

Article

Analysis of the Wind System Operation in the Optimal Energetic Area at Variable Wind Speed over Time

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Abstract: Due to high mechanical inertia and rapid variations in wind speed over time, at variable wind speeds, the problem of operation in the optimal energetic area becomes complex and in due time it is not always solvable. No work has been found that analyzes the energy-optimal operation of a wind system operating at variable wind speeds over time and that considers the variation of the wind speed over time. In this paper, we take into account the evolution of wind speed over time and its measurement with a low-power turbine, which operates with no load at the mechanical angular velocity ω_{MAX} . The optimal velocity is calculated. The energy that is captured by the wind turbine significantly depends on the mechanical angular velocity. In order to perform a function in the maximum power point (MPP) power point area, the load on the electric generator is changed, and the optimum mechanical velocity is estimated, ω_{OPTIM} , knowing that the ratio $\omega_{OPTIM}/\omega_{MAX}$ does not depend on the time variation of the wind speed.

Keywords: wind system; variable wind speeds; maximum power point; power; mechanical inertia; motion equation; mathematical model of wind turbine; maximum energy; maximum power point operation; optimum mechanical angular velocity

1. Introduction

In the literature [1–12], the operation of the wind turbine (WT) in the maximum energy area is studied at a constant wind speed; using various mathematical models, data from the manufacturing company and/or obtained in laboratory conditions, which are quite different from those in real conditions operation [13–15]. For this reason, the electrical energy obtained is less than the maximum value that was obtained in a maximum power point (MPP) operation, at optimum mechanical angular velocity (MAV), ω_{OPTIM} .

At variable wind speeds over time, the problem of operation in the MPP area becomes complex and not always solvable in due time due to high mechanical inertia and rapid variations in wind speed over time.

The WT power characteristic, i.e., the function $P_{WT}(\omega)$, at a certain constant velocity in time, shows a maximum at ω_{OPTIM} .

The optimal energy is at the velocity ω_{OPTIM} , where the power captured by the WT is the maximum, the MPP point on the power characteristic.

Important points on the WT power curve are:

- maximum power point—MPP—with MAV— ω_{OPTIM} ; and,
- null power point with maximum MAV— ω_{MAX} .

The correct determination of these points, in real operating conditions, assures the operation in the MPP area.

At variable wind speeds between V_{MIN} and V_{MAX} , the WT power takes values in the hash area of Figure 1 between:

P_{WT-MAX} —the power at V_{MAX}

and

P_{WT-MIN} —the power at V_{MIN}

The power at permanent magnets synchronous generator (PMSG) takes values between P_{G-MAX} and P_{G-MIN} , the green hazel area of Figure 1.

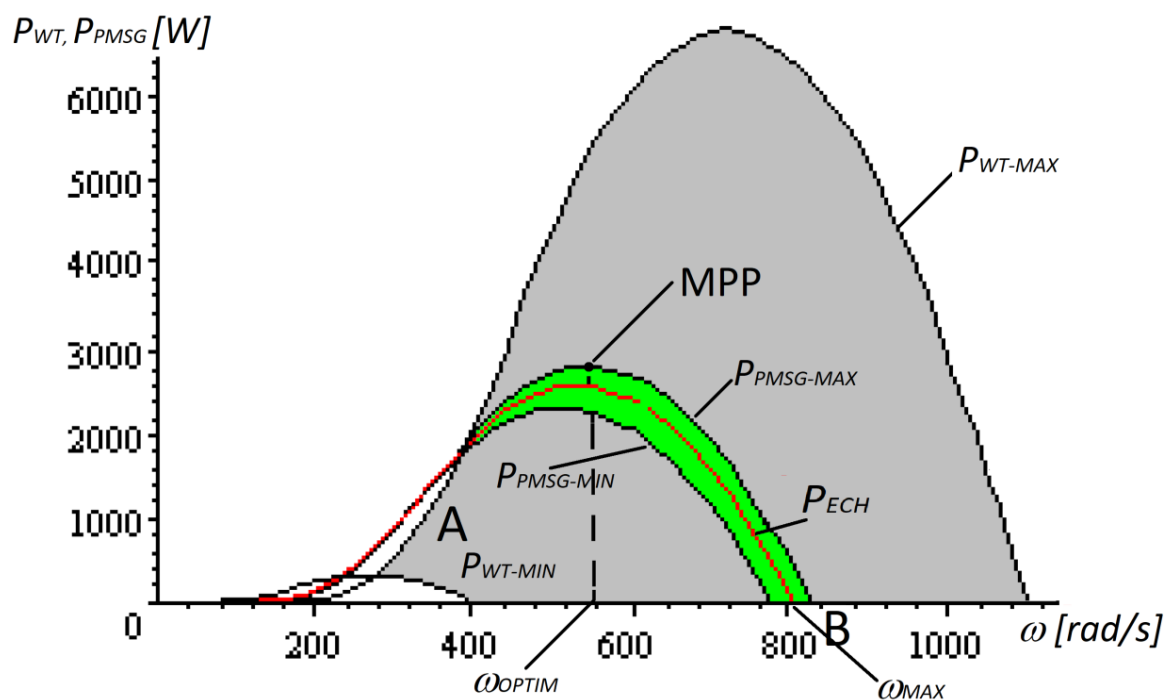


Figure 1. Power and ω_{OPTIM} characteristics.

PMSG power oscillations are much lower due to the high EOLIAN ELECTROENERGY SYSTEM (EES) equivalent moment of inertia. The equivalent moment of inertia attenuates PMSG's power oscillations from the WT as a shock absorber.

In the present paper, the results that were obtained by simulations are based on the usual mathematical models of WT and PMSG.

2. Mathematical Models

The simulations that presented in the paper are based on the classical mathematical models of WT and PMSG [16–19].

2.1. The Mathematical Model of WT, (MM-WT)

We will use a classical turbine model [19], which allows for the estimation of the reference angular velocity ω_{ref} . The mathematical model of the WT also allows for the calculation of the optimal velocity, so the captured energy will be a maximum one.

The power given by the WT can be calculated using the following equation:

$$P_{WT} = \rho \pi R_p^2 C_p(\lambda) V^3 \quad (1)$$

where: ρ —is the air density, R_p —the pales radius, $C_p(\lambda)$ —power conversion coefficient, $\lambda = R\omega/V$, V —the wind speed, and ω —mechanical angular velocity (MAV).

The power conversion coefficient, $C_p(\lambda)$, could be calculated, as follows:

$$C_p(\lambda) = c_1 \left(\frac{c_2}{\lambda} - c_3 \right) e^{-\frac{c_4}{\lambda}} \quad (2)$$

$$\frac{1}{\lambda} = \frac{1}{\lambda} - 0.0035 \quad (3)$$

c_1 – c_4 are data-book constants.

$$\frac{1}{\lambda} = \frac{1}{\lambda} - 0.0035 = \frac{V}{R\omega} - 0.0035 = \frac{V}{1.5\omega} - 0.0035 \quad (4)$$

By replacing, we can obtain the power conversion coefficient, as follows:

$$C_p(\lambda) = c_1 \left(\frac{c_2}{\lambda} - c_3 \right) e^{-\frac{c_4}{\lambda}} = c_1 \left(c_2 \left(\frac{V}{1.5\omega} - 0.0035 \right) - c_3 \right) e^{-c_4 \left(\frac{V}{1.5\omega} - 0.0035 \right)} \quad (5)$$

The power given by the wind turbine can be calculated, as follows:

$$P_{WT}(\omega, V) = \rho \pi R^2 C_p(\lambda) V^3 = 1.225\pi 1.5^2 c_1 \left(c_2 \left(\frac{V}{1.5\omega} - 0.0035 \right) - c_3 \right) e^{-c_4 \left(\frac{V}{1.5\omega} - 0.0035 \right)} V^3 \quad (6)$$

or

$$P_{WT}(\omega, V) = \rho \pi R^2 C_p(\lambda) V^3 = k_1 \left(k_2 \left(\frac{V}{\omega} - 0.0525 \right) - c_3 \right) e^{-k_3 \left(\frac{V}{\omega} - 0.0525 \right)} V^3 \quad (7)$$

where $k_1 = 1.225\pi 1.5^2$, $k_2 = c_2/1.5$, $k_3 = c_4/1.5$.

For the wind turbine WT, the producer gives the experimental power characteristics, $P_{WT}(\omega, V)$, or torque characteristics $T_{WT}(\omega, V)$, with the last ones being known as mechanical experimental characteristics.

$$T_{WT}(\omega, V) = P_{WT}(\omega, V)/\omega = k_1 \left(k_2 \left(\frac{V}{\omega} - 0.0525 \right) - c_3 \right) e^{-k_3 \left(\frac{V}{\omega} - 0.0525 \right)} V^3 / \omega \quad (8)$$

The maximum value of the function $P_{WT}(\omega, V)$ is achieved for a reference MAV ω_{ref} , and it yields:

$$\omega_{ref} = \omega_{OPTIM} = 400 \cdot k_3 \frac{k_2}{400 \cdot k_2 + 21 \cdot k_3 k_2 + 400 \cdot k_3 c_3} \cdot V = k_4 \cdot V \quad (9)$$

$$\omega_{OPTIM} = k_0 \cdot V \quad (10)$$

where k_0 is the constructive constant of the turbine.

This result proves the direct link between reference velocity and wind speed.

By replacing this result, it yields:

$$P_{WT-MAX}(V) = k_P \cdot V^3 \quad (11)$$

ie a cubic dependence of the maximum power value of the wind speed V .

In cases where the wind speed varies significantly over time, the result that is obtained must be re-analyzed and an equivalent wind speed is required.

For power wind turbines: $P_{TV-MAX} = 11$ KW the experimental power characteristics and $P_{WT}(\omega, V)$ are modeled by:

$$P_{WT}(\omega, V) = 1191.5 \cdot (V/\omega - 0.02) e^{-98.06(V/\omega)} \cdot V^3 \quad (12)$$

Obtaining the maximum WT power value.

Case 1

The maximum $P_{WT}(\omega)$ function is obtained by canceling the derivative

$$\frac{dP_{WT}(\omega, V)}{d\omega} = \frac{d(1191.5 \cdot (V/\omega - 0.02) e^{-98.06(V/\omega)} \cdot V^3)}{d\omega} = 0 \quad (13)$$

with the solution:

$$\omega_{OPTIM} = 33.115 V$$

By replacing this value in the function $P_{WT}(\omega)$, maximum velocity is obtained, at the wind speed V :

$$P_{WT-MAX-1} = 1191.5 \cdot (V/\omega_{OPTIM} - 0.02) e^{-98.06(V/\omega_{OPTIM})} \cdot V^3 = 1191.5 \cdot (1/33.115 - 0.02) e^{-98.06(1/33.115)} \cdot V^3 \quad (14)$$

or

$$P_{WT-MAX-1} = 0.62888 \cdot V^3 \quad (15)$$

Case 2

To obtain the maximum of $P_{WT}(\omega)$ function, we can also use the no-load MAV value from a low-power auxiliary WT (WT_{AUX}), operating at MAV, ω_{MAX} . The auxiliary turbine is a model of the main turbine; it is much smaller and models the main turbine power characteristic. The main and auxiliary turbines are in the same location and both are exposed to the same wind. This MAV value takes into account the evolution of wind speed in time. The value of the $\omega_{OPTIM}/\omega_{MAX}$ ratio is constant for a given WT and it does not depend on the time variation of the wind speed [11–14]

$$\begin{aligned} \omega_{OPTIM} &= 0.6623 \cdot \omega_{MAX} \\ P_{WT-MAX-2} &= 0.62888 \cdot (\omega_{MAX}/50)^3 \end{aligned} \quad (16)$$

This value is very easy to obtain during the operation of the EES.

2.2. The GSMP Mathematical Model, (MM-GSMP)

To analyze the behavior of the system WT-PMSG for the time-varying wind speeds, it uses the orthogonal mathematical model for permanent magnet synchronous generator (PMSG), which is given by the following equations [11–14]:

$$\begin{cases} -U\sqrt{3}\sin\theta = R_1 I_d - \omega L_q I_q \\ U\sqrt{3}\cos\theta = R_1 I_q + \omega L_d I_d + \omega \Psi_{PM} \\ M_{PMSG} = p_1 (L_d - L_q) I_d I_q + I_q \Psi_{PM} \\ P_G = R \cdot (I_d^2 + I_q^2) \end{cases} \quad (17)$$

where

U —stator voltage;

I_d, I_q —d-axis and q-axis stator currents;

θ —load angle;

R_1 —phase resistance of the generator;

L_d —synchronous reactance after d axis;

L_q —synchronous reactance after q axis;

Ψ_{PM} —flux permanent magnet;

M_{PMSG} —PMSG electromagnetic torque; and,

The two functions: $P_G(R, \omega)$ —electric power provided by the generator and $M_{PMSG}(R, \omega)$ —moment at the generator shaft depend on: R —load resistance and ω -MAV.

For $R = ct$ the moment M_{PMSG} depends linearly on ω , and the power depends squarely on ω .

From the nominal values of the PMSG [1], for the nominal power: $P_N = 5$ [kW], it yields $R_1 = 1.6$ [W], $L_d = 0.07$ [H], $L_q = 0.08$ [H], $\Psi_{PM} = 1.3$ [Wb].

From the equations of the PMSG, it obtains

$$\left\{ \begin{array}{l} -RI_d = 1.6I_d - \omega \cdot 0.08 \cdot I_q \\ -RI_q = 1.6I_q + \omega \cdot 0.07 \cdot I_d + \omega \Psi_{PM} \\ M_{PMSG} = -0.01 \cdot I_d I_q + I_q \Psi_{PM} \\ \Psi_{PM} = 1.3 \\ P_G = (I_d^2 + I_q^2) \end{array} \right. \quad (18)$$

$$P_G = 4225R\omega^2 \frac{4\omega^2 + 625R^2 + 2000R + 1600}{(1250R^2 + 4000R + 3200 + 7\omega^2)^2} \quad (19)$$

$$M_G = M_{PMSG} = -845\omega(5R + 8) \cdot \frac{4\omega^2 + 625R^2 + 2000R + 1600}{(1250R^2 + 4000R + 3200 + 7\omega^2)^2} \quad (20)$$

The analysis of the transient phenomena specific to variable wind speeds is done by simulation. The simulations are based on the motion equation:

$$J \frac{d\omega}{dt} = M_{WT} - M_{PMSG} \quad (21)$$

where J —equivalent moment of inertia; M_{WT} —Moment on WT; M_{PMSG} —Moment of PMSG.

3. Simulation Results at the Time Variable Wind Speed

This analyzes the EES operation at a time variation of wind speed of the form:

$$V(t) = 15 \cdot e^{-t/3600} - 2 \cdot \sin 0.17943t \quad (22)$$

as shown in Figure 2.

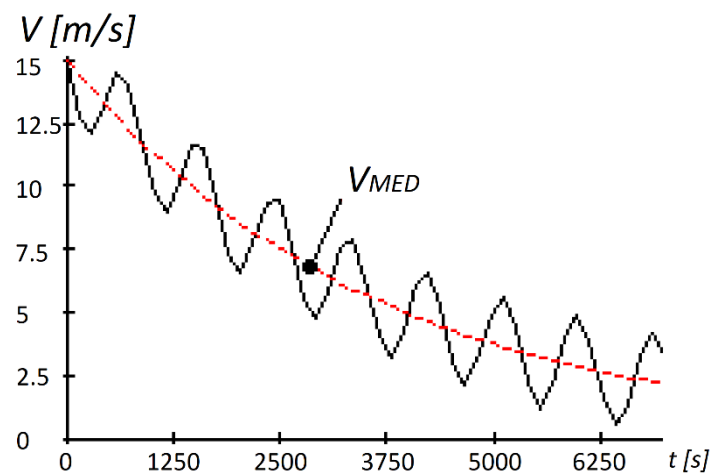


Figure 2. Wind speed variation over time.

3.1. No-Load Operation of See

For the no-load operation, the motion equation is:

$$\begin{cases} 45 \cdot \frac{d\omega}{dt} \omega = 1191 \cdot \left(\frac{(15 \cdot e^{-t/3600} - 2 \sin 017943t)}{\omega} - 0.02 \right) \cdot e^{-98.06((15 \cdot e^{-t/3600} - 2 \sin 017943t)/\omega)} \\ \omega(0) = 0 \end{cases} \quad (23)$$

The MAV over time is presented in Figure 3.

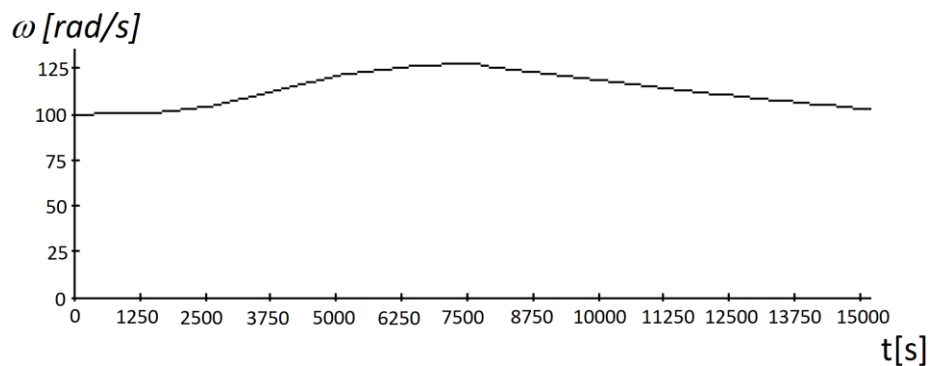


Figure 3. The mechanical angular velocity (MAV) over time for $t = 0$ –15000 s.

After two hours, the wind speed drops to 0 and the WT switches to fan mode. The power absorbed by it can be calculated with:

$$P_{FAN} = k \cdot \omega^3 \quad (24)$$

where k , the coefficient of proportionality, is determined by knowing that at:

$\omega = 571$ [rad/s] the equivalent power of WT is:

$$P_{ECH}(571) = 2055.5 \text{ [W]} \quad (25)$$

It yields:

$$k = \frac{P_{ECH}(571)}{\omega^3} = \frac{2055.5}{571^3} \quad (26)$$

and, therefore, power becomes:

$$P_{FAN} = 2055.5 \cdot \left(\frac{\omega}{571} \right)^3 \quad (27)$$

under these conditions and when considering friction losses as 5% of $P_{ECH}(571)$, the motion equation was obtained in the form of:

$$\begin{cases} 45 \cdot \frac{d\omega}{dt} \omega = -2055.5 \cdot \left(\frac{\omega}{571}\right)^3 - 102 \\ \omega(0) = 103.52 \end{cases} \quad (28)$$

After $t = 2258$ [s], MAV becomes:

$\omega(2258) = 0.11673$ [rad/s], and, therefore, it can be considered that WT stopped, as it can be seen in Figure 4.

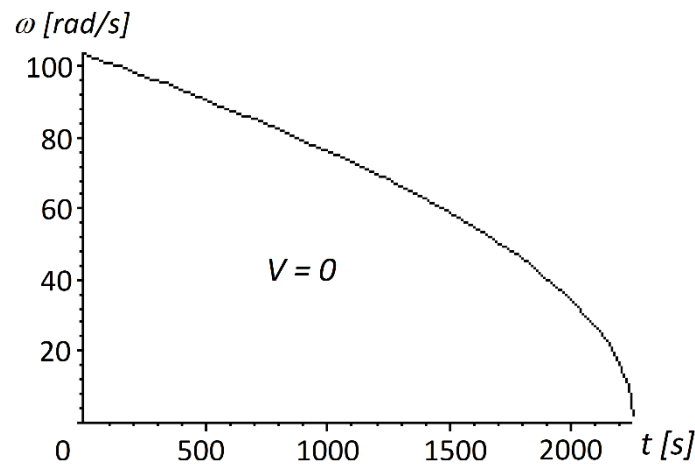


Figure 4. The variation of MAV for $t = 15000$ – 17500 [s].

The energy that is captured during this time period is significantly dependent on MAV values, which are load-dependent.

For a given time interval, the optimal MAV value, ω_{OPTIM} , is calculated based on the ratio:

$$\omega_{OPTIM} / \omega_{MAX} = 0.68$$

where the ω_{MAX} value is obtained by a no-load operation of a low power auxiliary WT_{AUX}.

3.2. No-Load Operation of WT_{AUX}

The moment of inertia moment, J , of the WT_{AUX} is much smaller than the power WT, for example:

$$J = 0.1 \text{ [kgm}^2\text{]}$$

For a time variation of wind speed of the form:

$$V(t) = 15 \cdot e^{-t/3600} - 2 \cdot \sin 0.17943t \quad (29)$$

the motion equation, for WT_{AUX}, becomes:

$$\begin{cases} 0.1 \cdot \frac{d\omega}{dt} \omega = 1.1 \cdot \left((15 \cdot e^{-t/3600} - 2 \sin 0.17943t) / \omega - 0.02 \right) \cdot e^{-98.06((15 \cdot e^{-t/3600} - 2 \sin 0.17943t) / \omega)} \\ \omega(0) = 756.85 \end{cases} \quad (30)$$

and the time variations of ω and ω_{OPTIM} are obtained over the interval 0–333 [s], as shown in Figure 5.

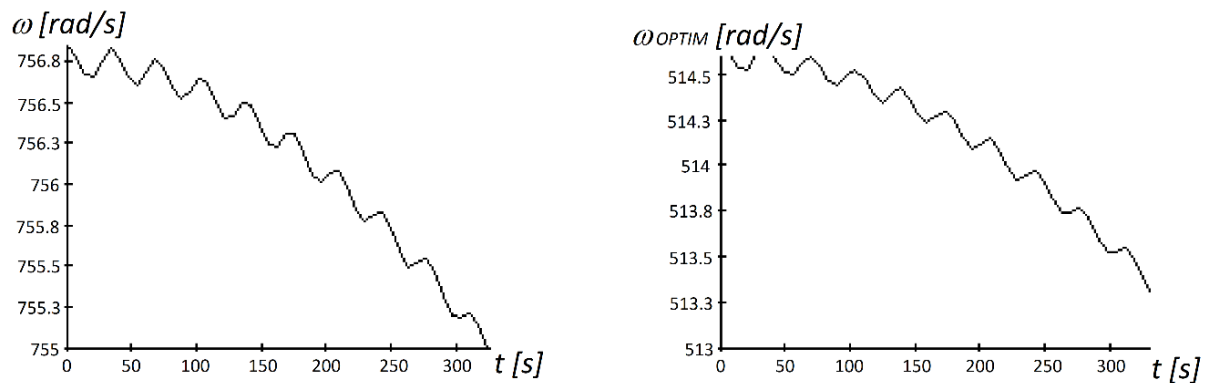


Figure 5. Variation of MAV ω and ω_{OPTIM} over the interval 0–333 [s].

Measuring MAV at WT_{AUX} , at 33 [s] time intervals, the optimal MAV values ω_{OPTIM} are obtained at time moments: $t = 33, 66, 99, \dots, 198$ [s], as follows:

$$\begin{aligned}
 \omega_{OPTIM}(33) &= 0.68\omega(33) = 514.64 \text{ [rad/s]} \\
 \omega_{OPTIM}(66) &= 0.68\omega(66) = 514.59 \text{ [rad/s]} \\
 \omega_{OPTIM}(99) &= 0.68\omega(99) = 514.51 \text{ [rad/s]} \\
 \omega_{OPTIM}(132) &= 0.68\omega(132) = 514.39 \text{ [rad/s]} \\
 \omega_{OPTIM}(165) &= 0.68\omega(165) = 514.25 \text{ [rad/s]} \\
 \omega_{OPTIM}(198) &= 0.68\omega(198) = 514.1 \text{ [rad/s]}
 \end{aligned} \tag{31}$$

3.3. Load Operation of the EES

Based on these values and measuring, at time t_k , the current MAV, ω_{tk} , from PMSG, we can estimate the amount of kinetic energy that is to be taken from PMSG, as follows:

$$\Delta W_{KINETIC} = J \cdot (\omega_{tk}^2 - \omega_{OPTIM-tk}^2) / 2 \tag{32}$$

This energy is added by the wind energy captured by WT, W_{WT} , during the time Δt :

$$\Delta t = t_k - t_{k-1}$$

When considering that, at the beginning, the operation of the EES is stable at the following MAV: $\omega(0) = 520$ [rad/s], in time interval $\Delta t = 33$ [s], the wind energy captured, W_{wind} , has the value:

$$W(520) = \int_0^{33} \left(\frac{1191.5 \cdot \left((15 \cdot e^{-t/3600} - 2 \sin 0.17943t) / 520 - 0.02 \right) \cdot e^{-98.06 \cdot ((15 \cdot e^{-t/3600} - 2 \sin 0.17943t) / 520)}}{(15 \cdot e^{-t/3600} - 2 \sin 0.17943t)^3} \right) dt = 67416 \text{ [J]} \tag{33}$$

and therefore, the average WT power is:

$$P_{WT-MED}(520) = \frac{67416}{33} = 2042.9 \text{ [W]} \tag{34}$$

the same as the PMSG, $P_{PMSG}(520)$.

PMSG load resistance is calculated from the algebraic system:

$$\begin{cases} P_G = 845\omega^2(5R + 8) \cdot \frac{4\omega^2 + 625R^2 + 2000R + 1600}{(1250R^2 + 4000R + 3200 + 7\omega^2)^2} \\ \omega = 520 \end{cases} \tag{35}$$

or:

$$\begin{cases} 2042.9 = 845\omega^2(5R + 8) \cdot \frac{4\omega^2 + 625R^2 + 2000R + 1600}{(1250R^2 + 4000R + 3200 + 7\omega^2)^2} \\ \omega = 520 \end{cases} \quad (36)$$

with the solution:

$$\omega = 520 \text{ [rad/s]} \text{ and } R = 216.12 \text{ } [\Omega]$$

For EES to operate in the MPP area, at:

$$\omega_{OPTIM}(33) = 514.64 \text{ [rad/s]}$$

must take the kinetic energy $\Delta W_{KINETIC}$ corresponding to the both MAV $\omega(0)$ and $\omega_{OPTIM}(33)$ and the wind energy that was captured by the WT in the time interval $\Delta t = 33$ [s].

The calculation of the wind energy captured by the WT over the analyzed time frame and at wind speed, $V(t)$, can be done through the integration of WT power over time:

$$P_{WT}(\omega) = 1.1 \cdot \left((15 \cdot e^{-t/3600} - 2 \sin 017943t) / \omega - 0.02 \right) \cdot e^{-98.06((15 \cdot e^{-t/3600} - 2 \sin 017943t) / \omega)} \left(15 \cdot e^{-t/3600} - 2 \sin 017943t \right)^3 \quad (37)$$

resulting:

$$W_{wind} = \int_0^{33} \left(\frac{1191.5 \cdot \left((15 \cdot e^{-t/3600} - 2 \sin 017943t) / \omega - 0.02 \right) \cdot e^{-98.06((15 \cdot e^{-t/3600} - 2 \sin 017943t) / \omega)}}{\left(15 \cdot e^{-t/3600} - 2 \sin 017943t \right)^3} \right) dt \quad (38)$$

Remark

Observation 1

In determining the value of wind energy captured by WT, two problems arise:

- (1) The estimated value of captured wind energy is based on the use of MM-WT, which is valid only under certain conditions, usually different from the operating conditions.
- (2) When calculating the integrity of the WT power, it is necessary to know the time variation of MAV, which is not known in advance. This can only be known later by solving the motion equation or by direct measurements. We can determine the maximum power of the TV corresponding to the wind speed at that time using the relationship:

$$P_{WT-MAX} = 0.62888 \cdot V^3$$

The wind speed varies, but between a maximum value: V_{MAX} and a minimum value: V_{MIN} , it is necessary to introduce the equivalent wind speed term, V_{ECH} , the speed at which the WT power has the same value as in the real case over a given time interval.

Determining the equivalent speed value in the graph of the function $V(t)$ that was obtained from the measurement of wind speed over time is complicated because V_{ECH} depends both on the evolution of wind speed and MAV, ω , on which WT works.

Considering the results of [11–14] the equivalent wind speed is calculated while using the relation [11–14]:

$$V_{ECH} = \sqrt[3]{\int_0^t (V(t))^3 dt / T} \quad (39)$$

or

$$V_{ECH} = \sqrt[3]{\int_0^{35} (15 \cdot e^{-t/3600} - 2 \sin 017943t)^3 dt / 35} = 15.054 \text{ [m/s]} \quad (40)$$

Based on this value, the maximum WT power can be obtained:

$$P_{WT-MAX} = 0.62888 \cdot V^3 = 0.62888 \cdot (15.054)^3 = 2145.5 \text{ [W]}$$

While considering the curve F->MPP from Figure 6, the medium WT power during Δt interval, it yields:

$$P_{WT-MED} = \frac{P_{WT-MAX} + P_{WT-F}}{2} \quad (41)$$

where P_{WT-F} is the power in point F. It results:

$$P_{WT-MED} = \frac{2145.5 + 2042.9}{2} = 2094.2 \text{ [W]} \quad (42)$$

The total win energy captured in this time interval is:

$$W_{eol} = P_{WT-MED} \cdot \Delta t = 2094.2 \cdot 33 = 69109 \text{ [J]} \quad (43)$$

The sum of energies: the kinetic energy $\Delta W_{KINETIK}$ corresponding to the two MAV $\omega(0)$ and $\omega_{OPTIM}(33)$ and the wind energy that is captured by the WT in the time interval $\Delta t = 33$ [s] implies the amount of energy that is required to be taken over by the generator on the F-MPP and it has the value

$$W_{G-REQ} = W_{eol} + W_{KINETIC} \quad (44)$$

The control is performed considering time intervals $\Delta t = 33$ [s].

Step 1; $t = 33 \div 66$ [s]

The value of the kinetic energy to be captured by the PMSG, between MAV

$$\omega(0) = 520 \text{ [rad/s]}$$

and $\omega_{OPTIM}(33) = 514.64$ [rad/s] is:

$$\Delta W_{KINETIC} = J \cdot (\omega_{t_k}^2 - \omega_{OPTIM-t_k}^2) / 2 = 45 \cdot (520^2 - 514.64^2) / 2 = 1.2478 \cdot 10^5 \text{ [J]} \quad (45)$$

The PMSG power, to reach the optimum MAV for the period Δt , is:

$$P_{G-REQ} = W_{G-REQ} / \Delta t = (W_{eol} + \Delta W_{KINETIC}) / 33 = (69109 + 1.2478 \cdot 10^5) / 33 = 5875.4 \text{ [W]} \quad (46)$$

$$1191.5 \cdot ((15)/\omega - 0.02) \cdot e^{-98.06 \cdot ((15)/\omega)} \cdot (15)^3$$

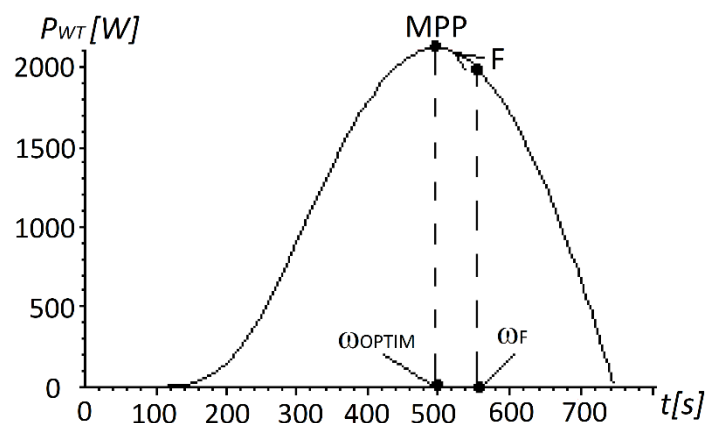


Figure 6. The movement of the operating point towards MPP.

The load resistance of the PMSG to reach the movement of the F point to MPP can be calculated from:

$$\begin{cases} P_{G-REQ} = 845\omega^2(5R + 8) \cdot \frac{4\omega^2 + 625R^2 + 2000R + 1600}{(1250R^2 + 4000R + 3200 + 7\omega^2)^2} \\ \omega = (\omega(0) + \omega_{OPTIM}(33))/2 \end{cases} \quad (47)$$

$$\begin{cases} 5875.4 = 845\omega^2(5R + 8) \cdot \frac{4\omega^2 + 625R^2 + 2000R + 1600}{(1250R^2 + 4000R + 3200 + 7\omega^2)^2} \\ \omega = 517.32 \end{cases} \quad (48)$$

with the solution

$$\omega = 517.32$$

$$R = 50.545$$

In these conditions, the motion equation for the EES becomes:

$$\begin{cases} 45 \cdot \frac{d\omega}{dt} = 1191.5 \cdot \left(\frac{(15 \cdot e^{-(t+33)/3600} - 2 \cdot \sin 0.17943 \cdot (t+33))}{e^{-98.06 \cdot ((15 \cdot e^{-(t+33)/3600} - 2 \cdot \sin 0.17943 \cdot (t+33))/\omega)}} - \left(15 \cdot e^{-(t+33)/3600} - 2 \cdot \sin 0.17943 \cdot (t+33) \right)^3 - \right. \\ \left. - 845\omega^2(5 \cdot 50.545 + 8) \cdot \frac{4\omega^2 + 625 \cdot 50.545^2 + 2000 \cdot 50.545 + 1600}{(1250 \cdot 50.545^2 + 4000 \cdot 50.545 + 3200 + 7\omega^2)^2} \right) \\ \omega(0) = 520 \end{cases} \quad (49)$$

The MAV over time is presented in Figure 7:

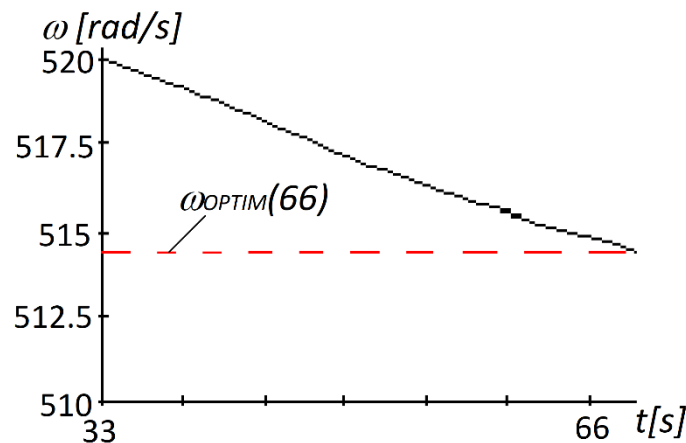


Figure 7. The MAV over time.

After $t = 33$ [s], MAV becomes

$$\omega(33 + 33) = 514.46 \text{ [rad/s]} \quad (50)$$

When comparing to the optimal one:

$$\omega_{OPTIM}(66) = 514.59 \text{ [rad/s]} \quad (51)$$

The differences are quite insignificant, below 0.025%.

Step 2; $t = 66 \div 99$ [s]

The value of the kinetic energy to be captured by the PMSG, between MAV

$\omega(33 + 33) = 514.46$ [rad/s] and $\omega_{OPTIM}(66) = 514.59$ [rad/s] is:

$$\Delta W_{KINETIC} = J \cdot (\omega_{t_k}^2 - \omega_{OPTIM-t_k}^2) / 2 = 45 \cdot (514.46^2 - 514.59^2) / 2 = -3010 \text{ [J]} \quad (52)$$

The wind energy captured by the WT during this time interval is:

$$W_{WIND} = \int_0^{33} \left(1191.5 \cdot \left(\left(15 \cdot e^{-(t+66)/3600} - 2 \cdot \sin 0.17943 \cdot t \right) / \omega - 0.02 \right) \cdot e^{-98.06 \cdot ((15 \cdot e^{-t/3600} - 2 \cdot \sin 0.17943 \cdot t) / \omega)} \cdot \left(15 \cdot e^{-t/3600} - 2 \cdot \sin 0.17943 \cdot t \right)^3 \right) \cdot dt \quad (53)$$

and at a medium value of MAV

$$\omega_{MED} = \frac{\omega(33 + 33) + \omega_{OPTIM}(66)}{2} = \frac{514.46 + 514.59}{2} = 514.53 \text{ [rad/s]} \quad (54)$$

The wind energy captured during the time interval $t = 66 \div 99$ [s] can be calculated as follows:

$$W_{WIND}(514.53) = \int_0^{33} \left(1191.5 \cdot \left(\left(15 \cdot e^{-(t+66)/3600} - 2 \cdot \sin 0.17943 \cdot (t + 66) \right) / 514.53 - 0.02 \right) \cdot e^{-98.06 \cdot ((15 \cdot e^{-(t+66)/3600} - 2 \cdot \sin 0.17943 \cdot (t+66)) / 514.53)} \cdot \left(15 \cdot e^{-(t+66)/3600} - 2 \cdot \sin 0.17943 \cdot (t + 66) \right)^3 \right) \cdot dt = 62467 \text{ [J]} \quad (55)$$

The sum of $\Delta W_{KINETIC}$ and $W_{WIND}(514.53)$ is:

$$W_{REQ} = W_{WIND}(514.53) + \Delta W_{KINETIC} = 62457 - 3010 = 59457 \text{ [J]} \quad (56)$$

The PMSG power, to reach the optimum MAV during Δt interval, is:

$$P_{G-REQ} = \frac{W_{G-REQ}}{dt} = \frac{59457}{33} = 1801 \text{ [W]} \quad (57)$$

The load resistance can be calculated, as follows:

$$\begin{cases} 1801.7 = 845\omega^2(5R + 8) \cdot \frac{4\omega^2 + 625R^2 + 2000R + 1600}{(1250R^2 + 4000R + 3200 + 7\omega^2)^2} \\ \omega = 514.53 \end{cases} \quad (58)$$

with the solution

$$\omega = 517.53$$

$$R = 241.5$$

In these conditions, the motion equation for the EES becomes:

$$\begin{cases} 45 \cdot \frac{d\omega}{dt} = 1191.5 \cdot \left(\left(15 \cdot e^{-(t+66)/3600} - 2 \cdot \sin 0.17943 \cdot (t + 66) \right) / \omega - 0.02 \right) \cdot e^{-98.06 \cdot ((15 \cdot e^{-(t+66)/3600} - 2 \cdot \sin 0.17943 \cdot (t+66)) / \omega)} \cdot \left(15 \cdot e^{-(t+66)/3600} - 2 \cdot \sin 0.17943 \cdot (t + 66) \right)^3 - \\ - 845\omega^2(5 \cdot 241.5 + 8) \cdot \frac{4\omega^2 + 625 \cdot 241.5^2 + 2000 \cdot 241.5 + 1600}{(1250 \cdot 241.5^2 + 4000 \cdot 241.5 + 3200 + 7\omega^2)^2} \\ \omega(0) = 514.46 \end{cases} \quad (59)$$

After $t = 99$ [s], MAV becomes

$$\omega(33 + 66) = 514.59 \text{ [rad/s]} \quad (60)$$

when comparing to the optimal one:

$$\omega_{OPTIM}(99) = 514.51 \text{ [rad/s]} \quad (61)$$

The differences are also insignificant, below 0.015% (Figure 8).

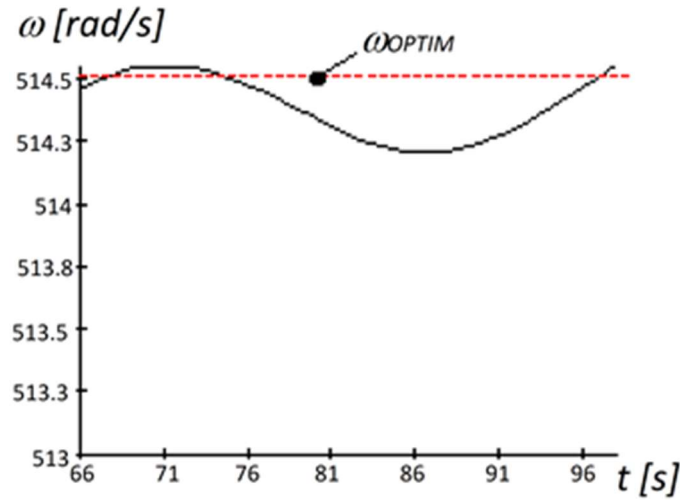


Figure 8. The evolution of the MAV during $\Delta t = 66 \div 99$ [s].

From the analysis of the above results, it can be observed that, in a very short time: $t = 99$ [s], EES reaches to operate in the optimal area of energy.

By estimating the wind speed, V , by measuring the maximum MAV, ω_{MAX-tk} from WT_{AUX} , it is possible to calculate $\omega_{OPTIM-tk}$ ensuring the optimal adjustment in the maximum energy area.

The adjustment algorithm is based on the optimal MAV determination based on the MAV value from the no-load operation of the WT_{AUX} .

Control algorithm

k -step at the moment t_k

- (1) measure the current MAV value, ω_{tk} , for the PMSG and maximum MAV, ω_{MAX-tk} , from WT_{AUX} ;
- (2) measure the power at PMSG at the operating point P and obtain the power value at WT, P_{WT-P} ; and,
- (3) MPP coordinates at power WT, optimum MAV, $\omega_{OPTIM-tk}$, and maximum power, P_{WT-MAX} , are obtained from ω_{MAX-tk} using the relations:

$$\omega_{OPTIM-tk} = 0.6623 \cdot \omega_{MAX-tk} \quad (62)$$

$$P_{WT-MAX} = 0.62888 \cdot \left(\frac{\omega_{MAX-tk}}{50} \right)^3 \quad (63)$$

- (4) With the measured MAV, ω_{tk} , measured and $\omega_{OPTIM-tk}$ values, the kinetics energy variations are obtained, over time intervals Δt , in the following form:

$$\Delta W_{KINETIC} = J \cdot (\omega_{tk}^2 - \omega_{OPTIM-tk}^2) / 2 \quad (64)$$

- (5) Calculate the value of the wind energy taken over by the WT, in the time interval Δt , knowing the value of the WT power at the operating point P and the maximum power, P_{WT-MAX} :

$$W_{WT} = ((P_{WT-P} + P_{WT-MAX}) / 2) \cdot (t_k - t_{k-1}) \quad (65)$$

- (6) Calculate the energy value, $W_{G-REQUIRED}$, which the generator should debit in the time interval Δt , to reach the optimum MAV, $\omega_{OPTIM-tk}$, in the time interval Δt :

$$W_{G-REQUIRED} = W_{WT} + \Delta W_{KINETIC} \quad (66)$$

- (7) The prescribed power value at the generator to reach the optimal MAV, $\omega_{OPTIM-tk}$, in the time interval Δt , is calculated from the energy value, $W_{G-REQUIRED}$, as:

$$P_{G-REQUIRED} = W_{G-REQUIRED} / \Delta t. \quad (67)$$

Regardless of the evolution of wind speed over time, we can estimate ω_{OPTIM} and thus achieve optimal energy control that is based on the value of the $\omega_{OPTIM}/\omega_{MAX}$ ratio, which does not change regardless of the wind speed evolution in time.

Based on the connection between ω_{OPTIM} and ω_{MAX} , and calculating the variation of kinetic energies, a simple and economically efficient driving system can be conceived, as seen in Figure 9.

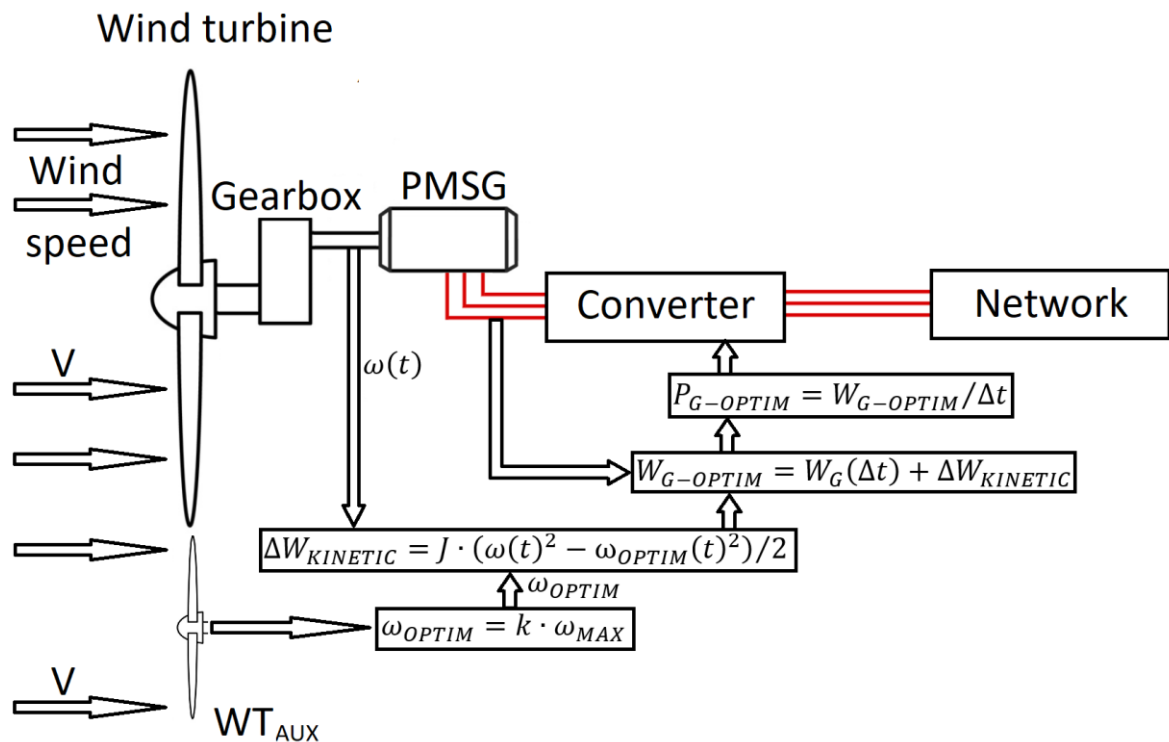


Figure 9. Optimal control of a wind system based on the connection between ω_{OPTIM} and ω_{MAX} .

4. Conclusions

This paper analyzes the operation of a wind power system so as to achieve optimal energy performance. By analyzing several cases, it was possible to obtain the basic parameters that lead to an optimal functioning from the energy point of view and the maintenance of the EES in the MPP area by measuring the speed and the estimation of the wind energy that is captured by the WT. The estimation of the wind energy captured by the WT is based on MM-WT, which requires a more accurate determination of it. The equivalent wind speed is defined and the optimal angular velocity is calculated, as a function of it. By knowing the optimal speed, the system can operate at the Maximal Power Point. The energy that is captured by the WT has maximum values in the MPP area. Through the simulations presented, the PMSG loads could be identified so that the WT + PMSG will reach the optimal area of in terms of energy in the shortest possible time. The method is based on the optimal MAV determination with an auxiliary WT operating in a no-load regime. The method is based on the fact that the optimal MAV value, ω_{OPTIM} , is directly proportional to ω_{MAX} . By measuring the maximum MAV, ω_{MAX} , using an auxiliary WT can determine, at any time, ω_{OPTIM} , regardless of the time variation of the wind speed. By calculating the variations of the kinetic energies and the measurement of the electric energy that is flowing by the generator, the value of the power to it is

determined, a value that is achieved by controlling the switching elements of the power converter between the generator and the network.

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