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A Probabilistic Multi-Objective Model for Phasor Measurement Units Placement in the Presence of Line Outage

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Abstract: Optimal phasor measurement units (PMU) placement was developed to determine the number and locations of PMUs on the premise of full observability of the whole network. In order to enhance reliability under contingencies, redundancy should also be considered beside the number of PMUs in optimal phasor measurement units placement problem. Thus, in this paper, a multi-objective model was established to consider the two conflicting components simultaneously, solved by ε-constraint method and the fuzzy satisfying approach. The redundancy here was formulated as average possibility of observability including random component outages, and full possibility formula was applied to calculate the average possibility of observability in the case of single line outage. Finally, the model was employed to the IEEE-57 bus system, and the results verified that the developed model could provide a placement scheme with higher reliability.

Keywords: average possibility of observability; ε-constraint; fuzzy satisfying approach average; multi-objective; PMU placement

1. Introduction

The wide-area measurement system (WAMS) was developed to provide real-time data for state estimator with high precision and great efficiency, in order to further guarantee the power system stability [1] and meet the requirement of a rapid increase in energy demand [2,3]. Phasor measurement units (PMU) can measure voltage phasor at the bus as well as current phasors of incident branches, thereby guaranteeing the observability of the bus where it is located and its adjacent buses [4–6]. In addition, due to restrictions of finance and technology, it is neither necessary nor economic to place PMU at each bus to assure the system becomes observable [7]. Thus, the optimal PMU placement (OPP) problem needs to be conducted to determine the location and number of PMU on the premise of full observability.

A great number of researches have contributed to the area of the OPP problem. The model consisted of the objective of minimum PMU number and constraint of full observability, was established, and solved by mixed-integer linear programming (MILP) and nonlinear programming (NLP) in [8], the branch-and-bound algorithm (BB) and binary-bonded genetic algorithm (BCGA) in [9]. To obtain a PMU placement scheme with higher redundancy, literature [10] formulated a generalized binary integer linear programming (ILP) model minimizing weighted degrees of nodes incorporating the topology changes. Besides, a multi-objective model was also proposed to simultaneously consider installation cost and measurement redundancy. Cellular learning automata (CLA) algorithm, artificial bee colony (ABC) algorithm, modified binary cuckoo optimization algorithm (MBCOA), weighted method, multi-objective evolutionary algorithm-based, were adopted to solve the multi-objective model in [11–15], respectively. In addition, to obtain the final solution from the Pareto front quickly and accurately, the fuzzy satisfying approach was applied in [16,17] and sparse neighborhood surroundings in [18]. Furthermore, the accuracy of state estimation was taken into consideration, subject to the constraint of supervisory control and data acquisition (SCADA) measurement data in [19] and the uncertainties of power system conditions in [20].

Literatures aforementioned analyzed the system observability from a deterministic view, including two contrary states, i.e., observable or unobservable. However, it was investigated in a probabilistic manner in [21–24]. In [21], to obtain a PMU placement scheme with unobservable probability resulted from 1st-order and 2nd-order failures of PMUs and line less than the pre-defined threshold, the unobservable probability was calculated iteratively by increasing PMU numbers gradually on the basis of initial PMU placement scheme, until the requirements were met. A multi-objective model was proposed, simultaneously minimizing the PMU cost and unobservable probability under PMU or line failures in [22]. However, random component outages were not taken into consideration in [21] and [22]. In [23], a probabilistic manner was adopted to differentiate the multiple solutions and showed significant priority comparing to measurement redundancy. A solution with maximum observable probability considering random component outages was selected as the final optimal scheme. In addition, when considering multistage PMU placement, the observable probability was regarded as the objective to optimize at each stage [24].

In this paper, firstly, *APO* of all buses in the network was calculated under two different cases, i.e., with and without consideration of single line outages. It is noticeable to consider the effect of single line outage. The *APO* was calculated by full probability formula due to the change of connectivity parameter. To better utilize data from PMUs, it is insufficient to depend on PMUs and transmission lines. Thus, in the process of calculating *APO*, the availability of random component outages was considered, and these components should be regarded as a series. Then, a multi-objective model was developed with objectives of minimization of the number of PMUs and maximization of the *APO* value. Furthermore, the Pareto front was obtained by ε-constraint, and fuzzy satisfying approach was applied to select the most preferred solution. Finally, the developed model was adopted for the IEEE-57 bus system, and results were compared to that in single-objective model.

2. Basic PMU Placement Model

In WAMS, PMUs transmit voltage phasors and current phasors to phasor data concentrator (PDC) through communication network, as shown in Figure 1. A PMU takes about 30 to 120 measurements per second. Therefore, if every bus is equipped with a PMU, not only the cost of PMU is high, but also a large amount of data generated will cause communication pressure and require powerful data processing capacity. Thus, for the consideration of economy and technology, the optimal PMU placement was proposed.

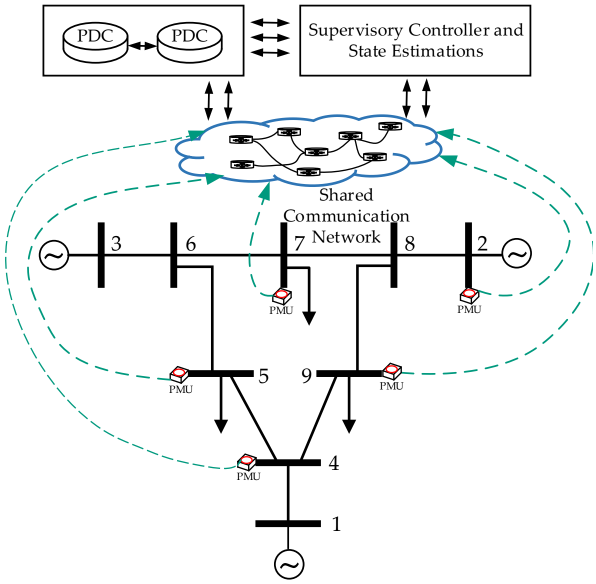

Figure 1. WAMS for the IEEE-9 bus system.

2.1. Optimal PMU Placement Model in Normal Condition

The basic target of the OPP problem is to minimize the cost and determine the locations of PMUs under the precondition of full observability of the whole system. To realize the system's full observability, each bus is observable by installing PMU at the bus or its adjacent buses. In addition, if the installation cost of each bus is regarded as the same, the objective could be substituted by minimizing the number of PMUs, and the OPP model could be described as follows:

$$\min F_1 = \sum_{i \in I} x_i \tag{1}$$

$$f_i \geq 1 \ \forall i \in I \tag{2}$$

$$f_i = \sum_{j \in I} a_{ij} x_j \tag{3}$$

$$x_i = \begin{cases} 1 \text{ if a PMU is placed at bus } i \\ 0 \text{ otherwise} \end{cases} \tag{4}$$

$$a_{ij} = \begin{cases} 1 \text{ if } i = j \text{ or bus } i \text{ is adjacent to } j \\ 0 \text{ otherwise} \end{cases} \tag{5}$$

where, F_1 is the objective representing the number of PMUs. I is the set of all the buses. x_i represents the installation status of PMU at bus i, and a_{ij} denotes the topology relationship between bus i and bus j. Observability function (f_i) in (2)–(3), as the constraints of the OPP problem, indicates that for each bus, it should be observed at least once to ensure its observability.

Above discussion is based on transmission lines in normal condition. However, line outages may affect system topology, and further change the observability function, namely Equations (2) and (3). Therefore, consideration of line outages will be discussed in the next section.

2.2. Optimal PMU Placement Model Considering Line Outage

Transmission lines are not fully reliable for a power system, in practice. Line outage contingency is one of the most common contingencies in a network. When line outage occurs, the topology of the network will change, meaning the change of connectivity parameters, i.e., a_{ij}, and the observability of the network may be destroyed. In such a case, more PMUs are required to guarantee complete observability. In addition, multiple line outages are similar to single line outage. Here, single line outage is discussed, similar to reference [25]. The difference is that zero injection bus (ZIB) is not taken into consideration here, and the reason will be introduced in the next section. The objective of the PMU placement model considering line outage is the same as that in normal condition. The difference between the two lies in the constraints:

$$f_i^l \geq 1 \; i \in I, l \in L \tag{6}$$

$$f_i^l = \sum_{j \in I} a_{ij}^l x_j \tag{7}$$

$$a_{ij}^l = \begin{cases} 0 \text{ if line } l \text{ is between bus } i \text{ and } j \\ a_{ij} \text{ otherwise} \end{cases} \tag{8}$$

where L is the set of all lines in a network; each element in L is denoted as l and regarded as a line outage contingency. It is noted that each line outage in single outage is mutually exclusive and they consist single line outage together. Apparently, Formulations (6)–(8) is similar to (2)–(3) and (5), and they express the same meaning. By comparing observability function in (2) and (6), it can be seen that the number of constraints increases, which is determined by the number of lines. Connectivity parameters a_{ij}, observability functions (f_i) in (2)–(3), (5) are replaced by a_{ij}^l and f_i^l in (6)–(8), respectively, corresponding to the l^{th} line outage.

3. Multi-Objective Model for OPP Problem

Most of the researches analyzed observability in a deterministic manner, namely observable or unobservable. In this paper, observability was analyzed from a probabilistic viewpoint. Possibility of observability was developed and defined for each bus and the whole system. The probabilistic index provides a quantitative description for observability.

In the process of calculation of possibility of observability, the effect of ZIB was contained in [23]. Results showed that the value of possibility of observability for buses whose observability is only from ZIB is quite lower compared to those from PMUs. Meanwhile, as Kirchhoff's current law equation of a ZIB may transmit deeper, the reliability of ZIB observation becomes weaker [26]. Therefore, despite with the consideration of ZIB effect in the OPP problem, the number of PMUs will be reduced [27], particularly for large-scale systems, in this paper, ZIBs are neglected.

3.1. Optimal Placement of PMU under Possibility

3.1.1. Possibility of Observability without Line Outage

As shown in Figure 2, the measurement system contains not only PMUs and their corresponding communication links transmitting data to higher level data concentrators, but also other critical components, i.e., current transformers (CT) measuring current phasors, potential transformers (PT) measuring voltage phasors. When calculating possibility of observability, the availability of these components should also be considered.

As is known, a bus can be observable by its own PMU or PMUs at its adjacent buses. Based on the types to guarantee observability are independent with each other, the possibility of observability can be expressed as:

$$PO_i = 1 - \prod_{j \in I} (1 - x_j A_{ij}) \; \forall i \in I \tag{9}$$

$$A_{ij} = a_{ij} A_j^{Vm} A_j^{PMU} A_j^{link} A_{ij}^{Cm} A_{ij}^{line} \tag{10}$$

PO_i is the possibility of observability for each bus. A_{ij} is a constant describing the possibility of observability of bus i resulted by PMU at bus j, and is calculated by expression (10). A_j^{Vm}, A_j^{PM}, A_j^{link}

describe the availability of voltage measurement, PMU, and communication line at bus j, respectively. A_{ij}^{Cm} and A_{ij}^{line} express the availability of current measurement and transmission line at line ij (A_{ii}^{Cm} = 1, A_{ii}^{line} = 1). Where: $A_j^{Vm} = (A_i^{PT})^3$ and $A_{ij}^{Cm} = (A_i^{CT})^3$, for three PTs and CTs are needed to measure corresponding phasor values, which are in series [28]. It is noted that when line outages are not considered in the OPP problem, the availability of transmission lines is regarded as 1.

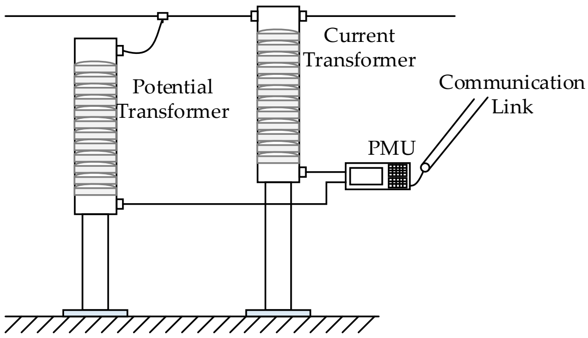

Figure 2. Measurement system installed in the substation.

3.1.2. Possibility of Observability with Line Outage

In the OPP problem under constraints (2)–(5), the possibility of observability can be calculated by formulas (9) and (10). However, when constraints (6)–(8) are considered, formulas (9) and (10) will not be adaptive due to the change of connectivity parameter a_{ij}. For this reason, the full possibility formula and conditional probability formula were applied to calculate possibility of observability under single line outage.

$$PO_i = \sum_{l \in L} PO_i^l P_l \tag{11}$$

$$PO_i^l = 1 - \prod_{j \in I} (1 - x_j A_{ij}^l)\ \forall i \in I \tag{12}$$

$$A_{ij}^l = a_{ij}^l x_j A_j^{Vm} A_j^{PMU} A_j^{link} A_{ij}^{Cm} A_{ij}^{line} \tag{13}$$

$$P_l = \frac{(1 - A_l) \prod\limits_{n \in L, n \neq l} A_n}{\sum\limits_{m \in L} \left((1 - A_m) \prod\limits_{n \in L, n \neq m} A_n\right)} = \frac{1/A_l - 1}{\sum\limits_{m \in L} (1/A_m - 1)} \tag{14}$$

PO_i^l is the possibility of observability for bus i, and A_l^{ij} is the possibility of observability of bus i resulted by PMU at bus j, when the l^{th} line is in outage. P_l is the outage possibility for l^{th} line, which is a conditional probability under the conditions of single line outage. Variables a_{ij}^l is introduced to show the distinction from normal condition. A_l is the availability of the l^{th} line. It is noted that when l^{th} line outage is assumed, the availability of other lines is equal to 1.

3.2. *Proposed Multi-Objective Model*

Possibility of observability discussed in Section 3.1 is an index related to each bus. To describe the characteristic of system observability in a probabilistic manner, *APO* is defined, which is calculated by the average value of possibility of observability of each bus, that is

$$APO = \frac{1}{n} \sum_{i \in I} PO_i \tag{15}$$

where n is the number of all buses.

To guarantee safe and stable operation of the network, the OPP problem should not only consider the minimum number of PMUs, but also measurement redundancy. Here, maximum measurement redundancy is addressed by maximum APO value, namely, minimum possibility of unobservability, which is considered as an additional objective function.

$$\min F_2 = 1 - APO = 1 - \frac{1}{n}\sum_{i \in I} PO_i \tag{16}$$

F_2 is the average possibility of unobservability (APUO) for all buses. For the two objective functions presented before, the first one in (1) is the number of PMUs representing the cost of PMUs to some extent. From the perspective of economical, the fewer the number of PMUs, the better. On the other hand, for the purpose of minimizing possibility of unobservability in (16), as many PMUs as possible should be employed. Therefore, the optimizing directions of two objective functions are the opposite. As a result, the multi-objective optimization is proposed. The multi-objective model is expressed as:

$$\begin{cases} \min F_1 = \sum\limits_{i \in I} x_i \\ \min F_2 = 1 - APO \\ \text{constraints (2)–(5) or (7)–(8)} \end{cases} \tag{17}$$

Apparently, formulation (17) contains the term formulated by the product of placement variables (x_i), thus it is a nonlinear model.

4. ε-Constraint Method and Fuzzy Satisfying Approach

The ε-constraint method is applied to effectively solve the proposed multi-objective problem proposed in Section 3.2. Based on a scalarization, the ε-constraint method optimizes one objective function, while others are transformed into additional constraints [28]. The mathematical description of ε-constraint method to optimize the two objectives can be presented as:

$$\begin{array}{l} \min F_2 \\ s.t. \\ \begin{cases} F_1 \le \varepsilon \\ \text{other constraints} \end{cases} \end{array} \tag{18}$$

Here, the first objective function F_1 is added to the constraints and limited by parameter ε representing the upper limit of F_1, for a minimization problem. Then, the model is converted into a single objective one. When ε varies from F_1^{min} to F_1^{max} with adjustable interval, a set of solutions are obtained, consisting of Pareto optimal sets for this multi-objective optimization problem. It is worthwhile to note that since the number of PMU is an integer, the interval is selected as 1, in addition, the solutions are a series of points and cannot be linked to lines.

After applying ε-constraint method to solve the problem, the Pareto front is obtained with a series of non-dominated solutions. Each non-dominated solution represents a PMU scheme. However, in practice, only one solution corresponding to one placement will be adapted to provide a win-win scheme. To select the best possible solution, a fuzzy satisfying approach is employed in which membership function is introduced.

Different objective functions have different units and range. To address this problem, the first step in the fuzzy satisfying approach is to normalize objective functions and map it to the interval

[0, 1] [29]. For an optimal problem with Q objective functions and P solutions, the membership function is defined as:

$$\mu_q^p = \begin{cases} 0, & f_q^{max} \le f_q^p \\ \frac{f_q^{max} - f_q^p}{f_q^{max} - f_q^{min}}, & f_q^{min} \le f_q^p \le f_q^{max} \\ 1, & f_q^p \le f_q^{min} \end{cases} \tag{19}$$

where f_q^{max} is the maximum value of the q^{th} objective function in the Pareto optimal sets, and relatively f_q^{min} is the minimum value. f_q^p is the objective value of the p^{th} solution for q^{th} objective function. μ_q^p means the optimality degree of q^{th} objective function subjecting to the p^{th} solution. It is noted that the closer the value of μ_q^p is to 1, the higher the optimality degree is. Thus, the objective functions are transferred to the same scale with a range of zero and unity. And then, calculate the optimality degree of the p^{th} solution as follow [30]:

$$\mu^p = \min\left\{\mu_1^p, \mu_2^p, \cdots, \mu_q^p\right\} \forall p = 1, 2, \cdots, P \tag{20}$$

The solution with maximum optimality degree will be selected as the most preferred solution:

$$\mu = \max\left\{\mu^1, \mu^2, \cdots, \mu^P\right\} \tag{21}$$

5. Numerical Study

In this paper, 2 cases for the IEEE-57 bus system (Figure 3) were implemented to verify the proposed approach. BONMIN in GAMS was applied to solve the multi-objective model in the form of mixed- integer nonlinear programing (MINLP).

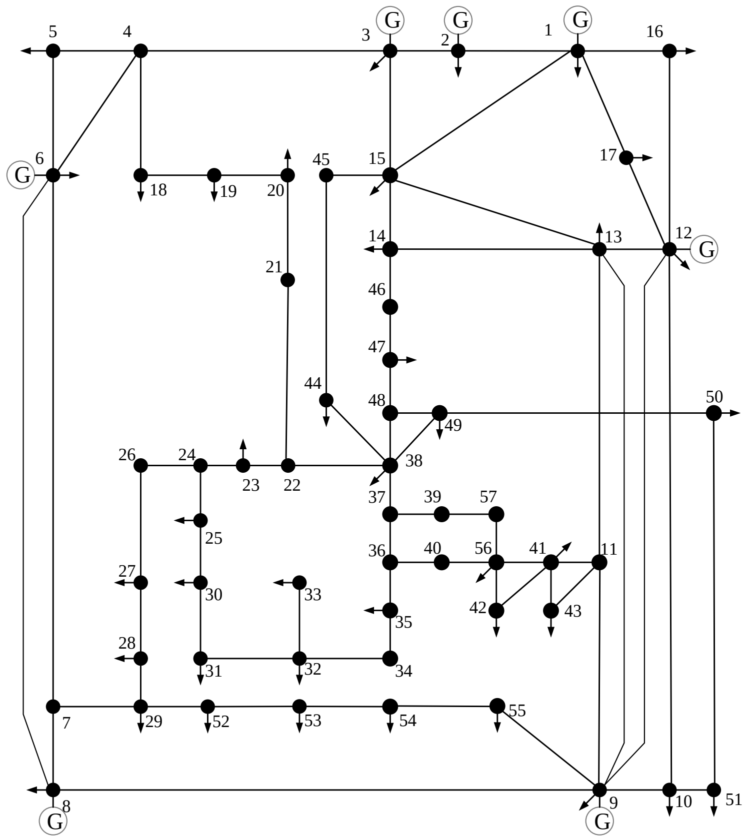

Figure 3. The IEEE-57 bus system.

When calculating the APUO value, the availability of PMUs, CTs, PTs, communication lines, transmission lines were all taken into consideration. Availability data of PMUs, CTs, PTs, transmission lines, and communication lines for the IEEE-57 bus system, which refers to literature [23], is shown in Table 1. To investigate the effect of single line outage on the possibility of unobservability in the OPP problem, two cases with and without single line outage are studied, and the results were compared.

Table 1. Availabilities of the IEEE-57 bus system.

Parameter	Value	Parameter	Value	Parameter	Value
A_i^{PMU}	0.99549768	$A_{12,16}$	0.9956	$A_{41,42}$	0.9968
A_i^{PT}	0.99854238	$A_{12,17}$	0.9929	$A_{41,43}$	0.9939
A_i^{CT}	0.99958447	$A_{14,15}$	0.9935	$A_{38,44}$	0.9927
A_i^{Link}	0.9990	$A_{18,19}$	0.9919	$A_{15,45}$	0.9960
$A_{1,2}$	0.9960	$A_{19,20}$	0.9948	$A_{14,46}$	0.9931
$A_{2,3}$	0.9944	$A_{21,20}$	0.9962	$A_{46,47}$	0.9938
$A_{3,4}$	0.9976	$A_{21,22}$	0.9974	$A_{47,48}$	0.9983
$A_{4,5}$	0.9923	$A_{22,23}$	0.9943	$A_{48,49}$	0.9943
$A_{4,6}$	0.9925	$A_{23,24}$	0.9931	$A_{49,50}$	0.9958
$A_{6,7}$	0.9929	$A_{24,25}$	0.9955	$A_{50,51}$	0.9922
$A_{6,8}$	0.9966	$A_{24,26}$	0.9966	$A_{10,51}$	0.9983
$A_{8,9}$	0.9944	$A_{26,27}$	0.9974	$A_{13,49}$	0.9953
$A_{9,10}$	0.9982	$A_{27,28}$	0.9965	$A_{29,52}$	0.9955
$A_{9,11}$	0.9955	$A_{28,29}$	0.9954	$A_{52,53}$	0.9953
$A_{9,12}$	0.9962	$A_{7,29}$	0.9948	$A_{53,54}$	0.9932
$A_{9,13}$	0.9973	$A_{25,30}$	0.9953	$A_{54,55}$	0.9962
$A_{13,14}$	0.9971	$A_{30,31}$	0.9925	$A_{11,43}$	0.9931
$A_{13,15}$	0.9955	$A_{31,32}$	0.9949	$A_{44,45}$	0.9952
$A_{1,15}$	0.9977	$A_{32,33}$	0.9920	$A_{40,56}$	0.9981
$A_{1,16}$	0.9943	$A_{34,32}$	0.9941	$A_{56,41}$	0.9972
$A_{1,17}$	0.9952	$A_{34,35}$	0.9919	$A_{56,42}$	0.9935
$A_{3,15}$	0.9937	$A_{35,36}$	0.9967	$A_{39,57}$	0.9952
$A_{4,18}$	0.9937	$A_{36,37}$	0.9938	$A_{57,56}$	0.9973
$A_{5,6}$	0.9981	$A_{37,38}$	0.9964	$A_{38,49}$	0.9961
$A_{7,8}$	0.9979	$A_{37,39}$	0.9939	$A_{38,48}$	0.9943
$A_{10,12}$	0.9962	$A_{36,40}$	0.9937	$A_{9,55}$	0.9971
$A_{11,13}$	0.9948	$A_{22,38}$	0.9979		
$A_{12,13}$	0.9940	$A_{11,41}$	0.9966		

Case 1: Multi-objective PMU placement without line outage (all the transmission lines are in normal condition, namely the availability of transmission lines is 1).

Case 2: Multi-objective PMU placement with single line outage.

For the two cases, the multi-objective optimal models are solved by ε-constraint method and the Pareto fronts are shown in Figure 4. By comparing the Pareto fronts for two cases in Figure 4, it can be seen that to maintain the full observability of the network, the minimum number of PMU required in case 2 is larger than in case 1. In addition, for the same PMU number, the APUO value in case 1 (ignoring line outages) is smaller than that in case 2 (considering single line outage) because availability of transmission lines in case 1 is assumed as 1, meaning that transmission lines are all in normal condition. In other words, to reach the same possibility of observability, more PMUs corresponding to higher cost are required in case 2 compared to case 1. In addition, it can be observed from Figure 4 that, the APUO value decreases with the number of PMUs increasing. However, when the number of PMUs reaches at a certain larger value, the APUO value is gradually near 0 but never equals to 0 as the availability of each component will never be 1. It means APUO value will not decrease a lot, even if more PMUs are added.

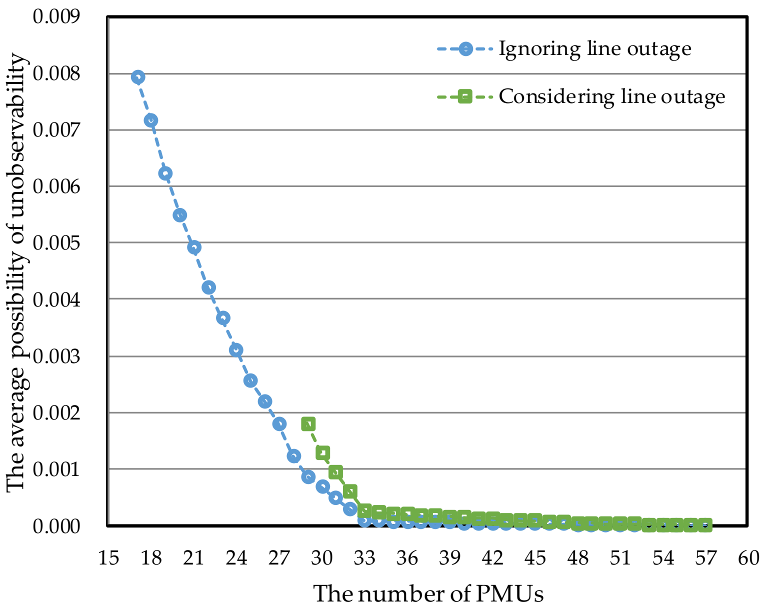

Figure 4. Obtained Pareto fronts for two cases.

To select the most preferred solution from Pareto solutions, the fuzzy satisfying approach was applied to calculate the optimality degree of each Pareto solution, which was described in the form of histogram in Figure 5. It is worthwhile that for the solutions with minimum PMUs and maximum PMUs, the optimality degree is equal to 0 according to equation (20). Apparently, the optimality degree increases firstly and then decreases with the change of PMU numbers. A compromised scheme was conducted, and the final solution was determined by the maximum optimality degree, which was expressed as solid ones in Figure 5. Detailed information for the final solution was depicted in Table 2, and N_{PMU} expresses the number of PMUs. For the solution under case 2 provided in Table 2, even if the PMU at bus 4 or transmission line between 4 and 6 is in outage, bus 6 is still observable due to the PMU at bus 6. Other buses can be analyzed similarly, thereby ensuring the adequacy of the system after implementation.

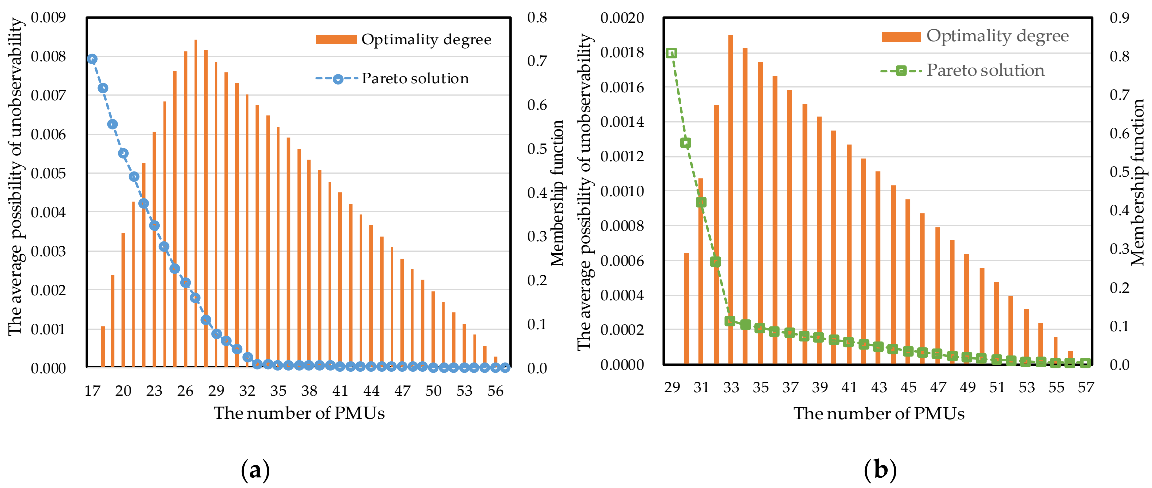

Figure 5. Pareto fronts and optimality degree. (**a**) Pareto fronts and optimality degree ignoring line outage; (**b**) Pareto fronts and optimality degree considering line outage.

To demonstrate the effectiveness of the developed model, single-objective model minimizing the number of PMUs was solved, and then APUO value was calculated by formulation (9) and (10) and (11)–(14) to compare with that in multi-objective model. Table 3 shows the details about the solutions for the two models under two cases. Figure 6 is the comparison between single-objective and multi-objective for the IEEE-57 bus system under the two cases. By analyzing the data in Table 3, it can be seen that the difference of the placement schemes between the two models lies in the locations of the PMUs. For example, 29 PMUs are needed in both models under case 2, and the majority of PMUs are placed at the same location, namely, bus 1,3,9,12,20,22,24,27,29,30,32,33,35,39,47,51,53,55,57; meanwhile, the rest few PMUs are placed at a different location, namely, bus 4,6,11,15,19,36,41,44,46,49 in multi-objective model and bus 5,7,14,18,38,40,42,43,45,50 in single-objective model. Obviously, the sum of adjacent buses of bus 4,6,11,15,19,36,41,44,46,49 is larger than bus 5,7,14,18,38,40,42,43,45,50. Thus, compared to single-objective models, parts of PMUs in multi-objective models are placed at the buses with more adjacent buses. In addition, with an equal number of PMUs, the APUO value reduced by 12.47% and 39.60% under the two cases, respectively. Apparently, the developed placement model can provide solutions with higher reliability.

Table 2. Detailed information for the most preferred solution.

Case	N_{PMU}	APUO	Membership Function	Location of PMUs
Case 1	27	0.00181	0.750	1,4,6,9,12,15,19,20,22,24,26,28,29,30, 32,35,36,38,39,41,44,46,47,50,53,54,56
Case 2	33	0.00025	0.857	1,3,4,6,9,11,12,15,19,20,22,24,26,28,29,30,31, 32,33,35,36,37,38,41,45,46,47,50,51,53,54,56,57

Table 3. Details about the solutions for the two model under two cases.

Case	Model	N_{PMU}	APUO	Location of PMUs
Case 1	Multi-objective	17	0.00793	1,4,6,9,15,20,24,25,28, 32,36,38,41,46,50,53,57
	Single-objective	17	0.00906	1,6,9,15,19,22,25,27,28, 32,36,41,45,47,50,53,57
Case 2	Multi-objective	29	0.00180	1,3,4,6,9,11,12,15,19,20,22,24,27,29,30, 32,33,35,36,39,41,44,46,47,49,51,53,55,57
	Single-objective	29	0.00298	1,3,5,7,9,12,14,18,20,22,24,27,29,30,32, 33,35,38,39,40,42,43,45,47,50,51,53,55,57

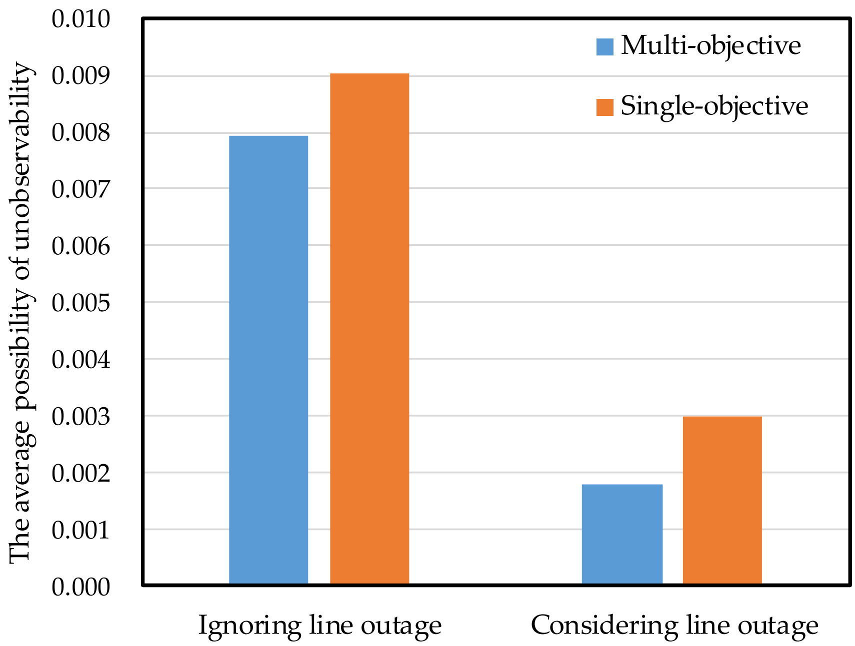

Figure 6. Comparison between single-objective and multi-objective.

6. Conclusions

In this paper, a multi-objective model for the OPP problem was developed with objectives of minimizing the number of PMUs and maximizing redundancy subjecting to complete observability to enhance the reliability of measurement system. Moreover, ε-constraint method was applied to solve the model by transforming the second objective into an extra constraint, and fuzzy satisfying approach is used to select the most compromised solution. Furthermore, the developed model is employed the IEEE-57 bus system and Pareto fronts under different cases were obtained. It can be concluded that most of the PMUs are placed at the bus with more adjacent buses in multi-objective model comparing with single-objective models. In other words, due to the different locations of PMUs, the APUO value in multi-objective model reduced by 12.47% and 39.60% than that in single-objective model under the two cases, respectively. Therefore, the developed placement model can provide solutions with higher reliability.

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