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Impact of Information Asymmetry on the Operation of Green Closed-Loop Supply Chain under Government Regulation

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Abstract: Recycling subsidy and carbon tax policies are ways to achieve energy and environmental sustainability. The implementation of these policies has changed the operating environment of traditional closed-loop supply chains, while the privacy of relevant information increases the difficulty of decision-making. Under the background, this paper considers the green closed-loop supply chain (GCLSC) under the hybrid policy of recycling subsidy and carbon tax where the manufacturer is in charge of recycling and the retailer invests in green marketing. Taking green marketing cost coefficient as the retailer's private information, this paper explores the influence of information asymmetry on optimal decisions and performance of the GCLSC. By constructing game models of information symmetry and asymmetry, the optimal decisions, economic and environmental performance, and social welfare are provided. Combined with numerical analysis, the influence of uncertainty of the manufacturer's estimation, subsidies and carbon tax on the GCLSC is proposed. The results indicate that the uncertainty in the manufacturer's estimation can improve the social welfare under certain conditions, but it cannot reduce carbon emissions. Recycling subsidy and carbon tax policies oppositely affect the manufacturer's optimal decisions and carbon emissions. Information asymmetry is beneficial to the retailer. However, less uncertainty in estimation is not always better for the manufacturer. The manufacturer needs to proactively adopt strategies to stimulate the retailer's information sharing.

Keywords: closed-loop supply chain; information asymmetry; carbon tax; government subsidy; social welfare

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1. Introduction

With the booming economy, people have gradually realized the limitation of natural resources and have begun to pay attention to conservation. A closed-loop supply chain (CLSC) is designed to collect used-products from consumers by reverse logistics and remanufactures the entire product or parts to create new value [1]. In April 2019, Apple announced the expansion of it's recycling program by quadrupling the number of recycling locations available to consumers in America. In 2020, Huawei processed more than 4500 tons of e-waste through its own recycling channels. It contributes to the carbon peak and carbon neutrality targets of China. By improving energy efficiency and reducing waste, CLSC extends traditional supply chains to the green supply chain.

In addition to improving energy efficiency, environmental sustainability has received increasing attention in recent years [2–4]. Green closed-loop supply chains (GCLSCs) have become a major trend in energy and environmental sustainability, as it reduces the use of raw materials and decreases energy consumption and associated carbon emissions [5]. Implementations of emission reduction regulations (such as carbon tax, cap-and-trade, mandatory cap) have enriched the significance of GCLSCs. Assuming that the information between CLSC members is symmetric, some literature concentrates on the optimal operations decisions for the CLSC under certain carbon emission regulation [6–9]. Other literature interests in comparing the impact of different carbon emission policies on CLSC [10–12]. In order to encourage the remanufacturing and recycling of products, many governments subsidize recycling programs during implementing carbon reduction policies. For example, in 2018, after Shanghai launched the carbon trading scheme in November 2013, the government gave enterprises that recycled batteries a subsidy of 1000 RMB per set. Under the carbon tax regulation, Japan spent approximately 100 billion yen on subsidies to support battery-related industries in 2021, such as the sorting and recycling of renewable battery materials. The interaction of carbon reduction policy and recycling subsidy policy challenges the GCLSC's operational decisions. Since the government charges taxes for each unit of carbon emissions emitted by enterprises under carbon tax policy [13,14]. For the CLSC, carbon tax increases the environmental cost of manufacturers while the subsidy policy reduces the costs of recycling and remanufacturing. The manufacturer has to balance the environmental cost and remanufacturing cost. Therefore, scientific guidance, with regard to the operation and decision-making, is needed for members of the GCLSC. However, there is little literature concentrates on it. Dou and Choi [15] compared the green investment and recycling decisions of CLSC under the subsidy for trade-in program. They thought that carbon tax and subsidy policies motivate the GCLSC and consumers to accept the trade-in program. Shang et al. [16] analyzed optimal operational decisions of CLSCs when the government subsidize manufacturer's emission reduction, recycling and the retailer's advertising investment. All above literature is studied based on the assumption of information symmetry among GCLSC members.

In reality, information asymmetry among members is also an important factor affecting the operations of GCLSCs apart from the operating environment and policy regulations. Every enterprise holds private information that may significantly affect supply chain operations, such as market demand, recovery and green marketing efforts. Information asymmetry means that one participant with information advantages does not share his private information with the others. As opposed to the uncertainty of information, the information asymmetry may result in different status for participants in GCLSC. It will further affect their operational decisions, economic profits, and environmental impacts. There is quite a lot of literature on optimal operation decisions of CLSCs under uncertainty market demand [17,18], recovery product quality [10,19], and carbon price [20]. All these literature assumes that the information among CLSC participants is symmetrical.

On the basis of such background, this paper studies the influence of the information asymmetry on a GCLSC under the hybrid policy of carbon tax and recycling subsidy. In the considered GCLSC, the retailer carries out green marketing promotion and the manufacturer is responsible for recycling. The manufacturer has to estimate the parameter of green marketing efforts because it is the retailer's private information. This paper constructs decision optimization models for both cases of information symmetry and asymmetry. The main contributions are three-fold: (1) This paper takes green marketing cost coefficient as retailer's private information, provides the closed-form solutions of the optimal pricing, recycling rate, and marketing promotion decisions of the GCLSC under the interaction of subsidy and carbon tax policies. (2) Via analyzing game behaviors, the difference of the optimal decisions and GCLSC's performance between information symmetry and asymmetry are displayed under the hybrid policy of carbon tax and recycling subsidy. (3) The impact of the hybrid policy and the uncertainty in the manufacturer's estimation on operation decisions, the economic and environmental performance of a GCLSC is revealed. The interesting results show that when the retailer keeps marketing cost coefficient as the private information under the hybrid policy of carbon tax and subsidy, it is not always adverse to the manufacturer. Under certain conditions, it is beneficial to the manufacturer.

The following sections are organized as follows. The related literature is reviewed in Section 2. The problem description and notations that will be used in the rest of this paper are described in Section 3. The optimal decisions and characters under both information symmetry and asymmetry scenarios are analyzed in Section 4. Numerical analysis is presented in Section 5 to complement theoretical results. Section 6 concludes the findings, managerial insights and the direction for further research.

2. Literature Review

This paper contributes to the following two research themes: CLSC management under the constraint of carbon emissions reduction and supply chain management with information asymmetry.

2.1. CLSC Management under the Constraint of Carbon Emissions Reduction

With increasing global attention to green supply chains, lots of scholars have studied how carbon emissions reduction affects the operational strategies of the supply chain from different angles [6,21–23]. The influence of emissions reduction on CLSCs has also received considerable attention because the remanufacturing process generates carbon emissions [24,25]. Growing literature studies the optimal remanufacturing and recycling strategies of an enterprise under carbon emission reduction regulations. For example, Chai et al. [16] explain how the adoption of carbon trading policy influences the optimal remanufacturing decisions of an enterprise. Chen et al. [26] focus on the optimal collection and remanufacturing decisions for a remanufacturing system under carbon cap and take-back policies. Bai et al. [27] employ a distributionally robust newsboy approach to propose the optimal production and collection decisions under a cap-and-trade policy. Other literature has studied the operational strategies of multi-echelon CLSCs under the constraint of carbon emissions reduction [28,29]. Recently, Dou and Cao [5] investigate the optimal operational strategies of the CLSC in two operational periods by considering three product collection channels under a carbon tax regulation. Yang et al. [30] study how the implementation of the cap-and-trade policy affects the collection model selection of a two-echelon CLSC. Jauhari et al. [31] consider two recovery processes in a three-echelon CLSC under a carbon trading regulation, and concentrate on the optimal decisions including green technology investment, product quality and selling price under five different scenarios. Shekarian et al. [32] explore the effect of remanufacturing and emissions on a dual-channel CLSC with competitive collection. Wang and Wu [33] study the recycling and carbon reduction investment decisions for two types of CLSCs under cap-and-trade regulation.

All above literature studies the deterministic environment in which the market demand and parameter information are known. Taking into account the universal existence of uncertainty in reality, some scholars investigate the low-carbon operation strategies of CLSCs in an uncertain scenario. For example, Jauhari et al. [7] reveal the operation decisions of CLSC with stochastic demand and return rate under carbon tax regulation. Xu et al. [20] formulate a stochastic model for a CLSC facing uncertain demand and carbon price under the carbon trading scenario to find the optimal operation decisions in a multi-period planning horizon. Guo et al. [19] consider a remanufacturing enterprise that faces uncertain recycled product quality and demand under subsidy and carbon tax policies. They employ heuristic and intelligent methods to find the approximate solutions of the provided discrete optimization models.

The main characteristics of above literature on CLSCs are that they study the impact of uncertainty on operation strategies by assuming that the relevant information is symmetrical among CLSC members. In contrast, this paper considers a GCLSC with information asymmetry under a hybrid policy of carbon emission reduction and recycling subsidy. When the green marketing cost coefficient is the retailer's private information, the manufacturer has to estimate it before making decision. Hence, the effect of information asymmetry and the uncertainty of estimation on optimal joint remanufacturing and carbon abatement strategies are studied.

2.2. Supply Chain Management with Information Asymmetry

In reality, it is difficult to realize information sharing among supply chain participants. A growing number of scholars studied the operational strategies for supply chains with information asymmetry [34–38]. Recently, some scholars pay close attention to the influence of information asymmetry on CLSCs [39–42]. Wang et al. [43] consider a dual-channel CLSC consisted of one retailer and one third party recycling institution under a government reward-penalty mechanism. By taking recycling efforts as private information, they design contracts for the manufacturer to obtain real information. Via analyzing 288 articles, Chen and Huang [44] deem that the information asymmetry in CLSC remains be solved. Wu et al. [45] explore how the government incentivizes the retailer to report his recovery information when the recovery technology-type is taken as the retailer's private information. Wang et al. [46] reveal the effect of information asymmetry and fairness concerns on the performance of CLSCs by taking fairness concerns as the manufacturer's private information.

Above literature on CLSC studies the impacts of different types of information asymmetry without considering the external factors. On the contrary, this paper concentrates on the comprehensive influence of both the information asymmetry and the hybrid policy of carbon reduction and subsidy on the GCLSC. Considering the green marketing cost coefficient as the retailer's private information, it compares the optimal equilibrium strategies of the GCLSC with information symmetry and asymmetry under carbon tax and subsidy policies, and provides some managerial insights. The differences of models between relevant literature and present work are summarized in Table 1.

Table 1. Comparison of models between relevant literature and present work.

literature	Echelons of CLSC	Carbon Policy	Recycling Subsidy	Uncertainty	Information Asymmetry
Dou and Cao [5]	Two	Carbon tax	×	×	×
Jauhari et al. [31]	Three	Cap-and-trade	×	×	×
Wang and Wu [33]	Three	Cap-and-trade	×	×	×
Xu et al. [20]	Two	Cap-and-trade	×	Demand and carbon price	×
Jauhari et al. [7]	Two	Carbon tax	×	Demand and return	×
Guo et al. [19]	One	Carbon tax	✓	Demand and recycled products quality	×
Gao et al. [42]	Two	×	×	Demand	✓
Wang et al. [43]	Three	×	×	Demand	✓
Wang et al. [46]	Two	×	×	Fairness concern	✓
Present work	Two	Carbon tax	✓	Marketing cost coefficient	✓

3. Problem Descriptions and Notations

3.1. Problem Description

This paper considers a two-echelon CLSC under both carbon tax and recycling subsidy regulations. The diagram of the considered GCLSC is shown in Figure 1. In this GCLSC, the manufacturer produces new products at a unit manufacturing cost c_m and collects used-products with collection rate τ from customers at a unit collection price f. The used-products are remanufactured at the same cost c_n. The remanufactured product does not differ from the new product in appearance and function. The retailer buys products from the manufacturer at unit price w, and sells them with green marketing efforts A_r at unit price p. The manufacturer's production process is the main source of carbon emissions. The unit carbon emissions generated during manufacturing and remanufacturing is e_0 and e_1, respectively. Under the carbon tax regulation, the government announces a tax to the manufacturer, who pays tax for the unit carbon emissions at price r. In addition, to encourage the remanufacturing of used-products, the government subsidizes G for used-products. The research approach of this paper is provided in Figure 2.

3.2. Notations and Assumtions

The symbols and notations that will be used in the rest of this paper are summarized in Table 2.

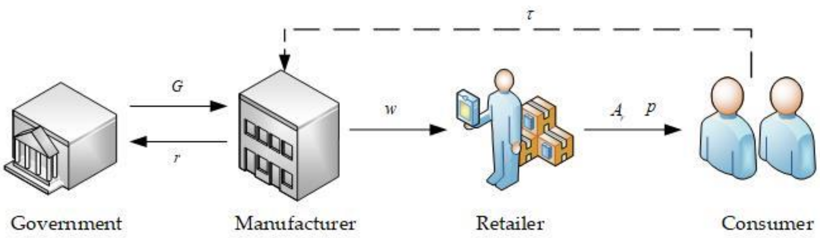


Figure 1. The diagram of GCLSC.

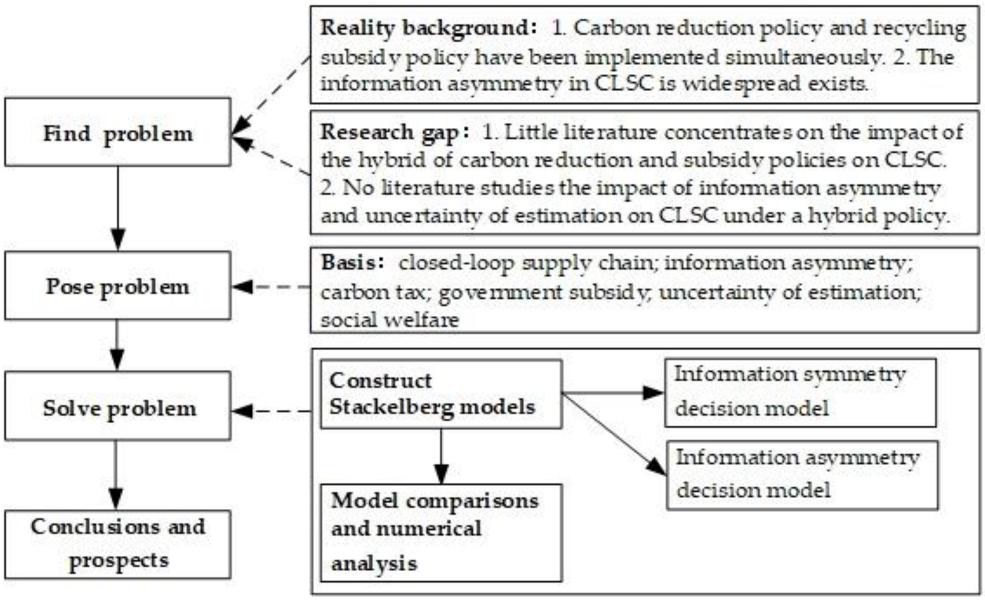


Figure 2. Flowchart of the research approach.

Table 2. The main parameters and notations.

Parameters	
D	The total market demand (unit)
ϕ	Basic market scale of products (unit)
β	Price-sensitive parameter of demand (unit/$)
b	Elasticity coefficient of the demand to green marketing efforts (unit/unit effort)
c_m	manufacturing cost per unit new product ($/unit)
c_n	remanufacturing cost per unit used-product ($/unit)
μ_r	Coefficient of the retailer's green marketing effort cost ($)
c_l	Coefficient of the manufacturer's collection cost ($)
f	Unit collection price of the manufacturer ($/unit)
G	Unit government subsidy for used-products ($/unit)
r	Unit carbon tax price ($/unit emission)
e_0	Carbon emissions generated during manufacturing one product (kg/unit)
e_1	Carbon emissions generated during remanufacturing one used-product (kg/unit)
Decision variables	
w	Wholesale price charged by the manufacturer ($/unit)
τ	Collection rate of the manufacturer
p	Selling price of retailer ($/unit)
A_r	Green marketing efforts of the retailer (index of efforts level)
Objective functions	
$\prod_R^{sym}, \prod_M^{sym}$	Profit of the retailer and manufacturer under information symmetry ($)
$E(\prod_R^{asy}), E(\prod_M^{asy})$	Expected profit of the retailer and manufacturer under information asymmetry ($)

The following assumptions are used to make this research closer to reality and concentrate on the key points.

Assumption 1. *The corresponding collection cost and green marketing efforts cost is* $\frac{1}{2}c_l\tau^2$ *and* $\frac{1}{2}\mu_r A_r^2$*, where* c_l *is the collection cost coefficient, and* μ_r *is the green marketing cost coefficient [13,35]. It accords with the economic principle of increasing marginal cost.*

Assumption 2. *The demand for products is linearly influenced by both the retail price and green marketing efforts, i.e.,* $D = \phi - \beta p + bA_r$*, where* ϕ *is the basic market scale,* β *is the price-sensitive parameter, and* b *is the elastic coefficient of demand to the green marketing efforts [35].*

Assumption 3. $c_m > c_n + f + re_1$*. It means that the manufacturer pays more for manufacturing than remanufacturing. It ensures the sustainability of the collection activity [26].*

Assumption 4. $c_l > \frac{\beta^2\mu_r(\Delta - re_1)^2}{2(2\beta\mu_r - b^2)} > 0$*, where* $\Delta = c_m - c_n + G - f$*. It indicates a higher cost for collection used-products, and guarantees the feasibility of the developed models [13].*

According to above notations and assumptions, we can formulate the profits, carbon emissions and social welfare as follows.

The profit functions of the retailer and the manufacturer are expressed as follows:

$$\prod\nolimits_R = (p - w)D - \frac{1}{2}\mu_r A_r^2 \tag{1}$$

$$\prod\nolimits_M = (w - c_m)D + (c_m - c_n)\tau D + (G - f)\tau D - \frac{1}{2}c_l\tau^2 - r(e_0 D + e_1\tau D) \tag{2}$$

Equation (1) is composed by the profit on sale and the green marketing effort cost. In Equation (2), the first two terms are the profit on sale, the third and fourth terms are the profit earned from collecting used-products, and the fifth term is the carbon tax, where e_0D is the carbon emissions from manufacturing new products and $e_1\tau D$ is the carbon emissions from remanufacturing used-products. To facilitate the analysis, the total carbon emissions are expressed as

$$J = e_0 D + e_1\tau D \tag{3}$$

The government subsidizes the manufacturer's collection activity from a social welfare perspective. Referring to previous studies [2,47], social welfare (SW) consists of three components: total profits of GCLSC participants, consumer surplus (CS) and government subsidy. CS is the difference between the maximum unit price and the actually unit paid price. Then, CS and SW can be expressed as follows:

$$CS = \int_{\frac{\phi + bA_r - D}{\beta}}^{\frac{\phi + bA_r}{\beta}} (\phi - \beta p + bA_r)dp = \frac{D^2}{2\beta} \tag{4}$$

$$SW = \Pi_M + \Pi_R + CS - G\tau D \tag{5}$$

4. Problem Formulation and Solutions

Considering the power of the retailer and the manufacturer, the Stackelberg game is used to establish a decision model in which the manufacturer first announces the wholesale price and the collection rate as the leader, then the retailer decides the selling price and the green marketing efforts. Under this decision model, the decision-making of supply chain members in the cases of information symmetry and information asymmetry are considered.

4.1. Information Symmetry Decision Model

In this model (represented as the "sym" model), all information of the retailer is shared with the manufacturer. The retailer's profit is first analyzed, and the manufacturer makes decisions based on the response function of the retailer, solving Equations (1) and (2) leads to the following conclusions.

Theorem 1. *There exist optimal solutions that maximize the GCLSC members' profits under information symmetry.*

(1) The retailer's optimal retail price and optimal green marketing efforts are

$$p^{sym*} = \frac{\mu_r \phi}{2\beta\mu_r - b^2} + \frac{\beta\mu_r - b^2}{2\beta\mu_r - b^2} \frac{c_l(2\beta\mu_r - b^2)[\phi + \beta(c_m + re_0)] - \beta^2\mu_r\phi(\Delta - re_1)^2}{2\beta c_l(2\beta\mu_r - b^2) - \beta^3\mu_r(\Delta - re_1)^2},$$

$$A_r^{sym*} = \frac{\mathrm{b}c_l[\phi - \beta(c_m + re_0)]}{2c_l(2\beta\mu_r - b^2) - \beta^2\mu_r(\Delta - re_1)^2}$$

(2) The manufacturer's optimal wholesale price and collection rate are

$$w^{sym*} = \frac{c_l(2\beta\mu_r - b^2)[\phi + \beta(c_m + re_0)] - \beta^2\mu_r\phi(\Delta - re_1)^2}{2\beta c_l(2\beta\mu_r - b^2) - \beta^3\mu_r(\Delta - re_1)^2},$$

$$\tau^{sym*} = \frac{\beta\mu_r(\Delta - re_1)[\phi - \beta(c_m + re_0)]}{2c_l(2\beta\mu_r - b^2) - \beta^2\mu_r(\Delta - re_1)^2}.$$

The proof is shown in Appendix A.

Theorem 1 shows that there are optimal solutions for the manufacturer and retailer when information is symmetric. Substituting p^{sym*}, A_r^{sym*}, w^{sym*} and τ^{sym*} into the relevant functions, the optimal profits of both retailer and manufacturer, total carbon emissions and social welfare under information symmetry are given as follows:

$$\Pi_R^{sym*} = \frac{\mu_r c_l^2[\phi - \beta(c_m + re_0)]^2(2\beta\mu_r - b^2)}{2[2c_l(2\beta\mu_r - b^2) - \beta^2\mu_r(\Delta - re_1)^2]^2}, \tag{6}$$

$$\Pi_M^{sym*} = \frac{\mu_r c_l[\phi - \beta(c_m + re_0)]^2}{2[2c_l(2\beta\mu_r - b^2) - \beta^2\mu_r(\Delta - re_1)^2]}, \tag{7}$$

$$J^{sym*} = \frac{e_0\beta\mu_r c_l[\phi - \beta(c_m + re_0)]}{2c_l(2\beta\mu_r - b^2) - \beta^2\mu_r(\Delta - re_1)^2} + \frac{e_1\beta^2\mu_r^2 c_l(\Delta - re_1)[\phi - \beta(c_m + re_0)]^2}{[2c_l(2\beta\mu_r - b^2) - \beta^2\mu_r(\Delta - re_1)^2]^2}, \tag{8}$$

$$SW^{sym*} = \frac{\mu_r c_l[\phi - \beta(c_m + re_0)]^2[3c_l(2\beta\mu_r - b^2) - \beta^2\mu_r(\Delta - re_1)^2 + 2\beta\mu_r(c_l - g\beta(\Delta - re_1))]}{2[2c_l(2\beta\mu_r - b^2) - \beta^2\mu_r(\Delta - re_1)^2]^2} \tag{9}$$

4.2. Information Asymmetry Decision Model

Since the participants in the decentralized GCLSC make decisions independently, the retailer may not share all information with other participants that results in information asymmetry. In this section, the green marketing effort cost coefficient μ_r is the retailer's private information, which the manufacturer lacks full information about it (denoted as model "asy"). It is assumed that μ_r is uniformly distributed, that is, $\mu_r \sim U[\overline{\mu}_r - \varepsilon, \overline{\mu}_r + \varepsilon]$, where ε, $0 < \varepsilon < \overline{\mu}_r$, denotes the degree of information uncertainty that reflects the uncertainty in the manufacturer's estimation of the green marketing cost coefficient. When the value of ε increases, the uncertainty in manufacturer's estimation on the green marketing cost coefficient increases.

The retailer has the same information under asymmetric information and keeps the private information about the green marketing effort cost coefficient μ_r. Given the retailer's decisions, the manufacturer decides the optimal collection rate and wholesale price by maximizing its expected profit $E(\prod_M^{asy})$ which is given as follows:

$$\begin{aligned} E(\prod_M^{asy}) &= \int_{\overline{\mu_r}-\varepsilon}^{\overline{\mu_r}+\varepsilon} [(w-c_m+\Delta\tau)\frac{\beta\mu_r(\phi-\beta w)}{2\beta\mu_r-b^2} - \frac{1}{2}c_l\tau^2 - r(e_0+e_1\tau)\frac{\beta\mu_r(\phi-\beta w)}{2\beta\mu_r-b^2}]\frac{1}{2\varepsilon}d\mu_r \\ &= (\phi-\beta w)[w-c_m+\Delta\tau-r(e_0+e_1\tau)][\frac{1}{2}+\frac{b^2}{8\beta\varepsilon}\ln\frac{2\beta(\overline{\mu_r}+\varepsilon)-b^2}{2\beta(\overline{\mu_r}-\varepsilon)-b^2}] - \frac{1}{2}c_l\tau^2. \end{aligned} \tag{10}$$

Let $h(\varepsilon) = \frac{1}{2} + \frac{b^2}{8\beta\varepsilon}\ln\frac{2\beta(\overline{\mu_r}+\varepsilon)-b^2}{2\beta(\overline{\mu_r}-\varepsilon)-b^2}$. Solving Equation (10) leads to the following conclusion.

Theorem 2. *There exist optimal solutions that maximize the GCLSC participants' profits under information asymmetry.*

(1) The retailer's optimal selling price and green marketing efforts are

$$p^{asy*} = \frac{\mu_r\phi}{2\beta\mu_r-b^2} + \frac{\beta\mu_r-b^2}{2\beta\mu_r-b^2}\frac{c_l[\phi+\beta(c_m+re_0)]-\beta\phi h(\varepsilon)(\Delta-re_1)^2}{\beta[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]},$$

$$A_r^{asy*} = \frac{bc_l[\phi-\beta(c_m+re_0)]}{(2\beta\mu_r-b^2)\left[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2\right]}$$

(2) The manufacturer's optimal collection rate and wholesale price are

$$\tau^{asy*} = \frac{h(\varepsilon)(\Delta-re_1)[\phi-\beta(c_m+re_0)]}{2c_l-\beta h(\varepsilon)(\Delta-re_1)^2},$$

$$w^{asy*} = \frac{c_l[\phi+\beta(c_m+re_0)]-\beta\phi h(\varepsilon)(\Delta-re_1)^2}{\beta[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]}.$$

The proof is shown in Appendix A.

The above process proves the existence of optimal solutions for GCLSC participants when information is asymmetric. Substituting the optimal decisions into the correlation function yields the optimal expected profits of the GCLSC participants, total carbon emissions and social welfare as follows:

$$E(\prod_R^{asy*}) = \frac{\mu_r c_l^2[\phi-\beta(c_m+re_0)]^2}{2(2\beta\mu_r-b^2)[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^2}, \tag{11}$$

$$E(\prod_M^{asy*}) = \frac{c_l[\phi-\beta(c_m+re_0)]^2[2\mu_r c_l-h^2(\varepsilon)(\Delta-re_1)^2(2\beta\mu_r-b^2)]}{2(2\beta\mu_r-b^2)[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^2}, \tag{12}$$

$$J^{asy*} = \frac{e_0\beta\mu_r c_l[\phi-\beta(c_m+re_0)]}{(2\beta\mu_r-b^2)[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]} + \frac{e_1\beta\mu_r c_l h(\varepsilon)(\Delta-re_1)[\phi-\beta(c_m+re_0)]^2}{(2\beta\mu_r-b^2)[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^2}, \tag{13}$$

$$\begin{aligned} SW^{asy*} &= \frac{c_l[\phi-\beta(c_m+re_0)]^2[3\mu_r c_l-h^2(\varepsilon)(\Delta-re_1)^2(2\beta\mu_r-b^2)]}{2(2\beta\mu_r-b^2)[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^2} \\ &+ \frac{\beta\mu_r c_l[\phi-\beta(c_m+re_0)]^2[\mu_r c_l+2gh(\varepsilon)(\Delta-re_1)(2\beta\mu_r-b^2)]}{2(2\beta\mu_r-b^2)^2[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^2}. \end{aligned} \tag{14}$$

4.3. Model Comparisons and Analysis

This section reveals the effect of ε, r and G on optimal decisions, profits, carbon emissions, and social welfare, and further compares the optimal decisions between the models with information symmetry and asymmetry.

Corollary 1. *Uncertainty in estimation and government policies affect the manufacturer's decisions, and the following conclusions hold when* $\phi > \beta(c_m+re_0)$ *is satisfied.*

(1) As ε and G increase, the optimal wholesale price w^{asy*} decreases while the optimal collection rate τ^{asy*} increases.

(2) As r increases, the optimal wholesale price w^{asy*} increases while the optimal collection rate τ^{asy*} decreases.

The proof is shown in Appendix A.

As seen from Corollary 1, when the basic market scale is large, the manufacturer's uncertainty about the green marketing effect coefficient increases with the increases of ε, and the manufacturer's dominant position in the game weakens to reduce the wholesale price and improve the collection rate. The optimal wholesale price decreases with G while it increases with r. However, the optimal collection rate increases with G while it decreases with r. It implicates that the impact of recycling subsidy and carbon tax on the manufacturer's optimal decisions is opposite. The carbon tax and subsidy policies are complementary to each other.

Corollary 2. *Uncertainty in estimation affects the manufacturer's profit, and the following conclusions hold.*

(1) If $\frac{\beta\mu_r}{2\beta\mu_r-b^2} < h(\varepsilon) < \frac{2c_l}{\beta(\Delta-re_1)^2}$, $E(\prod_M^{asy*})$ decreases while $\Pi_M^{sym*} - E(\prod_M^{asy*})$ increases with ε.

(2) If $\frac{1}{2} < h(\varepsilon) < \frac{\beta\mu_r}{2\beta\mu_r-b^2}$, $E(\prod_M^{asy*})$ increases while $\Pi_M^{sym*} - E(\prod_M^{asy*})$ decreases with ε.

Corollary 3. *Both $E(\prod_R^{asy*}$ and $E(\prod_R^{asy*}) - \Pi_R^{sym*}$ increase as ε increases.*

The proofs of Corollaries 2 and 3 are shown in Appendix A.

Corollaries 2 and 3 show that the retailer's expected profit increases with ε, indicating that keeping the green marketing effort cost coefficient as private information facilitates the increase of profit and improves the disadvantageous position in the game. In contrast, the uncertainty of estimation has more complex effect on the manufacturer's profit. When $h(\varepsilon) < \frac{2c_l}{\beta(\Delta-re_1)^2}$, as the uncertainty in estimation increases, the manufacturer's expected profit first increases and then decreases. The difference of the manufacturer's profit in the cases of information symmetry and asymmetry first decreases and then increases. It indicates that the retailer prefers a large uncertainty in the manufacturer's estimation. From the aspect of the manufacturer, it is not true that the less uncertainty in estimation is the better.

Corollary 4. *If $\frac{1}{2} < h(\varepsilon) < \frac{\beta\mu_r}{2\beta\mu_r-b^2}$, the social welfare SW^{asy*} increases with ε.*

Corollary 5. *When $\phi > \beta(c_m + re_0)$ holds, carbon emissions J^{asy*} increase with ε and G, and decrease with r.*

The proofs of Corollaries 4–5 are shown in Appendix A.

Corollaries 4–5 provide the conditions in which the social welfare and carbon emissions increase with respect to the uncertainty in the manufacturer's estimation. Corollary 5 further reveals that when the basic market scale is large, the carbon tax and subsidies oppositely affect carbon emissions. It indicates that the uncertainty in the manufacturer's estimation can improve the social welfare under certain conditions, but it cannot reduce carbon emissions.

Corollary 6. *The optimal decisions of the GCLSC are affected by information asymmetry. When $\phi > \beta(c_m + re_0)$, the following conclusions hold.*

(1) If $h(\varepsilon) < \frac{\beta\mu_r}{2\beta\mu_r-b^2}$, $\tau^{sym*} > \tau^{asy*}$ and $A_r^{\mathrm{sym}*} > A_r^{\mathrm{asy}*}$ hold.

(2) If $\frac{2c_l\left[1-(2\beta\mu_r-b^2)\right]}{\beta(\Delta-re_1)^2}+\beta\mu_r<h(\varepsilon)<\frac{c_l\left[\phi+\beta(c_m+re_0)\right]\left[1-(2\beta\mu_r-b^2)\right]}{\beta\phi(\Delta-re_1)^2}+\beta\mu_r$, $w^{sym*}<w^{asy*}$ *and* $p^{sym*}<p^{asy*}$ *hold.*

The proof is shown in Appendix A.

Corollary 6 compares the optimal decisions between the model with information symmetry and asymmetry. Conclusion (1) proposes the condition under which the collection rate and green marketing efforts in the information symmetry scenario are greater than those in the information asymmetry scenario. Conclusion (2) proposes the condition under which the wholesale price and selling price in the information asymmetry scenario are greater than those under information symmetry. It indicates that the influence of information asymmetry on the optimal decisions depends on the uncertainty in the manufacturer's estimation.

5. Numerical Analysis

This section illustrates the theoretical results and draws several managerial insights by the numerical analysis. Referring to the data in literature [35,48], we use the following values of parameters: $c_m=15$, $c_n=10$, $c_l=20$, $f=2.5$, $r=1.1$, $G=0.5$, $e_0=5$, $e_1=2$, $\phi=55$, $b=1$, $\beta=1.2$, $\mu_r=2$. When information is symmetric, the optimal decisions and profits of GCLSC members, social welfare and carbon emissions are as follows: $w^{sym*}=33.0112$, $\tau^{sym*}=0.3887$, $p^{sym*}=41.1094$, $A_r^{sym*}=4.0491$, $\Pi_R^{sym*}=62.3018$, $\Pi_M^{sym*}=123.0927$, $SW^{sym*}=222.8543$, $J^{sym*}=56.1441$. Let $\overline{\mu}_r=2$ and $\varepsilon=1$. When information is asymmetric, the optimal decisions and profits of GCLSC members, social welfare and carbon emissions are as follows: $w^{asy*}=33.0053$, $\tau^{asy*}=0.4033$, $p^{asy*}=41.1072$, $A_r^{asy*}=4.0509$, $E(\prod_R^{asy}*)=62.3586$, $E(\prod_M^{asy}*)=127.7171$, $SW^{asy*}=227.4995$, $J^{asy*}=56.4537$. The comparison reveals that information asymmetry has small impacts on the retailer's optimal decisions and profit, while it has significant impacts on the manufacturer's collection decision and profit, carbon emissions and social welfare. In this case, information asymmetry is not conducive to reducing carbon emissions but has positive impacts on GCLSC's economic performance and social welfare.

Next, the effect of the main parameters on the GCLSC is examined. Figure 3 proposes the effect of ε on GCLSC members' decisions and profits, carbon emissions, and social welfare.

Figure 3 shows that for the manufacturer, the collection rate and the expected profit increases while the wholesale price decreases as ε increases. For the retailer, the green marketing efforts and the expected profit increases while the retail price decreases as ε increases, but the trend is not significant. Carbon emissions and social welfare increase as ε increases. The reasons are as follows. The manufacturer's dominant position is weakened under asymmetric green marketing effort cost information. As the uncertainty grows, the manufacturer reduces the wholesale price and increases the collection rate, leading to an increase in the expected profit. According to the manufacturer's decisions, the retailer reduces the retail price and increases green marketing efforts. The retailer's expected profit increases under information asymmetry, but the private information does not fundamentally change the retailer's disadvantageous position as a follower. So the changes in the retailer's decisions and profit are not significant. On the other hand, lower selling price and higher green marketing efforts lead to higher product demand, and a higher collection rate leads to higher collection quantities. Therefore, the number of products increases, as well as carbon emissions. Meanwhile, the increase in product demand and supply chain members' profits lead to an increase in social welfare.

Based on the information asymmetry model, the influence of carbon tax price r and government subsidy G on the GCLSC is further analyzed. Figure 4 shows the influence of r and G on GCLSC members' decisions, profits, carbon emissions, and social welfare.

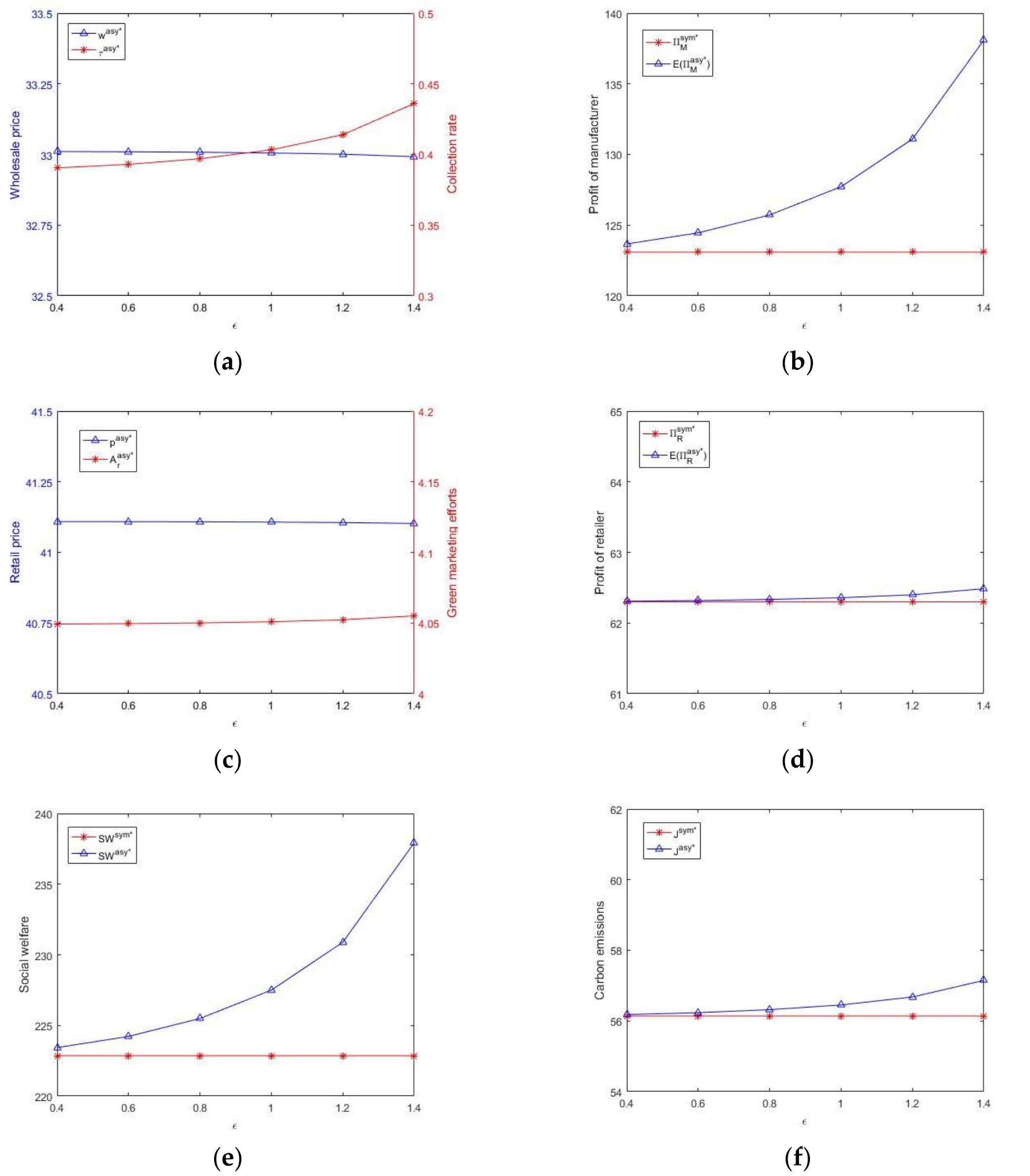
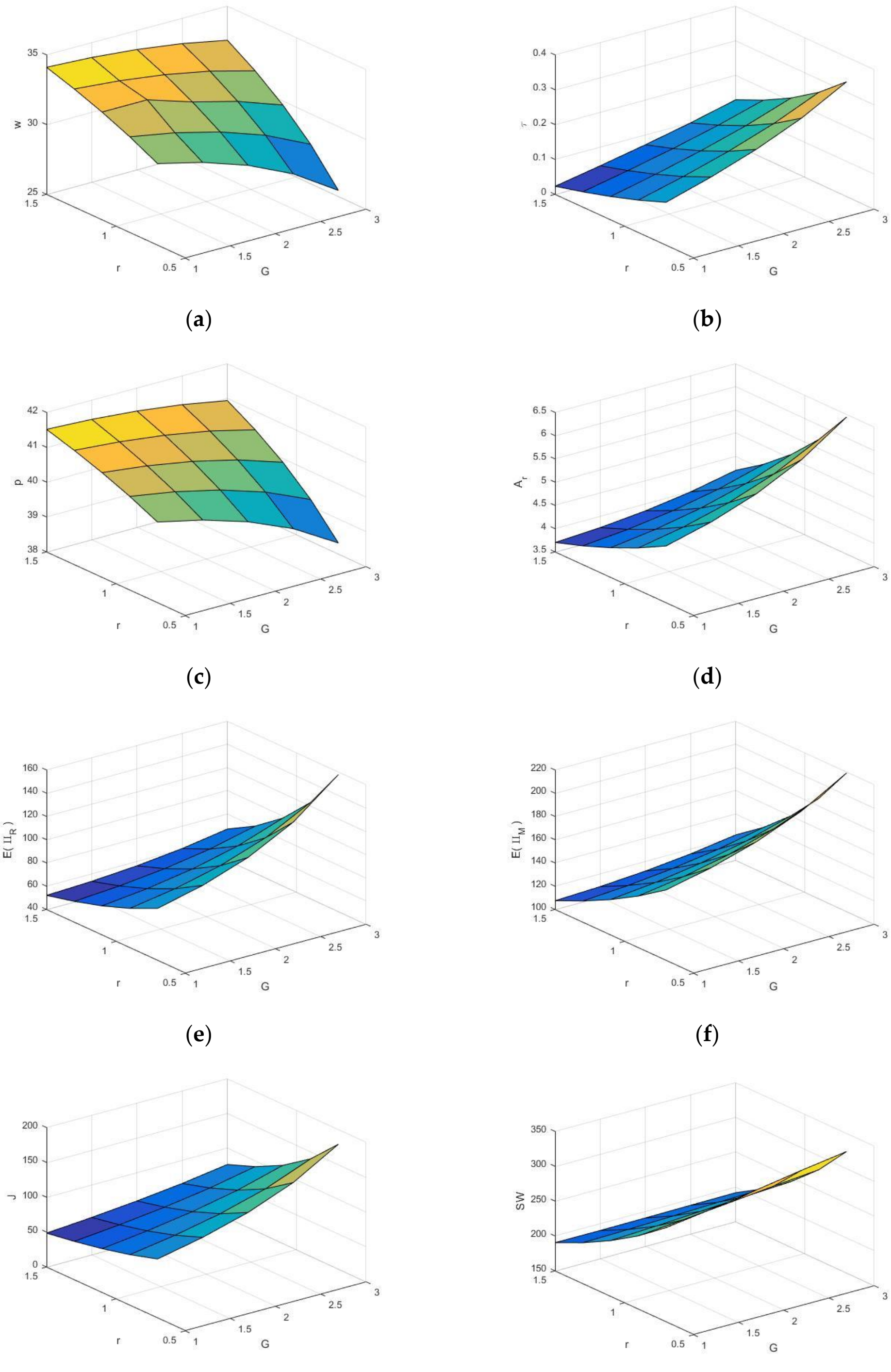


Figure 3. The effect of ε on the GCLSC. (**a**) The effect on w^{asy*} and τ^{asy*}, (**b**) The effect on Π_M^{sym*} and $E(\prod_M^{asy}*)$, (**c**) The effect on p^{asy*} and A_r^{asy*}, (**d**) The effect on Π_R^{sym*} and $E(\prod_R^{asy}*)$, (**e**) The effect on SW^{sym*} and SW^{asy*}, (**f**) The effect on J^{sym*} and J^{asy*}.

As seen in Figure 4, for the manufacturer, the collection rate and the expected profit decrease while the wholesale price increases as r increases. For the retailer, the green marketing efforts and the expected profit decrease while the retail price increases as r increases. Carbon emissions and social welfare decrease as r increases. The reasons are as follows. The increase in the carbon tax price means an increase in costs for the manufacturer. Hence, the manufacturer compensates for the loss by raising the wholesale price and reduces collection cost by decreasing the collection rate. Specifically, when $r = 1.5$, $\tau = 0$ holds and the manufacturer no longer collects used-products. The increase in wholesale price causes a higher selling price and fewer green marketing efforts. Subsequently, the market demand and carbon emissions of manufacturing new products decrease. The decrease in collection rate leads to fewer remanufactured products, so the carbon emissions of remanufactured products decrease. As market demand and supply chain members' profits decline, social welfare decreases as well.



(**a**) (**b**)

(**c**) (**d**)

(**e**) (**f**)

(**g**) (**h**)

Figure 4. The effect of r and G on the GCLSC. (**a**) The effect on w^{asy*}, (**b**) The effect on τ^{asy*}, (**c**) The effect on p^{asy*}, (**d**) The effect on A_r^{asy*}, (**e**) The effect on $E(\prod_R^{asy}*)$. (**f**) The effect on $E(\prod_M^{asy}*)$, (**g**) The effect on J^{asy*}, (**h**) The effect on SW^{asy*}.

From Figure 4, we can also observe that for the manufacturer, the expected profit and the collection rate increase while the wholesale price decreases as G increases. For the retailer, the expected profit and the green marketing efforts increase while the retail price decreases as G increases. Carbon emissions and social welfare increase as G increases. The reasons are as follows. Because of the government subsidies, the manufacturer obtains more subsidies by reducing wholesale price and increasing collection rate. A decrease in wholesale price causes a lower retail price and higher green marketing efforts. The market demand and the carbon emissions of manufacturing new products increase. The increase in the collection rate rises the quantity of remanufactured products, which causes an increase in the carbon emissions from remanufacturing process. As market demand and total profits increase, social welfare increases as well.

According to the above analysis, it can be seen that information asymmetry does not give the retailer greater advantages since the increase in the retailer's profit is not significant when the green marketing effort cost information is asymmetric. But the manufacturer's decisions are greatly affected by information asymmetry. Under certain conditions, the expected profit of the manufacturer increases with uncertainty ε. Otherwise, the manufacturer's expected profit decreases. Overall, information asymmetry brings great uncertainty to the supply chain, which has negative impacts on social welfare and environmental benefits. The collection rate increases when the green marketing effort cost coefficient is asymmetric, but with increasing uncertainty, carbon emissions increase subsequently, which is not conducive to the environmental protection and emission reduction policies. As the price of carbon tax increases, carbon emissions and the collection rate decrease together with the social welfare. However, carbon emissions and social welfare increase as G increases. Hence, determining the carbon tax price and subsidies that will both control carbon emissions and effectively collect products is a matter for careful decision-making.

6. Conclusions

This paper considers a two-echelon GCLSC under a hybrid of carbon tax and subsidy regulations, where the manufacturer is in charge of production and collection, and the retailer sells products and makes green marketing efforts. Viewing the marketing effort cost coefficient as the private information of the retailer, we derive optimal decisions, profits, carbon emissions and social welfare under the game models with information symmetry and asymmetry. The results obtained by theoretical and numerical analysis indicate that: (1) The influence of information asymmetry on the optimal decisions relates to the uncertainty in the manufacturer's estimation. Information asymmetry is beneficial to the retailer. But from the aspect of the manufacturer, it is not true that the less uncertainty in estimation is the better. (2) The uncertainty in the manufacturer's estimation can improve the social welfare under certain conditions, but it cannot reduce carbon emissions. (3) Recycling subsidy and carbon tax policy oppositely affect the manufacturer's optimal decisions and carbon emissions.

The following suggestions are putting forward to governments and closed-loop supply chains. (1) The government needs to determine a reasonable carbon tax price and subsidies for the GCLSC to tradeoff the economic and environmental sustainability although subsidy and carbon tax policies are complementary to each other. (2) From the perspective of the CLSC participants, the manufacturer needs to proactively adopt strategies to stimulate the retailer's information sharing, which is beneficial to improve the accuracy of decision-making and reduce profit loss caused by the information asymmetry. Because less uncertainty in estimation is not always better for the manufacturer when the retailer has the private information. Besides, measures such as collecting used-products, introducing green technology and improving green marketing efforts should be taken to reduce carbon emissions and realize sustainable development.

In this paper, optimal decisions for the GCLSC with asymmetric information are explored by taking the green marketing cost coefficient as the retailer's private information. It assumes that the manufacturer's estimation of the retailer's green marketing cost coefficient follows uniform distribution. However, another distribution may result in different conclusion. Hence, whether the probability distribution of the estimation affects the research findings is another interesting issue. The analysis results of this paper indicate that the manufacturer should proactively adopt strategies to stimulate the retailer's information sharing. Then, the optimal decisions under other information asymmetry cases and how to encourage information sharing among GCLSC participants will be interesting research topics.

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Appendix A

Proof of Theorem 1. Simplifying Equation (1) and taking the first-order and second-order derivatives of Π_R, we can obtain the Hessian matrix of $\prod_R$ on variables p and A_r as follows:

$$H_1 = \begin{pmatrix} \frac{\partial^2\Pi_R}{\partial p^2} & \frac{\partial^2\Pi_R}{\partial p\partial A_r} \\ \frac{\partial^2\Pi_R}{\partial A_r\partial p} & \frac{\partial^2\Pi_R}{\partial A_r^2} \end{pmatrix} = \begin{pmatrix} -2\beta & b \\ b & -\mu_r \end{pmatrix}$$

Hence, when $\beta > \frac{b^2}{2\mu_r}$, Π_R is a concave function of p and A_r. According to the first-order condition, that is, $\frac{\partial\Pi_R}{\partial p} = -2\beta p + \phi + bA_r + \beta w = 0$ and $\frac{\partial\Pi_R}{\partial A_r} = b(p-w) - \mu_r A_r = 0$, the following equations can be deduced: $p(w) = \frac{\mu_r\phi + (\beta\mu_r - b^2)w}{2\beta\mu_r - b^2}$ and $A_r(w) = \frac{b(\phi-\beta w)}{2\beta\mu_r - b^2}$. Substituting $p(w)$ and $A_r(w)$ into the demand function, we have $D = \frac{\beta\mu_r(\phi-\beta w)}{2\beta\mu_r - b^2}$. Substituting it into $\prod_M$, the Hessian matrix of $\prod_M$ on variables w and τ is given as follows:

$$H_2 = \begin{pmatrix} \frac{\partial^2\Pi_M}{\partial w^2} & \frac{\partial^2\Pi_M}{\partial w\partial\tau} \\ \frac{\partial^2\Pi_M}{\partial\tau\partial w} & \frac{\partial^2\Pi_M}{\partial\tau^2} \end{pmatrix} = \begin{pmatrix} \frac{-2\beta^2\mu_r}{2\beta\mu_r - b^2} & \frac{\beta^2\mu_r(re_1-\Delta)}{2\beta\mu_r - b^2} \\ \frac{\beta^2\mu_r(re_1-\Delta)}{2\beta\mu_r - b^2} & -c_l \end{pmatrix}$$

According to the assumption, we can conclude that H_2 is negative definite. Solving $\frac{\partial\Pi_M}{\partial w} = \frac{\beta\mu_r[\phi - 2\beta w + \beta(c_m - \Delta\tau) + \beta r(e_0 + \tau e_1)]}{2\beta\mu_r - b^2} = 0$ and $\frac{\partial\Pi_M}{\partial\tau} = \frac{\beta\mu_r(\phi-\beta w)(\Delta - re_1)}{2\beta\mu_r - b^2} - \tau c_l = 0$, we can obtain w^{sym*} and τ^{sym*}. After substituting them into $p(w)$ and $A_r(w)$, we get the expression of p^{sym*} and A_r^{sym*}, and Theorem 1 is proved. □

Proof of Theorem 2. By solving the Hessian matrix of $E(\prod_M^{asy})$, it is found that when $h(\varepsilon) < \frac{2c_l}{\beta(\Delta - re_1)^2}$, $E(\prod_M^{asy})$ is a concave function of w and τ. Solving the equations

$$\frac{\partial E(\prod_M^{asy})}{\partial\tau} = h(\varepsilon)(\phi - \beta w)(\Delta - re_1) - \tau c_l = 0$$

and

$$\frac{\partial E(\prod_M^{asy})}{\partial w} = h(\varepsilon)[\phi - 2\beta w + \beta(c_m - \Delta\tau) + \beta r(e_0 + \tau e_1)] = 0,$$

We obtain w^{asy*} and τ^{asy*}. Substituting them into $p(w)$ and $A_r(w)$, analytical expressions of p^{asy*} and A_r^{asy*} are obtained. Theorem 2 is proved. □

Proof of Corollary 1. Let $g(\varepsilon)=\frac{4\beta\varepsilon(2\beta\overline{\mu_r}-b^2)}{[2\beta(\overline{\mu_r}+\varepsilon)-b^2][2\beta(\overline{\mu_r}-\varepsilon)-b^2]}-\ln\frac{2\beta(\overline{\mu_r}+\varepsilon)-b^2}{2\beta(\overline{\mu_r}-\varepsilon)-b^2}$, then $g(0)=0$ and $g\prime(\varepsilon)=\frac{32\beta^3\varepsilon^2(2\beta\overline{\mu_r}-b^2)}{[2\beta(\overline{\mu_r}+\varepsilon)-b^2]^2[2\beta(\overline{\mu_r}-\varepsilon)-b^2]^2}>0$ hold. Because $0<\varepsilon<\overline{\mu_r}$, we have $g(\varepsilon)>0$ and $h'(\varepsilon)=\frac{b^2}{8\beta\varepsilon^2}\cdot g(\varepsilon)>0$. Furthermore, we obtain $\frac{\partial w^{asy*}}{\partial\varepsilon}=-\frac{c_l h'(\varepsilon)(\Delta-re_1)^2[\phi-\beta(c_m+re_0)]}{[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^2}<0$ and $\frac{\partial\tau^{asy*}}{\partial\varepsilon}=\frac{2c_l h'(\varepsilon)(\Delta-re_1)[\phi-\beta(c_m+re_0)]}{[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^2}>0$.

After taking the first derivatives of w^{asy*} and τ^{asy*} with respect to r and G, it is found that

$$\frac{\partial w^{asy*}}{\partial r}=\frac{c_l e_0}{2c_l-\beta h(\varepsilon)(\Delta-re_1)^2}+\frac{2e_1c_l h(\varepsilon)(\Delta-re_1)[\phi-\beta(c_m+re_0)]}{[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^2}>0,$$

$$\frac{\partial\tau^{asy*}}{\partial r}=-\frac{\beta e_0 h(\varepsilon)(\Delta-re_1)}{2c_l-\beta h(\varepsilon)(\Delta-re_1)^2}-\frac{e_1 h(\varepsilon)[\phi-\beta(c_m+re_0)][2c_l+\beta h(\varepsilon)(\Delta-re_1)^2]}{[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^2}<0,$$

$$\frac{\partial w^{asy*}}{\partial G}=-\frac{2c_l h(\varepsilon)(\Delta-re_1)[\phi-\beta(c_m+re_0)]}{[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^2}<0,$$

$$\frac{\partial\tau^{asy*}}{\partial G}=\frac{h(\varepsilon)[\phi-\beta(c_m+re_0)][2c_l+\beta h(\varepsilon)(\Delta-re_1)^2]}{[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^2}>0.$$

Proof completed. □

Proof of Corollary 2. Based on Equations (7) and (12), we have

$$\begin{aligned}\Delta E(\textstyle\prod_M)&=\Pi_M^{sym*}-E(\textstyle\prod_M^{asy*})\\&=\frac{\mu_r c_l[\phi-\beta(c_m+re_0)]^2}{2[2c_l(2\beta\mu_r-b^2)-\beta^2\mu_r(\Delta-re_1)^2]}-\frac{c_l[\phi-\beta(c_m+re_0)]^2[2\mu_r c_l-h^2(\varepsilon)(\Delta-re_1)^2(2\beta\mu_r-b^2)]}{2(2\beta\mu_r-b^2)[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^2}.\end{aligned}$$

When $\frac{\beta\mu_r}{2\beta\mu_r-b^2}<h(\varepsilon)<\frac{2c_l}{\beta(\Delta-re_1)^2}$, we have $\beta\mu_r-h(\varepsilon)(2\beta\mu_r-b^2)<0$. Hence, $\frac{\partial E(\prod_M^{asy*})}{\partial\varepsilon}=\frac{2c_l^2h'(\varepsilon)(\Delta-re_1)^2[\phi-\beta(c_m+re_0)]^2[\beta\mu_r-h(\varepsilon)(2\beta\mu_r-b^2)]}{(2\beta\mu_r-b^2)[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^3}<0$ and $\frac{\partial\Delta E(\prod_M)}{\partial\varepsilon}=-\frac{2c_l^2h'(\varepsilon)(\Delta-re_1)^2[\phi-\beta(c_m+re_0)]^2[\beta\mu_r-h(\varepsilon)(2\beta\mu_r-b^2)]}{(2\beta\mu_r-b^2)[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^3}>0$ hold. That is, as ε increases, $E(\prod_M^{asy*})$ decreases while $\Delta E(\prod_M)$ increases. Similarly, it can be proved that when $\frac{1}{2}<h(\varepsilon)<\frac{\beta\mu_r}{2\beta\mu_r-b^2}$, $\frac{\partial E(\prod_M^{asy*})}{\partial\varepsilon}>0$ and $\frac{\partial\Delta E(\prod_M)}{\partial\varepsilon}<0$ hold. That is, when the value of ε increases, $E(\prod_M^{asy*})$ increases while $\Delta E(\prod_M)$ decreases. Proof completed. □

Proof of Corollary 3. Based on Equations (6) and (11), we have

$$\begin{aligned}\Delta E(\textstyle\prod_R)&=E(\textstyle\prod_R^{asy*})-\Pi_R^{sym*}\\&=\frac{\mu_r c_l^2[\phi-\beta(c_m+re_0)]^2}{2(2\beta\mu_r-b^2)[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^2}-\frac{\mu_r c_l^2[\phi-\beta(c_m+re_0)]^2(2\beta\mu_r-b^2)}{2[2c_l(2\beta\mu_r-b^2)-\beta^2\mu_r(\Delta-re_1)^2]^2}.\end{aligned}$$

Recall that $h'(\varepsilon)>0$ holds. So we have $\frac{\partial E(\prod_R^{asy*})}{\partial\varepsilon}=\frac{\beta h'(\varepsilon)\mu_r c_l^2(\Delta-re_1)^2[\phi-\beta(c_m+re_0)]^2}{(2\beta\mu_r-b^2)[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^3}>0$ and $\frac{\partial\Delta E(\prod_R)}{\partial\varepsilon}=\frac{\beta h'(\varepsilon)\mu_r c_l^2(\Delta-re_1)^2[\phi-\beta(c_m+re_0)]^2}{(2\beta\mu_r-b^2)[2c_l-\beta h(\varepsilon)(\Delta-re_1)^2]^3}>0$. That is, when the value of ε increases, both $E(\prod_R^{asy*})$ and $\Delta E(\prod_R)$ increase. Proof completed. □

Proof of Corollary 4. When $\frac{1}{2} < h(\varepsilon) < \frac{\beta\mu_r}{2\beta\mu_r - b^2}$, we have $\beta\mu_r - h(\varepsilon)(2\beta\mu_r - b^2) > 0$ and $\frac{\partial SW^{asy*}}{\partial\varepsilon} = \frac{h'(\varepsilon)c_l^2(\Delta - re_1)^2[\phi - \beta(c_m + re_0)]^2}{(2\beta\mu_r - b^2)[2c_l - \beta h(\varepsilon)(\Delta - re_1)^2]^3}\{2[\beta\mu_r - h(\varepsilon)(2\beta\mu_r - b^2)] + \beta\mu_r\} + \frac{\beta h'(\varepsilon)\mu_r c_l(\Delta - re_1)[\phi - \beta(c_m + re_0)]^2}{(2\beta\mu_r - b^2)^2[2c_l - \beta h(\varepsilon)(\Delta - re_1)^2]^3}\left\{\beta\mu_r c_l(\Delta - re_1) + g(2\beta\mu_r - b^2)[2c_l + \beta h(\varepsilon)(\Delta - re_1)^2]\right\} > 0$. Together with Corollaries 2 and 3 and the fact that consumer surplus and total subsidies increase with ε, we conclude that the social welfare increases with ε. Proof completed. □

Proof of Corollary 5. Based on Equation (13), we obtain that

$$\frac{\partial J^{asy*}}{\partial\varepsilon} = \frac{\beta\mu_r c_l h'(\varepsilon)(\Delta - re_1)[\phi - \beta(c_m + re_0)]}{(2\beta\mu_r - b^2)[2c_l - \beta h(\varepsilon)(\Delta - re_1)^2]^2}\left\{e_0\beta(\Delta - re_1) + \frac{e_1[\phi - \beta(c_m + re_0)][2c_l + \beta h(\varepsilon)(\Delta - re_1)^2]}{2c_l - \beta h(\varepsilon)(\Delta - re_1)^2}\right\}$$

$$\frac{\partial J^{asy*}}{\partial r} = -\frac{\beta^2\mu_r c_l e_0}{(2\beta\mu_r - b^2)[2c_l - \beta h(\varepsilon)(\Delta - re_1)^2]} - \frac{4\mu_r c_l\beta^2 e_1^2 h^2(\varepsilon)(\Delta - re_1)^2[\phi - \beta(c_m + re_0)]^2}{(2\beta\mu_r - b^2)[2c_l - \beta h(\varepsilon)(\Delta - re_1)^2]^3} - \frac{\beta\mu_r c_l e_1 h(\varepsilon)[\phi - \beta(c_m + re_0)]\{e_1[\phi - \beta(c_m + re_0)] + 2\beta e_0(\Delta - re_1)\}}{(2\beta\mu_r - b^2)[2c_l - \beta h(\varepsilon)(\Delta - re_1)^2]^2}.$$

$$\frac{\partial J^{asy*}}{\partial G} = \frac{\beta\mu_r c_l eh1(\varepsilon)[\phi - \beta(c_m + re_0)]^3[2c_l + 3\beta h(\varepsilon)(\Delta - re_1)^2]}{(2\beta\mu_r - b^2)[2c_l - \beta h(\varepsilon)(\Delta - re_1)^2]^3} + \frac{2\beta^2\mu_r c_l e_0 h(\varepsilon)(\Delta - re_1)[\phi - \beta(c_m + re_0)]}{(2\beta\mu_r - b^2)[2c_l - \beta h(\varepsilon)(\Delta - re_1)^2]^2}.$$

According to the assumptions, we have $\frac{\partial J^{asy*}}{\partial\varepsilon} > 0, \frac{\partial J^{asy*}}{\partial G} > 0$ and $\frac{\partial J^{asy*}}{\partial r} < 0$. That is, when the basic market scale is large, the carbon emissions increase with ε and G but decrease with carbon tax. □

Proof of Corollary 6. According to Theorems 1 and 2, we have

$$\tau^{sym*} - \tau^{asy*} = \frac{2c_l(\Delta - re_1)[\phi - \beta(c_m + re_0)][\beta\mu_r - h(\varepsilon)(2\beta\mu_r - b^2)]}{[2c_l(2\beta\mu_r - b^2) - \beta^2\mu_r(\Delta - re_1)^2][2c_l - \beta h(\varepsilon)(\Delta - re_1)^2]},$$

$$A_r^{\mathrm{sym}*} - A_r^{\mathrm{asy}*} = \frac{\beta bc_l(\Delta - re_1)^2[\phi - \beta(c_m + re_0)][\beta\mu_r - h(\varepsilon)(2\beta\mu_r - b^2)]}{(2\beta\mu_r - b^2)[2c_l(2\beta\mu_r - b^2) - \beta^2\mu_r(\Delta - re_1)^2][2c_l - \beta h(\varepsilon)(\Delta - re_1)^2]},$$

$$w^{sym*} - w^{asy*} = \frac{c_l(2\beta\mu_r - b^2)[\phi + \beta(c_m + re_0)] - \beta^2\mu_r\phi(\Delta - re_1)^2}{2\beta c_l(2\beta\mu_r - b^2) - \beta^3\mu_r(\Delta - re_1)^2} - \frac{c_l[\phi + \beta(c_m + re_0)] - \beta\phi h(\varepsilon)(\Delta - re_1)^2}{\beta[2c_l - \beta h(\varepsilon)(\Delta - re_1)^2]}$$

$$p^{sym*} - p^{asy*} = \frac{\beta\mu_r - b^2}{2\beta\mu_r - b^2}\cdot\left\{\frac{c_l(2\beta\mu_r - b^2)[\phi + \beta(c_m + re_0)] - \beta^2\mu_r\phi(\Delta - re_1)^2}{2\beta c_l(2\beta\mu_r - b^2) - \beta^3\mu_r(\Delta - re_1)^2} - \frac{c_l[\phi + \beta(c_m + re_0)] - \beta\phi h(\varepsilon)(\Delta - re_1)^2}{\beta[2c_l - \beta h(\varepsilon)(\Delta - re_1)^2]}\right\}.$$

It can be seen that if $h(\varepsilon) < \frac{\beta\mu_r}{2\beta\mu_r - b^2}$, there are $\tau^{sym*} > \tau^{asy*}$ and $A_r^{\mathrm{sym}*} > A_r^{\mathrm{asy}*}$. If $\frac{2c_l[1 - (2\beta\mu_r - b^2)]}{\beta(\Delta - re_1)^2} + \beta\mu_r < h(\varepsilon) < \frac{c_l[\phi + \beta(c_m + re_0)][1 - (2\beta\mu_r - b^2)]}{\beta\phi(\Delta - re_1)^2} + \beta\mu_r$, $w^{sym*} < w^{asy*}$ and $p^{sym*} < p^{asy*}$ hold. Proof completed. □

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