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Study of the Effect of Rescuers and the Use of a Massive Alarm in a Population in a Disaster Situation

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Abstract: Faced with an ongoing or imminent danger, crisis managers must do their utmost to protect the exposed population and limit the extent of the disaster. More than during the pre- and post-disaster phases, time is of the essence. This temporal specificity of the disaster is essential compared to the risk. It requires a perfect coordination and a quick response in a context of uncertainty. It is important to intervene rapidly on the scene of the disaster while ensuring there are enough first responders. Crisis managers must also quickly alert the population at risk in order to favor the adoption of protective behaviors and limit inappropriate reactions, panic phenomena, and the spread of rumors. In France, in the event of a danger affecting the population, the intervention of law enforcement and emergency services is relatively rapid, even though there may be differences depending on the territories (urban or rural). On the contrary, the triggering of the alert by institutional actors (the mayor or the prefect, depending on the extent of the disaster) must follow a strict procedure that imposes longer delays and may limit or even neutralize its effectiveness. This article proposes a theoretical reflection on the effectiveness of these two types of intervention (relief and warning) with affected populations in the case of rapid kinetic or unpredictable events affecting people with a low risk culture. This reflection is based on the mathematical model "alert, panic, control" (APC) inspired by models used in epidemiology. It enables the modeling of behavior dynamics by distinguishing control and panic behaviors resulting from the difficulty or incapacity to regulate emotions. Several scenarios are proposed to identify the phases during which these two kinds of intervention have an optimal effect on the population by limiting panic phenomena.

Keywords: catastrophic event; emergency and risk management; human behavior; mathematical model; simulations; optimal control

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1. Introduction

A disaster is a severe and exceptional disturbance that profoundly destabilizes the functioning of a society and more broadly of a territory. It causes human, material, and economic losses as well as damages to the environment, which are increasingly better quantified. Since 1990, natural hazards have caused more than 1.6 million fatalities in the world, with economic losses averaging approximately USD 260–310 billion per year [1]. If the death tolls and economic losses from man-made disasters are lower, the psychological consequences of events such as an industrial explosion or a terrorist attack are often underestimated. In order to attempt to reduce the toll of disasters, several actions can be implemented, such as modeling the hazard component [2], creating spatially explicit datasets for quantifying vulnerability and its dynamics [3], improving people's awareness of risk and education [4,5], increasing the availability of early warning systems [6] and preparing crisis management by analyzing feedback from previous accidents or exercises and training for emergency planning and preparedness. This paper deals with this last parameter, which is crucial in terms of human tolls. More than during the phase preceding the event (risk reduction, prevention, and preparedness) and the one following it (recovery and mitigation), the immediate response phase must be rapid and adapted [7] in a context where uncertainty prevails. Crisis managers must make decisions based on imperfect data that are received in a disordered manner and are delayed in time [8]. The sources of uncertainty are numerous. They include the hazard itself (such as its causes, diffusion modes, etc.), its real effects (destruction, number of victims, pollution), and the reactions of the affected populations.

In order to better cope with these unprecedented and anxiety-provoking situations, anticipation remains the main response to enable communities to be more resilient. This requires planning, strategic monitoring, feedback reports, and simulation of disaster scenarios via tabletop or field exercises. However, while these exercises are essential for preparedness, they often do not take into account the diversity of people's possible reactions, particularly how those reactions may evolve during the event. This is why the use of numerical simulations is necessary, especially to model complex situations integrating different behavioral scenarios. Many factors can have an effect on human behaviors during a disaster, but it appears that the risk culture, the experience of similar events, and the perception of the event are decisive. While it is not possible to change the risk culture during the event, it is possible to influence the perception of danger, which in turn influences behaviors. The rapid and sufficient intervention of law enforcement agencies and rescuers, as well as rapid and regular information toward the population, are decisive to encourage the adoption of safeguarding behaviors, to limit inappropriate behaviors, and to avoid the possible phenomena of panic and the propagation of false rumors.

In France, although the average response time for law enforcement and emergency services is about fifteen minutes, the warning of the population is often delayed for unpredictable events. Indeed, only the competent authorities, the mayor or the prefect, who have the 'police powers', can assume the role of "Director of Emergency Operations" (DOS in French) in charge of managing the crisis unit and deciding whether or not to alert the population. This centralized management of the alert has been criticized on several occasions (such as the Nice attacks in 2016 and the fire at the Lubrizol factory in Rouen in 2019) because the lack of information or its late release generates fear [9], suspicion of a deliberately hidden truth, mistrust and rumors. This is why the deployment of the new FR-ALERT system must be an opportunity to reconsider existing doctrines and to broaden the list of actors having competencies in alert matters [6]. It is also necessary to systematize the information on current events, whether major or more moderate, in order to acculturate the population on the new information vectors. This implies a reflection on the content of the messages transmitted to the population [10].

This paper presents a theoretical analysis to highlight the crucial role by the response times of the different actors involved in crisis management. The analysis is based on the "alert, panic, control" (APC) model, which is a mathematical model designed to simulate the behaviors of the population during a crisis, taking into account their capacity to regulate their emotions or stress level (for more details, see [11,12]). This model also pays particular attention to the possible evolution of behaviors by distinguishing intrinsic transitions from transitions due to imitation. The objective is to study under which conditions the intervention of law enforcement agencies and emergency services on the one hand, and the warning and information on the other hand, have an optimal efficiency toward the population.

The paper is organized as follows. Section 2 provides a definition of human behavior and its specificities during disasters. It also emphasizes the role of warning and the intervention of law enforcement and emergency responders in crisis management and their effects on population behavior. In Section 3, the mathematical framework of the APC model is presented. People's behaviors are modeled using a set of differential equations that take into account the emotions (or stress level) of individuals, as well as their tendency to imitate (or not) the behavior of others. Two control variables representing the actions of law enforcement and emergency services are added, as well as the alert and information process. The objective is then to study how these control variables can be optimized to reduce panic and promote control behaviors in the population during a crisis. Section 4 analyzes the effectiveness of rescue and law enforcement in terms of response times and the number of first responders required. Section 5 focuses on the effectiveness of the alert and regular information in reducing panic and promoting state of control. Finally, in Section 6, we test the combination of these two types of action on the affected population by solving an optimal control problem. Its resolution gives numerically the best configurations with respect to the costs of controls and the efficiency of rescuers. Section 7 concludes the paper highlighting the contributions of our study to theory, organizational management, and society by a cross-analysis.

2. Theoretical and Empirical Background and Introduction of Crisis Management Strategies

2.1. Theoretical and Empirical Background

The term behavior refers to all the observable manifestations of human activity. According to Kurt Lewin [13], these manifestations depend on the personality of the individual (or the group) and the surrounding environment, which are considered in a broad sense (physical and social). Therefore, behavior during catastrophic events is the result of an interaction between the following elements:

- The characteristics of the individual: personal history, personality, emotions, beliefs, desires, motivations, risk culture and experience of similar events.
- The social context: people's behavior in the vicinity, available information, actions of crisis managers
- The physical environment, taking into account the specific nature of the disaster: predictability, modalities of diffusion, temporality, distance from the source of hazard, but also from shelter [14–17].

The research in psychology and geography on behavior in disaster situations emphasizes that the reactions of people involved in such events can be diverse. This variability is explained by the multitude of factors that influence how those affected perceive and understand the situation [18]. However, despite this apparent diversity, the reactions observed during disasters are strongly determined by the feeling of control over the situation, which depends on the individual's ability to regulate his or her emotions in order to preserve or recover an "emotional balance".

In disaster situations, recovery is influenced by risk culture, the experience of similar events, as well as the perception of danger. When the emotional charge (or the level of stress) is too intense or prolonged, the capacity of regulation can be altered and give rise to specific behaviors such as disordered agitation, panic flight, freeze response and inhibition that can disrupt the organization of the disaster relief but also cause post-trauma disorders. Conversely, when this charge is regulated, individuals are able to analyze the situation, socially share their emotions, and thus to better face the situation by adopting behaviors that seem more adapted to the situation.

The APC model presented in detail in [11,12] is inspired by research on emotional regulation. It is based on five meta-behaviors that allow the description of human reactions during sudden events, without any precursor sign, or if they exist, are not detected by the population. In this model, the classification of the behaviors is based on two parameters used in emotional psychology: the emotional charge and the capacity of the individuals or groups to regulate this emotional charge. In addition to the everyday behaviors that precede the onset of the disaster and a pseudo-daily state that marks the end of the period of immediate response to the danger (acute phase), three meta-behaviors make it possible to represent the diversity of behaviors potentially observable during a disaster:

- The set of alert behaviors corresponds to the first reaction in the face of danger. It can consist of looking around or moving to search for information, paying attention to specific noises or unusual smells, and so on. The duration of alert behaviors varies from less than a second to several minutes. People in a state of alert usually have a low emotional charge and therefore low emotional regulation.
- The set of panic behaviors (for example, panic flight, freeze, trampling, pushing, etc.) corresponds to all the behaviors with a high emotional charge that can be hardly regulated. People exhibit a high level of stress that partially or completely distorts their faculty of reasoning when faced with a terrifying situation. The duration of panic behavior varies from a few minutes to around an hour [19]. It usually resolves spontaneously; however, an energetic external intervention can help panic-stricken people regain a state of control.
- The set of control behaviors (such as evacuation, mutual aid, search for help, etc.) is characterized by the capacity to regulate emotional charge and evaluate the situation before reacting. However, this rational thinking or reasoning does not mean that people systematically adopt behavior adapted to the dramatic event. The duration of the control behavior varies from a few minutes to several hours, depending on the intervention of the institutional and emergency actors.

The behaviors are not static and can change depending on the evolution of the situation during the simulations. Two kinds of transitions between alert, panic, and control behaviors are taken into account: intrinsic motivation and imitation (contagion) processes [11,12]. Intrinsic motivation occurs when individuals adopt a behavior on their own initiative without being influenced by people around them. This situation is more observed in communities or societies with a strong risk culture. Individuals know what life-saving behaviors to adopt. The imitation (contagion) process is present in all types of disasters. Individuals tend to rely on people in their vicinity and adopt the same behavior. This situation is frequent when the population is dense [20,21]. The more the crowd is dense, the more people tend to mimic or are forced to act similarly to those around them. It is also observed when the population has a low culture of risk.

2.2. Introduction of Crisis Management Strategies

The APC model allows simulating various disaster scenarios by taking into account the risk culture and the population density. The objective of this paper is to introduce parameters related to the management of the crisis by the authorities in order to limit the dramatic consequences of the disaster. Results of surveys conducted in 2018–2019 among actors involved in disaster management (firefighters, civil society organizations, emergency workers, "gendarmerie" and police services, DREAL (regional department of the environment, planning and housing), ARS (regional management of the health system), etc.) show that the management of a crisis is complex because of the need to provide rapid responses in a context where there are many uncertainties [22]. A wide variety of actions must be undertaken to deal with the situation, such as securing the impacted area, evacuating the injured, managing material and human resources in real time, exchanging information with operators of vital importance, etc. The objective is also to limit inappropriate behaviors, the spread of false rumors, and panic, which can disrupt the management of the crisis. To achieve this, two elements are particularly important: public warning and on-site intervention of emergency services and police forces.

In France, the average response time for first responders is slightly less than 15 min, whether it involves the firefighters, the SAMU (French emergency medical services), or the police. However, this delay can vary greatly, and multiple factors (location, disaster size, intensity, etc.) should be taken into account. Concerning the public warning by the institutional actors in charge of crisis management, it is quite impossible to define an average timeframe. Even if warning and information systems have the objective to support populations in times of crisis by disseminating behavioral instructions allowing them to take an active part in their protection (French Ministry of the Interior, 2013, https://www.interieur.gouv.fr/content/download/65308/473026/file/GUIDE%20ORSEC%202013.pdf), victims of disasters frequently regret that these warning systems remain poorly effective for unpredictable and sudden events, even if the authorities can rely on new warning and information vectors based on mobile phones (SMS, cell broadcast), variable message signs or the Internet (e-mails, official accounts on social networks) that complement the sirens. The French doctrine stipulates that alerting the population is the responsibility of the mayor or the prefect, depending on the magnitude of the event, due to their administrative police power. The alert of the population implies the prior triggering of the communal safeguard plan or ORSEC plan (regional emergency response plan) if the event extends over several municipalities. Actors or services that are involved in risk monitoring, such as the French National Hydrometeorological and Flood Forecasting Centre (SCHAPI) or the French Tsunami warning center for the Mediterranean and North-East Atlantic (CENALT), that could directly alert the population are not currently allowed [6]. Furthermore, warning and information procedures are far from being automated. The messages to be disseminated are not systematically prepared in advance and thought out according to local realities. The instructions disseminated are therefore sometimes aberrant, which is often explained by the "copy and paste" reproduction of standard instructions [23].

This raises the question of the benefit of the alert when we assume that informing is reassuring the population. Too long delays encourage the spread of rumors likely to generate irrational behavior, a feeling of abandonment, or even panic. "During an emergency, the right message, from the right person, at the right time can save lives" (CERC, 2018). This problem concerning the event alert notification time is not specific to France. This is why the European directive of 11 December 2018, establishing the European Electronic Communications Code, imposes the obligation to provide a population alert system to the 27 Member States of the European Union via cell phones in addition to existing systems (Article 110). Since 21 June 2022, France has launched a new alert system, "FR_Alert", based both on location-based SMS and cell broadcast which permits sending notifications to the cell phones of individuals present in an area facing a serious danger. The purpose is to inform them directly about the nature of the danger and the appropriate actions to take for self-protection. Beyond this new information vector, many operational actors hope that this new methodology will stimulate a reflection on the ways to reduce warning times in emergency situations and on the need for regular information to the population.

3. Mathematical Model

3.1. Setup of the Mathematical Model

The intervention of the emergency services and the police forces as well as the warning of the population appear to be determining factors in the human toll of disasters. In addition to the life-saving dimension, we hypothesize that these interventions with victims reassure the population and thus limit panic and trauma. Our aim is to estimate the effectiveness of these two protective measures according to the intervention time frame from a theoretical point of view. For this, we use the mathematical model named APC (alert–panic–control) presented in [12]. The APC model is a compartmental model inspired by the classical epidemic mathematical models such as the SIR (susceptible–infected–recovered) models (see for example [24]). In this paper, the model has been modified to incorporate the effect of the rescuers' intervention and the massive alert defined in Section 2.2 in the form of control variables (see Figure 1).

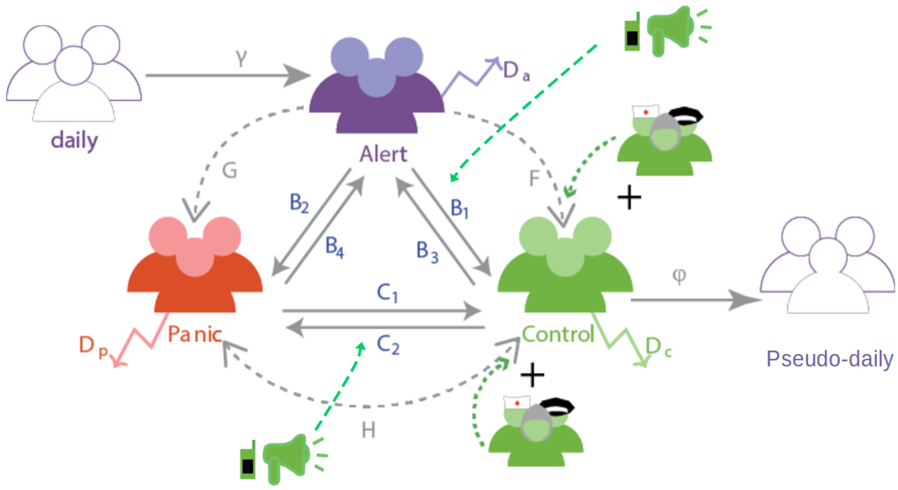


Figure 1. Transfer diagram for the APC (alert, panic and control) model in which intervention of rescuers and institutional information are included. The intrinsic transitions are represented in solid lines, while the imitation ones are represented in dashed lines.

The diversity of possible behaviors observed during disasters has been synthesized in the form of three main behaviors defined according to the emotional charge and the ability of individuals or groups to regulate it. Two other behaviors (everyday and pseudo-everyday) delimit the moment of the disaster (impact and immediate post phase). At the beginning of the simulation, all the population is supposed to be in a daily behavior before the disaster whose effects on the population are described by the function γ. Once catastrophe occurs, people adopt a state of alert, searching for information. According to their experiences and their ability to handle danger, they adopt either a state of control or a state of panic. Then, after a certain time, they can return to a pseudo-daily behavior only being in a state of control. This behavioral transition is represented by function φ. A new behavioral state (among alert, panic and control) can be achieved by two types of transitions: intrinsic transitions (solid arrows in Figure 1) and imitation processes (dashed arrows). The possible death of the three populations can be integrated into the model. The green dashed arrows represent the controls acting on the system and will be explained below.

In the simulations, functions γ and φ are chosen so that they model a sudden event, such as a tsunami earthquake, terrorist attack, or explosion in a chemical plant. An example of such functions is given in Figure 2 where people become aware of the disaster between 3 and 20 min. They turn back to a pseudo-daily behavior between 60 and 240 min.

The variables of the APC model are (for $t \geq 0$):

- $a(t)$ the density of individuals in a state of alert;
- $p(t)$ the density of individuals in a state of panic;
- $c(t)$ the density of individuals in a state of control.

In addition, we consider

- $q(t)$ the density of individuals in the everyday behaviors;
- $b(t)$ post-emergency behaviors and pseudo-daily behavior after the disaster;
- $v(t)$ the density of individuals who lose their lives during the disaster.

The non-negative parameters B_i, $i = 1, \ldots, 4$ and C_i, $i = 1, 2$ define intrinsic transitions. F, G and H describe the imitation processes (see Table A1 for more explanations and [11] for details about the APC model without control).

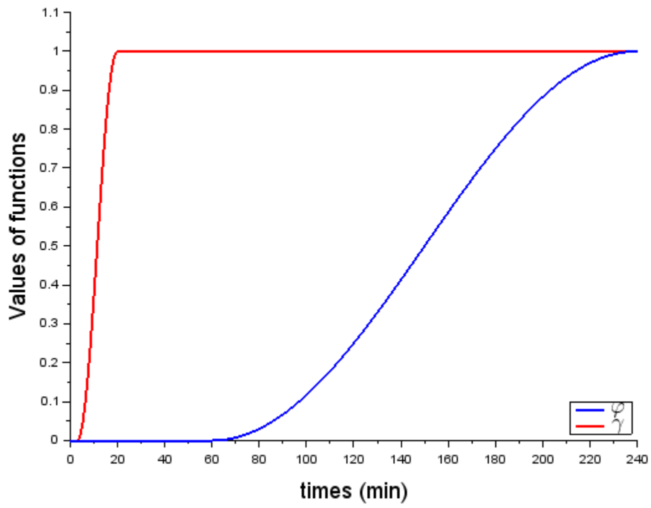


Figure 2. Representation of the functions γ and φ in the case of a sudden event. People become aware of the disaster between 3 and 20 min, and they return to a back to daily or pseudo-daily behavior between 60 and 240 min.

Mathematically, controls $\tilde{u}_i$, $i = 1$, 2, whose aim is to reduce the phenomenon of panic in favor of control, are defined in the following manner:

1. Control 1: $\tilde{u}_1 = \kappa u_1$, where u_1 represents rescuers intervening in the disaster-affected area. This staff is supposed to exhibit a control behavior throughout the event and therefore is not subjected to the behavioral panel (alert, panic, control). Thus, this external population considered as an additional population to the one in a control state has a reassuring effect and favors the process of imitation toward control. Therefore, control $\tilde{u}_1$ acts on the imitation functions F and H, and the other populations a and p interact with $c + \tilde{u}_1$. κ is a constant modeling the effect of the rescuers. For example, $\kappa = 1$ means that rescuers interact with alert and panic populations as other individuals in a state of control, whereas a value greater than 1 means that the rescuers have an above average effect. Afterwards, we will suppose that $\kappa \in [1,3]$.
2. Control 2: $\tilde{u}_2 = \rho u_2$ where u_2 represents the percentage of individuals in a state of alert and control that will be concerned by the institutional information transmitted during the event such as the transmission of a message or the triggering of a siren. This function is adaptive and time-dependent. Parameter ρ takes into account the effect of the institutional information on the population hearing the alert. People's reactions during a catastrophic event depend on their past experiences. For example, people who have been trained on how to react to the arrival of a tsunami will adopt appropriate reactions when they hear the specific sound of the siren for this type of event. Thus, the control $\tilde{u}_2$ acts on the intrinsic transition between alert–control and control–panic, that is B_1 and C_2. We suppose to have $B_1 + \rho u_2 \in [0,1]$ and $C_2 - \rho u_2 \in [0,1]$ where $\rho \in [0;1]$.

The complete APC model with controls reads as ($t \geq t_0$ and see Table A1 for the parameter definitions) :

$$
\begin{cases}
\dfrac{da(t)}{dt} = \gamma(t)\, q(t) - (B_1 + \tilde{u}_2 + B_2 + D_a)\, a(t) + B_3\, c(t) + B_4\, p(t) \\
\quad -F(a(t), c(t), \tilde{u}_1(t))\, a(t)\, (c(t) + \tilde{u}_1(t)) - G(a(t), p(t))\, a(t)\, p(t), \\
\dfrac{dp(t)}{dt} = B_2 a(t) + (C_2 - \tilde{u}_2(t))c(t) - (B_4 + C_1 + D_p)\, p(t) \\
\quad +G(a(t), p(t))\, a(t) p(t) - H(c(t), p(t), \tilde{u}_1(t))\, (c(t) + \tilde{u}_1(t))\, p(t), \\
\dfrac{dc(t)}{dt} = (B_1 + \tilde{u}_2) a(t) + C_1 p(t) - (B_3 + (C_2 - \tilde{u}_2(t)) + D_c)\, c(t) \\
\quad +F(a(t), c(t), \tilde{u}_1)\, a(t)(c(t) + \tilde{u}_1(t)) + H(c(t), p(t), \tilde{u}_1(t))\, (c(t) + \tilde{u}_1(t)) p(t) \\
\quad -\varphi(t) c(t), \\
\dfrac{dq(t)}{dt} = -\gamma(t) q(t), \\
\dfrac{db(t)}{dt} = \varphi(t) c(t), \\
\dfrac{dv(t)}{dt} = D_a a(t) + D_c c(t) + D_p p(t)
\end{cases}
\tag{1}
$$

The imitation functions are defined to take into account the dominant mass principle: that is, when populations with different behaviors meet, depending on the ratio among the populations, imitation transitions can take place.

Thus, they have the following form:

$$F(a,\ c,\ \tilde{u}_1) = \alpha \times \xi\left(\frac{c + \tilde{u}_1}{a + \varepsilon}\right),\ G(a,\ p) = \beta \xi\left(\frac{p}{a + \varepsilon}\right) \tag{2}$$

and

$$H(c,\ p,\ \tilde{u}_1) = \gamma_{p \to c} \times \xi\left(\frac{c + \tilde{u}_1}{p + \varepsilon}\right) - \gamma_{c \to p} \times \xi\left(\frac{p}{c + \tilde{u}_1 + \varepsilon}\right). \tag{3}$$

The variable ϵ is such that $0 < \epsilon << 1$ to avoid singularities, and the function $\xi(w) = \dfrac{w^2}{1 + w^2}$ $w \in \mathbb{R}$ is a function that asymptotically tends to 1. Function ξ (see Figure 3) has been chosen in order to model the fact that individuals in an alert state adopt a control state only if there is a majority of people in a control state.

Remark that the difference between the imitation process H and the two others, F and G, is that it can be in both directions. Indeed, according to [25], alert behaviors are not imitable.

Afterwards, we suppose that the parameters B_1, B_2, B_3, B_4, C_1, C_2 are non-negative and the parameters D_c, D_p, D_a, α, β, γ_1, γ_2 are positive. Let there be the following set of admissible controls (a_i and b_i are constants in $[0, 1]$):

$$\mathcal{U}_T := \{(u_1, u_2) \mid u_i \text{ piecewise continuous function, } a_i \leq u_i(t) \leq b_i,\ i = 1, 2,\ \forall t \in [0, T]\}.$$

Clearly, the controls are bounded and locally integrable on the interval $I := [0, T]$. According to Cauchy's theorem, the following proposition ensures the existence of controls when considering System (1) with the initial condition $(0, 0, 0, 1, 0, 0)$.

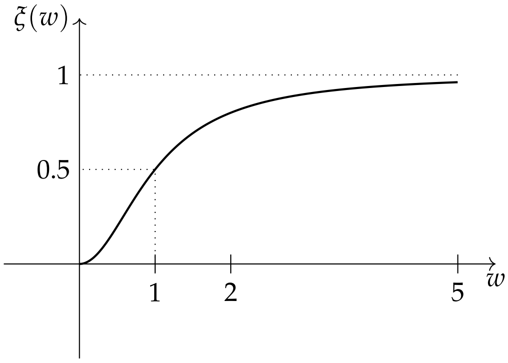


Figure 3. Representation of the sygmoidal function ξ used in the imitation interaction terms defined in Equations (2) and (3). w represents the ratio between two populations exhibiting different behaviors. This function has been chosen to model the fact that the behavior of the majority is the most imitated one.

Proposition 1. *For any control $u \in \mathcal{U}_T$, the Cauchy problem defined by System (1) with the initial conditions $(0,0,0,1,0,0)$ admits a unique non-negative solution. Furthermore, the set*

$$\mathcal{K} := \{(a,p,c,q,b,v) \in (\mathbb{R}^+)^6 \mid a+p+c+q+b+v \leq 1\}$$

is positively invariant for the flow induced by System (1) with the initial conditions $(0,0,0,1,0,0)$.

3.2. Scenario Construction and Model Parametrization for Simulations

The scenarios we propose in this paper are based on the following three postulates:

- Postulate 1: warning and mass information of the population have a reassuring effect on the population, if the alert is issued quickly and information on the situation and its evolution is issued at regular intervals;
- Postulate 2: the intervention of law enforcement agencies and rescuers has a beneficial role on the levels of stress observed within the population;
- Postulate 3: In a society with a strong risk culture, people faced with a catastrophic or potentially dangerous event are able to regulate their stress level and adopt control behaviors to cope with the situation. This is less true in societies marked by a low-risk culture.

Throughout the paper, we assume that the population is marked by a lack of risk culture when faced with a sudden and unforeseeable hazard due to a lack of training or education on how to behave. This difficulty in defining the right strategy to adopt generates high or even acute levels of stress that are often difficult to regulate and therefore favorable to different forms of panic. Furthermore, the selected case study concerns a dense crowd where the process of imitation influences the evolution of behaviors.

We also propose testing three scenarios with different variants highlighting the importance of intervention or warning delays with this type of population.

- The first scenario focuses on the response times of firefighters and/or law enforcement actors and the number of teams needed to reassure the population and then limit panic phenomena.
- The second scenario aims to analyze the effectiveness of a massive alert depending on when it occurs but also the interest of having regular communication with the population.
- Finally, the last results from the combination of the first two. It is intended to highlight an ideal strategy for minimizing the development of unsuitable behaviors due to acute stress levels.

For the simulations, the catastrophe is supposed to be sudden so people become aware of the disaster between 3 and 20 min and they return back to daily behavior between 60 and 240 min (see Figure 2).

In [11], different scenarios of behavioral dynamics concerning levels of risk culture and population density have been designed and for each scenario, a set of parameter values of the APC model has been selected. In this paper, to model a dense population with a low culture of risk, we set the following parameter values:

$$B_1 = 0.28,\ B_2 = 0.3,\ C_1 = 0.25,\ C_2 = 0.3,\ \alpha,\ \beta,\ \gamma_{p\to c},\ \gamma_{c\to p} \geq 0.5. \tag{4}$$

Since the values of the imitation parameters α, β, $\gamma_{p\to c}$, $\gamma_{c\to p}$ will depend on our scenario, they will be calculated afterwards.

Remark 1. *We recall that in [11], the effects of the population density are represented by the imitation parameters: a dense (resp. sparse) population means high (resp. low) imitation parameters. A population with a low culture risk is taken into account by choosing $B_1 < B_2$, $C_1 < C_2$.*

In order to measure the impact of a control, the mean value of the population in a state of control during the interval time $[0, T]$ is calculated, in which T is the time when everybody returns back to daily behavior. Its expression is equal to

$$Mean(c) = \int_0^T c(s)ds. \tag{5}$$

4. Effects of Emergency Rescuers

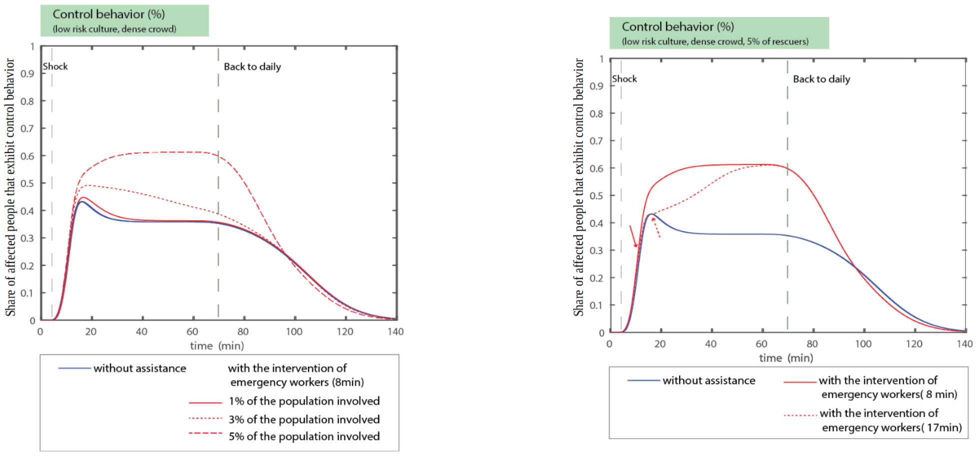
In this part, we aim at studying the effects of the control $\tilde{u}_1 = \kappa u_1$ and more precisely the effects of the combination of response time and the proportion of rescuers and law enforcement actors on the percentage of persons in a control state.

The quantity $u_1(t)$ represents a percentage of rescue workers present on site at time t and κ represents their positive effect. In contrast to those affected by the disaster, they remain controlled as long as there is no major domino effect and enhance the transition from alert to control by reinforcing the imitation process. Upon contact with rescuers, individuals are reassured and adopt a control state. However, if rescuers and policemen are not numerous enough, they are overtaken, and their intervention has little influence on the imitation process. In the present case, to observe the real effect of their action, the parameters involved in the imitation processes from panic to control and from control to panic have all the same value: that is, we set $\alpha = \beta = \gamma_{p\to c}$, $\gamma_{c\to p} = 0.6$. Furthermore, we assume that rescuers have the same effect as other control persons so that the parameter κ, corresponding to their action effect, is taken to be equal to 1 during all the simulations.

4.1. Analysis on a Dense Population

Figure 4a,b give the percentage of persons in a control state during the simulations in the case of a dense population (see set of parameters 2 in Appendix A).

Figure 4a compares the effect of the proportion of rescuers varying from 1% to 5% and arriving at t = 8 min. The presence of 5% of rescuers among the affected population increases the maximum number of people in a control state from 35% to 60%, while the effect is really limited in the case of 1% of rescuers compared to a situation without assistance (blue line). Moreover, when their size reaches 5% of the population, the benefits persist over time. When the proportion of rescuers is low, we can see that the positive effect on the victims decreases quickly in the case of 1% and more slowly with 3%. In fact, the stress spreads among the population when they realize that the number of rescue workers is too small to help them to cope. Figure 4b shows that the arrival time of emergency services is also a key factor in reassuring the population and favoring control behavior. An intervention of 5% of first responders after 17 min requires more time to decrease the stress level of the population: it takes about 40 min to drive 60% of people in a control state versus around 10 min when the rescuers arrive after 8 min.



(**a**) (**b**)

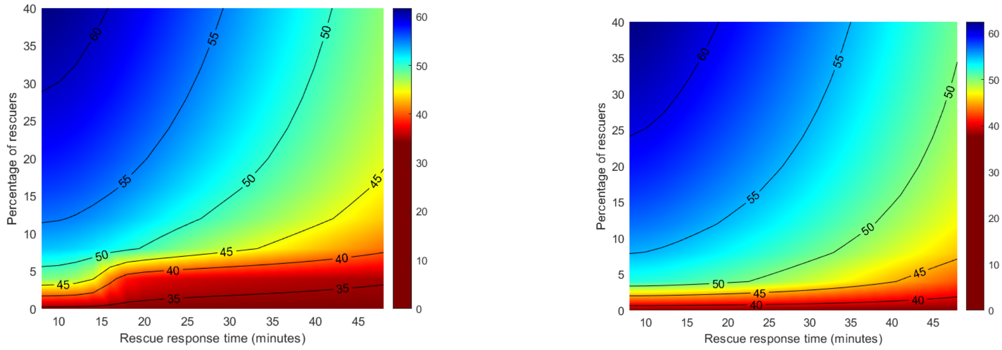
Figure 4. Effects of the response time and the proportion of the rescuers in a dense crowd. The imitation parameters have been chosen to be equal to 0.6 to reflect a dense crowd and favor the imitation process. (**a**) Comparison of the effect of the rescuer percentage arriving at $t = 8$ min: the blue solid line represents the case without assistance; the red solid line represents the case where the percentage of rescuers is 1% of the involved population; the red dotted line represents the case where the percentage of rescuers is 3% of the involved population and the red dashed line represents the case where the percentage of rescuers is 5% of population involved. (**b**) Comparison of the response time effect of the rescuers when there is no assistance (blue solid line) with the intervention of emergency workers at 8 min (red solid line) and at 17 min (red dotted line). Here, we suppose that 5% of rescuers arrive on site.

4.2. Summary of the Results

To evaluate the effects of rescuers, the average of the density of people in a state of control, c (see Equation (5)), is calculated as a function of the rescuers density and their arrival time. Figure 5 depicts the results of two cases when the crowd is dense: (a) without rescuers at the moment when the disaster takes place (b) with the presence of 1% of rescuers on site. If we set a goal of having 55% of people in a control behavior, then around 12% of rescuers are needed if they arrive within 10 min and around 20% are required if they arrive in 20 min.

If we can only rely on less than 5% of rescuers with regard to the involved population, they should arrive in less than 14 min to maintain 45% of the population in a control state. Moreover, a threshold effect can be observed. For the same amount of people in a control behavior, in the case of a bigger response time, significantly more rescuers are required.

In the simulations results represented in Figure 5b, we suppose that 1% of rescuers are already on site before the catastrophe. This is a reasonable hypothesis in the case where the disaster takes place during an event involving a dense population (for example, on many beaches of the Mediterranean, during summer, there are lifeguards). The on-site rescuers have a real positive effect on the control behavior of the people, allowing the external rescuers to arrive later and in a smaller number compared to the previous situation.



(**a**) (**b**)

Figure 5. Effect of the presence of rescuers and their response time on the control behavior. The left figure shows the case when there is no rescuer at the beginning of the catastrophe whereas in the right figure, 1% of rescuers (compared to the total population) are already on site. (**a**) Average of people in a control behavior depending on rescuer percentage and rescue response time in the case of a dense crowd. (**b**) Average of people in a control behavior as function of rescuer percentage and rescuer response time when 1% of rescuers are already present in the impact zone at the beginning of the catastrophe.

5. Effect of a Massive Alert

This part focuses on the effects of the alert signals represented by the control $\tilde{u}_2 = \rho u_2$ on the percentage of the population in a control state. The values of imitation parameters are set to slightly favor panic and are equal to

$$\alpha = \gamma_{p \to c} = 0.6; \, \beta = \gamma_{c \to p} = 0.62.$$

We also suppose that $\rho = 0.05$ and $\tilde{u}_2 = 0.5$. The choice for ρ and u_2 means that only 50% of the population is concerned by the alert (for example, in the case of the use of phones, only 50% are called) and that the effect on this alerted population is low ($\rho = 0.05$).

5.1. Analysis of the Case of a Dense Population

Unlike emergency workers, the positive effect of the warning is supposed to decrease over time. Without new information, the stress level of the population tends to increase. For example, control $\tilde{u}_2$ in Figure 6 represents a massive alert diffused 15 min after the beginning of the catastrophe. The curves at 30 and 60 min are similar.

To investigate the effect of prevention, two different parameter values for ρ are chosen: first, ρ is set to 0.05, and then, it is increased to 0.3. Figure 7b shows the effects on the evolution of the share of people in a control behavior. The alert has a very positive effect on disaster-affected populations at 15 and 30 min. The effect is smaller and does not last as long when the warning is sent after 60 min. However, these simulations show that the positive effect, while still high, decreases over time, even when the warning is sent after 15 min. In the absence of new official information, the population begins to have doubts about what is happening, which leads to an increase of stress levels (panic behavior).

To measure the effectiveness of an alert followed by official public information messages, control $\tilde{u}_2$ represented in Figure 8a is proposed. Here, we suppose that information is provided three times, every 15 min. According to Figure 8b, there is a cumulative effect. Even if the second information does not generate an increase of control behavior as important as the alert and the first information message, it has a critical role in keeping the share of individuals in a control state at over 70% over time.

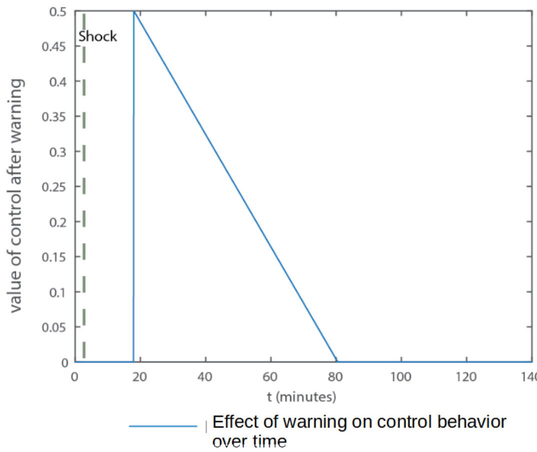


Figure 6. Example of a representation of control $\tilde{u}_2$ that models the effect of a massive alert diffused 15 min after the beginning of the catastrophe and with a limited effect on time.

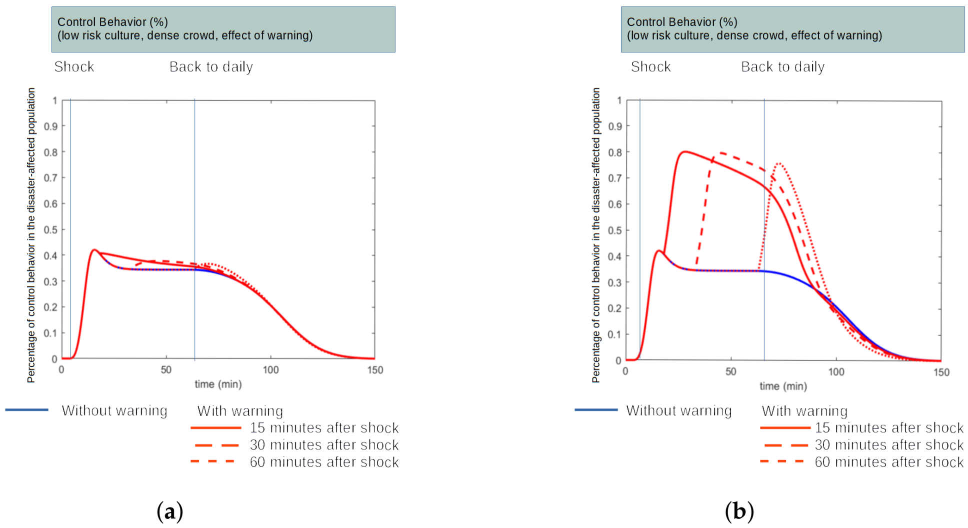


Figure 7. Effect of a message on control behavior when it is diffused 15, 30 and 60 min after the beginning of the catastrophe and its effect is limited in time. In this scenario, we suppose that the message is sent to 50% of the population with a low effect ($\rho = 0.05$) in the left figure and a high effect in the right figure ($\rho = 0.3$). (**a**) Effect of a message described by Figure 6 on control behavior when the population is low sensitive to the control effect ($\rho = 0.05$). (**b**) Effect of a message described by Figure 6 on control behavior when the population is sensitive to the control effect ($\rho = 0.3$).

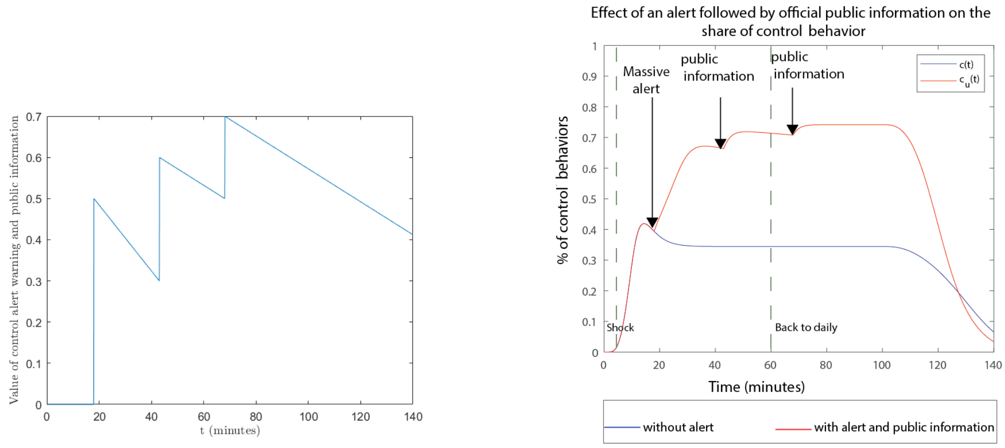
5.2. Summary of the Effect of Warning Decision Time

Figure 9 sums up the results of warning messages on control behaviors as a function of the percentage of alerted people and the time at which they are alerted. The control $\tilde{u}_2$ has the form given at Figure 6.

For example, if an alert or a warning is addressed to 50% of the population, the proportion of people in control is

- Around 50% if the alert is triggered before 20 min;
- 45% if the alert is triggered at 40 min;
- Under 35% after 70 min.

Under 35% of alerted people, the benefit is weak and it does not vary over time until there is a return to daily life.



(**a**) (**b**)

Figure 8. The left figure represents the control $\tilde{u}_2$ on control behavior. It models an alert broadcast after 15 min followed by two official public information messages. The right figure represents its effect on the population in a control state. In this scenario, we suppose that the message is sent to 50% of the population. (**a**) Example of a representation of the control $\tilde{u}_2$ modeling the effect of a massive alert followed the spread of two public information messages. (**b**) Effect of a message followed by two public information alerts described by Figure 8a on the percentage of people in a control behavior.

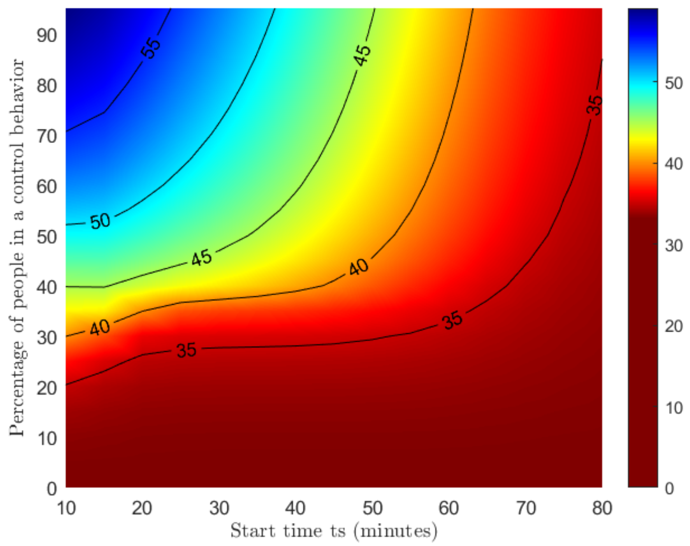


Figure 9. Percentage of people in a control state with respect to the percentage of people reached by the warning messages and the time at which they are alerted.

6. Combination of the Two Controls $\tilde{u}_1$ and $\tilde{u}_2$

6.1. Problem Formulation

This section is devoted to the study of the control model (1) when rescuers intervene and institutional information is transmitted during the event over a fixed time window. In this aim, an objective functional, J, is defined (see Equation (6)). Our goal is to minimize the number of panic behaviors while at the same time keeping the cost of these two controls very low. For a fixed terminal time T, the problem is to minimize the objective functional

$$J(u_1, u_2) = \int_0^T [C_p p(s) + A_1 u_1^2(s) + A_2 u_2^2(s)] ds. \tag{6}$$

The constants A_i, $i = 1, 2$ and C_p are positive. A_i, $i = 1, 2$ are weights that permit regulating the costs of the controls: that is, the presence of rescuers and the diffusion of a massive alert. For $i = 1, 2$, a low value of A_i means that the control u_i is cheap (thus, the strategy mainly aims to increase efforts on control u_i) and a great value of A_i means that the control u_i is expensive. The total cost on the interval $[0, T]$ is the sum of the cost induced by the panic itself and the cost induced by the control interventions.

The existence and uniqueness of an optimal solution and the characterization of optimal controls are given in Appendix A.2.

Afterwards, we will consider the quantity $I_{\Delta c}(t)$:

$$I_{\Delta c}(t) = \int_0^t (c_u(s) - c(s))ds, \tag{7}$$

where c is the density of individuals in a control behavior when there is no external intervention and c_u is the density of individuals in a control behavior when rescuers act and institutional information is diffused. Thus, $I_{\Delta c}(t)$ measures the difference between the mean of c_u and the mean of c during the time interval $[0, t]$.

The higher the value of $I_{\Delta c}$, the higher the population in control due to control interventions.

6.2. Parameter Values

The parameters are those defined in Table A1 of Appendix A, with the imitation parameters defined by the set of parameters 2 that favors the panic behaviors. The choice of the parameter values A_i, $i = 1, 2$ will be explained in Section 6.3 and will serve to elaborate our scenarios.

We assume that the controls $\tilde{u}_1 = \kappa u_1$ and $\tilde{u}_2 = \rho u_2$ are positive and there are practical limitations on their maximum value. For $\tilde{u}_1$, we suppose that the number of rescuers (i.e., u_1) intervening in the disaster impact zone cannot be greater than 30% of the total population, and no more than 50% of the individuals in a state of alert and control will be concerned by the institutional information. Consequently, u_1 will be smaller than 0.3 and u_2 will be smaller than 0.5.

Other constraints are added on the controls due to their definition. Since no more than 100% of individuals can change their alert behavior in a control one, $B_1 + \tilde{u}_2$ must be smaller than 1. Moreover, $C_2 - \tilde{u}_2$ must be non-negative. These constraints imply that the maximal value of u_2 is equal to $\min((1 - B_1)/\rho, C_2/\rho, 0.5)$.

We recall that the parameter ρ describes the effect of the institutional information on the population, and it is chosen to be equal to 0.3 in this section. Thus, among individuals that are concerned by the information, no more than 30% will adopt a control behavior. Control u_1 represents the density of operational staff, and the parameter κ represents their effect. If their effect must be strengthened due to their efficiency, κ can be taken to be greater than 1; otherwise, it is equal to 1.

6.3. Scenarios

Four scenarios are proposed. The first three scenarios aim to compare the effect of the two controls: the rescuers and the institutional information with respect to their costs. For this purpose, the parameters A_i, $i = 1, 2$ are either set to the value 0.01 to favor the strategy or set to the value 1 otherwise. For example, a management strategy involving mainly rescuers corresponds to the pair of parameters $(A_1, A_2) = (0.01, 1)$, whereas setting $(A_1, A_2) = (1, 0.01)$ corresponds to a management strategy based on institutional information. Consequently, to measure the effect of each control, the pairs of parameters $(A_1, A_2) = (1, 1)$, $(A_1, A_2) = (0.01, 1)$ and $(A_1, A_2) = (1, 0.01)$ are chosen to construct the three first sub-scenarios.

In the case of the two combined controls in favor of rescuers ($(A_1, A_2) = (0.01, 1)$), a fourth scenario, compared to the second one, aims to test the impact of rescuers efficiency by considering either the parameter value $\kappa = 3$ or the value $\kappa = 1$.

6.4. Numerical Results and Interpretations

Table 1 summarizes the results of the four scenarios. The objective functional J (see Equation (6)) corresponds to the total cost of the two controls (rescuers and alert/information), and its values in Table 1 correspond to a choice of controls giving the best compromise between a low cost, that is a low value of J, and a maximal number of individuals in a state of control. The real values given by $I_{\Delta_c}(T)$ represent the difference between the averages of individuals in a control state when there is no control and individuals in a control state in the presence of rescuers and a massive alert during all the simulations (see Equation (7)). For example, for scenario 1, the value 35.5 means that during the entire catastrophic event, that is between 3 and 240 min, there was an increase of 35.5 points of individuals in a control state with the presence of rescuers and a massive alert. Its evolution over time can be seen in Figure 10.

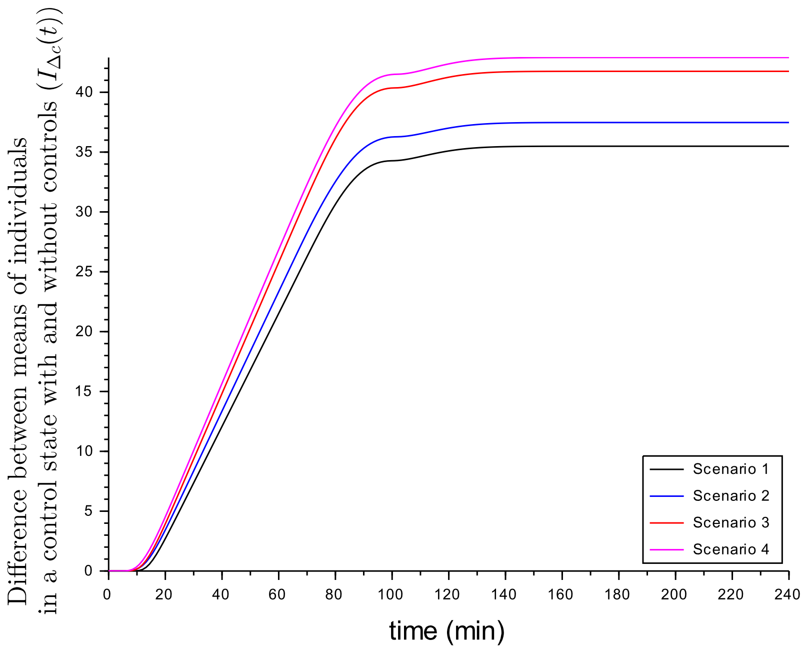


Figure 10. The figure represents, for each scenario, the difference $I_{\Delta_c}(t)$ in time between the mean of individuals in a control state when there is no control and the mean of individuals in a control state in presence of rescuers and a massive alert.

For each scenario, controls u_1 and u_2 are plotted in Figure 11a–d. They represent the best control values to obtain a maximal effect on the reduction of panic behavior. It is important to note that in each scenario, control u_2 is continuous over time. This condition can correspond to a scenario with an alert followed by regular information allowing to maintain a stable effect over time.

Table 1. Four scenarios are constructed by varying the costs of controls u_1 and u_2 and by varying the efficiency of rescuers with the parameter κ. The other parameters are defined as shown in Section 6.2.

Scenario	Parameter Values (A_1, A_2)	$I_{\Delta c}(T)$	J
1	$\kappa = 1$ and $(A_1, A_2) = (1, 1)$	35.5	70.4
2	$\kappa = 1$ and $(A_1, A_2) = (0.01, 1)$	37.5	68.6
3	$\kappa = 1$ and $(A_1, A_2) = (1, 0.01)$	41.7	70.2
4	$\kappa = 3$ and $(A_1, A_2) = (0.01, 1)$	42.9	74.0

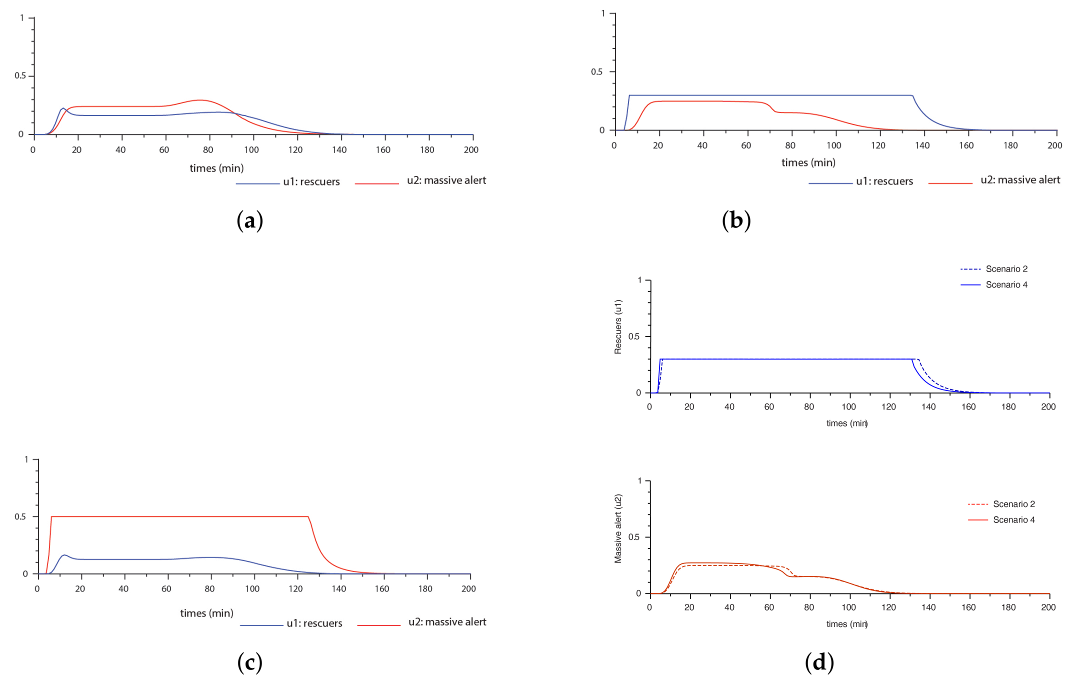


Figure 11. (**a**–**d**) represent the controls u_1 and u_2 obtained in the case of scenarios 1, 2, 3 and 4. They were obtained by solving the optimal control problem with the parameters defined in Section 6.2. To compare respectively the efficiency of operational staff, controls obtained in Scenario 2 were added to (**d**). (**a**) Controls obtained in the case of scenario 1, when $(A_1, A_2) = (1, 1)$ and $\kappa = 1$. (**b**) Controls obtained in the case of scenario 2, when $(A_1, A_2) = (0.01, 1)$ and $\kappa = 1$. (**c**) Controls obtained in the case of scenario 3, when $(A_1, A_2) = (1, 0.01)$ and $\kappa = 1$. (**d**) Controls obtained in the case of scenarios 2 and 4, when $(A_1, A_2) = (1, 1)$ and $\kappa = 1$ or $\kappa = 3$.

As we mentioned before, the simulations are based on the case of sudden onset disasters affecting a dense population (concert, cultural event, etc.) with a low-risk culture. People are concerned by the disaster between 3 and 20 min and slowly begin to adopt post-emergency behaviors (pseudo-daily behavior in the model) between 60 and 240 min.

According to the second (based on a massive alert) and third scenarios (favoring a massive intervention of rescuers), the values of the functional cost J and the mean of individuals in a control state, $I_{\Delta c}(T)$, are close with a slight advantage for a massive alert that should have a constant effect on the population during the entire scenario. That means that the two strategies can produce similar effects on the population with this choice of parameter values. However, the temporality is not the same.

When the crisis management strategy emphasizes regular warnings and information (see Figure 11c), warning must be delivered quickly and followed by regular information up to 120 min. In addition, it must be addressed to at least 50% of the affected population. Rescuers can arrive a little later, within 10 to 12 min (which is empirically possible), and begin to leave the place from the 90th minute, as the information of the population allows them to manage the share of people who have difficulty controlling their emotions. The maximum number of required rescuers is about 20% at the beginning, but then it mainly represents 12% of the population. For this scenario, $I_{\Delta_c} = 41.7$. It means that the crisis management strategy increases the proportion of people in a control behavior by 41.7 points compared to a situation where the affected populations would not receive any assistance.

Conversely, for scenario 2, when law enforcement and rescue workers respond very quickly (less than 5 min) and make up to 30 percent of the affected population, the warning system can be triggered a bit later. It must be addressed to about 25% of the affected population (twice less than in the previous scenario) until 70 min. Beyond 70 min, information plays a progressively less important role until the 120th minute. Conversely, the proportion of rescuers should remain unchanged until at least the 140th minute, even if most of the population has begun to adopt post-emergency behaviors by the 60th minute.

Scenario 4 partially replicates scenario 2. The main difference is that law enforcement agencies and rescuers are more experienced in disaster management. This efficiency allows them to respond slightly faster (Figure 11d), which encourages the adoption of control behaviors among the population favoring post-emergency behaviors. Their intervention time is slightly reduced. Values of I_{Δ_c} and J in Table 1 show that this experience is useful in the management of the crisis since it increases the average number of individuals in a state of control by 5.4 points between scenarios 2 and 4.

Finally, Scenario 1 combines both controls (alert and rescuers) to find the least costly solution. In order to foster control behavior, crisis managers must favor a rapid intervention of rescuers and law enforcement officers (between 10 and 12 min) followed by an alert of at least 25% of the population. When the warning is effective, it enables the population to regulate its emotions (control behaviors), which allows a very slight reduction in the proportion of law enforcement and rescue workers. However, this information must be maintained and even slightly reinforced when the population starts to adopt "post-emergency" behaviors from the 60th minute. In this scenario, both controls decrease progressively from the 90th minute. This diminution is faster for the information, as law enforcement and rescuers play an important role in the management of the end of the crisis. The advantage of this scenario lies in the fact that the transition to a post-emergency phase is much shorter than in the other scenarios, even if it is a little less optimal than in scenario 4 (Table 1) concerning panic behaviors.

7. Discussion and Conclusions

This paper examines the effect of two complementary crisis management strategies on people's behavior: intervention by law enforcement and emergency services on the one hand, and warning and information dissemination on the other. We postulate that these two variables have a beneficial effect on populations and can limit panic behavior. Most existing studies focus primarily on the resources to be deployed in crisis situations, often neglecting the significant role of the affected population [26]. However, the population is far from being a passive entity, and its reaction can have a considerable impact on crisis management. Depending on their training, knowledge, and awareness, behaviors can either disrupt the efforts of stakeholders and operational actors or, conversely, enhance and support their actions.

To measure the effect of emergency services and warning combined with information dissemination, we carried out analyses using a mathematical model developed from the APC model which allows simulating different human behaviors during a catastrophic event [11]. The APC model offers an innovative approach to the study of disaster-related crisis management, as it takes into account changes in people's behavior as the situation evolves. Indeed, it is difficult to carry out crisis management exercises involving the population and to determine their behavior. When they do take part, it is mainly to play the role of casualties to be evacuated. This is why simulation can be a useful tool for theoretical reflection on crisis management, including the different reactions of individuals and their evolution. By explicitly integrating the human dimension into the model, researchers and decision-makers can gain valuable insights into the complex dynamics of crisis management. They can assess the potential impact of different population behaviors on the overall effectiveness of crisis response efforts by identifying areas of improvement in communication and coordination and developing more tailored and effective strategies to mitigate the consequences of disasters. Therefore, the model provides a framework for analyzing and optimizing crisis management strategies that actively engage and account for the actions and responses of the affected population.

In this paper, law enforcement and emergency services and warning have been integrated into the APC model in the form of control variables that are likely to increase the proportion of people in a control behavior if the strategies implemented by crisis managers are adapted to the situation. This increase is due to imitation processes when the intervention of emergency services is considered (reassuring effect, feeling of being taken care of) and to intrinsic processes when an alert/information is massively disseminated to the population, informing them of the safeguards to adopt (reassuring effect of information on what to do). We built different scenarios based on a sudden disaster, with no warning signs, affecting a dense population with no risk culture. These factors are likely to generate intense stress if the population feels abandoned and unable to cope, rising to various forms of panic (inhibition, freeze reaction, panic flight, agitation), and disrupting crisis management.

The first results consider these two variables individually. It appears that in the absence of a risk culture, law enforcement and rescuers face two major challenges: response times combined with sufficient numbers of on-site responders. The latter is more important than the timing of emergency first responders in reassuring the population (Figures 4 and 5). It should be noted, however, that in crisis management, the role of first responders is not necessarily to assist those affected but rather to provide information on the resources to be mobilized and the needs according to the specific nature of the disaster. Warnings and information messages to the population should play an important role in limiting the panic phenomena that can disrupt the organization of relief efforts. However, for warnings to be effective, the challenges are more numerous: the population must be aware of the content of the message (Figure 7), which must not be considered a fake; the message must be disseminated relatively quickly (Figure 7), and, above all, it must be followed by regular information (Figure 8). Countries such as Belgium and Australia have greatly simplified warning procedures to respond rapidly to specific disasters. In Australia, the Emergency services in Victoria and New South Wales have developed Standard Operating Procedures for multiple life-at-risk scenarios beyond large-scale disasters. They are able to authorize, construct and disseminate alerts within 15 min of the decision to activate [27]. The combination of these two factors shows that rescue operations can be facilitated when the population is given regular warnings and information (Figure 11). With the population aware of how to behave, law enforcement and emergency services can focus on the injured and secure the affected area. In the absence of an effective alert, more human resources need to be mobilized to assist people who are shocked, helpless, under great stress, and unsure of how to behave.

In the recent literature, several spatio-temporal models using diffusion processes on networks have been developed to describe the diffusion of multiple behaviors within populations [28–30]. However, it is worth noting that these existing studies have not yet explored the specific inclusion of emergency rescuers and the impact of massive alerts in their models. A future extension of the current paper will be to investigate the spatial effects of these two controls on a spatio-temporal model in which the dynamic of each node is governed by the APC model and the edges symbolize the interactions between nodes. This extension would lead to a more comprehensive understanding of how the presence of emergency responders and the communication of large-scale alerts influence the diffusion and dynamics of different behaviors within populations.

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Appendix A

Appendix A.1. Recapitulation of the Parameter Values for the Simulations

For the simulations, the catastrophe is supposed to be sudden so that people become aware of the disaster between 3 and 20 min and they return to a back to daily behavior between 60 and 240 min.

In all the simulations, the parameter values are those shown in Table A1.

Table A1. Parameters of the APC model and their value in the simulations.

Parameters	Notation	Value
Intrinsic evolution from alert to control	B_1	0.28
Intrinsic evolution from alert to panic	B_2	0.3
Intrinsic evolution from control to alert	B_3	0.001
Intrinsic evolution from panic to alert	B_4	0.001
Intrinsic evolution from panic to control	C_1	0.25
Intrinsic evolution from control to panic	C_2	0.3
Mortality rates	D_a, D_p, D_c	0
Imitation from alert to control	α	$\geq$0.5
Imitation from alert to panic	β	$\geq$0.5
Imitation from panic to control	$\gamma_{p \to c}$	$\geq$0.5
Imitation from control to panic	$\gamma_{c \to p}$	$\geq$0.5
Effect of emergency rescuers	κ	1
Effect of massive alert	ρ	0.05 or 0.3

According to the scenario, the following values for the imitation parameters were chosen:

1. Set of parameters 1: $\alpha = \gamma_{p \to c} = \beta = \gamma_{c \to p} = 0.6$.
2. Set of parameters 2: $\alpha = \gamma_{p \to c} = 0.6$; $\beta = \gamma_{c \to p} = 0.62$.

To test the effects of emergency rescuers, the first set of parameters was chosen so that imitation processes are the same between the states of alert, panic and control, whereas the second one promotes slightly the state of panic.

Appendix A.2. Existence and Uniqueness of an Optimal Solution

The Pontryagin Maximum principle permits characterizing the optimal controls [31]. We suppose that there is no victim: that is, $v(t) = 0$ for all $t \geq 0$.

Let $\phi(t)$ represent the second member of (1). The Hamiltonian permits to derive necessary conditions.

$$\begin{aligned} H(t,(a,p,c,q,b),(u_1,u_2),\lambda) = \quad & C_p p + A_1 u_1^2 + A_2 u_2^2 + \lambda_1 \phi_1(t) + \lambda_2 \phi_2(t) + \lambda_3 \phi_3(t) \\ & + \lambda_4 \phi_4(t) + \lambda_5 \phi_5(t) \end{aligned} \tag{A1}$$

where $\lambda = (\lambda_i)_{i=1,\ldots,5} \in \mathbb{R}^5$ is the adjoint variable.

Theorem A1. *Given an optimal control (u_1^*, u_2^*) and the corresponding solution vector $(a^*, p^*, c^*, q^*, b^*)$ that minimizes the objective functional (6), there exist adjoint variables λ_1, λ_2, λ_3, λ_4, λ_5 satisfying*

$$
\begin{cases}
\dot{\lambda}_1 &= (\lambda_1-\lambda_3)(B_1+\rho u_2+\dfrac{\partial \tilde{F}}{\partial a}a(c+\kappa u_1)+\tilde{F}(c+\kappa u_1))+(\lambda_1-\lambda_2)(B_2+\dfrac{\partial G}{\partial a}ap+Gp) \\
\dot{\lambda}_2 &= -C_p+(\lambda_2-\lambda_1)(B_4-\dfrac{\partial G}{\partial p}ap-Ga)+(\lambda_2-\lambda_3)(C_1+\dfrac{\partial \tilde{H}}{\partial p}(c+\kappa u_1)p+\tilde{H}(c+\kappa u_1))), \\
\dot{\lambda}_3 &= (\lambda_3-\lambda_1)(B_3-\dfrac{\partial \tilde{F}}{\partial c}a(c+\kappa u_1)-\tilde{F}a)+(\lambda_3-\lambda_2)(C_2-\rho u_2-\dfrac{\partial \tilde{H}}{\partial c}(c+\kappa u_1)p+\tilde{H}p) \\
 & +(\lambda_3-\lambda_5)\varphi, \\
\dot{\lambda}_4 &= (\lambda_4-\lambda_1)\gamma, \\
\dot{\lambda}_5 &= 0
\end{cases} \tag{A2}
$$

with transversality conditions

$$\lambda_1(T)=0,\ \lambda_2(T)=0,\ \lambda_3(T)=0,\ \lambda_4(T)=0,\ \lambda_5(T)=0. \tag{A3}$$

The optimal control pair is characterized by the piecewise continuous functions

$$
\begin{aligned}
u_1^* &= \max(0,\min(b_1^\theta,S_1)), \\
u_2^* &= \max\left(0,\min\left(\rho\frac{(\lambda_1-\lambda_3)a+(\lambda_2-\lambda_3)c}{2A_2},b_2^\theta\right)\right),
\end{aligned} \tag{A4}
$$

where $b_1^\theta=0.3$, $b_2^\theta=\min((1-B_1)/\rho,C_2/\rho,0.5)$ *and* S_1 *is solution of*

$$2A_1S_1+(\lambda_3-\lambda_1)\frac{\partial(\tilde{F}(a,c,\kappa S_1)a(c+\kappa S_1))}{\partial S_1}+(\lambda_3-\lambda_2)\frac{\partial(\tilde{H}(c,p,\kappa S_1)p(c+\kappa S_1))}{\partial S_1}=0. \tag{A5}$$

Remark A1. b_1^θ *was introduced to take into account that the rescuers must not be more than 30% of the total population. The parameter* b_2^θ *was defined from the constraints on the model parameters. Indeed, in the mathematical model,* $B_1+\rho u_2$ *and* $C_2-\rho u_2$ *represent transition rates. Consequently, they cannot be greater than 1.*

Proof. Relations (A2) and (A3) are a consequence of the Pontryagin Maximum principle and the optimality conditions on the set $\{t\in[0,T]|0<u_1^*(t)<b_1^\theta$ and $0<u_2^*(t)<b_2^\theta\}$. Conditions $\dfrac{\partial H}{\partial u_1}(u_1^*)=\dfrac{\partial H}{\partial u_2}(u_2^*)=0$ are required on the interior of the control set. These last equations lead to

$$\left(2A_1u_1+(\lambda_3-\lambda_1)\frac{\partial(\tilde{F}(a,c,\kappa u_1)a(c+\kappa u_1))}{\partial u_1}+(\lambda_3-\lambda_2)\frac{\partial(\tilde{H}(c,p,\kappa u_1)p(c+\kappa u_1))}{\partial u_1}\right)\Big|_{u_1=u_1^*}=0. \tag{A6}$$

and

$$2A_2u_2^*-\rho(\lambda_1-\lambda_3)a-\rho(\lambda_2-\lambda_3)c=0 \tag{A7}$$

The boundary conditions on $u_1^*(t)$ and $u_2^*(t)$ impose that

$$u_1^*(t)=\begin{cases} 0 & \text{if } S_1<0 \\ b_1^\theta & \text{if } S_1>b_1^\theta \\ S_1 & \text{otherwise} \end{cases}$$

and

$$u_2^*(t)=\begin{cases} 0 & \text{if } S_1<0 \\ b_2^\theta & \text{if } S_2>b_2^\theta \\ S_2 & \text{otherwise} \end{cases}$$

where S_1 is solution of (A5) and $S_2=\dfrac{(\lambda_1-\lambda_3)\rho a+(\lambda_2-\lambda_3)\rho c}{2A_2}$.

Rewriting the expressions of $u_1^*(t)$ and $u_2^*(t)$ gives the theorem result. □

The state system (1) with the initial conditions $x_0 = (0,0,0,1,0)^T$ completed with the adjoint system (A2), the boundary conditions (A3) and the optimal control pair (A4) represent the optimality system to be solved.

Starting from a null control, the resolution of the state system (1) gives a solution that permits solving the adjoint system (A2). Then, from the solutions of the state and adjoint systems, the new control is calculated from (A4) and compared with the previous one. The process is reiterated as soon as the difference of the two controls is as small as desired.

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