Supplementary Materials：Proofs of the Propositions

Lemma 1:

Proof. It is easy to show that the coefficients of Y in e^S and ω^S are positive when $m > \frac{1}{4}\gamma^2$. Moreover, as we assume $m > \frac{1}{3}\gamma^2$, so the coefficients of e^S and ω^S are always positive for the rest of our paper.

Proposition 1:

Proof. Let the retailer's profit under the information sharing case be less than that without information sharing, i.e., $\pi_R^S > \pi_R^N$; it is easy to derive that the condition $m < \frac{1}{2}\gamma^2$ must be satisfied. Similarly, we can derive the conditions where information sharing benefits the manufacturer and supply chain by comparing the manufacturer's and supply chain's profits under the two cases, respectively.

Proposition 2:

Proof. It is easy to prove proposition 2 by comparing the optimal investment level of the manufacturer under the two cases.

Proposition 3:

Proof. As the demand becomes more variable, that is, a larger σ, we can obtain the following results by taking derivations of the retailer's, manufacturer's and supply chain's profits under the information sharing or non-information sharing cases with respect to σ^2, respectively, as follows: $\frac{d\pi_R^S}{d\sigma^2} > 0$; $\frac{d\pi_R^N}{d\sigma^2} > 0$; $\frac{d\pi_M^S}{d\sigma^2} > 0$; $\frac{d\pi_M^N}{d\sigma^2} = 0$; $\frac{d\pi^S}{d\sigma^2} > 0$; $\frac{d\pi^N}{d\sigma^2} > 0$.

Proposition 4:

Proof. (a) When $m < \frac{1}{2}\gamma^2$, we have $\pi_R^S > \pi_R^N$ and $\pi_M^S > \pi_M^N$. Hence, the retailer will share the private information voluntarily without any payment, and $X^* = S$ is the equilibrium decision.

(b) When $\frac{1}{2}\gamma^2 < \mathrm{m} < \frac{(3+\sqrt{5})}{4}\gamma^2$, we have $\pi_R^S < \pi_R^N$, and $\pi_M^S - (\pi_R^N - \pi_R^S) \geq \pi_M^N$, so the manufacturer will offer T=$\pi_R^N - \pi_R^S$ to obtain the retailer's private information, and $X^* = S$ is the equilibrium decision.

(c) When $m > \frac{(3+\sqrt{5})}{4}\gamma^2$, we have $\pi_R^S < \pi_R^N$, $\pi_M^S - \left(\pi_R^N - \pi_R^S\right) < \pi_M^N$. The manufacturer will not pay for the information, and $X^* = N$ is the equilibrium decision.

Lemma 2:

Proof. For Cournot competition, when the information sharing arrangement is (X_1, X_2), we can prove that $q_i^{X_iX_j} = q_i^{X_i}(q_j^{X_jX_i})$ by substituting the equations of $q_j^{X_jX_i}$ into $q_i^{X_i}$, where $q_i^{X_i}(q_j)$ is given by (26) and (30), $X = S$ or N and $i \neq j$, respectively. Similarly, we can verify that $\omega_i^{X_iX_j} = \omega_i^{X_i}(q_j^{X_jX_i})$ and $e_i^{X_iX_j} = e_i^{X_i}(q_j^{X_jX_i})$, where $\omega_i^{X_i}(q_j)$ is given by (24) and (28), and the $e_i^{X_i}(q_j)$ is given by (25) and (29). Thus $(q_i^{X_iX_j},\ \omega_i^{X_iX_j}, e_i^{X_iX_j})$ is an equilibrium. The proof of the uniqueness of the equilibrium is similar to that of Ha et al. (2011) and is omitted.

Lemma 3:

Proof. For part (a), we can verify $C_i^{X_iX_j} < 1$. So, $\alpha_i^{SX_j} > 0$ and $\beta_i^{SX_j} > 0$ if and only if $m > \frac{1}{4}\gamma^2$. Moreover, as we assume that $m > \frac{1}{3}\gamma^2$ to ensure interior equilibrium solutions; in this case, we can say $e_i^{X_iX_j}$ and $\omega_i^{X_iX_j}$ are increasing in Y_i.

For part (b), we can derive the following:

$$C_i^{SS} - C_i^{NS} = \frac{t\sigma^2(1+t\sigma^2)^2(4m-\gamma^2)(\gamma^2-2m)}{[(4+4t\sigma^2+\lambda t\sigma^2)m-(1+t\sigma^2)\gamma^2][[8(1+t\sigma^2)^2-\lambda^2t^2\sigma^4]m-2\gamma^2(1+t\sigma^2)^2]} > 0 \quad \text{when} \quad m < \frac{1}{2}\gamma^2$$

$$C_i^{SN} - C_i^{NN} = \frac{2t\sigma^2(1+t\sigma^2)^2(\gamma^2-2m)}{[(4+4t\sigma^2+\lambda t\sigma^2)m-(1+t\sigma^2)\gamma^2][2(1+t\sigma^2)+\lambda t\sigma^2]} > 0 \text{ when } m < \frac{1}{2}\gamma^2$$

$$C_j^{SS} - C_j^{SN} = \frac{mt^2\sigma^4\lambda(1+t\sigma^2)(2m-\gamma^2)}{[(4+4t\sigma^2+\lambda t\sigma^2)m-(1+t\sigma^2)\gamma^2][[8(1+t\sigma^2)^2-\lambda^2t^2\sigma^4]m-2\gamma^2(1+t\sigma^2)^2]} > 0 \quad \text{when} \quad m \geq \frac{1}{2}\gamma^2$$

$$C_j^{NS} - C_j^{NN} = \frac{t^2\sigma^4\lambda(1+t\sigma^2)(2m-\gamma^2)}{[[8(1+t\sigma^2)^2-\lambda^2t^2\sigma^4]m-2\gamma^2(1+t\sigma^2)^2][2(1+t\sigma^2)+\lambda t\sigma^2]} > 0 \text{ when } m \geq \frac{1}{2}\gamma^2$$

As a result, information sharing in supply chain i makes q_i less responsive to Y_i and makes q_j more responsive to Y_j when $m \geq \frac{1}{2}\gamma^2$.

Proposition 5:

Proof. For part (a), we have the following:

$$\pi_{R_j}^S(C_i^{SS}) - \pi_{R_j}^S(C_i^{NS}) = \frac{m^2t\sigma^4\lambda(C_i^{NS}-C_i^{SS})(2-\lambda C_i^{SS}-\lambda C_i^{NS})}{(4m-\gamma^2)^2(1+t\sigma^2)} \geq 0 \text{ when } m \geq \frac{1}{2}\gamma^2$$

$\pi_{M_j}^S\left(C_i^{SS}\right)-\pi_{M_j}^S\left(C_i^{NS}\right)=\frac{mt\sigma^4\lambda(C_i^{NS}-C_i^{SS})(2-\lambda C_i^{SS}-\lambda C_i^{NS})}{2(4m-\gamma^2)(1+t\sigma^2)}\geq 0$ when $m\geq\frac{1}{2}\gamma^2$

For part (b), we have the following:

$\pi_{R_j}^N\left(C_i^{SN}\right)-\pi_{R_j}^N\left(C_i^{NN}\right)=\frac{t\sigma^4\lambda(C_i^{NN}-C_i^{SN})(2-\lambda C_i^{SN}-\lambda C_i^{NN})}{4(1+t\sigma^2)}\geq 0$ when $m\geq\frac{1}{2}\gamma^2$

$\pi_{M_j}^N\left(C_i^{SN}\right)-\pi_{M_j}^N\left(C_i^{NN}\right)=0$

Proposition 6:

Proof. $\pi_i^S\left(C_j^{X_jN}\right)-\pi_i^N\left(C_j^{X_jN}\right)=\frac{t\sigma^4\left(1-\lambda C_j^{X_jN}\right)^2(6m\gamma^2-4m^2-\gamma^4)}{4(4m-\gamma^2)^2(1+t\sigma^2)}>0$ when $m<\frac{(3+\sqrt{5})}{4}\gamma^2$

$\pi_i^S\left(C_j^{X_jS}\right)-\pi_i^S\left(C_j^{X_jN}\right)=\frac{m(6m-\gamma^2)t\sigma^4\lambda(C_j^{X_jN}-C_j^{X_jS})(2-\lambda C_j^{X_jS}-\lambda C_j^{X_jN})}{2(4m-\gamma^2)^2(1+t\sigma^2)}>0$ when $m<\frac{1}{2}\gamma^2$

Proposition 7:

Proof. For part (a), we can show that $\beta_i^{SX_j}-\beta_i^{NX_j}>0$ and $\beta_i^{SX_j\,2}-\beta_i^{NX_j\,2}>0$ regardless of $X_j=S$ or N; For part (b), we can show that $\Delta E\left[I_i^S\right]-\Delta E\left[I_i^N\right]>0$ when $m<\frac{1}{2}\gamma^2$.

Proposition 8:

Proof. Information sharing benefits supply chain i when $V_i^S>0$, which is equivalent to $g>0$, where

$$g=m^3\frac{t^4\sigma^8\lambda^4}{(1+t\sigma^2)^4}-8m^3\frac{t^2\sigma^4\lambda^2}{(1+t\sigma^2)^2}-2(4m-\gamma^2)(-6m\gamma^2+4m^2+\gamma^4)$$

Let $x=\frac{t^2\sigma^4\lambda^2}{(1+t\sigma^2)^2}$; we have $0<x<1$; rewrite g as $g(x)=m^3x^2-8m^3x-2(4m-\gamma^2)(-6m\gamma^2+4m^2+\gamma^4)$, and $g(x)$ has two roots, as follows:

$$x_1(m)=\frac{1}{m^3}(4m^3+\gamma^2\sqrt{2m^3(6m-\gamma^2)}-2m\sqrt{2m^3(6m-\gamma^2)})$$

$$x_2(m)=\frac{1}{m^3}(4m^3-\gamma^2\sqrt{2m^3(6m-\gamma^2)}+2m\sqrt{2m^3(6m-\gamma^2)})$$

We can prove that when $m>\frac{1}{3}\gamma^2$, $x_2(m)>1$, and $x_1(m)$ is decreasing with m. Therefore, we do not need to consider $x_2(m)$ when $0<x<1$. Given t and λ, we have the unique m^S such that $x_1(m^S)=\frac{t^2\sigma^4\lambda^2}{(1+t\sigma^2)^2}$. We can prove that $g>0$ if and only if $m<m^S$. Since $x_1(m)>1$ when $m<\frac{1}{2}\gamma^2$ and $x_1(m)>0$ when $m>\frac{(3+\sqrt{5})}{4}\gamma^2$, we can obtain that $\frac{1}{2}\gamma^2<m^S<\frac{(3+\sqrt{5})}{4}\gamma^2$. Note that $x_1(m^S)$ is increasing in t, λ and γ (i.e., $\frac{dx_1(m^S)}{dt}=$

$\frac{2t\sigma^4\lambda^2}{(1+t\sigma^2)^3} > 0$, $\frac{dx_1(m^S)}{d\lambda} = \frac{2t^2\sigma^4\lambda}{(1+t\sigma^2)^2} > 0$, $\frac{dx_1(m^S)}{d\gamma} = \frac{1}{(m^S)^3}\left[2\gamma\sqrt{2(m^S)^3(6m^S-\gamma^2)} + (2m - \gamma^2)\frac{2\gamma(m^S)^3}{\sqrt{2(m^S)^3(6m^S-\gamma^2)}}\right] = \frac{2\gamma(14m^S-\gamma^2)}{\sqrt{2(m^S)^3(6m^S-\gamma^2)}} > 0$), $x_1(m)$ is decreasing with m, so m^S is decreasing in t, λ, and increasing in γ.

We can also prove that $V_i^N > 0$ if and only if $h > 0$, where

$$h = m(4m-\gamma^2)\frac{t^4\sigma^8\lambda^4}{(1+t\sigma^2)^4} - 16m^2\frac{t^2\sigma^4\lambda^2}{(1+t\sigma^2)^2} - 8(-6m\gamma^2 + 4m^2 + \gamma^4)$$

Let $x = \frac{t^2\sigma^4\lambda^2}{(1+t\sigma^2)^2}$, and we have $0 < x < 1$; rewrite h as $h(x) = m(4m-\gamma^2)x^2 - 16m^2x - 8(-6m\gamma^2 + 4m^2 + \gamma^4)$, and $h(x)$ has two roots as follows:

$$x_3(m) = \frac{1}{m(4m-\gamma^2)}(8m^2 + 2\gamma^2\sqrt{2m(6m-\gamma^2)} - 4m\sqrt{2m(6m-\gamma^2)})$$

$$x_4(m) = \frac{1}{m(4m-\gamma^2)}(8m^2 - 2\gamma^2\sqrt{2m(6m-\gamma^2)} + 4m\sqrt{2m(6m-\gamma^2)})$$

We can prove that when $m > \frac{1}{3}\gamma^2$, $x_4(m) > 1$ and $x_3(m)$ is decreasing with m. Therefore, we do not need to consider $x_4(m)$ when $0 < x < 1$. Given t and λ, we have the unique m^N such that $x_3(m^N) = \frac{t^2\sigma^4\lambda^2}{(1+t\sigma^2)^2}$. We can prove that $h > 0$ if and only if $m < m^N$. Since $x_3(m) > 1$ when $m < \frac{1}{2}\gamma^2$ and $x_3(m) > 0$ when $m > \frac{(3+\sqrt{5})}{4}\gamma^2$. We can obtain that $\frac{1}{2}\gamma^2 < m^N < \frac{(3+\sqrt{5})}{4}\gamma^2$. Note that $x_3(m)$ is increasing in t, λ and γ (i.e., $\frac{dx_3(m^N)}{dt} = \frac{2t\sigma^4\lambda^2}{(1+t\sigma^2)^3} > 0$, $\frac{dx_3(m^N)}{d\lambda} = \frac{2t^2\sigma^4\lambda}{(1+t\sigma^2)^2} > 0$, $\frac{dx_3(m^N)}{d\gamma} = \frac{8m^N\gamma\sqrt{2m^N(6m^N-\gamma^2)}}{m^N(4m^N-\gamma^2)^2} + \frac{(4m^N-2\gamma^2)2m^N\gamma}{m^N(4m^N-\gamma^2)\sqrt{2m^N(6m^N-\gamma^2)}} = \frac{4\gamma[32(m^N)^2-10m^N\gamma^2+\gamma^4]}{(4m^N-\gamma^2)^2\sqrt{2m^N(6m^N-\gamma^2)}} > 0$), $x_3(m)$ is decreasing with m. Therefore, m^N is decreasing in t, λ, and increasing in γ. In addition, $m^N < m^S$ because $x_1(m) > x_3(m)$ if $\frac{1}{2}\gamma^2 < m < \frac{(3+\sqrt{5})}{4}\gamma^2$.

Proposition 9:

Proof. For part (a), we divide the expression $\pi_{R_i}^{SX_j} - \pi_{R_i}^{NX_j}$ into two parts, i.e., $\pi_{R_i}^S\left(C_j^{X_jN}\right) - \pi_{R_i}^N\left(C_j^{X_jN}\right)$ and $\pi_{R_i}^S\left(C_j^{X_jS}\right) - \pi_{R_i}^S\left(C_j^{X_jN}\right)$, that is $\pi_{R_i}^{SX_j} - \pi_{R_i}^{NX_j} = \pi_{R_i}^S\left(C_j^{X_jN}\right) - \pi_{R_i}^N\left(C_j^{X_jN}\right) + \pi_{R_i}^S\left(C_j^{X_jS}\right) - \pi_{R_i}^S\left(C_j^{X_jN}\right)$. We find that $\pi_{R_i}^S\left(C_j^{X_jN}\right) - \pi_{R_i}^N\left(C_j^{X_jN}\right) =$

$\frac{t\sigma^4\left(1-\lambda C_j^{X_jN}\right)^2(8m\gamma^2-8m^2-\gamma^4)}{4(4m-\gamma^2)^2(1+t\sigma^2)} \geq 0$ when $m \leq \frac{(2+\sqrt{2})}{4}\gamma^2$, and $\pi_{R_i}^S\left(C_j^{X_jS}\right) - \pi_{R_i}^S\left(C_j^{X_jN}\right) = \frac{m^2 t\sigma^4\lambda\left(C_j^{X_jN}-C_j^{X_jS}\right)\left(2-\lambda C_j^{X_jS}-\lambda C_j^{X_jN}\right)}{(4m-\gamma^2)^2(1+t\sigma^2)} \geq 0$ when $m \leq \frac{1}{2}\gamma^2$. Therefore, we can prove that $\pi_{R_i}^{NX_j} \leq \pi_{R_i}^{SX_j}$ $(i = 1, 2)$ when $m \leq \frac{1}{2}\gamma^2$, so retailer i will share the information with manufacturer i without any payment, and the (S, S) is the unique equilibrium; For part (b), when $\frac{1}{2}\gamma^2 < m < m^N$, we can obtain both that $V_i^S > 0$ and $V_i^N > 0$ for $i = 1,\ 2$, so S is the dominant strategy for both manufacturers and (S, S) is the unique equilibrium; For part (c), when $m^N < m < m^S$, we can obtain that $V_i^N < 0$ and $V_i^S > 0$ for $i = 1, 2$, so (N, N) and (S, S) are possible equilibria. However, we can show that $\pi_{M_i}^{NS} + V_i^S > \pi_{M_i}^{NN}$ for $i = 1, 2$. Hence, (S, S) is optimal; For part (d), when $m > m^S$, we can show that $V_i^S < 0$ and $V_i^N < 0$. Therefore, (N, N) is the unique equilibrium.