the need for changes in the building's appearance or an augmentation of column cross-sectional dimensions. This approach harmonizes effectively with the principles of sustainability.

6. Conclusions

A new method of wind-resistant design optimization for twin towers, called the modal substructure method (MSS), is introduced. The MSS method consists of three major steps: (1) to establish the control objectives based on the wind-induced response characteristics of the twin towers; (2) to determine the control strategies by analyzing the modal accelerations characteristics of the twin towers; and (3) to perform parameter optimization of the link bridge, which includes evaluating the damping ratio, mass ratio, and frequency ratio of the bridge. This optimization process helps identify optimal configurations for the link bridge to achieve the desired vibration reduction effects.

It is found that the wind-induced responses of the twin towers exhibit three distinct types: predominance of out-of-phase response, predominance of in-phase response, and equal importance of in-phase and out-of-phase responses. Due to these diverse response characteristics, it is essential to implement specific control strategies tailored to each type.

When the out-of-phase response is predominant, the recommended approach involves the design of a link bridge using the "stiffness + viscous dampers" configuration to effectively constrain the relative vibration between the two towers. For situations characterized by in-phase response predominance, the mass of the link bridge is harnessed to create a spring-damping constraint system at both ends of the bridge, adhering to the principles of tuned mass dampers (TMD). In cases where both in-phase and out-of-phase responses carry equal significance, the link bridge is configured to independently control each response.

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