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A Heuristic Algorithm Based on Travel Demand for Transit Network Design

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Abstract: This study proposes a simultaneous optimization model that considers flow assignment and vehicle capacity for the problem of transit network design to determine the route structure and frequencies simultaneously. The problem is focused on reducing the total travel time and the number of transfers. A heuristic algorithm is developed to solve this problem. In the proposed algorithm, the initial routes are generated according to a changing demand matrix, which can reflect the real-time demand with transfers and ensure that the direction of route generation maximizes the percentage of direct service. A regulating method for a sequence of stops is used during route generation to guarantee the shortest trip time for a formed route. Vehicles are allocated to each route according to the flow share. The concept of vehicle difference is introduced to evaluate the distinction between actual allocated vehicles and required vehicles for each route. The optimization process of frequencies based on vehicle difference can ensure that the solution meets the constraints. Two scale networks are used to illustrate the performances of the proposed method. Results show that route structure and frequencies can be optimized simultaneously through the proposed method. Different scenarios are created to test the algorithm properties via various parameter values. The test result indicates that the upper bound is a key parameter to balance the proportion of direct service and average in-vehicle travel time (AIVTT), and the increased number of planning routes can improve the proportion of direct service.

Keywords: heuristics; transportation; transit network design; simultaneous optimization

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1. Introduction

Transit network design has received major attention because of its significance in alleviating urban traffic congestion and air pollution. Public transport plays a key role in urban resident travel. For example, in Chengdu, China, over 3 million travelers complete their trips by bus, suggesting the necessity of a well-designed transit network for urban development. Studies on public transit service design have mainly focused on route structure design [1–7], frequency determination [8–11], timetable setting [12–15], vehicle scheduling [16–19], crew scheduling [20], fare policy [21,22], and data mining [23–29], or a combination of these topics [30–33].

Previous studies about transit network design can be classified into two groups: optimizing the route structure and service frequencies separately [2,34–36] and optimizing the route structure and service frequencies simultaneously [30–33,37]. For example, Kılıç and Gök et al. [2] focused on route structure optimization, whereas Huang et al. [10] concentrated on frequency setting based on an existing transit network. Combining these works, Szeto and Jiang et al. [31] simultaneously optimized the route structure and frequency setting.

The optimization objective of this problem is associated not only with transit route structure but also with corresponding service frequencies. Transit route structure and corresponding service frequencies determine flow assignment, and whether service capacity can meet travel demand is also closely related to both. These factors indicate the significance of simultaneously determining the route structure and corresponding service frequencies.

The transfer issue cannot be ignored in the optimization objective. Passengers always complain that no direct service exists to complete their travels because of the discomfort and inconvenience caused by transfers [30]. Some passengers even find alternative travel modes, such as car, taxi, and motorcycle, when the number of transfers exceeds their expectations. Baaj and Mahmassani [38] considered the existence of transfers but ignored the number of transfers in their proposed network design method. Kılıç and Gök et al. [2] introduced the penalty of time cost for each transfer to reduce the number of transfers. A similar method was used by Szeto and Wu [32], who emphasized the importance of improving the direct service when the transit service is designed.

In addition to the transfer issue, the assignment problem among different routes occurs when more than one route can serve passengers. Two flow types must be assigned during the design process, including the flow on familiar routes and the flow on multiple routes. Common routes indicate more than one route serving the same pair of stops with the same path, whereas multiple routes suggest more than one route serving the same pair of stops with different paths. Different network representations, such as hyperpath graph representation and route-section representation, are applied to deal with the flow assignment among competition routes [39–43]. Hyperpath graph representation highlights the attractive routes, but considerable dummy boarding and alighting stops must be added, complicating the network. Route-section representation can reduce the number of links, simplify the network complexity, and ease the handling of the assignment problem. However, the attractive routes should be determined in advance when the route-section representation is used.

In-vehicle congestion issue is another factor for determining whether the optimized results can be applied to practice. To simplify the transit network design problem, some studies assumed that the bus load is unlimited [2,44,45]; however, the optimized results are challenging to apply to actual planning. Two modeling streams, namely, capacity constraint and congestion cost function, are adopted to improve the practical performances of the optimized results. The capacity constraint is set to avoid the flow on the link of routes exceeding the maximum capacity, and the congestion cost function is a penalty function to decrease the congestion on vehicles. Capacity constraint is more realistic to ensure that capacity meets travel demand. Nevertheless, we cannot obtain a feasible solution when the total capacity is insufficient.

Newell and Salzborn et al. [18,46] used an analytical method to solve single-route design and optimization, but the method was difficult to apply to the network problem. Metaheuristics, such as genetic algorithm [7,19,44], simulated annealing [47–49], and artificial bee colony [31,32,50], are popular for solving the problem.

A heuristic algorithm is accordingly developed in the current study. Routes are generated according to a changing demand matrix, which can maximize the direct service. The sequence of stops for each route is regulated to ensure the shortest path for the formed route. The initial frequencies are set in terms of the flow share, and a neighborhood search heuristic is proposed to tackle the frequency optimization. In the frequency-determining process, a concept of vehicle difference is introduced to ensure the solution satisfies the capacity constraint.

The main contributions of this study are as follows:

(1) A simultaneous optimization model is formulated for the transit network structure and frequency problem considering flow assignment and vehicle capacity.
(2) A heuristic algorithm is developed to solve the transit network design problem, in which routes are generated based on accumulated flow, and the frequency for each route is set according to flow share. A new concept of vehicle difference is also introduced to reflect the difference between actually assigned vehicles and required vehicles for guiding the process of regulating frequencies.

(3) The model and algorithm are applied to different scale networks. The properties of the proposed model and algorithm are examined, and the different performances for various scenarios through different parameter values are discussed.

The remainder of this paper proceeds as follows. Section 2 presents the simultaneous optimization model, following the notations and assumptions used in this study. Section 3 describes the proposed algorithm procedure and numerical experiments. Section 4 discusses the performance evaluation. Finally, Section 5 concludes the study and discusses future research.

2. Problem Formulation

2.1. Assumptions

Several classical assumptions are made for the model setting according to previous studies on the transit network problem as follows: (i) Passengers arrive at stops randomly. (ii) Passengers select a route from attractive routes to their destinations, and we assume that all routes that pass through an origin and destination (OD) pair are the attractive routes for the OD pair to simplify the problem. (iii) Passengers prefer selecting a path without transfer, and they board the first available bus to their destinations. (iv) The headways of vehicles follow the exponential distribution. (v) Travel demand is predefined and fixed during the planning period. (vi) For simplicity, the capacity of each vehicle is the same.

2.2. Model Setting

2.2.1. Objective Function

As in the literature [32], the objective function is formulated in terms of the number of passengers without direct service and the total travel time for the passengers with direct service.

$$\min_{x\,f} z = w_1 \sum_{i\in N, j\in N, i\neq j} d_{ij} M_{ij} + w_2 \sum_{i\in N, j\in N, i\neq j} d_{ij} t_{ij} \left(1 - M_{ij}\right) \tag{1}$$

Equation (1) is the representation of the objective function, which is the weighted sum of the passengers without direct service and the total travel time for the passengers with direct service.

2.2.2. Constraints

(i) Calculations of variables

The calculations of some variables for the objective function are as follows:

$$t_{ij} = t_{ij}^{w} + t_{ij}^{v} \quad i,j \in N \tag{2}$$

$$t_{ij}^{w} = 0.5 \cdot \frac{1}{\sum_{r\in R_{ij}} f_r} \quad i,j \in N \tag{3}$$

$$t_{ij}^{v} = \sum_{r\in s_{ij}} \delta_{ij}^{r} t_{ij}^{rv} \tag{4}$$

$$t_{ij}^{rv} = x_{ij}^{r} c_{ij} + \sum_{i,j,k\in N} x_{ik}^{r} th_{kj}^{r} \left(t_{kj}^{r} + c_{ik} + s\right) \quad i,j \in N_r, r \in R_{ij} \tag{5}$$

$$M_{ij} = \prod_{r=1}^{num} \left(1 - th_{ij}^{r}\right) \quad i,j \in N \tag{6}$$

$$th_{ij}^{r} = x_{ij}^{r} + \sum_{i\in N, j\in N, k\in N, i\neq j\neq k} x_{ik}^{r} th_{kj}^{r} \quad r = 1 \textit{ to } num \tag{7}$$

Constraint (2) is the calculation of t_{ij}, which is the sum of AIVTT and waiting time from stop i to stop j, and constraints (3) and (4) are the calculations of waiting time and AIVTT from stop i to stop j, respectively. Constraint (5) is the calculation of in-vehicle travel time from stop i to stop j through route r. Constraint (6) is the calculation of M_{ij}, which estimates the existence of a direct service for passengers from stop i to stop j. The value of M_{ij} is 0 if at least one route passes through stop i and stop j simultaneously, and 1 otherwise. Constraint (7) is the calculation of th_{ij}^r, which estimates whether route r can provide the direct service for the demand from stop i to stop j. The value of th_{ij}^r is 1 if the direct service can be supplied by route r, and 0 otherwise.

(ii) Constraints for route structure

The constraints for route structure are as follows:

$$\sum_{i \in N \cup o, i \neq j} x_{ij}^r - \sum_{i \in N \cup o, i \neq j} x_{ji}^r = 0 \quad j \in N,\ r = 1 \ to \ num \tag{8}$$

$$\sum_{i \in N \cup o, i \neq j} x_{ij}^r \leq 1 \quad j \in N,\ r = 1 \ to \ num \tag{9}$$

$$\sum_{j \in N \cup o, j \neq i} x_{ij}^r \leq 1 \quad j \in N,\ r = 1 \ to \ num \tag{10}$$

$$\sum_{i \in N, j \in N, i \neq j} x_{ij}^r \leq S_{max} \quad r = 1 \ to \ num \tag{11}$$

Constraint (8) ensures that each stop on a route, excluding the dummy stop, has preceding and following stops. Constraints (9) and (10) guarantee that any stop in the network can be visited by a determined route at most once. Constraint (11) limits the number of stops for a route.

(iii) Constraints for frequencies

$$\sum_{r \in R} \frac{2 f_r t_r}{\eta} \leq n_{fleet} \tag{12}$$

$$t_r = \sum_{i,j \in N_r} x_{ij}^r (c_{ij} + s) - s \tag{13}$$

Constraint (12) describes that the fleet size of the determined routes cannot exceed the given maximum fleet size. Constraint (13) is the calculation of single-trip time of route r.

(iv) Constraints for capacity

$$CS_{i(i+1)}^r \leq f_r Cap \quad i \in (N_r^m \cup N_r^o),\ r \in R \tag{14}$$

$$CS_{i(i+1)}^r = \begin{cases} board_i^r & i \in N_r^o \\ CS_{(i-1)i}^r + board_i^r - alight_i^r & i \in N_r^m \end{cases} \tag{15}$$

Constraint (14) depicts the flow in the segment from stop i to stop $i+1$ for route r per unit time. The flow in each segment can be calculated by constraint (15).

(v) Constraint of flow assignment

The flow assignment is based on the method proposed by De Cea and Szeto et al. [32,40]. A normal network is first converted into a route-section network. Different routes serving the same pair of stops, called attractive routes, are then aggregated as one section. Competition among different routes can be handled by this method. Flow assignment can be described by

$$d_{ij}^r = \frac{f_r}{\sum_{r' \in R_{ij}} f_{r'}} \quad r, r' \in R \ and \ i, j \in N \tag{16}$$

Equation (16) is the expression of the assignment to each route from the flow between each demand pair.

3. Solution

3.1. General Scheme of the Proposed Algorithm

The general scheme of the proposed algorithm is presented in Figure 1. The steps of this algorithm can be described as follows.

(a) The parameters, including those for the planning area, such as number of stops, number of routes, maximum available fleet size, bus capacity, maximum number of stops for a planning route, shortest path matrix, travel demand matrix, and those for the algorithm, such as stopping criteria, are initialized.
(b) The initial route structure is generated according to the predefined parameters.
(c) The vehicles for each determined route are allocated with the limit of maximum available fleet size.
(d) After determining the route structure and corresponding frequencies, the travel demand can be assigned to each route in terms of the principle presented by the flow assignment.
(e) The efficiency of the solution to meet the capacity constraint (14) is evaluated; if the answer is yes, the next step is performed; otherwise, the vehicles are regulated among routes until the solution satisfies the constraint.
(f) The objective function value for the feasible solution is calculated according to Equation (1).
(g) The frequencies of the determined transit network are optimized.
(h) The stop criterion is checked; if the algorithm meets the stop criterion (the iteration reaches 500 generations), then the algorithm procedure is terminated, and the best solution is the output; otherwise, the next step is performed.
(i) The route structure is optimized by adding and deleting stops in the routes.

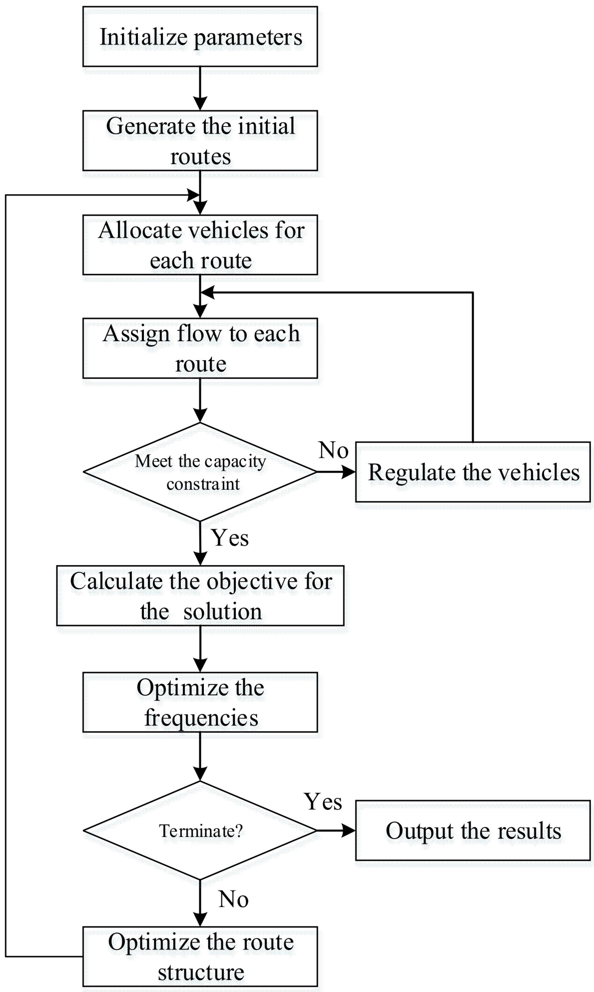


Figure 1. Flowchart of the proposed algorithm.

After the general scheme for the proposed algorithm is introduced, the procedure for each step is elucidated.

3.2. Procedure of Initial Route Generation

The initial route generation is completed in two steps: stop selection for unformed routes and insertion of selected stops into unformed routes.

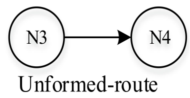
3.2.1. Stop Selection

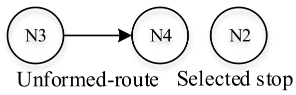
Each stop is selected to maximize the direct service during the procedure of route generation. A travel demand matrix is changed continuously to reflect the demand without direct service during the stop selection procedure; that is, a new stop is selected according to the changing matrix.

Table 1 presents the process of stop selection and the corresponding changes for the travel demand matrix by using four stops that are defined as N1–N4 ∈ N. A pair of stops is first determined to maximize the passengers without transfer. N3 and N4 become unformed routes, and the demand matrix is changed from Table 1a to Table 1b. The stop N2 is then selected to maximize the direct service for the current demand matrix, and the changing matrix is changed from Table 1b to Table 1c.

Table 1. Illustration of changing demand matrix: (**a**) Original demand matrix; (**b**) Changing matrix; (**c**) Changing matrix.

(a)					**(b)**					**(c)**				
Stops	**N1**	**N2**	**N3**	**N4**	**Stops**	**N1**	**N2**	**N3**	**N4**	**Stops**	**N1**	**N2**	**N3**	**N4**
N1	0	1	2	3	N1	0	1	2	3	N1	0	1	2	3
N2	1	0	4	5	N2	1	0	4	5	N2	1	0	0	0
N3	2	4	0	6	N3	2	4	0	0	N3	2	0	0	0
N4	3	5	6	0	N4	3	5	0	0	N4	3	0	0	0





3.2.2. Combination and Sequence

A descent search heuristic is used for each selected stop to obtain the improved combination of a new stop and the unformed route. This process aims to optimize the sequence of stops for the minimum trip time, which does not depend on frequencies. The steps are outlined as follows:

For each selected stop
Combine the selected stop with an unformed route to form a new unformed route R_n, set the selected stop as the first stop for the new unformed route, and set R_n as the optimal structure R_n^o.
Calculate the length of R_n^o and define the length as Le_s
Calculate the number of stops of the unformed route N_u
Set $i' = 1$
While $i' \leq N_u$
$i' = i' + 1$
Change the sequence of R_n^o and set the selected stop as the i'-th stop in the new unformed route to form a different new unformed route R_n'
Calculate the length of R_n' and define the length as Le
If $Le < Le_s$
Make R_n' as R_n^o and Le as Le_s
endif
endwhile
output the optimal structure R_n^o and the shortest path Le_s for the new combined unformed route
Next selected stop

Until the number of stops meets the predefined number, an unformed route becomes a formed route, which indicates that the route generation is complete.

Different from routes generated randomly, the initial routes obtained by the above-proposed procedure can improve the direct service and minimize the trip time for the determined stops.

3.3. Allocating Vehicles for the Initial Routes

Vehicles are allocated to the designed routes according to the flow share, which can be mathematically expressed as follows:

$$n_{veh}^{r} = \lceil \frac{\sum_{i \in N_r, j \in N_r} d_{ij}}{\sum_{i \in N, j \in N} d_{ij}} \cdot n_{fleet} \rceil \quad r \in R. \tag{17}$$

The vehicles for each route accord to the flow share, and the rule of round down is used to make the number of vehicles an integer. The total vehicles assigned to routes are less than the predefined available fleet size because the integral principle is round down. Therefore, the unassigned vehicles are allocated randomly to routes.

3.4. Flow Assignment

The flow assignment is based on the principle described by the constraint of flow assignment in Section 2. The flow is assigned to the attractive routes according to the frequency share.

3.5. Judgment of Constraints

After determining the travel demand of each route, a vehicle difference is introduced to evaluate the difference between the actual allocated vehicles and the required vehicles to meet the capacity constraint (14).

$$VD_r = n_{veh}^{r} - \lceil \frac{\max(CS_{i(i+1)}^{r})}{d_o} \rceil \quad r \in R \; and \; i \in (N_r^o \cup N_r^m) \tag{18}$$

The positive value of VD_r suggests that the vehicles for the route r are sufficient to meet the capacity constraint (14), whereas the negative value of VD_r indicates that the capacity constraint (14) is not satisfied.

3.6. Vehicle Regulation

Vehicle regulation is necessary to change the vehicles of routes when the capacity constraint (14) is violated. The regulating procedure is outlined as follows:

```
For each route
if the VD_r for route r is negative
define a new variable TD = 0
while TD ≤ 0
r1 with maximum VD_r1
TD as sum of VD_r and VD_r1
if TD > 0
move −VD_r + 1 vehicles from route r1 to route r and update −VD_r for all routes
else
move VD_r1 vehicles from route r1 to route r and update −VD_r for all routes
endif
endwhile
endif
next route in the proposed solution
```

The virtue of using VD_r to determine the number of vehicles to move among routes is that VD_r can reflect the difference between actual allocated vehicles and required vehicles. Thus, the number of vehicles for moving directly can be determined.

In the proposed vehicle regulation, we will add $-VD_r + 1$ vehicles to the route r if the value of VD_r is negative, which is one more vehicle than required. The constraint will be met by adding $-VD_r$ vehicles if the assignment to route r is invariable. However, the frequency of route r increases with additional vehicles, thereby increasing the assignment to the route r. In this condition, the addition of $-VD_r$ vehicles will violate the constraint with the increasing flow assignment.

3.7. Objective Calculation

The objective function value is calculated via Equation (1), following the step of vehicle regulation.

3.8. Frequency Optimization

The frequency optimization procedure is developed according to the method proposed by Szeto and Wu (2011), in which the vehicles are moved one by one among routes. The operation of the vehicle movement will be accepted if the solution becomes more optimal, whereas the operation will be abandoned if the solution worsens or is unchanged.

3.9. Termination

Two stopping criteria are used in this algorithm.

Criterion (i): the tolerance of objective values during successive iterations < preset tolerance

Criterion (ii): iteration > set stall iteration

Criterion (i) is used to stop the algorithm when the objective values for the successive several iterations are sufficiently close, which implies that the latest solution is probably optimal.

Criterion (ii) is used to stop the algorithm when the iterations reach the preset stall iteration. Criterion (ii) is a complement for criterion (i), which avoids long computation time.

3.10. Route Structure Optimization

Different neighborhood search operations, namely, (a) stop removal and (b) stop insertion, are developed to generate neighbor solutions for the route structure optimization.

3.10.1. Operation of Stop Removal

The operation of stop removal is used to delete some stops from the determined route structure in order to search for a more appropriate transit network. Stops from the routes are randomly selected, and new transit networks are obtained by removing the selected stops. A new route structure will replace the original transit network if the objective is more optimal than the old one. Otherwise, the new route structure will be abandoned.

3.10.2. Operation of Stop Insertion

Different from the operation of removing a stop based on random rules, stop insertion aims to decrease the travel demand without a direct service, which is similar to route generation. Each inserted stop is imported to the shortest route with consideration of constraint (11). The new route structure will be abandoned if the performance of the transit network service cannot be improved.

4. Numerical Examples

Two scale networks are used to evaluate the properties of the proposed method through analyzing the results.

4.1. *Small-Scale Network*

4.1.1. Data

A small-scale network composed of 15 stops and the corresponding travel demand are presented in Figure 2 and Table 2 respectively. This classic case has been used by many researchers [2,4,37,51] to examine the attributes of the proposed model and algorithm for the transit network problem.

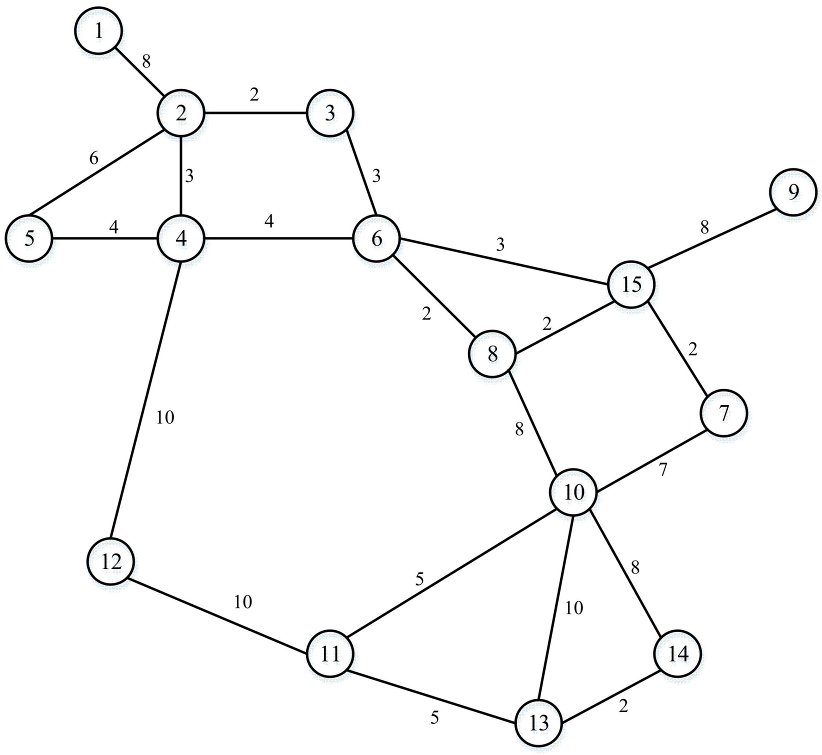


Figure 2. Small-scale network.

Table 2. Travel demand of the small-scale network.

Stops	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	400	200	60	80	150	75	75	30	160	30	25	35	0	0
2	400	0	50	120	20	180	90	90	15	130	20	10	10	5	0
3	200	50	0	40	60	180	90	90	15	45	20	10	10	5	0
4	60	120	40	0	50	100	50	50	15	240	40	25	10	5	0
5	80	20	60	50	0	50	25	25	10	120	20	15	5	0	
6	150	180	180	100	50	0	100	100	30	880	60	15	15	10	0
7	75	90	90	50	25	100	0	50	15	440	35	10	10	5	0
8	75	90	90	50	25	100	50	0	15	440	35	10	10	5	0
9	30	15	15	15	10	30	15	15	0	140	20	5	0	0	0
10	160	130	45	240	120	880	440	440	140	0	600	250	500	200	0
11	30	20	20	40	20	60	35	35	20	600	0	75	95	15	0
12	25	10	10	25	15	15	10	10	5	250	75	0	70	0	0
13	35	10	10	10	5	15	10	10	0	500	95	70	0	45	0
14	0	5	5	5	0	10	5	5	0	200	15	0	45	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

4.1.2. Parameters

The planning route number is set to 4, the maximum bus load is 100, the average dwell time at a stop is 1.5 min, the number of stops in a route ranges from 4 to 8, and the fleet size is 60. Criterion (i) sets the tolerance of successive 10 generations as 0.01; criterion (ii) sets 1000 generations as the stall iteration.

4.1.3. Results

The optimized results are presented in Table 3. Columns 2 and 3 are the route structure and the vehicles for each route, respectively. The trip time and headway of each route are calculated according to the route structure and the vehicle assignment.

Table 3. Optimized route structure.

Route Label	Route Structure	Vehicles	Trip Time	Headway
1	13-11-10-7-6	17	60	3.53
2	10-8-6-3-4-2-1	22	82	3.73
3	1-5-12-11-10-14-13	15	130	8.67
4	11-7-8-3-2-4-5	6	84	14.00

The proportion of direct service can reach 92.42%. The value of the optimized objective function value is 333,000. In Table 3. the route structure indicates the stop sequence of a route. Each bus line is connected according to the shortest path in the network, the path 13-11-10-7-6 is the shortest path from 13 to 11, the shortest path from 11 to 10, the shortest path from 10 to 7, and the shortest path from 7 to 6, thus forming 13-11-10-7-6 bus line.

4.2. Medium-Scale Network

4.2.1. Data

A medium-scale network with 127 stops is shown in Figure 3 to evaluate the properties of the proposed method for the transit network problem [2,52]. The travel demand is different from the original data and is 5% of the data because the planning time is an hour.

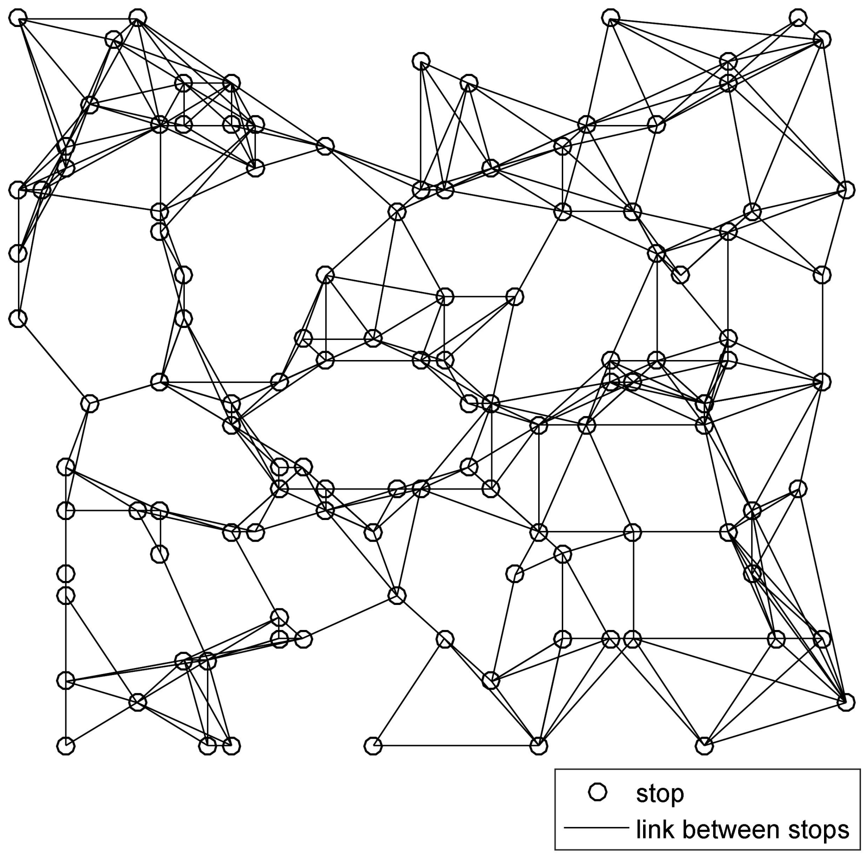


Figure 3. Medium-scale network.

4.2.2. Parameters

The planning number of routes is set to 60, the maximum bus load is 100, the average dwell time at a stop is 1.5 min, the number of stops in a route ranges from 12 to 25, and the fleet size is 900. Criterion (i) sets the tolerance of successive 10 generations as 0.01; criterion (ii) sets 5000 generations as the stall iteration.

4.3. Analysis of the Proposed Algorithm

4.3.1. Initial Route Structure

The proposed generation is compared with the random generation in Table 4 to illustrate the properties of the proposed initial route structure. The proposed generation procedure can achieve a higher proportion of direct service with lesser total trip time than random generation.

Table 4. Comparison of initial route structure.

Methods of Generation	Proportion of Direct Service	Total Trip Time
Random generation	70.26%	27,695
Proposed generation	94.86%	11,919

4.3.2. Initial Frequencies

In Table 5, the initial frequency is compared with the optimized frequency in a determined route structure to test the properties of the proposed initial frequency setting. The initial frequency is close to the optimized frequency for a determined route.

Table 5. Comparison of initial and optimized frequencies.

	Initial Frequencies Scenario	Optimized Frequencies Scenario
Objectives value (mins)	33,250,015.89	33,153,646.8

4.3.3. Computation Time

In the proposed algorithm, a bus difference is introduced to guide the vehicle regulation, in which the precise number of regulated vehicles can be determined directly. Otherwise, the vehicles need to move one by one to avoid violating the capacity constraint. Two routes are selected to illustrate the computation time with and without bus difference in Table 6. The computation time can be decreased by 35.7% with the application of bus difference.

Table 6. Comparison of computation time with and without bus difference.

	With Bus Difference	Without Bus Difference
Computation time (s)	0.09	0.14

4.3.4. Comparison with Genetic Algorithm

In order to evaluate the proposed algorithm, we compare the computation times and the objective values for the genetic algorithm and the proposed one in Table 7 using the small-scale network. It is observed the proposed algorithm can obtain a better solution with less computation time, which indicates the efficiency of the proposed algorithm.

Table 7. Comparison of computation time with and without bus difference.

	Genetic Algorithm	Proposed Alogrithm
Computation time (s)	150 s	90 s
Objective values (min)	353,000	333,000

4.4. Analysis of Results

The performances of the proposed algorithm to solve the transit network problem are summarized as follows:

(i) The proportion of direct service is high because the route generation is based on the changing matrix of travel demand.
(ii) The route is the shortest path because of the sequential optimization for route structure.
(iii) The initial frequency setting is close to the optimal solution for the determined transit network by the principle of vehicle assignment, which is based on the flow share.
(iv) Computation time can be saved by setting a bus difference to determine the direction and step size and reassigning the vehicles to routes.
(v) The proportion cannot be increased to 100% because of the following reasons: (1) the number of routes is insufficient to accomplish the goal, and (2) the distances are too long and the flow is limited for some pairs of OD, leading to inefficiency in the design of direct service for these OD.

4.5. Analysis of Parameters

Route generation is based on a changing demand matrix, which can reflect the demand without direct service. A pair of OD will be 0 on the changing matrix when direct service is developed during the route generation. We only focus on the existence of direct service and ignore the in-vehicle travel time between the formed path and the shortest path, which will cause long in-vehicle travel for some passengers in spite of the short distances between origins and destinations. An upper bound is set to adjust the changing rule for the changing demand matrix to solve the problem. The changing demand matrix for a pair of OD will be unaltered if the in-vehicle travel time exceeds the set upper bound. Although direct service is available, the route generation will regard the pair of OD as a direct service OD. The upper bound can be mathematically expressed by

$$UB_{ij} = \gamma \cdot t_{ij}^{short} \quad i, j \in N \tag{19}$$

where UB_{ij} is the upper bound for passengers from stop i to stop j, t_{ij}^{short} is the in-vehicle travel time from stop i to stop j by the shortest path, and γ is the coefficient for UB_{ij}.

An interesting phenomenon is that the in-vehicle travel time may be saved when UB_{ij} is close to the shortest path. However, saving is obtained at the cost of reducing the proportion of direct service. Figure 4 depicts the proportion of direct service for different UB_{ij} in different scale networks. Figure 5 presents the corresponding AIVTT for the passengers with a direct service.

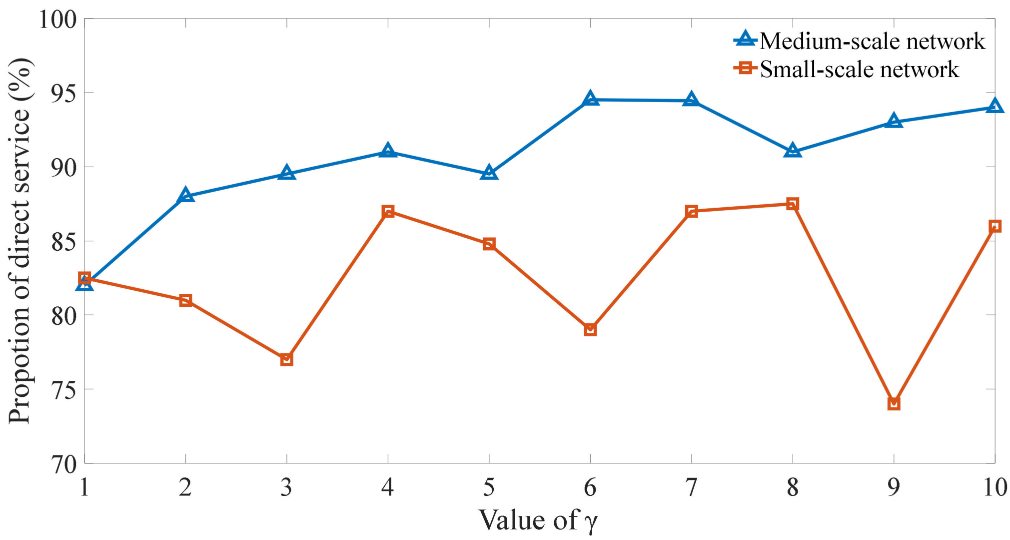


Figure 4. Proportion of direct service for different UB_{ij} in different scale networks.

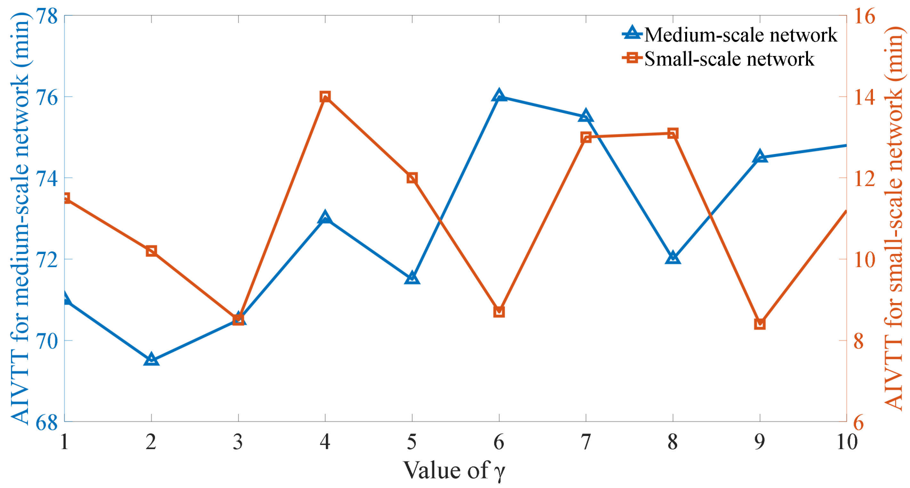


Figure 5. AIVTT for the passengers with direct service.

Figure 4 indicates that the performances of setting UB_{ij} are different for different scale networks. In the medium-scale network, the percentage of direct service exhibits an increasing tendency when γ ranges from 1 to 4, whereas fluctuation occurs in the interval from 5 to 10. In the small-scale network, the percentage keeps fluctuating in the entire value range. Figure 5 shows a similar shape of the AIVTT with direct service, suggesting a contradiction to optimize the percentage of direct service and in-vehicle travel time.

The results indicate that UB_{ij} actively balances the in-vehicle travel time and the percentage of direct service for a medium-scale network that ranges from 1 to 4 of γ. UB_{ij} has no effect when the value of γ is more than four. The actual in-vehicle travel time in the determined route structure is rarely greater than UB_{ij}, because UB_{ij} is too large to affect the route structure. For the small-scale network, the feasible region of solutions is small, causing the in-vehicle travel time route structure to be close to the time on the shortest path.

Possible solutions to regulate the contradiction between the proportion of direct service and the AIVTT include expanding the fleet size and increasing the number of routes. Figure 6 presents the relationship between the percentage of direct service and the number of planning routes with the same other parameters for the medium-scale network. The small-scale network has a similar property throughout our experiment. The transit network can offer efficient service by expanding the predefined number of routes. However, an additional subsidy is required.

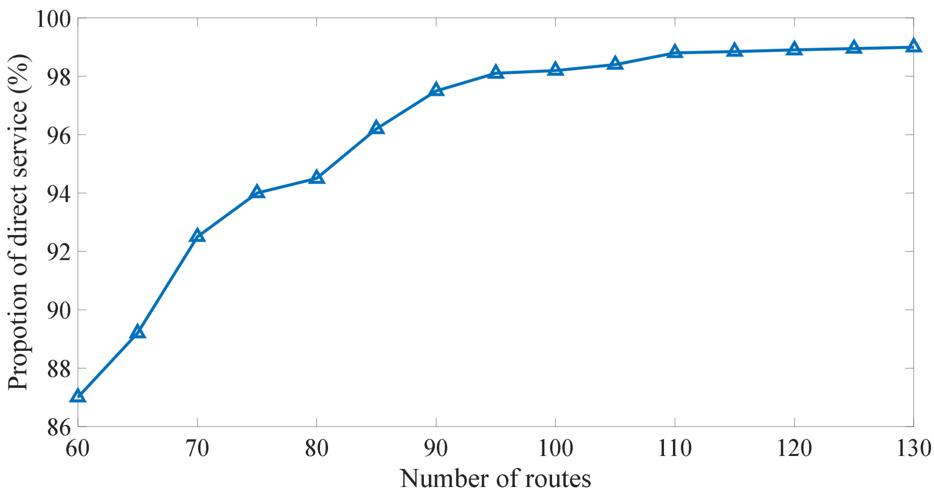


Figure 6. Proportions of direct service for different numbers of planning routes.

5. Conclusions

In this study, a model and a heuristic algorithm are developed to optimize the route structure and its corresponding frequencies simultaneously. Flow assignment and bus capacity are considered. Two scale networks are introduced to examine the properties of the proposed heuristic algorithm. The significant conclusions are as follows:

(i) The proposed model and heuristic algorithm can simultaneously optimize the route structure and its corresponding frequencies with minimum travel time and transfer for different scale networks.
(ii) Although the neighborhood search method is used to improve the route structure with the predefined number of routes, the proportion of direct service cannot reach 100%. An absolute direct service for all travel demands is consequently unreasonable to pursue. Providing direct service for the OD pairs with long distances and limited flow is inefficient and costly when the resource is limited.
(iii) The introduction of an upper bound can improve the algorithm. For a medium-scale network, the upper bound is a useful parameter to balance the two contradictory sub-objectives in the objective function. However, a small-scale network has a minimal response to the parameter, thus suggesting that different performances may appear with the same model and algorithm.
(iv) The proportion of direct service is not only sensitive to the upper bound but is also related to the number of routes. Direct service can be improved by increasing the number of routes if the subsidy is sufficient.
(v) This algorithm currently tests an ideal hypothetical network. In the actual network, the algorithm can be applied to practice in combination with GIS tools.

In this study, we assume that the travel demand, in-vehicle travel time, and dwell time are predefined. However, the transit demand varies within a day, week, month, or year. Numerous attributes such as fare, travel time, frequency, walking time, routing and transferring, stop location, comfort and inconvenience elements, information, socio-economic factors, land use, and security affect transit demand [22]. In-vehicle travel time is associated with road traffic situations, driving behavior, and vehicle performance. Dwell time at stops is an increasing function of the number of passengers boarding and alighting at stops [53–55]. Hence, an interesting project for future studies should consider the stochastic factors in the transit network, including the running time and the arrival of passengers.

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Abbreviations

The main symbols in this work are defined as follows:

N	set of stops in the network;
N_r	set of stops of route r;
N_r^o	starting stop of route r;
N_r^d	ending stop of route r;
N_r^m	intermediate stops of route r;
i, j, k	induces of stops;
r, r'	induces of routes;
R	set of routes;
z	objective function of the problem;
w_1	weight of the travel without direct service;
w_2	weight of the travel time;
d_{ij}	travel demand from stop i to stop j per unit time;
M_{ij}	binary variable, in which $M_{ij} = 0$ if the passengers from stop i to stop j can complete their travels without transfer and $M_{ij} = 1$ otherwise;
t_{ij}	sum of AIVTT and waiting time from stop i to stop j;
t_{ij}^w	average waiting time from stop i to stop j;
t_{ij}^v	AIVTT from stop i to stop j;
t_{ij}^{rv}	in-vehicle travel time from stop i to stop j through route r;
f_r	frequency of route r during the planning period;
δ_{ij}^r	proportion of the travel demand assigned to route r from stop i and stop j per unit time;
x_{ij}^r	binary variable, in which $x_{ij}^r = 1$ if the route r passes through stop i and stop j continuously and $x_{ij}^r = 0$ otherwise;
c_{ij}	in-vehicle travel time from stop i to stop j by the shortest path;
th_{ij}^r	binary variable, in which $th_{ij}^r = 1$ if the route r passes through stop i and stop j, that is, passengers from stop i to stop j can complete their travels without transfer by route r; and $th_{ij}^r = 0$ otherwise;
s	average dwell time at each intermediate stop for all routes;
R_{ij}	set of routes that can provide the direct service from stop i to stop j
num	number of routes for the network;
S_{max}	maximum number of stops for a route;
t_r	single-trip time of route r;
η	time conversion unit in this study, in which $\alpha = 60$ min/h;
n_{fleet}	maximum given fleet size for the planning network;
$CS_{i(i+1)}^r$	flow on the segment from stop i to stop $i + 1$ for route r per unit time;
Cap	maximal load of a bus;
$board_i^r$	number of passengers who board a bus at stop k through route r after assignment per unit time;
$alight_i^r$	number of passengers who alight from a bus at stop k through route r after assignment per unit time;
d_{ij}^r	flow assignment to route r from stop i to stop j;
s_{ij}	section from stop i to stop j; and
S_{max}	maximum number of stops for a route.

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