Article

Managing Procurement for a Firm with Two Ordering Opportunities under Supply Disruption Risk

Kelei Xue[iD]**, Ya Xu * and Lipan Feng**

Business School, Nankai University, Tianjin 300071, China; xkl7111@163.com (K.X.); hnnuflp@163.com (L.F.)
* Correspondence: xuya@nankai.edu.cn

check for updates

Abstract: Supply disruption is a common phenomenon in industry, which brings destructive effects to downstream firms and damages the sustainability of the supply chain. To mitigate the supply disruption risk, the authors investigate two types of procurement strategies for a firm with two ordering opportunities. Through establishing Stackelberg game models, the authors drive the supplier's optimal production, and the firm's optimal procurement and replenishment strategies under the option purchase (OP) strategy and the procurement commitment (PC) strategy, respectively. The findings show that, under both types of strategies, the firm's procurement follows a "threshold" principle. Moreover, the firm's procurement quantity can be represented by two newsvendor solutions. A lower option price or option exercise price benefits the firm, while it damages the supplier. The supplier benefits from a higher mean value (MV) of emergency procurement price and the firm benefits from a lower market demand variability. Counter-intuitively, a lower MV of the emergency procurement price is not always beneficial to the firm. A higher market demand variability could be beneficial to the supplier under the PC strategy. The firm should first choose the PC strategy and then change to the OP strategy as the disruption risk increases.

Keywords: supply disruption; option purchase strategy; procurement commitment strategy; random emergency procurement price; strategy selection

1. Introduction

The recent trade disputes between the US and China and the increasing frequency of natural and anthropogenic catastrophes indicate the need for organizations to hedge their supply chains against major disruptions [1,2]. Supply chain disruption might result in severe problems in many aspects, such as enterprise bankruptcy, job losses, stock market crash, panic buying, and more. All of these not only damage the reputation of the related firm and the interests of shareholders, but also lead to social instability, which affects the sustainability of the supply chain [3,4]. The recent U.S. ban on exports of Chinese telecommunications equipment and smartphone maker ZTE, which employs roughly 75,000 workers and had been worth about $20 billion before the ban, immediately put the company out of business and in a state of shock (https://www.scmp.com/tech/article/2142557/zte-calls-us-government-ban-extremely-unfair-vows-fight-its-rights, accessed date 20 June 2018), for example. The Great East Japan Earthquake in March 2011, and the subsequent Fukushima nuclear accident, impacted some highly concentrated industrial facilities on which many industries worldwide depended, so numerous supply chains across many industries were disrupted [5]. A large number of firms, including those in other countries such as the US, South Korea, Thailand, and France, had to cease operations due to shortage of supply (https://www.rieti.go.jp/en/columns/v01_0085.html, 25 April 2017).

To manage the supply disruption risk and maintain the sustainability of the supply chain, many strategies have been proposed, such as multi-sourcing, emergency sourcing/production, holding

inventory, and supplier process improvement [6–12]. The authors investigate two types of procurement strategies, namely, the option purchase (OP) strategy and the procurement commitment (PC) strategy, for a firm with two ordering opportunities to mitigate the supply disruption risk. This study is motivated by the procurement risk management practice at Hewlett-Packard (HP) company [13,14]. To manage the uncertainties in product demand, component price, and component availability, HP adopted a combination of traditional long-term contracts and more flexible short-term contracts. The PC strategy and OP strategy used in this paper, in fact, belong to the traditional long-term contract and flexible short-term contract, respectively. Under the PC strategy, the firm can buy a fixed quantity at a relatively low and fixed price before the demand is realized. Using the OP strategy, the firm can reserve a pre-determined amount of stock beforehand and adjust its ordering quantity at selling season. Additionally, the authors also consider the situation where the firm has two ordering opportunities. That is to say, the firm can order from the supplier before and after the demand is realized, respectively. Actually, adopting two or more ordering opportunities is a common phenomenon for a firm [15]. Regarding the case of HP, for example, it could procure from its suppliers in advance or postpone its procurement after the demand uncertainty is resolved [13,14]. International Business Machines Corporation (IBM) and Oracle also use such a flexible approach to purchasing [16].

Supply disruption usually brings about an increase and uncertainty in price. The devastating shortage of flu vaccine occurred in the US in 2004, which was caused by bacterial pollution, and led to a dramatic increase in vaccine price from $60 to $800 [17], for example. The earthquake in Japan led to the increasing prices of the affected products [12,18,19]. Moreover, even if the disruption does not occur, the procurement price can also exhibit high volatility. The price of Dynamic Random Access Memory (DRAM) adopted by HP dropped by over 90% in 2001 and more than tripled in 2002 [13] for example, in the semiconductor industry. Due to this, the current work also takes the uncertainty of procurement price into account. The authors, in other words, assume that the procurement price at selling season (or emergency procurement price) is stochastic. Moreover, it is assumed that the supplier can begin emergency production with the help of the downstream firm when the disruption happens. The emergency production cost is higher than the normal production cost, but it is not necessarily lower than the procurement price at selling season due to its high volatility (refer to the case of HP). Therefore, the supplier needs to trade-off the normal production and emergency production to satisfy the demand of the firm with two ordering opportunities. Generally speaking, the randomness of the procurement price at selling season has an important influence on the production and procurement strategies.

To summarize, the authors consider a decentralized supply chain consisting of a supplier and a firm in the context of frequent disruptions. Based on the real-world cases, the authors consider the uncertainties in supply, demand, and the emergency procurement price. The authors explore the supplier's optimal production and the firm's optimal procurement and replenishment strategies under the option purchase strategy and procurement commitment strategy, respectively. The authors also give the optimal strategy selection of the firm. Sensitivity analysis is conducted also to study the influences of the system's parameters. The main findings are as follows: (1) Under both types of strategies, the firm's procurement follows a "threshold" principle. Moreover, the firm's procurement quantity can be represented by two newsvendor solutions; (2) Using the OP strategy, the firm tends to reserve more options to deal with the increasing supply disruption risk. A lower option price or option exercise price benefits the firm, while it damages the supplier; (3) The supplier benefits from a higher mean value (MV) of emergency procurement price and the firm benefits from a lower market demand variability. Counter-intuitively, a lower MV of emergency procurement price is not always beneficial to the firm. A higher market demand variability could be beneficial to the supplier under the PC strategy; (4) The firm should first choose the PC strategy and then change to the OP strategy as the disruption risk increases.

The rest of the paper is structured as follows. Section 2 is the literature review. Section 3 describes the problem and model. Section 4 investigates the equilibrium decisions of the supplier and the firm under the OP strategy and PC strategy, respectively. In Section 5, the authors compare the optimal

decisions and profits of the supplier and firm under the two types of strategies and give the firm's optimal strategy selection. Section 6 draws some concluding remarks and gives the future research directions. All the proofs of lemmas, theorems, and corollaries are given in the Appendix A.

2. Literature Review

There are two main streams of literature which are most related to this paper. One stream is about procurement strategies for managing supply risks. The other stream is about adopting option contracts to deal with supply risks. The authors also refer readers to Tomlin [6,20], Tang [21], Snyder et al. [22], and references therein for more approaches to managing supply risk.

The first stream discusses the procurement strategies for managing supply risk. Feng et al. [11] study a retailer's procurement strategy when supply reliability is endogenously determined. Yan et al. [23] study a buy-back contract coordination strategy when facing both disruption and demand risks. Wang et al. [7] study the dual sourcing strategy and process improvement strategy to mitigate supply risk, respectively. They characterize the optimal improvement efforts and procurement quantities and generate many managerial insights. Considering the interdependency between disruption risks, Zhao and Freeman [24] propose a robust sourcing strategy to manage the ambiguously correlated major disruption risks that arise in a supply chain. Yin and Wang [25] study the optimal sourcing decisions for a firm with a dedicated supplier and a backup supplier. They consider three kinds of cooperation strategies: advance purchase, reservation, and contingency purchase, and give the optimal strategy selection of the firm. Looking at a following study, Wang and Yin [26] consider the risk-averse property of the firm and study the optimal sourcing and pricing problem in a similar setting. Unlike the current paper, they explore how a firm cooperates with a reliable supplier, but not an unreliable supplier, to mitigate disruption risk. He et al. [27] compare the emergency procurement (EP) strategy and the optimal allocation procurement (OAP) strategy in the presence of supply disruption risks. However, they do not consider the uncertainties of the demand and emergency procurement price. Bolandifar et al. [28] find that a simple linear contract can assure supply under a noncontractible capacity and asymmetric cost information. Hwang et al. [29] show that wholesale price contracts can perform well in inducing reliable supply and they also identify when and why they perform well. The above two papers show the importance of the wholesale price contact and linear contract, which are both explored in the current paper. However, they investigate them in an asymmetric environment or in random capacity/yield risk; they also do not consider them with two ordering opportunities under supply disruption risk.

The second stream of related research discusses option contracts to deal with supply risks. Ma et al. [15] consider a risk averse retailer who has two opportunities to make order decisions with demand forecast updating. Considering their study, the buyer makes an initial purchase decision with a preliminary demand forecast. Once the demand information is updated, the retailer can order again. However, they do not consider the supply risk. Examining a random yield problem, Xu [30] considers the uncertainty of instant ordering price and studies how to manage the production and procurement through an option contract in a decentralized supply chain consisting of one supplier and one manufacturer. Found in Xu's paper, the unreliable supplier can adopt normal and emergency production to satisfy the option ordering and instant ordering of the manufacturer. Li et al. [31] investigate the optimal production and ordering strategies of the manufacturer and the retailer under the commitment unidirectional call option and the commitment bidirectional option contracts, respectively. However, the above two papers study the random yield risk, which is different from the disruption risk in the current research. Köle et al. [32] establish three models depending on the level of information available when the options from the reliable supplier are exercised. They investigate the effectiveness of the option contract. Different from this paper, they consider the cooperation between a reliable supplier and a firm to mitigate the possible disruption risk. Xia et al. [33] study two kinds of contract mechanisms (option contract and firm order contract) to share demand and supply risks in a decentralized supply chain. Unlike their study, this work explores the situation where

a third-party emergency source is non-existent and the emergency procurement price is uncertain. Recently, Xue et al. [34] also explored how to mitigate the supply disruption risk through contract selection. However, they do not consider the uncertainty of emergency procurement price and the firm's two ordering opportunities.

Additionally, there are also literature which consider multi-sourcing, holding inventory, financial instruments and supply chain network design when facing disruption risk. Li and Li [35] study a dual sourcing problem when facing supply disruption risk and consider the loss-averse behavior of the firm. Dong and Tomlin [8] and Zhen et al. [9] both consider how to use business interruption insurance and other operational strategies, for example, holding inventory, backup sourcing, to manage disruption risk. Mizgier et al. [36] use an agent-based modeling (ABM) approach and analyze how the local disruptions form and propagate through the dynamic supply chain network. Viewing another study, Mizgier et al. [37] propose a generic model for calculating the loss distribution due to disruptions in the supply chain network (SCN) and generate valuable insights which can help to implement SCN designs. Wagner et al. [38] conduct an exploratory research to investigate the relationship between business cycles and disruption risk in financial services and manufacturing industries. They find there exists a positive lagged relationship between operational disruptions and business cycles.

Finally, to the best of the authors' knowledge, this paper is one of the first to investigate the firm's strategy selection problem among different types of procurement strategies with random emergency procurement price under both supply disruption and demand risks. The main contributions are given as follows.

(1) The authors study how a firm should take full advantage of multiple ordering opportunities and different procurement strategies to mitigate supply disruption and demand risks and improve the reliability of the supply chain;

(2) Beside the supply risk and demand risk, the authors also consider the uncertainty of the emergency procurement price at selling season in this model and explore the impacts of random supply, demand, and emergency procurement price on the firm's procurement strategy and supply chain performance;

(3) Under the constantly changing operational environment, the authors investigate how a firm should adjust the procurement strategy to maintain the sustainability of the supply chain and even improve supply chain performance.

3. Model Descriptions

Consider a decentralized supply chain consisting of a supplier (he) and a firm (she) who sources key components or products from the supplier to satisfy the random market demand ζ. The mean value of random demand is μ and the probability density function (PDF) and cumulative distribution function (CDF) are $f(\cdot)$ and $F(\cdot)$, respectively. The retail price is assumed to be r per unit. The supplier is subject to a random disruption (exogenous) whose probability of occurrence is β. When disruption occurs, no products are delivered to the firm; otherwise, all the ordering of the firm will be satisfied. This "all-or-nothing" type of disruption is widely used in the operations management field [6,10,20,25,32–35,39].

Concerning the firm, there are two types of procurement strategies. One is the option purchase (OP) strategy and the other is the procurement commitment (PC) strategy. Moreover, the firm has two ordering opportunities to replenish her inventory under the OP or PC strategy. One is before the selling season and the other is at the selling season. The authors call the second procurement opportunity emergency procurement or instant ordering. The authors assume that the emergency procurement price P is random, and its CDF and PDF are $G(p)$ and $g(p)$ with support $P \in [A, B]$.

All the notations used in this paper are concluded as follows:

C	Supplier's production cost per unit
S	Product's salvage value per unit
c_e	Supplier's emergency production cost per unit
β	Probability of disruption occurrence
$\zeta, f(\cdot), F(\cdot)\ \mu$	Random market demand and its PDF, CDF and mean value
c_o	Option price per unit
w	Option exercise price or wholesale price per unit
r	Product's retail price per unit
$P, g(p), G(p)$	Random emergency procurement price and its PDF and CDF, $P \in [A, B]$
π_j^i, Π_j^i	The profit, and expected profit of j under i strategy, $i = \{O, W\}$, O = option purchase strategy, W = procurement commitment strategy, $j = \{S, R\}$, S = supplier, R = firm
Decision variables	
Z_i	Supplier's production quantity under i strategy
Q_i	Firm's (option) order quantity under i strategy

Under the OP strategy (Figure 1), before the selling season, at time t_0, the firm decides the option order quantity Q_o and the unreliable supplier decides his production quantity Z_o afterwards. When the selling season (t_1) begins, after the firm observes the realized demand and the emergency procurement price, she exercises the options and decides whether to instant order from the supplier with price p. The option price and exercise price are c_o and w, respectively. Regarding the unreliable supplier, if he is disrupted or his inventory is not enough, the supplier can carry out emergency production to satisfy the option ordering or emergency ordering. The emergency production here represents the situation where the supplier can work overtime, begin using the backup production line, dig spare capacity, or just use the same or high-end product in inventory for alternative supply [10,30,40]. The supplier's production cost is c and the emergency production cost is c_e ($c_e > c$). All unsold product will be salvaged with price s per unit.
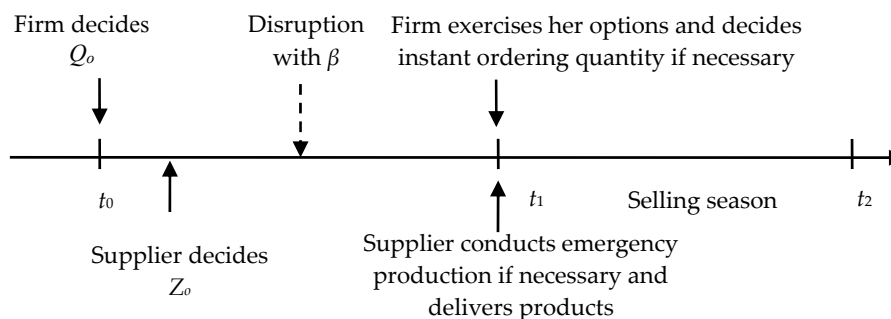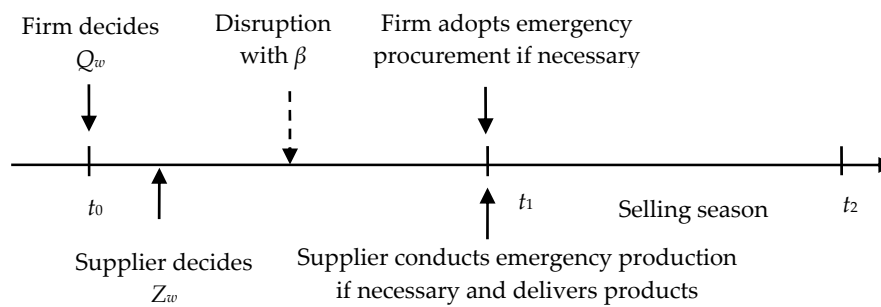

Figure 1. Sequence of events under the option purchase strategy.

Under the PC strategy (Figure 2), at time t_0, the firm first decides the order quantity Q_w with wholesale price w. Subsequently, the unreliable supplier decides the production quantity Z_w. Once the firm observes the realized demand and emergency procurement price at time t_1, she decides whether to instant order from the supplier. When the remnant inventory is enough, her emergency ordering will be satisfied. The authors assume when the instant price is relatively high ($p > c_e$) and the disruption does not happen, the supplier can adopt emergency production to satisfy the firm's instant ordering.

The authors assume the supplier and firm are both risk-neutral and they have complete information about each other. The relationship between the supplier and the firm is characterized by a Stackelberg game, where the firm acts as the leader and the supplier is the follower. The subscripts "S", "R", "o", and "w" denote the supplier, the firm, the OP strategy, and the PC strategy, respectively. The superscript "*" denotes the optimum.

Figure 2. Sequence of events under the procurement commitment strategy.

Throughout this paper, the authors make the following assumptions:

(1) $s < c$, which ensures the salvage value is lower than the production cost;
(2) $c < c_o + w$ and $c_o + w < r$, which ensure the supplier and the firm both have positive profits under the OP strategy;
(3) $c < c_e$, which represents the emergency production cost is higher than the normal production cost;
(4) $A < c_e < B < r$, which ensures the firm can achieve positive profit through instant ordering;
(5) $c_o + w < A$, which denotes the emergency purchase price is higher than the option purchase price;
(6) $(1 - \beta)\hat{p} - c \geq 0$, which ensures the supplier has a positive profit if he produces.

The authors define: $\hat{p} := \underline{p}(c_e) + \overline{G}(c_e)c_e$, $\widetilde{p} := E(p) = \underline{p}(c_e) + \overline{p}(c_e)$, $p_1 := \beta r G(c_e) + (1 - \beta)\underline{p}(c_e) + \overline{p}(c_e)$, $p_2 := r G(c_e) + \overline{p}(c_e)$. Then, it is easy to derive the following condition: $A < \hat{p} < \widetilde{p}$, c_e; $\widetilde{p} < p_1 < p_2 < B$.

4. Model Analyses

The authors explore the equilibrium production and procurement decisions of the unreliable supplier and the firm under the option purchase (OP) strategy and procurement commitment (PC) strategy, respectively.

4.1. Option Purchase Strategy

4.1.1. Supplier's Production Decision

Given the firm's option ordering quantity Q_o, let $\pi_S^O(Z_o|Q_o)$ denote the supplier's profit. When the realized emergency procurement price is lower than the emergency production cost, $p < c_e$, then the supplier's expected profit is:

$$\pi_S^O(Z_o|Q_o, p < c_e) = -cZ_o + c_oQ_o + wE[\min\{Q_o, \zeta\}] + (1 - \beta)pE[\{\min\{Z_o, \zeta\} - Q_o\}^+] \\ +(1 - \beta)sE[\{Z_o - \zeta\}^+] - \beta c_e E[\min\{Q_o, \zeta\}] - (1 - \beta)c_e E[\{\min\{Q_o, \zeta\} - Z_o\}^+] \tag{1}$$

where the first three terms denote the production cost, the option ordering and final option exercise revenues, respectively; the fourth term is the revenue for satisfying the firm's emergency procurement; the fifth term represents the salvage value; and the final two terms are the emergency production investments. Due to the low emergency procurement price, the firm's emergency procurement demand will be met only with leftover inventory. The supplier's emergency production only is used to satisfy the firm's possible exercised option ordering. $p \geq c_e$, then the supplier's expected profit is:

$$\pi_S^O(Z_o|Q_o, p \geq c_e) = -cZ_o + c_oQ_o + wE[\min\{Q_o, \zeta\}] + pE[\{\zeta - Q_o\}^+] + \\ (1 - \beta)sE[\{Z_o - \zeta\}^+] - \beta c_e E[\zeta] - (1 - \beta)c_e E[\{\zeta - Z_o\}^+]. \tag{2}$$

Due to the high emergency procurement price in this situation, all the firm's emergency procurement demand will be met by the supplier with normal production and emergency production.

The authors, for convenience, define $\underline{p}(c_e) = \int_A^{c_e} xg(x)dx$, $\overline{p}(c_e) = \int_{c_e}^B xg(x)dx$, $G(c_e) = \int_A^{c_e} g(x)dx$, $\overline{G}(c_e) = \int_{c_e}^B g(x)dx$. Therefore, considering the randomness of the emergency procurement price, the supplier's expected profit can be given by:

$$\Pi_S^O(Z_o|Q_o) = -cZ_o + c_oQ_o + wE[\min\{Q_o, \zeta\}] + (1-\beta)\underline{p}(c_e)E[\{\min\{Z_o, \zeta\} - Q_o\}^+]$$
$$+ \overline{p}(c_e)E[\{\zeta - Q_o\}^+] + (1-\beta)sE[\{Z_o - \zeta\}^+] - \beta c_e G(c_e)E[\min\{Q_o, \zeta\}] \tag{3}$$
$$- (1-\beta)c_e G(c_e)E[\{\min\{Q_o, \zeta\} - Z_o\}^+] - \beta c_e \overline{G}(c_e)E[\zeta] - (1-\beta)c_e \overline{G}(c_e)E[\{\zeta - Z_o\}^+],$$

where the first three terms have the same meaning as above; the fourth and fifth terms are the revenues for the possible instant ordering when the emergency procurement price is relatively low and high, respectively; the last four terms are the emergency procurement costs when the disruption occurs and not, and the emergency procurement price is low and high, respectively. The supplier's expected profit can also be formulated as:

$$\Pi_S^O(Z_o|Q_o) = -cZ_o + c_oQ_o + w(\int_0^{Q_o} xf(x)dx + \int_{Q_o}^{+\infty} Q_of(x)dx)$$
$$+ (1-\beta)\underline{p}(c_e)(\int_{Q_o}^{\max\{Z_o, Q_o\}} (x - Q_o)f(x)dx + \int_{Z_o}^{+\infty} \{Z_o - Q_o\}^+ f(x)dx)$$
$$+ \overline{p}(c_e)\int_{Q_o}^{+\infty} (x - Q_o)f(x)dx + (1-\beta)s\int_0^{Z_o} (Z_o - x)f(x)dx -$$
$$\beta c_e G(c_e)(\int_0^{Q_o} xf(x)dx + \int_{Q_o}^{+\infty} Q_of(x)dx) - \tag{4}$$
$$(1-\beta)c_e G(c_e)\left(\begin{array}{c} \int_{Z_o}^{\max\{Z_o, Q_o\}} (x - Z_o)f(x)dx \\ + \int_{Q_o}^{+\infty} \{Q_o - Z_o\}^+ f(x)dx \end{array} \right) - \beta c_e \overline{G}(c_e)\int_0^{+\infty} xf(x)dx - (1-$$
$$\beta)c_e \overline{G}(c_e)\int_{Z_o}^{+\infty} (x - Z_o)f(x)dx.$$

Taken from Equation (4), it can be found that $\Pi_S^O(Z_o|Q_o)$ has different forms when $Z_o > Q_o$ or $Z_o < Q_o$. Exploring the properties of $\Pi_S^O(Z_o|Q_o)$, Lemma 1 can be derived.

Lemma 1. *The supplier's profit function, $\Pi_S^O(Z_o|Q_o)$ is continuous for $Z_o \in [0, +\infty)$. Additionally, it is differentiable and concave in Z_o for $Z_o \in [0, Q_o]$ and $Z_o \in [Q_o, +\infty)$, respectively.*

Let $Z_o^*(Q_o)$ denote the best response function. Based on Lemma 1, by the first order condition, the supplier's optimal production decision can be derived. Define Z_o^l and Z_o^h satisfying $F(Z_o^l) = \frac{(1-\beta)\hat{p} - c}{(1-\beta)(\hat{p} - s)}$ and $F(Z_o^h) = \frac{(1-\beta)c_e - c}{(1-\beta)(c_e - s)}$, respectively.

Theorem 1. *For any option ordering quantity Q_o the supplier's best response function is*

$$Z_o^*(Q_o) = \begin{cases} Z_o^l, & if \quad 0 \le Q_o < Z_o^l, \\ Q_o, & if \quad Z_o^l \le Q_o < Z_o^h, \\ Z_o^h, & if \quad Q_o \ge Z_o^h. \end{cases}$$

When the option order is moderate (refer to Figure 3), $Z_o^l \le Q_o < Z_o^h$, the supplier's optimal production quantity is equal to firm's option ordering quantity. When the option order is relatively big, $Q_o \ge Z_o^h$, the supplier's optimal production quantity has an upper bound, which is independent of the emergency procurement price. When the option order is relatively small, $0 < Q_o < Z_o^l$, the supplier's optimal production quantity has a lower bound, which is dependent on the emergency procurement price. Furthermore, the upper and lower bounds are decreasing in both disruption risk and production cost, while they are increasing in both emergency production cost and salvage value.
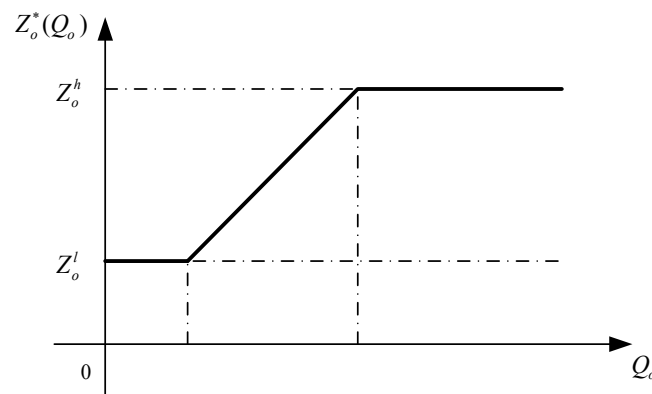
Figure 3. The supplier best response curve.

4.1.2. Firm's Procurement Decision

Let $\pi_R^O(Q_o)$ be the firm's expected profit. $Z_o^*(Q_o)$ is the supplier's best response when the firm's option order is Q_o. When the realized emergency procurement price is lower than the emergency production cost, $p < c_e$, the firm's expected profit is:

$$\pi_R^O(Q_o|p < c_e) = rE[\min\{Q_o, \zeta\}] + (1-\beta)(r-p)E[\{\min\{Z_o^*(Q_o), \zeta\} - Q_o\}^+] - wE[\min\{Q_o, \zeta\}] - c_o Q_o, \quad (5)$$

where the first term is the revenue from satisfying the demand with exercised option; the second term is the revenue from emergency procurement; the last two terms are the option exercising and purchasing cost. When the emergency procurement price is relatively low, the firm's emergency ordering demand can be met only when the disruption does not occur, and the inventory is sufficient.

Similarly, if the realized emergency procurement price is higher than the emergency production cost, $p \geq c_e$, then the firm's expected profit is:

$$\pi_R^O(Q_o|p \geq c_e) = rE[\zeta] - pE[\{\zeta - Q_o\}^+] - wE[\min\{Q_o, \zeta\}] - c_o Q_o, \quad (6)$$

where the first two terms are the revenue and last two terms are the costs. Note that, in the condition of $p \geq c_e$, the firm's emergency procurement will always be satisfied by the supplier's normal and emergency production. Using Equation (6), it can be seen that the firm's profit is independent of the supplier's production quantity and disruption risk.

Considering the randomness of the emergency procurement price and the best response from the supplier, the firm's expected profit can be expressed by:

$$\Pi_R^O(Q_o) = rG(c_e)E[\min\{Q_o, \zeta\}] + (1-\beta)rG(c_e)E[\{\min\{Z_o^*(Q_o), \zeta\} - Q_o\}^+] + r\overline{G}(c_e)E[\zeta] - \\ (1-\beta)\underline{p}(c_e)E[\{\min\{Z_o^*(Q_o), \zeta\} - Q_o\}^+] - \overline{p}(c_e)E[\{\zeta - Q_o\}^+] - wE[\min\{Q_o, \zeta\}] - c_o Q_o, \quad (7)$$

where the first two terms are the revenues when $p < c_e$ and the third term is the revenue when $p \geq c_e$; the last four terms are the emergency procurement, option exercising, and ordering costs. The firm's expected profit can also be formulated as:

$$\Pi_R^O(Q_o) = (rG(c_e) - w)\left(\int_0^{Q_o} x f(x)dx + \int_{Q_o}^{+\infty} Q_o f(x)dx\right) \\ + (1-\beta)\left(rG(c_e) - \underline{p}(c_e)\right)\left(\begin{array}{l} \int_{Q_o}^{\max\{Z_o^*(Q_o), Q_o\}} (x - Q_o)f(x)dx \\ + \int_{\max\{Z_o^*(Q_o), Q_o\}}^{+\infty} \{Z_o^*(Q_o) - Q_o\}^+ f(x)dx \end{array}\right) \\ + r\overline{G}(c_e)\mu - \overline{p}(c_e)\int_{Q_o}^{+\infty}(x - Q_o)f(x)dx - c_o Q_o. \quad (8)$$

Taken from Equation (8), $\Pi_R^O(Q_o)$ has different forms when $Z_o^*(Q_o) > Q_o$ or $Z_o^*(Q_o) < Q_o$. Exploring the properties of $\Pi_R^O(Q_o)$, Lemma 2 can be derived as follows.

Lemma 2. *The firm's profit function, $\Pi_R^O(Q_o)$ is continuous for $Q_o \in [0, +\infty)$. Additionally, it is differentiable and concave in Q_o for $Q_o \in [0, Z_o^l]$ and $Q_o \in [Z_o^l, +\infty)$, respectively.*

As per Lemma 2, through comparing the firm's optimal profits when $Q_o \in [0, Z_o^l]$ and $Q_o \in [Z_o^l, +\infty)$, the optimal decision of the firm can be obtained. To avoid a trivial case, the authors assume $\frac{(\hat{p}-s)c_o-(p_2-w)(c-s)}{(p_2-w)s+(p_1-s)c_o} < \frac{\hat{p}-c}{\hat{p}}$. Defining Q_o^l and Q_o^h satisfying $F(Q_o^l) = \frac{p_1-c_o-w}{p_1-w}$ and $F(Q_o^h) = \frac{p_2-c_o-w}{p_2-w}$, the optimal decisions of the supplier and the firm can be derived as follows.

Theorem 2. *Under the OP strategy, the firm's optimal option procurement quantity Q_o^* and the supplier's optimal production quantity Z_o^* satisfy the following conditions:*

(i) *when $0 < \beta \le \hat{\beta}_o$, then $Q_o^* = Q_o^l$, $Z_o^* = Z_o^l$;*

(ii) *when $\hat{\beta}_o < \beta < \frac{\hat{p}-c}{\hat{p}}$, $Q_o^* = Q_o^h$, $Z_o^* = \left\{ \begin{array}{ll} Q_o^h, & if \quad \beta \le \beta_2 \\ Z_o^h, & if \quad \beta > \beta_2 \end{array} \right.$;*

where $\beta_2 = \frac{(c_e-s)c_o-(p_2-w)(c-s)}{(p_2-w)s+(c_e-s)c_o}$, $\hat{\beta}_o$ satisfies $F(t_0) = \frac{(1-\hat{\beta}_o)\hat{p}-c}{(1-\hat{\beta}_o)(\hat{p}-s)}$, t_0 satisfies $H_1(t_0) = 0$, and $H_1(t)$ can be formulated as:

$$H_1(t) = (rG(c_e) + \overline{p}(c_e) - w)(Q_o^h - \int_0^{Q_o^h} F(x)dx) - c_o(Q_o^h - Q_o^l) - (\beta rG(c_e) +$$
$$(1-\beta)\underline{p}(c_e) + \overline{p}(c_e) - w)\left(Q_o^l - \int_0^{Q_o^l} F(x)dx\right) - (1-\beta)\left(rG(c_e) - \underline{p}(c_e)\right)\left(t - \int_0^t F(x)dx\right).$$

Corollary 1. $\frac{\partial Q_o^h}{\partial \beta} = 0$, $\frac{\partial Q_o^l}{\partial \beta} > 0$, $\frac{\partial Z_o^h}{\partial \beta} < 0$, $\frac{\partial Z_o^l}{\partial \beta} < 0$; $\frac{\partial Q_o^h}{\partial c_o} < 0$, $\frac{\partial Q_o^l}{\partial c_o} < 0$, $\frac{\partial Z_o^h}{\partial c_o} = \frac{\partial Z_o^l}{\partial c_o} = 0$; $\frac{\partial Q_o^h}{\partial w} < 0$, $\frac{\partial Q_o^l}{\partial w} < 0$, $\frac{\partial Z_o^h}{\partial w} = \frac{\partial Z_o^l}{\partial w} = 0$; $\frac{\partial Q_o^h}{\partial c_e} > 0$, $\frac{\partial Q_o^l}{\partial c_e} > 0$, $\frac{\partial Z_o^h}{\partial c_e} > 0$, $\frac{\partial Z_o^l}{\partial c_e} > 0$.

Figure 4 gives a graphic description of Theorem 2. Found in Figure 4, $\beta_1 = \frac{(\hat{p}-s)c_o-(p_2-w)(c-s)}{(p_2-w)s+(\hat{p}-s)c_o}$ and $\beta_3 = \frac{(\hat{p}-s)c_o-(p_1-w)(c-s)}{(p_1-w)s+(\hat{p}-s)c_o}$ are the critical points when $Z_o^l > Q_o^h$ and $Z_o^l < Q_o^l$, respectively. Note that whether β_2 is bigger than $\hat{\beta}_o$ or not is uncertain. Figure 4 describes the cases of $\beta_2 > \hat{\beta}_o$. Looking at Theorem 2 and Figure 4, there also exist upper and lower bounds for Q_o^*. When $\beta < \hat{\beta}_o$, Q_o^* and Z_o^* take their lower bounds Q_o^l and Z_o^l, respectively. Accompanying the increase of the disruption risk, Q_o^* increases, whereas Z_o^* decreases (refer to Corollary 1). When $\beta > \hat{\beta}_o$, Q_o^* takes its upper bound Q_o^h, and Z_o^* takes its upper bound Z_o^h or is equal to Q_o^*. Occurring in this situation (refer to Corollary 1), Q_o^* is a constant and is independent of disruption risk. However, Z_o^* is non-increasing in the disruption risk.
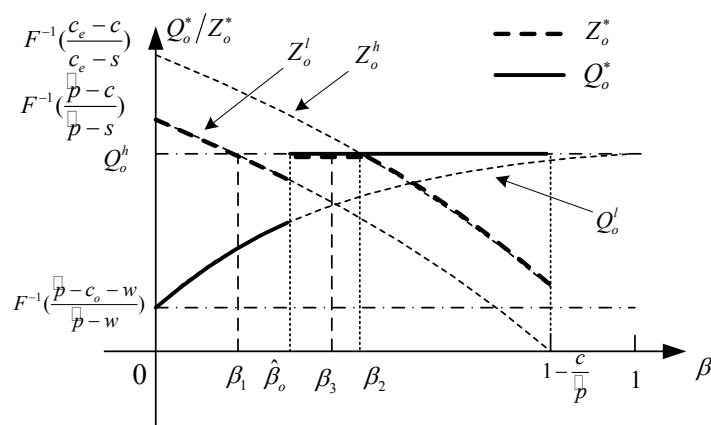
Figure 4. Supplier's and firm's optimal decisions with different disruption risks under the option purchase (OP) strategy.

The following managerial insights can be derived. Using the OP strategy, the firm's optimal procurement quantity can be represented by two newsvendor solutions (p_1 and p_2 can be considered as the retail prices). The firm's procurement follows a "threshold" principle. When the disruption risk is less than the threshold, the firm's optimal procurement quantity takes the lower newsvendor solution (or lower bound); otherwise, it takes the higher newsvendor solution (or upper bound). Similarly, for the unreliable supplier, his optimal normal production quantity takes the production lower bound when the disruption risk is lower than the threshold; it is equal to the firm's option ordering quantity or takes the production upper boundary otherwise. Moreover, the firm's optimal procurement quantity is non-decreasing, whereas the supplier's optimal production quantity is non-increasing in the disruption risk when the disruption risk is lower and higher than the threshold, respectively.

4.2. Procurement Commitment Strategy

4.2.1. Supplier's Production Decision

Given the firm's order quantity Q_w, let $\pi_S^W(Z_w|Q_w)$ denote the supplier's expected profit. When the realized emergency procurement price is lower than the emergency production cost, $p < c_e$, then the supplier's expected profit is:

$$\pi_S^W(Z_w|Q_w, p < c_e) = -cZ_w \quad +(1-\beta)wE[\min\{Z_w, Q_w\}] + (1-\beta)pE[\{\min\{Z_w, \zeta\} - Q_w\}^+]$$
$$+(1-\beta)sE[\{\{Z_w - Q_w\}^+ - \{\zeta - Q_w\}^+\}^+], \tag{9}$$

where the first term is the production cost; the second and third terms are the revenues from normal ordering and emergency ordering when disruption does not occur; and the last term is the salvage value. Due to low emergency procurement price, the supplier only will use its leftover inventory to satisfy the firm's instant ordering. It is easy to know that the supplier will pick $Z_w^* (\geq Q_w)$ to maximize the following profit:

$$\pi_S^W(Z_w|Q_w, p < c_e) = -cZ_w + (1-\beta)wQ_w + (1-\beta)\left(\begin{array}{c} \int_{Q_w}^{Z_w}(x - Q_w)f(x)dx \\ +\int_{Z_w}^{+\infty}(Z_w - Q_w)f(x)dx \end{array} \right)$$
$$+(1-\beta)s(\int_0^{Q_w}(Z_w - Q_w)f(x)dx + \int_{Q_w}^{Z_w}(Z_w - x)f(x)dx). \tag{10}$$

Similarly, if the realized emergency procurement price is no lower than the emergency production cost, $p \geq c_e$, the supplier will satisfy all the firm's instant ordering when he is not disrupted, then the supplier's expected profit is:

$$\pi_S^W(Z_w|Q_w, p \geq c_e) = -cZ_w + (1-\beta)wE[\min\{Z_w, Q_w\}] + (1-\beta)pE[\{\zeta - Q_w\}^+]$$
$$-(1-\beta)c_eE[\{\zeta - Z_w\}^+] + (1-\beta)sE[\{\{Z_w - Q_w\}^+ - \{\zeta - Q_w\}^+\}^+]. \tag{11}$$

Due to the high emergency procurement price, the supplier will fully satisfy the firm's instant ordering if he is not disrupted. Concerning the supplier, he will always choose $Z_w^* \geq Q_w$, then the supplier's expected profit can be formulated as:

$$\pi_S^W(Z_w|Q_w, p \geq c_e) = -cZ_w + (1-\beta)wQ_w + (1-\beta)p\int_{Q_w}^{+\infty}(x - Q_w)f(x)dx-$$
$$(1-\beta)c_e\int_{Z_w}^{+\infty}(x - Z_w)f(x)dx + (1-\beta)s(\int_0^{Q_w}(Z_w - Q_w)f(x)dx + \int_{Q_w}^{Z_w}(Z_w - x)f(x)dx). \tag{12}$$

Considering the randomness of the emergency procurement price, the supplier's expected profit can be given by:

$$\Pi_S^W(Z_w|Q_w) = -\quad cZ_w + (1-\beta)wQ_w + (1-\beta)\underline{p}(c_e)E[\{\min\{Z_w, \zeta\} - Q_w\}^+]$$
$$+(1-\beta)\overline{p}(c_e)E[\{\zeta - Q_w\}^+] - (1-\beta)c_e\overline{G}(c_e)E[\{\zeta - Z_w\}^+] \tag{13}$$
$$+(1-\beta)sE[\{\{Z_w - Q_w\}^+ - \{\zeta - Q_w\}^+\}^+].$$

Equation (13) can also be expressed by:

$$\Pi_S^W(Z_w|Q_w) = \quad -cZ_w + (1-\beta)wQ_w + (1-\beta)\underline{p}(c_e)\left(\begin{array}{l} \int_{Q_w}^{Z_w}(x-Q_w)f(x)dx \\ + \int_{Z_w}^{+\infty}(Z_w-Q_w)f(x)dx \end{array}\right)$$
$$+(1-\beta)\overline{p}(c_e)\int_{Q_w}^{+\infty}(x-Q_w)f(x)dx - (1-\beta)c_e\overline{G}(c_e)\int_{Z_w}^{+\infty}(x-Z_w)f(x)dx$$
$$+(1-\beta)s\left(\int_0^{Q_w}(Z_w-Q_w)f(x)dx + \int_{Q_w}^{Z_w}(Z_w-x)f(x)dx\right). \tag{14}$$

Let $Z_w^*(Q_w)$ denote the supplier's best response function. It is easy to verify that $\Pi_S^W(Z_w|Q_w)$ is concave in Z_w. Define Z_w^l satisfying $F(Z_w^l) = \frac{(1-\beta)\hat{p}-c}{(1-\beta)(\hat{p}-s)}$. Using the first order condition, the supplier's optimal production decision can be derived.

Theorem 3. *For any firm's order quantity Q^w, the supplier's best response function is:*

$$Z_w^*(Q_w) = \begin{cases} Z_w^l, & if \quad 0 < Q_w < Z_w^l, \\ Q_w, & if \quad Q_w \geq Z_w^l. \end{cases}$$

Corollary 2. $Z_w^l = Z_o^l$.

Regarding the supplier, his optimal production quantity has a lower bound, which is equal to his optimal production lower bound under the OP strategy. Similarly, the lower bound is decreasing in disruption risk. When the firm's ordering is less than the lower bound, the supplier's optimal production quantity takes the lower bound, otherwise, it is equal to the firm's ordering quantity.

4.3. Firm's Procurement Decision

Let $\pi_R^W(Q_w)$ be the firm's expected profit. When the realized emergency procurement price is lower than the emergency production cost, $p < c_e$, then the firm's expected profit is:

$$\pi_R^W(Q_w|p < c_e) = \quad -(1-\beta)wQ_w + (1-\beta)rE[\min\{Z_w, \zeta\}]$$
$$-(1-\beta)pE[\{\min\{Z_w, \zeta\} - Q_w\}^+] + (1-\beta)sE[\{Q_w - \zeta\}^+]. \tag{15}$$

Here, the first term is the normal ordering cost. The second term is the revenue for satisfying the market demand. The third term is the expense for emergency procurement. The last term is the salvage value. Note that the firm's emergency ordering is only satisfied with the leftover inventory of the supplier.

Similarly, if the realized emergency procurement price is no lower than the emergency production cost, $p \geq c_e$, then the firm's expected profit is:

$$\pi_R^W(Q_w|p \geq c_e) = -(1-\beta)wQ_w + (1-\beta)rE[\zeta] - (1-\beta)pE[\{\zeta - Q_w\}^+] + (1-\beta)sE[\{Q_w - \zeta\}^+]. \tag{16}$$

The difference between Equations (15) and (16) is, due to the high emergency procurement price, all the firm's emergency procurement demand always will be satisfied by the supplier if he is not disrupted.

Considering the randomness of the emergency procurement price, the firm's expected profit can be formulated as:

$$\Pi_R^W(Q_w) = -(1-\beta)wQ_w + (1-\beta)rG(c_e)E[\min\{Z_w, \zeta\}] + (1-\beta)r\overline{G}(c_e)E[\zeta]$$
$$- (1-\beta)\underline{p}(c_e)E[\{\min\{Z_w, \zeta\} - Q_w\}^+] - (1-\beta)E[\{\zeta - Q_w\}^+] \tag{17}$$
$$+ (1-\beta)\overline{s}E[\{Q_w - \zeta\}^+].$$

where the term is the normal purchase expense; the second and third terms are the revenue when the emergency procurement price is lower and higher than the emergency production cost, respectively; the fourth and fifth terms are the emergency procurement costs when the emergency procurement

price is relatively lower and higher, respectively; and the last term is the salvage value. The firm's expected profit can also be expressed by:

$$\Pi_R^W(Q_w) = -(1-\beta)rG(c_e)(\int_0^{z_w} xf(x)dx + \int_{z_w}^{+\infty} z_w f(x)dx) +$$
$$(1-\beta)r\overline{G}(c_e)\mu - (1-\beta)\underline{p}(c_e)(\int_{Q_w}^{Z_w}(x - Q_W)f(x)dx + \int_{z_w}^{+\infty}(Z_w - Q_w)f(x)dx) - \tag{18}$$
$$(1-\beta)r\overline{p}(c_e)\int_{Q_w}^{+\infty}(x - Q_w)f(x)dx + (1-\beta)s\int_0^{Q_w}(Q_w - x)f(x)dx.$$

Define Q_w^l and Q_w^h satisfying $F(Q_w^l) = \frac{\widetilde{p}-w}{\widetilde{p}-s}$ and $F(Q_w^h) = \frac{p_2-w}{p_2-s}$, respectively. The optimal decisions of the supplier and the firm under the PC strategy can be derived as follows.

Theorem 4. *Under the PC strategy, the firm's optimal procurement quantity Q_w^* and the supplier's optimal production quantity Z_w^* satisfy the following conditions:*

(i)   *if $0 \leq \beta \leq \hat{\beta}_w$, then $Q_w^* = Q_w^l$, $Z_w^* = Z_w^l$;*

(ii) *if $\hat{\beta}_w < \beta \leq \frac{\hat{p}-c}{\hat{p}}$, then $Q_w^* = Z_w^* = Q_w^h$, where $\hat{\beta}_w$ satisfies $F(t_1) = \frac{(1-\hat{\beta}_w)\hat{p}-c}{(1-\hat{\beta}_w)(\hat{p}-s)}$, t_1 satisfies $H_2(t_1) = 0$ and $H_2(t)$ can be formulated as:*

$$H_2(t) = -(1-\beta)w(Q_w^h - Q_w^l) + (1-\beta)(rG(c_e) + \overline{p}(c_e))\left(Q_w^h - t - \int_t^{Q_w^h} F(x)dx\right)$$
$$+(1-\beta)\underline{p}(c_e)\left(t - Q_w^l - \int_{Q_w^l}^t F(x)dx\right) + (1-\beta)s\int_{Q_w^l}^{Q_w^h} F(x)dx.$$

Corollary 3. $\frac{\partial Q_w^h}{\partial \beta} = \frac{\partial Q_w^l}{\partial \beta} = 0$, $\frac{\partial Z_w^l}{\partial \beta} < 0$; $\frac{\partial Q_w^h}{\partial w} < 0$, $\frac{\partial Q_w^l}{\partial w} < 0$, $\frac{\partial Z_w^l}{\partial w} = 0$; $\frac{\partial Q_w^h}{\partial c_e} > 0$, $\frac{\partial Q_w^l}{\partial c_e} = 0$, $\frac{\partial Z_w^l}{\partial c_e} > 0$.

Figure 5 gives a graphic description of Theorem 4. $\beta_4 = \frac{(\hat{p}-c)(p_2-s)-(\hat{p}-s)(p_2-w)}{\hat{p}(p_2-s)-(\hat{p}-s)(p_2-w)}$ and $\beta_5 = \frac{(\hat{p}-c)(\widetilde{p}-s)-(\hat{p}-s)(\widetilde{p}-w)}{\hat{p}(\widetilde{p}-s)-(\hat{p}-s)(\widetilde{p}-w)}$ are the critical points when $Z_w^l > Q_w^h$ and $Z_w^l < Q_w^l$, respectively. Looking at Theorem 4 and Figure 5, there also exist upper and lower bounds for Q_w^*. The two bounds are the newsvendor solutions ($\widetilde{p}$ and p_2 can be considered as the retail prices) and they are independent of the disruption risk. When $\beta < \hat{\beta}_w$, Q_w^* and Z_w^* take their lower bounds, Q_w^l and Z_w^l, respectively. When the disruption risk increases, Z_w^* decreases (refer to Corollary 3). When $\beta > \hat{\beta}_w$, Q_w^* takes its upper bound Q_w^h, and Z_w^* is equal to Q_w^*. Considering this situation (refer to Corollary 3), Q_w^* and Z_w^* are both constants and are independent of disruption risk.
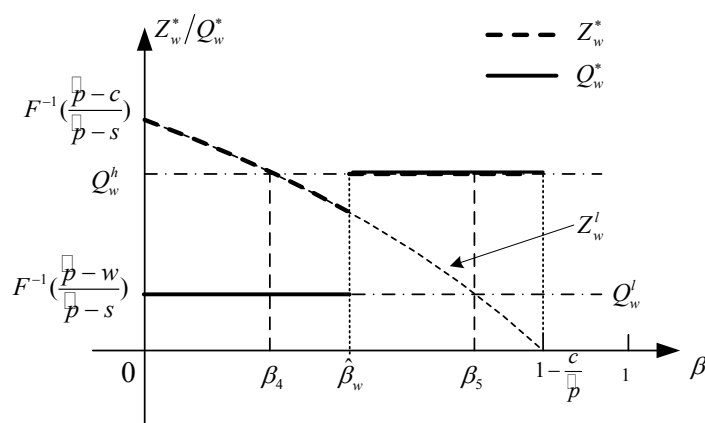
Figure 5. Supplier's and firm's optimal decisions with different disruption risk under the procurement commitment (PC) strategy.

Using the PC strategy, the firm's procurement also follows a "threshold" principle. When the disruption risk is less than the threshold, the firm's optimal procurement quantity takes the lower newsvendor solution (or lower bound); otherwise, it takes the higher newsvendor solution (or upper

bound). Similarly, for the unreliable supplier, his optimal normal production quantity takes the production lower bound when the disruption risk is lower than the threshold; and it is equal to the firm's normal ordering quantity otherwise.

5. Comparisons

The authors compare the supplier's and the firm's optimal decisions and profits under option purchase (OP) strategy and procurement (PC) strategy, respectively and also explore the impacts of some important model parameters. Furthermore, the firm's optimal strategy selection decision is given.

Due to the complexity of the optimal decisions (refer to Theorems 2 and 4), it is hard to analyze the optimal profits and explore the impacts of model parameters under the two types of strategies, in theory. To achieve more managerial insights, the authors conduct numerical simulation to compare the two types of strategies. The numerical examples are based on the following combination of parameters: $s = 0.5$, $c = 3$, $c_o = 3$, $w = 5$, $A = 6$, $B = 14$, $c_e = 10$, $r = 16$, $P \sim U(6, 14)$, and demand $\zeta \sim U(0, 600)$. The values of these parameters are set based on assumptions (1)–(6) presented in Section 3 and previous literature in the area (refer to [33–35]). The authors also conduct numerical experiments with other combinations of parameters, the results of which show that the main conclusions and analytical results in this study are robust.

5.1. Optimal Decisions and Profits of the Supplier and Firm under the Two Types of Strategies

5.1.1. Impact of β

Using the OP strategy (Figures 6 and 7), the firm's option ordering quantity is non-decreasing, while her profit is non-increasing in disruption risk. This shows that, with the increase of the disruption risk, the benefit of the emergency procurement decreases, and the firm tends to reserve more options to deal with the increasing disruption risk. When the disruption risk is above a threshold, the firm's option ordering and profit become a constant. When in this situation, the OP strategy can protect the firm from any disruption risk. For the supplier, his production quantity and profit both decrease with the disruption risk, either when the disruption risk is lower or higher than a threshold. This implies that the supplier tends to produce less to mitigate the increasing supply disruption risk. Counterintuitively, the supplier's production quantity and profit might rise when the disruption risk exceeds a threshold. This is mainly because the supplier needs a rise in production to deal with the firm's changes in option ordering when the disruption risk exceeds a threshold. The supplier will benefit from this increase in production, thus his profit will rise too.
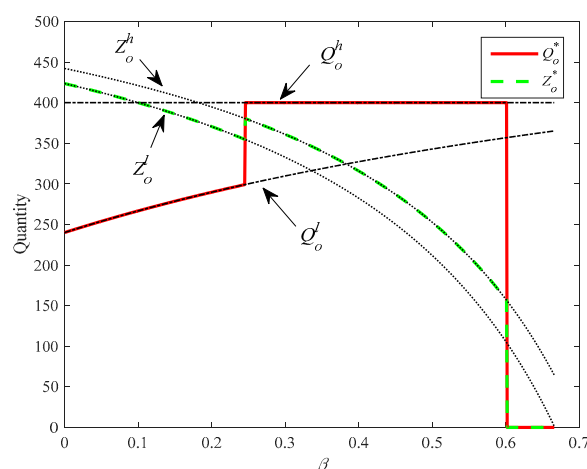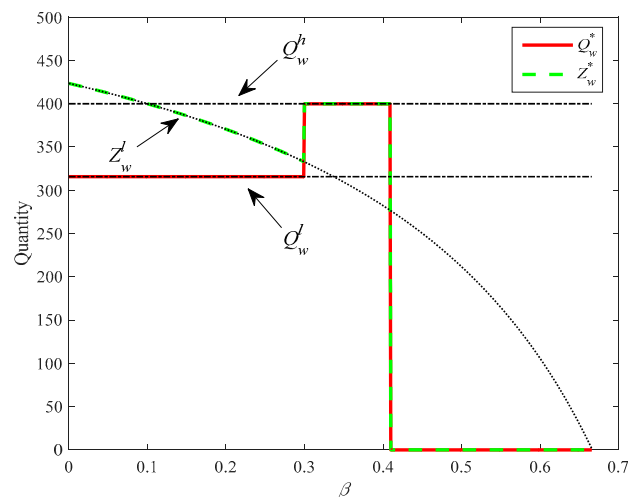

Figure 6. The optimal procurement and production decisions of the firm and supplier Q_o^* and Z_o^* under option purchase (OP) strategy with different disruption risks β.
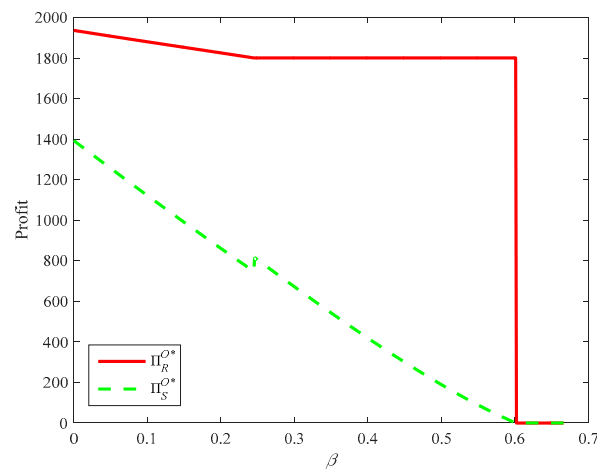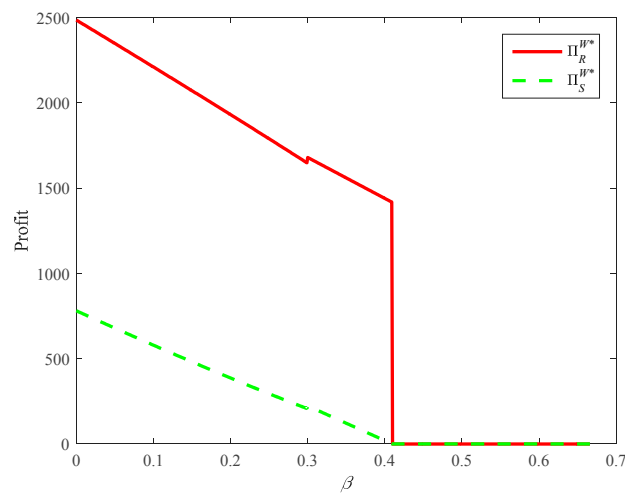
Figure 7. The optimal profits of the firm and supplier Π_R^{O*} and Π_S^{O*} under OP strategy with different β.

Using the PC strategy (Figures 8 and 9), when the disruption risk is relatively low, the supplier has an incentive to produce more products to satisfy the firm's possible instant ordering, but the motivation becomes lower with the increase of disruption risk. Regarding the firm, her normal ordering is relatively low, and she can rely more on the emergency procurement to satisfy market demand. When the disruption risk exceeds a threshold, the firm's order quantity changes from a low newsvendor quantity to a high newsvendor quantity. When in this situation, the supplier has no incentive to produce more products than that ordered. Concerning the firm, she raises her normal ordering to satisfy the demand. Generally, the disruption risk has a negative influence on the supplier's and firm's profits. However, the firm's profit might rise when the disruption risk exceeds a threshold. This implies that the firm will benefit from the rise of her ordering quantity.

Figure 8. The optimal procurement and production decisions of the firm and supplier Q_w^* and Z_w^* under procurement commitment (PC) strategy with different β.

Figure 9. The optimal profits of the firm and supplier Π_R^{W*} and Π_S^{W*} under PC strategy with different β.

5.1.2. Impacts of c_o, w, and c_e

Figures 10–15 describe the impacts of option price, c_o, and exercise price, w, on the supplier's and firm's optimal decisions, and the profits under the two types of strategies. Under the OP strategy, the option price and exercise price have a similar influence on the optimal decisions and profits of the supplier and firm. The option price and exercise price have no influence on the supplier's production upper bound and lower bound, but a lower option price or option exercise price can raise the upper and lower bounds of the firm's option ordering. Moreover, a lower option price or exercise price benefits the firm, while it damages the supplier. Accompanying the increase of the option price or exercise price, the supplier has an incentive to cooperate with the firm in a riskier operational environment.
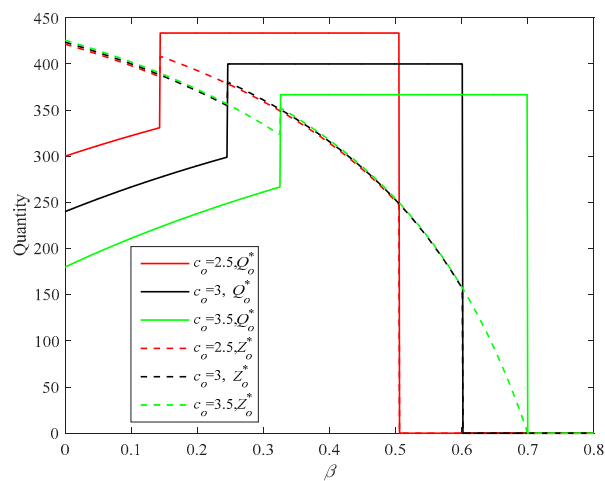
Figure 10. Q_o^* and Z_o^* with different option price c_o.
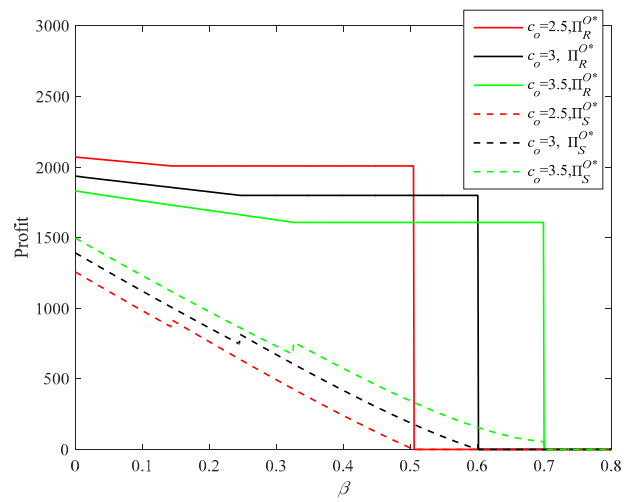
Figure 11. Π_R^{O*} and Π_S^{O*} with different c_o.
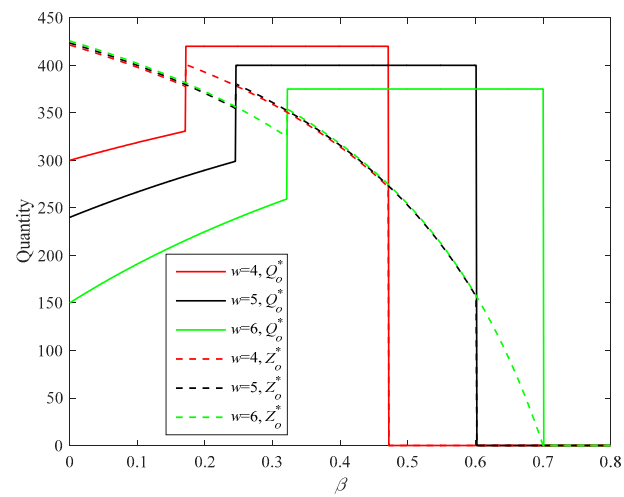
Figure 12. Q_o^* and Z_o^* with different option exercise price w.
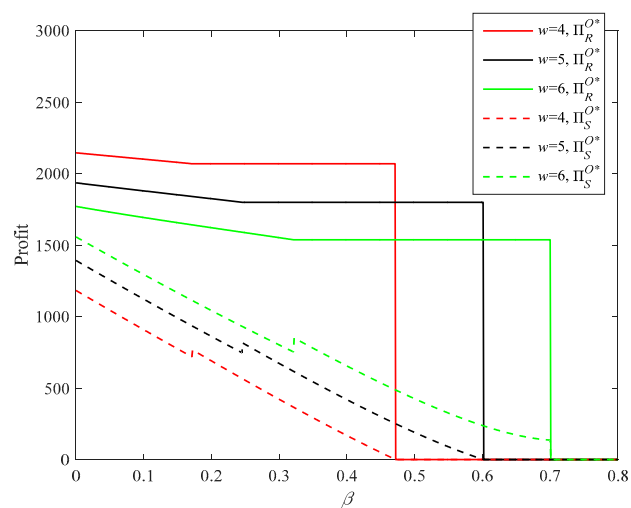
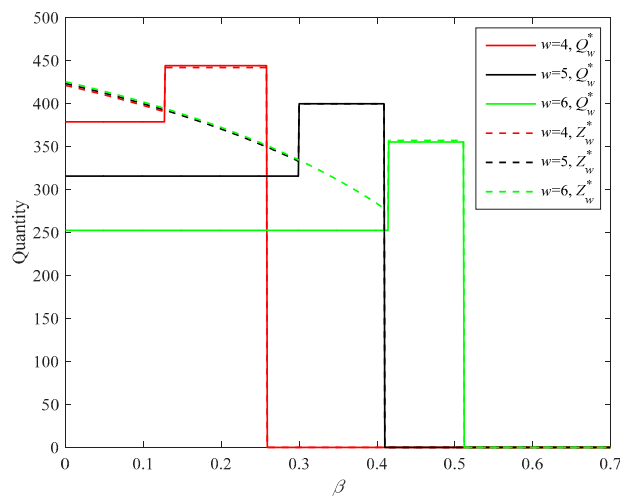**Figure 13.** Π_R^{O*} and Π_S^{O*} with different wholesale price w.

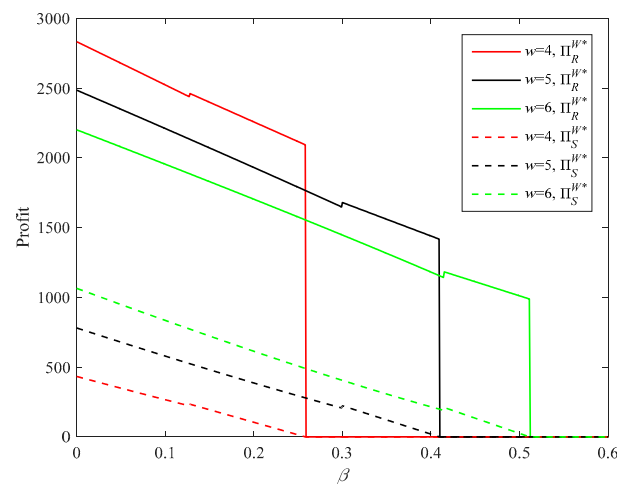**Figure 14.** Q_w^* and Z_w^* with different w.

Figure 15. Π_R^{W*} and Π_S^{W*} with different w.

Similarly, under the PC strategy, the wholesale price has no impact on the production lower bound of the supplier, but a lower wholesale price will raise the upper and lower bounds of the firm's normal ordering. Alongside the increase of the wholesale price, the supplier also has incentive to cooperate with the firm in a riskier operational environment.

Figures 16–19 describe the impacts of emergency production cost, c_e, on the optimal decisions and profits under different strategies. Under the OP strategy, a higher emergency production cost will raise the upper and lower bounds of the firm's option ordering and the supplier's production. Both the supplier's and the firm's profits are decreasing in the emergency production cost. This implies that a higher emergency production cost will impel the supplier and firm to rely more on normal production and normal option ordering, respectively. A higher emergency production cost always has a negative impact on the unreliable supplier and firm.

Under the PC strategy, a higher emergency production cost will raise the lower bounds of the supplier's production and the upper bound of the firm's normal ordering. It has no impact on the lower bound of the firm's normal ordering. That is to say, with the increase of the emergency production cost, the supplier relies more on the normal production. Regarding the firm, only when the disruption risk is relatively high does she rely more on the first ordering opportunity. Similarly, a higher emergency production cost will damage both the supplier's and the firm's profits. Therefore, reducing the emergency production cost can improve the supply chain member's profit.
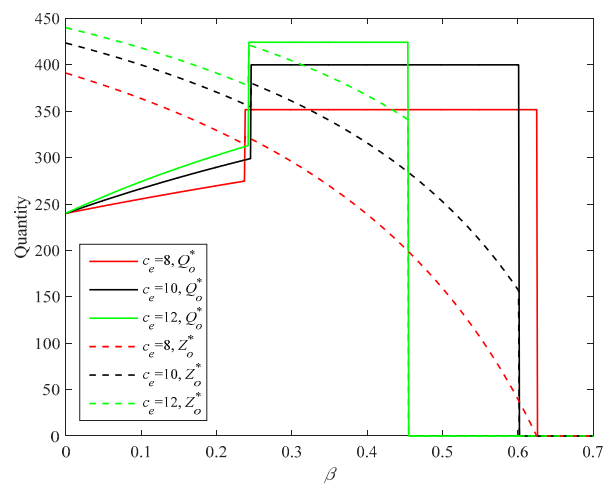
Figure 16. Q_o^* and Z_o^* with different emergency production cost c_e.
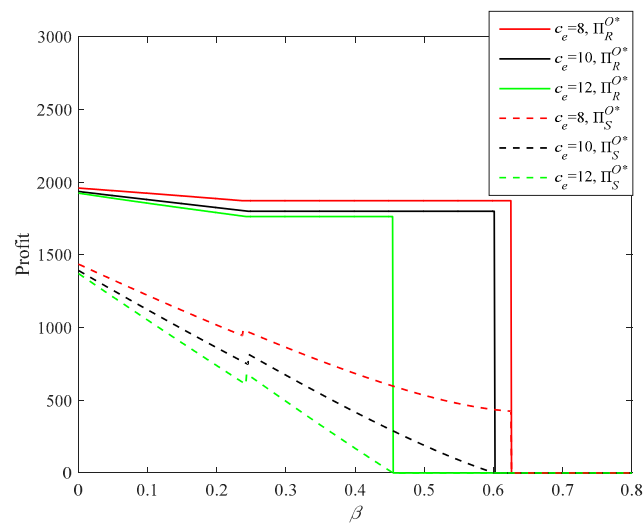

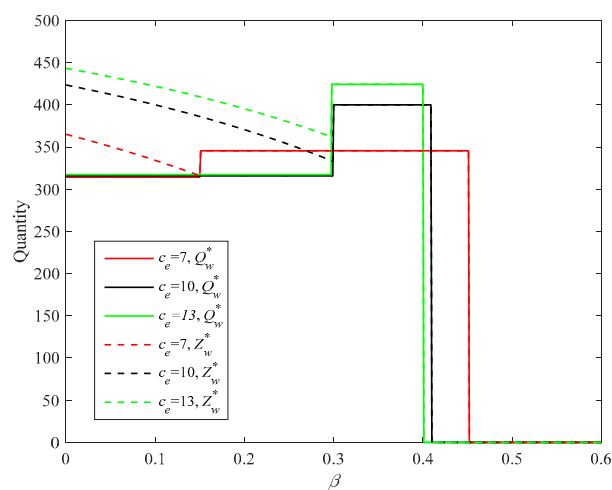**Figure 17.** Π_R^{O*} and Π_S^{O*} with different c_e.

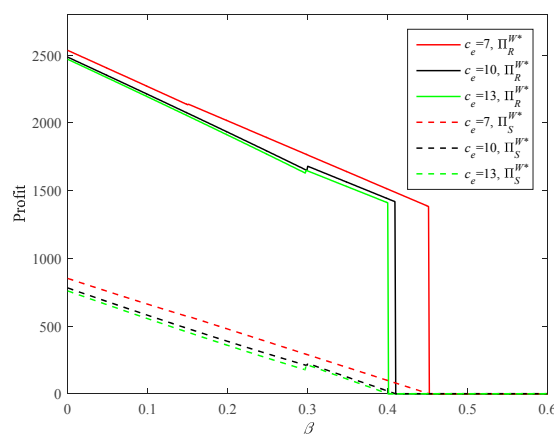**Figure 18.** Q_w^* and Z_w^* with different c_e.

Figure 19. Π_R^{W*} and Π_S^{W*} with different c_e.

5.1.3. Impacts of P and ζ

Figures 20–23 describe the impacts of the mean value (MV) of emergency procurement price on the optimal decisions and profits under two types of strategies. Under the OP strategy, with the increase of the MV of emergency procurement price, the lower bounds of the supplier's production and the firm's option ordering increase; the upper bound of the supplier's production remains unchanged; whereas the upper bound of the firm's option ordering decreases. Similarly, under the PC strategy, with the increase of the MV of emergency procurement price, the lower bounds of the supplier's production and the firm's normal ordering increase, while the upper bounds of the supplier's production and the firm's normal ordering decrease.

Under both strategies, it is natural to find that a higher MV of emergency procurement price is beneficial to the supplier. This is mainly because a higher MV of emergency procurement price often means the firm must pay more to the supplier. However, a lower MV of emergency procurement price is not always beneficial to the firm. When disruption risk is relatively low, the firm benefits from a lower MV of emergency procurement price; otherwise, a higher MV of emergency procurement price is better for the firm. This is counterintuitive and the authors provide an explanation as follows. When the disruption risk is relatively high, the supplier's optimal production quantity is no more than the firm's optimal option or normal ordering quantity. The firm has no possibility to adopt emergency procurement due to no leftover inventory. Then the firm pays nothing for the emergency procurement. Moreover, a higher MV of emergency procurement price will impel the firm to reduce her option or normal ordering quantity. The firm will benefit from dealing with demand risk efficiently. Therefore, when disruption risk is relatively high, the firm can benefit from a higher MV of emergency procurement price.
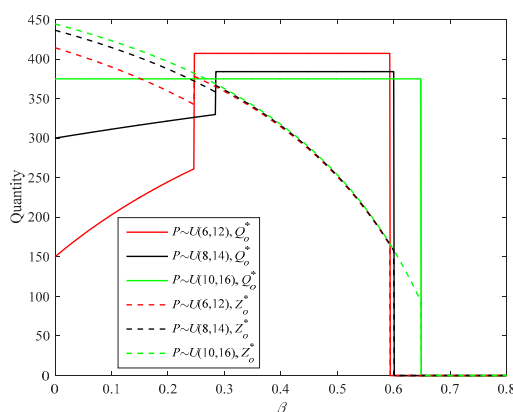

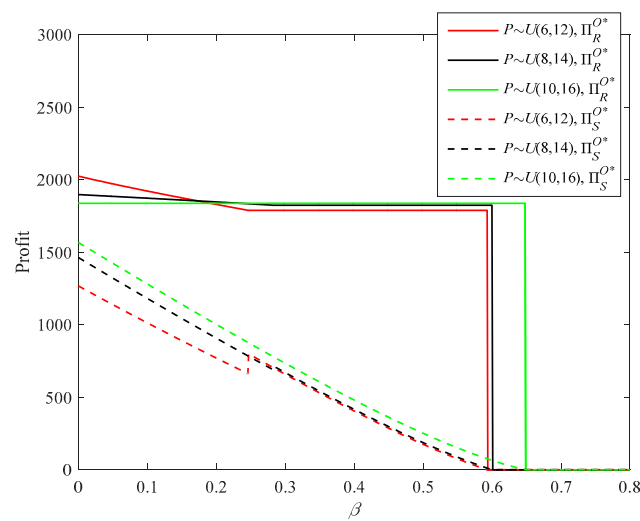**Figure 20.** Q_o^* and Z_o^* with different mean value (MV) of emergency procurement price P.

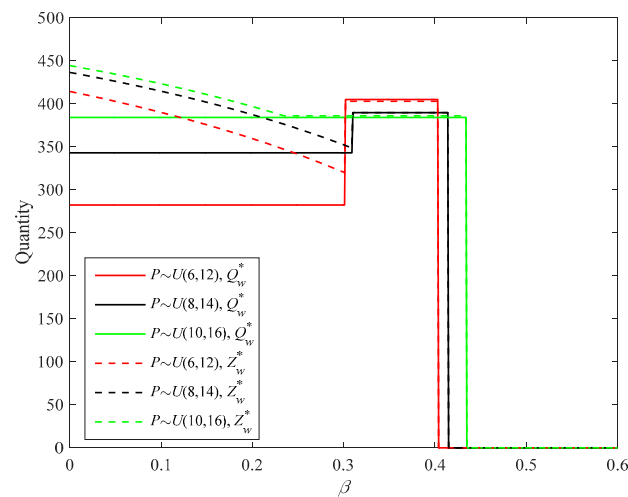**Figure 21.** Π_R^{O*} and Π_S^{O*} with different MV of P.

Figure 22. Q_w^* and Z_w^* with different MV of P.
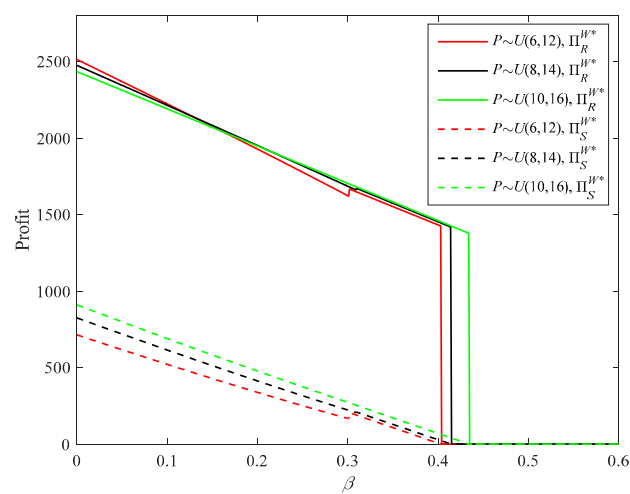

Figure 23. Π_R^{W*} and Π_S^{W*} with different MV of P.

Figures 24–27 describe the impacts of market demand variability (VAR) on the optimal decisions and profits under two types of strategies. Using the OP strategy, with the increase in the market demand variability, the lower boundary of the firm's option ordering decreases, whereas her upper

bound increases; the lower and upper bounds of the supplier's production both increase. Generally, a higher market demand variability is harmful to both the supplier and firm.
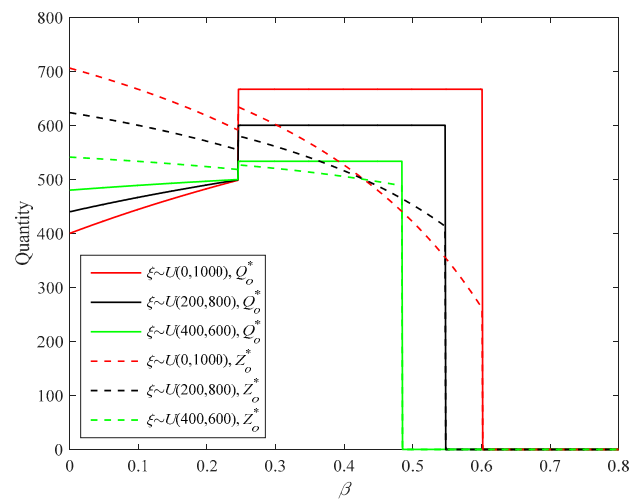

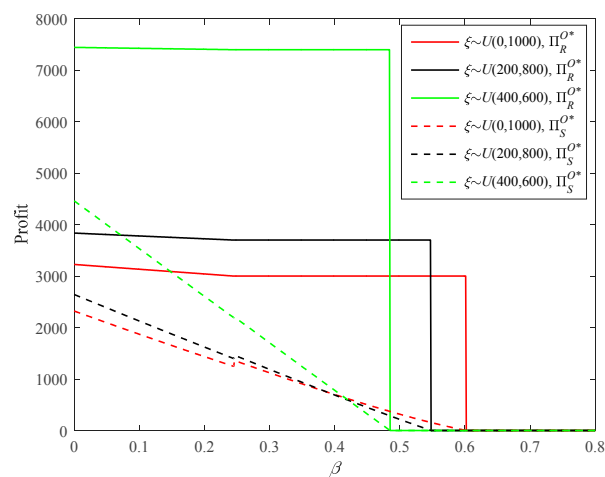**Figure 24.** Q_o^* and Z_o^* with different VAR of ζ.

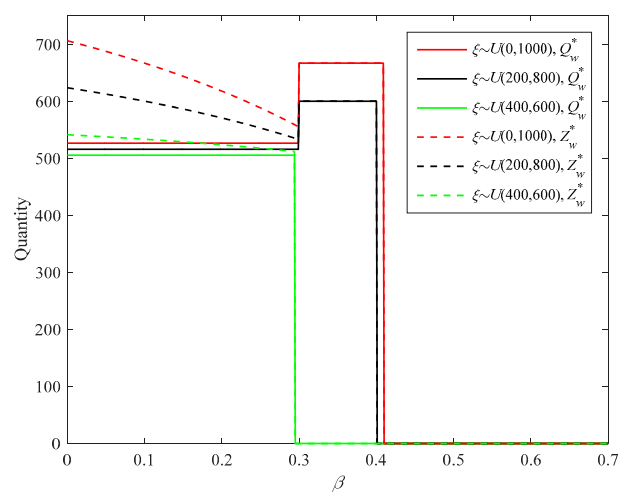**Figure 25.** Π_R^{O*} and Π_S^{O*} with different variability VAR of market demand ζ.

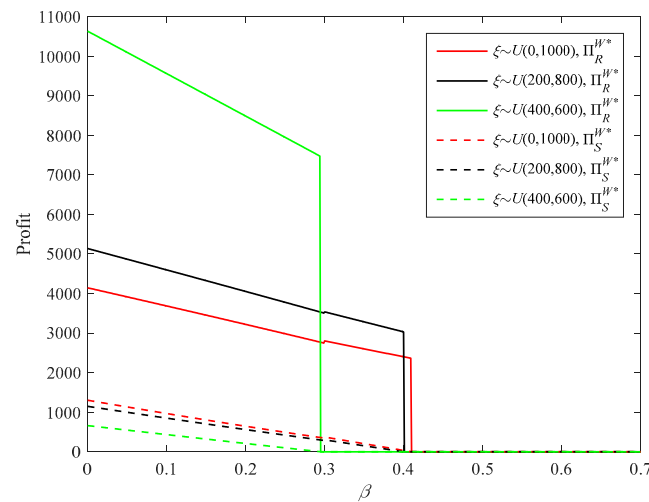**Figure 26.** Q_w^* and Z_w^* with different VAR of ζ.

Figure 27. Π_R^{W*} and Π_S^{W*} with different VAR of ζ.

Using the PC strategy, with the increase in the market demand variability, the lower bound of the supplier's production and the lower and upper bounds of the firm normal ordering all increase. A higher market demand variability is harmful to the firm. However, it is interesting to find that the supplier could benefit from a higher market demand variability. This implies that the supplier benefits more from over-production than the damages from the increase in the market demand variability.

5.2. Firm's Strategy Selection Decision

Figure 28 gives the firm's optimal strategy selection with different disruption risks. When the disruption risk is relatively low, the firm would like to choose the PC strategy. When the disruption risk increases, the firm switches to choose the OP strategy. Moreover, the firm's profit is non-increasing in disruption risk. Regarding the supplier, his profit decreases with the increase of the disruption risk, in general. However, when the firm switches from the PC strategy to the OP strategy, the supplier's profit has an increase.
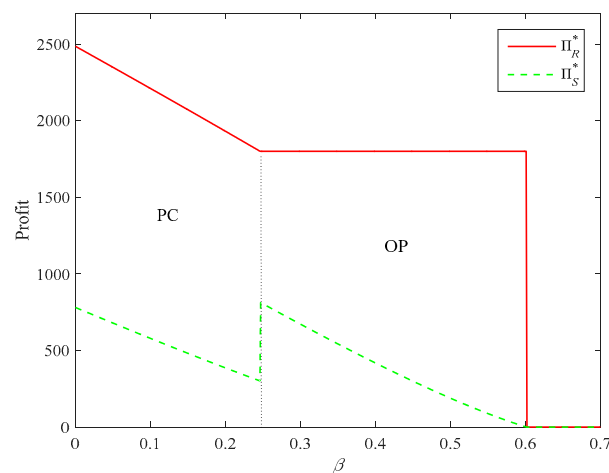

Figure 28. Firm's strategy selection.

Figures 29–31 describe the impacts of the option price and exercise price, as well as the emergency production cost on the firm's strategy selection. Alongside the increase of the option price, the firm is more likely to choose the PC strategy when the disruption risk is relatively low. A higher option price can assure the supplier to cooperate with the firm under the OP strategy, even if the disruption risk is relatively high. The varying of option exercise price and emergency production cost influences

the supplier's and firm's profits, whereas it has no influence on the firm's strategy selection from the risk perspective.
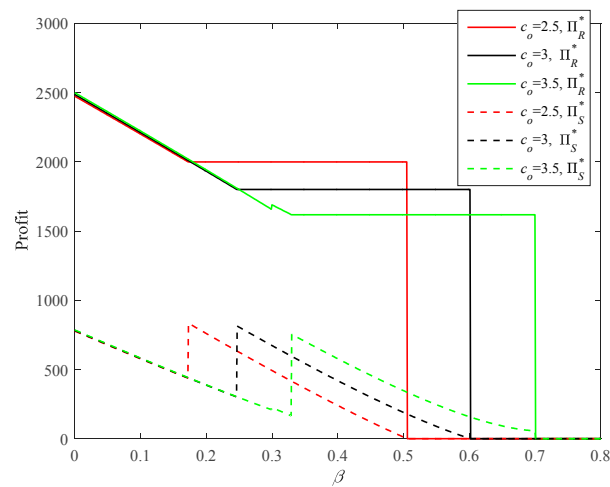
Figure 29. Impacts of different option price c_o.
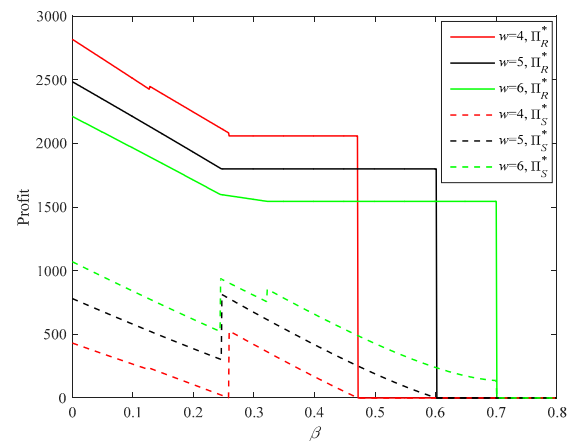
Figure 30. Impacts of different w.
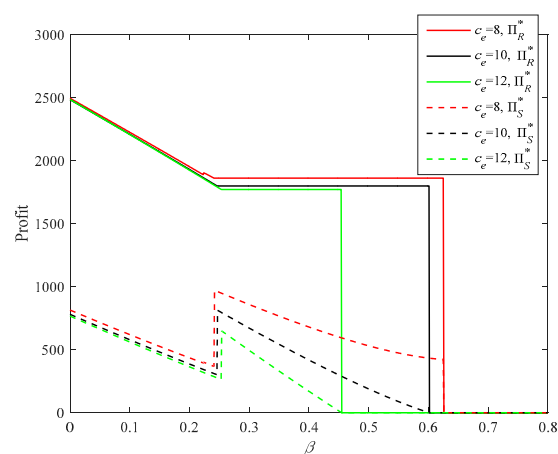
Figure 31. Impacts of different c_e.

Figures 32 and 33 describe the impacts of the uncertainty of the emergency procurement price and the market demand variability on the firm's strategy selection. Viewing Figure 32, the varying of MV of the emergency procurement price also does not influence the firm's strategy selection. Looking at Figure 33, with the increase in the market demand variability, the firm is more likely to choose the

OP strategy with the increase of the disruption risk. This is mainly because the OP strategy can help to mitigate both the supply risk and demand risk [34]. Moreover, a higher market demand variability can impel the supplier to cooperate with the firm, even if the disruption risk is relatively high.
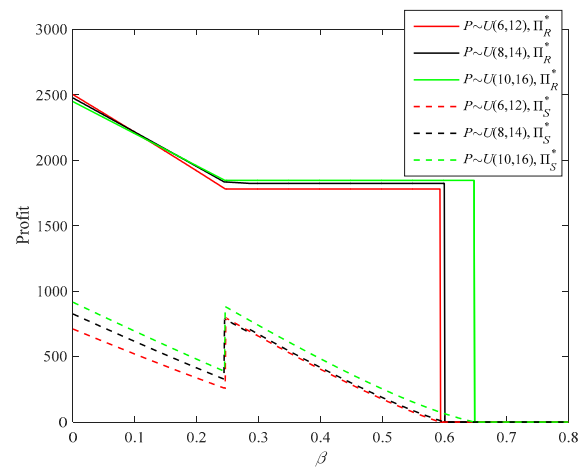

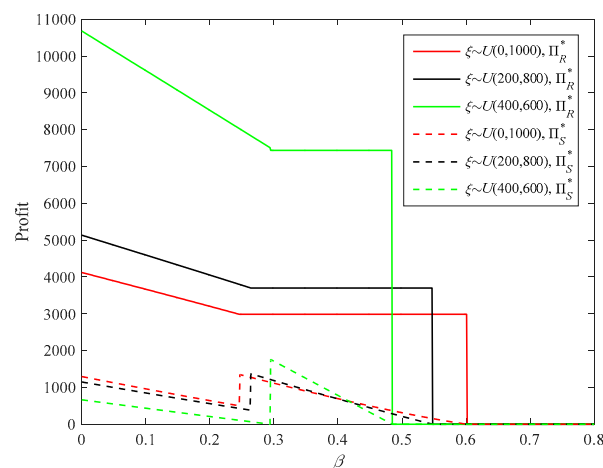**Figure 32.** Impacts of different MV of *P*.

Figure 33. Impacts of different VAR of ζ.

6. Conclusions

Supply disruption is becoming more and more common and brings serious damage to the firm's operational performance, even leading to the unsustainability of the supply chain, in practice. How to manage supply disruption risk has attained increasing attention from industry and academia. Motivated by real-world cases, in a supply chain faced with supply disruption risk, the authors investigated two types of procurement strategies for a firm with two ordering opportunities to mitigate the supply disruption risk. Considering the uncertainties in supply, demand, and the emergency procurement price simultaneously, the authors established the Stackelberg game models and derived the supplier's optimal production, and the firm's optimal procurement and replenishment strategies under the option purchase (OP) strategy and procurement commitment (PC) strategy, respectively. The authors also conducted sensitivity analysis to study the impacts of model parameters on the optimal decisions, profits and the firm's strategy selection. The following are this work's important conclusions and managerial insights which can provide a theoretical reference for decision makers, in practice.

(1) Under both types of strategies, the firm's procurement quantity can be represented by two newsvendor solutions and her procurement strategy follows a "threshold" principle. That is

to say, when the disruption risk is less than the threshold, the firm's optimal procurement quantity takes the lower newsvendor solution, otherwise, it takes the higher newsvendor solution. Moreover, when the disruption risk is relatively low, the supplier tends to produce more products than the ordered or option reserved. Using the OP strategy, the firm tends to reserve more options to deal with the increasing supply risk. Generally speaking, the disruption risk has a negative impact on the supplier's and firm's profits.

(2)　A lower option price or option exercise price benefits the firm, while it damages the supplier. The increase of the emergency production cost will damage both the supplier's and the firm's profits under both strategies. When the option price or the option exercise price is relatively high, the supplier has an incentive to cooperate with the firm in a risky operational environment under both types of strategies.

(3)　Under two types of strategies, a higher mean value (MV) of emergency procurement price is beneficial to the supplier. However, it is counterintuitive that a lower MV of emergency procurement price is not always beneficial to the firm. Concerning the uncertainty of demand, a higher market demand variability is harmful to the firm and supplier, while it could be beneficial to the supplier under the PC strategy.

(4)　Accompanying the increase in the disruption risk, the firm first chooses the PC strategy and then changes to the OP strategy. When the option price and market demand variability increases, the firm is more likely to choose the OP strategy with the increase of the disruption risk. A higher option price and market demand variability can impel the supplier to cooperate with the firm even in a risky operational environment.

This study also can be extended in several directions. The authors considered the situation where a firm deals with and mitigates its supply disruption risk through different procurement strategies in a forward supply chain. Later, the authors can consider the supply risk issues in a closed supply chain [41]. Moreover, this paper only explores two types of procurement strategies. Future research can consider other types of procurement strategies which are usually adopted by firms in practice, for example, the advance purchase strategy or contingency purchase strategy [25,26]. Finally, considering the firm's risk preference or the competition between the firms will also be interesting issues.

Author Contributions: K.X. proposed the main idea, developed the model and wrote the draft of the paper; Y.X. and L.F. contributed the cases and provided advice on the revision.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Proof of Lemma 1. It is easy to know $\Pi_S^O(Z_o|Q_o)$ is continuous for $Z_o \in [0, +\infty)$. The authors explore the concavity and convexity of $\Pi_S^O(Z_o|Q_o)$ when $0 \le Z_o \le Q_o$ and $Z_o > Q_o$, respectively.

(1) When $0 \le Z_o \le Q_o$, then:

$$\begin{aligned}
\Pi_S^O(Z_o|Q_o) &= -cZ_o + c_oQ_o + (\overline{p}(c_e) - c_e\overline{G}(c_e))\mu \\
&\quad + (w - \overline{p}(c_e) - G(c_e)c_e)(Q_o - \int_0^{Q_o} F(x)dx) \\
&\quad + (1 - \beta)c_e(Z_o - \int_0^{Z_o} F(x)dx) + (1 - \beta)s\int_0^{Z_o} F(x)dx.
\end{aligned}$$

Taking the first-order and second-order derivatives:

$$\frac{d\Pi_S^O(Z_o|Q_o, 0 \le Z_o \le Q_o)}{dZ_o} = -c + (1 - \beta)c_e - (1 - \beta)(c_e - s)F(Z_o),$$

$$\frac{d^2\Pi_S^O(Z_o|Q_o, 0 \le Z_o \le Q_o)}{d(Z_o)^2} = -(1-\beta)(c_e - s)f(Z_o).$$

Therefore, in the case of $0 \le Z_o \le Q_o$, $\Pi_S^O(Z_o|Q_o)$ is differentiable and concave in Z_o for $Z_o \in [0, Q_o]$.

(2) When $Z_o \ge Q_o$, then:

$$\begin{aligned}
\Pi_S^O(Z_o|Q_o) &= -cZ_o + c_oQ_o + (\overline{p}(c_e) - c_e\overline{G}(c_e))\mu + (1-\beta)s\int_0^{Z_o} F(x)dx \\
&\quad + (w - (1-\beta)\underline{p}(c_e) - \overline{p}(c_e) - \beta c_e G(c_e))(Q_o - \int_0^{Q_o} F(x)dx) \\
&\quad + (1-\beta)(\underline{p}(c_e) + c_e\overline{G}(c_e))(Z_o - \int_0^{Z_o} F(x)dx).
\end{aligned}$$

Taking the first-order and second-order derivatives:

$$\frac{d\Pi_S^O(Z_o|Q_o, Z_o \ge Q_o)}{dZ_o} = -c + (1-\beta)(\underline{p}(c_e) + c_e\overline{G}(c_e)) - (1-\beta)(\underline{p}(c_e) + c_e\overline{G}(c_e) - s)F(Z_o),$$

$$\begin{aligned}
\frac{d^2\Pi_S^O(Z_o|Q_o, Z_o \ge Q_o)}{d(Z_o)^2} &= (1-\beta)(s - \int_A^{c_e} xg(x)dx - c_e\int_{c_e}^B g(x)dx)f(Z_o) \\
&< (1-\beta)(s - A\int_A^{c_e} g(x)dx - A\int_{c_e}^B g(x)dx)f(Z_o) \\
&= (1-\beta)(s - A)f(Z_o) < 0.
\end{aligned}$$

Therefore, in the case of $Z_o \ge Q_o$, $\Pi_S^O(Z_o|Q_o)$ is differentiable and concave in Z_o for $Z_o \in [Q_o, +\infty)$.

Now exploring the continuity of $\frac{d\Pi_S^O(Z_o|Q_o)}{dZ_o}$ at point $Z_o = Q_o$,

$$\frac{d\Pi_S^O(Z_o|Q_o, Z_o \ge Q_o)}{dZ_o}\Big|_{Z_o = Q_o^+} = -c + (1-\beta)sF(Q_o) + (1-\beta)(\underline{p}(c_e) + c_e\overline{G}(c_e))(1 - F(Q_o)),$$

$$\frac{d\Pi_S^O(Z_o|Q_o, 0 \le Z_o \le Q_o)}{dZ_o}\Big|_{Z_o = Q_o^-} = -c + (1-\beta)sF(Q_o) + (1-\beta)c_e(1 - F(Q_o))$$

Hence, $\frac{d\Pi_S^O(Z_o|Q_o, Z_o \ge Q_o)}{dZ_o}\Big|_{Z_o = Q_o^+} \ne \frac{d\Pi_S^O(Z_o|Q_o, 0 \le Z_o \le Q_o)}{dZ_o}\Big|_{Z_o = Q_o^-}$ and $\Pi_S^O(Z_o|Q_o)$. is not differentiable at $Z_o = Q_o$.

Therefore, the Lemma 1 can be derived. □

Proof of Theorem 1. Based on the Lemma 1, the authors solve the optimal solutions in the cases of $[0, Q_o]$ and $[Q_o, +\infty)$, respectively.

(1) When $0 \le Z_o \le Q_o$, by the first order condition:

$$\frac{d\Pi_S^O(Z_o|Q_o, 0 \le Z_o \le Q_o)}{dZ_o} = -c + (1-\beta)c_e - (1-\beta)(c_e - s)F(Z_o) = 0.$$

This implies $F(Z_o^*) = \frac{(1-\beta)c_e - s}{(1-\beta)(c_e - s)}$. Due to $(1-\beta)c_e - c > (1-\beta)(\underline{p}(c_e) + c_e\overline{G}(c_e)) - c \ge 0$, then $F(Z_o^*) > 0$. Considering the condition of $0 \le Z_o \le Q_o$, then $F(Z_o^*) = \min\{\frac{(1-\beta)c_e - s}{(1-\beta)(c_e - s)}, F(Q_o)\}$.

(2) When $Z_o \ge Q_o$, by the first order condition,

$$\begin{aligned}
\frac{d\Pi_S^O(Z_o|Q_o, Z_o \ge Q_o)}{dZ_o} &= -c + (1-\beta)(\underline{p}(c_e) + c_e\overline{G}(c_e)) - (1-\beta)(\underline{p}(c_e) + c_e\overline{G}(c_e) - s)F(Z_o) \\
&= 0
\end{aligned}$$

then $F(Z_o^*) = \frac{(1-\beta)\hat{p} - c}{(1-\beta)(\hat{p} - s)}$. Considering the condition of $Z_o \ge Q_o$, $F(Z_o^*) = \max\{\frac{(1-\beta)\hat{p} - c}{(1-\beta)(\hat{p} - s)}, F(Q_o)\}$ is derived.

Define $F(Z_o^h) = \frac{(1-\beta)c_e - s}{(1-\beta)(c_e - s)}$ and $F(Z_o^l) = \frac{(1-\beta)\hat{p} - c}{(1-\beta)(\hat{p} - s)}$. Then, it is known $Z_o^l < Z_o^h$. Based on the different firm's option order quantity Q_o, try to obtain the supplier's optimal decision under the following three situations:

(1) If $Q_o \geq Z_o^h$, the optimal solution $Z_o^*(Q_o)$ must be obtained when $Z_o \in [0, Q_o]$. Then, $Z_o^*(Q_o) = Z_o^h$ can be derived.

(2) If $Q_o \leq Z_o^l$, the optimal solution $Z_o^*(Q_o)$ must be obtained when $Z_o \in [Q_o, +\infty)$. Then, $Z_o^*(Q_o) = Z_o^l$ can be derived.

(3) If $Z_o^l \leq Q_o \leq Z_o^h$, the optimal solution $Z_o^*(Q_o)$ must be obtained when $Z_o = Q_o$. Then, $Z_o^*(Q_o) = Q_o$ can be derived.

To conclude, the Theorem 1 can be derived. □

Proof of Lemma 2. The proof of Lemma 2 is similar to the proof of Lemma 1, then the proof of Lemma 2 is omitted. □

Proof of Theorem 2. Based on Lemma 2, first solve the firm's optimal solutions when $Q_o \in [0, Z_o^l]$ and $Q_o \in [Z_o^l, +\infty)$, respectively. Then obtain the optimal solution for $Q_o \in [0, +\infty)$.

(1) When $0 \leq Q_o \leq Z_o^l$, $Z_o^*(Q_o) \geq Q_o$. By the first order condition:

$$\frac{d\Pi_R^O(Q_o)}{dQ_o} = (\beta r G(c_e) + (1-\beta)\underline{p}(c_e) + \overline{p}(c_e) - w)(1 - F(Q_o)) - c_o = 0,$$

then $F(Q_o^*) = \frac{p_1 - c_o - w}{p_1 - w}$. Due to $p_1 - w \geq c_o$, then $F(Q_o^*) > 0$. Considering the condition of $0 \leq Q_o \leq Z_o^l$, $F(Q_o^*) = \min\{\frac{p_1 - c_o - w}{p_1 - w}, F(Z_o^l)\}$ can be derived.

(2) When $Q_o \geq Z_o^l$, $Z_o^*(Q_o) \leq Q_o$. By the first order condition:

$$\frac{d\Pi_R^O(Q_o)}{dQ_o} = (r G(c_e) + \overline{p}(c_e) - w)(1 - F(Q_o)) - c_o = 0$$

then $F(Q_o^*) = \frac{p_2 - c_o - w}{p_2 - w}$. Due to $p_2 \geq c_o + w$, then $F(Q_o^*) > 0$. Considering the condition of $Q_o \geq Z_o^l$, $F(Q_o^*) = \min\{\frac{p_2 - c_o - w}{p_2 - w}, F(Z_o^l)\}$ can be derived.

Define $F(Q_o^h) = \frac{p_2 - c_o - w}{p_2 - w}$ and $F(Q_o^l) = \frac{p_1 - c_o - w}{p_1 - w}$. Then, it is known that $Q_o^l < Q_o^h$. Next, try to obtain the firm's optimal decision under the following three situations:

(1) If $Z_o^l \leq Q_o^l$ (that is also $\beta > \frac{(\hat{p}-s)c_o - (p_1-w)(c-s)}{(p_1-w)s + (\hat{p}-s)c_o}$), then the firm's optimal decision is $Q_o^* = Q_o^h$. Regarding the supplier, if $Q_o^h \leq Z_o^h$ ($\beta \leq \frac{(c_e-s)c_o - (p_2-w)(c-s)}{(p_2-w)s + (c_e-s)c_o}$), his optimal decision is $Z_o^* = Q_o^h$; if $Q_o^h > Z_o^h$ ($\beta > \frac{(c_e-s)c_o - (p_2-w)(c-s)}{(p_2-w)s + (c_e-s)c_o}$), then $Z_o^* = Z_o^h$.

(2) If $Q_o^l \leq Z_o^l \leq Q_o^h$ ($\frac{(\hat{p}-s)c_o - (p_2-w)(c-s)}{(p_2-w)s + (\hat{p}-s)c_o} \leq \beta \leq \frac{(\hat{p}-s)c_o - (p_1-w)(c-s)}{(p_1-w)s + (\hat{p}-s)c_o}$), then when $0 \leq Q_o \leq Z_o^l$, $Q_o^* = Q_o^l$; when $Q_o > Z_o^l$, $Q_o^* = Q_o^h$. When in this situation, comparison of the optimal profits when $Q_o^* = Q_o^l$ and $Q_o^* = Q_o^h$ to obtain the firm's optimal solution is needed.

A. When $Q_o^* = Q_o^l$, then $Z_o^*(Q_o) = Z_o^l$,

$$\Pi_{R1}^O(Q_o^*) = (\beta r G(c_e) + (1-\beta)\underline{p}(c_e) + \overline{p}(c_e) - w)\left(Q_o^l - \int_0^{Q_o^l} F(x)dx\right)$$
$$+ (1-\beta)\left(r G(c_e) - \underline{p}(c_e)\right)(Z_o^l - \int_0^{Z_o^l} F(x)dx) + r\overline{G}(c_e)\mu - \overline{p}(c_e)\mu - c_o Q_o^l.$$

B. When $Q_o^* = Q_o^h$, then $0 < Z_o^*(Q_o) \leq Q_o^*$,

$$\Pi_{R2}^O(Q_o^*) = (r G(c_e) + \overline{p}(c_e) - w)\left(Q_o^h - \int_0^{Q_o^h} F(x)dx\right) + r\overline{G}(c_e)\mu - \overline{p}(c_e)\mu - c_o Q_o^h.$$

Then, it can be derived:

$$
\begin{aligned}
&\Pi_{R2}^O(Q_o^*) - \Pi_{R1}^O(Q_o^*) \\
&= (rG(c_e) + \overline{p}(c_e) - w)\left(Q_o^h - \int_0^{Q_o^h} F(x)dx\right) - c_o(Q_o^h - Q_o^l) \\
&\quad - (\beta rG(c_e) + (1-\beta)\underline{p}(c_e) + \overline{p}(c_e) - w)\left(Q_o^l - \int_0^{Q_o^l} F(x)dx\right) \\
&\quad - (1-\beta)(rG(c_e) - \underline{p}(c_e))(Z_o^l - \int_0^{Z_o^l} F(x)dx).
\end{aligned}
$$

Assume that:

$$
\begin{aligned}
H_1(t) &= (rG(c_e) + \overline{p}(c_e) - w)\left(Q_o^h - \int_0^{Q_o^h} F(x)dx\right) - c_o(Q_o^h - Q_o^l) \\
&\quad - \left(\beta rG(c_e) + (1-\beta)\underline{p}(c_e) + \overline{p}(c_e) - w\right)\left(Q_o^l - \int_0^{Q_o^l} F(x)dx\right) \\
&\quad - (1-\beta)\left(rG(c_e) - \underline{p}(c_e)\right)\left(t - \int_0^t F(x)dx\right),
\end{aligned}
$$

where $Q_o^l \le t \le Q_o^h$, then $H_1(Z_o^l) = \Pi_{R2}^O(Q_o^*) - \Pi_{R1}^O(Q_o^*)$. Taking the first-order derivative of $H_1(t)$,

$$
\frac{dH_1(t)}{dt} = -(1-\beta)\left(rG(c_e) - \underline{p}(c_e)\right)(1 - F(t)) < 0,
$$

then $H_1(t)$ is decreasing for $t \in [Q_o^l, Q_o^h]$. Because of:

$$
\begin{aligned}
H_1(Q_o^l) &= (rG(c_e) + \overline{p}(c_e) - w - c_o)(Q_0^h - Q_0^l) - (rG(c_e) + \overline{p}(c_e) - w)\int_{Q_0^l}^{Q_0^h} F(x)dx \\
&> (rG(c_e) + \overline{p}(c_e) - w - c_o)(Q_o^h - Q_o^l) \\
&\quad - (rG(c_e) + \overline{p}(c_e) - w)(Q_o^h - Q_o^l)(1 - \tfrac{c_o}{rG(c_e) + \overline{p}(c_e) - w}) = 0,
\end{aligned}
$$

$$
\begin{aligned}
H_1(Q_o^h) &= \left(\beta rG(c_e) + (1-\beta)\underline{p}(c_e) + \overline{p}(c_e) - w - c_o\right)(Q_o^h - Q_o^l) \\
&\quad - (\beta rG(c_e) + (1-\beta)\underline{p}(c_e) + \overline{p}(c_e) - w)\int_{Q_o^l}^{Q_o^h} F(x)dx \\
&< (\beta rG(c_e) + (1-\beta)\underline{p}(c_e) + \overline{p}(c_e) - w - c_o)(Q_o^h - Q_o^l) \\
&\quad - (\beta rG(c_e) + (1-\beta)\underline{p}(c_e) + \overline{p}(c_e) - w)(Q_o^h - Q_o^l)(1 \\
&\quad - \tfrac{c_o}{\beta rG(c_e) + (1-\beta)\underline{p}(c_e) + \overline{p}(c_e) - w} = 0,
\end{aligned}
$$

there exists a unique t_0 which satisfies $H_1(t_0) = 0$. Due to Z_o^l is decreasing in β. Hence there also exists a unique $\hat{\beta}_o$ which satisfies $F(t_0) = \frac{(1-\hat{\beta}_o)\hat{p} - c}{(1-\hat{\beta}_o)(\hat{p} - s)}$. Next, consider the following two cases:

A. If $Z_o^l < t_0$ (that is also $\beta > \hat{\beta}_o$), then $\Pi_{R2}^O(Q_o^*) - \Pi_{R1}^O(Q_o^*) > 0$, then $Q_o^* = Q_o^h$. Similarly, when $Q_o^h \le Z_o^h$ ($\beta \le \frac{(c_e - s)c_o - (p_2 - w)(c-s)}{(p_2 - w)s + (c_e - s)c_o}$), $Z_o^*(Q_o) = Q_o^h$; when $Q_o^h = Z_o^h$ ($\beta > \frac{(c_e - s)c_o - (p_2 - w)(c-s)}{(p_2 - w)s + (c_e - s)c_o}$), $Z_o^*(Q_o) = Z_o^h$.

B. If $Z_o^l > t_0$ ($\beta < \hat{\beta}_o$), then $\Pi_{R2}^O(Q_o^*) - \Pi_{R1}^O(Q_o^*) < 0$, then $Q_o^* = Q_o^l$ and $Z_o^*(Q_o) = Z_o^l$.

(3) If $Z_o^l \ge Q_o^h$ ($\beta \le \frac{(\hat{p} - s)c_o - (p_2 - w)(c-s)}{(p_2 - w)s + (\hat{p} - s)c_o}$), then $Q_o^* = Q_o^l$ and $Z_o^*(Q_o) = Z_o^l$.

To summarize, (1) when $0 < \beta \le \hat{\beta}_o$, then $Q_o^* = Q_o^l$ and $Z_o^* = Z_o^l$; (2) when $\hat{\beta}_o < \beta \le \frac{\hat{p} - c}{\hat{p}}$, then

$$
Q_o^* = Q_o^h \text{ and } Z_o^* = \begin{cases} Q_o^h & if \quad \beta \le \beta_2 \\ Z_o^h & if \quad \beta > \beta_2 \end{cases}, \text{ where } \beta_2 = \frac{(c_e - s)c_o - (p_2 - w)(c-s)}{(p_2 - w)s + (c_e - s)c_o}.
$$

Therefore, the Theorem 2 is derived. □

Proof of Theorem 3. The supplier's expected profit is:

$$
\begin{aligned}
\Pi_S^W(Z_w | Q_w) &= -cZ_w + (1-\beta)wQ_w + (1-\beta)\underline{p}(c_e)\left(Z_w - Q_w - \int_{Q_w}^{Z_w} F(x)dx\right) \\
&\quad + (1-\beta)\overline{p}(c_e)\left(\mu - Q_w + \int_0^{Q_w} F(x)dx\right) \\
&\quad - (1-\beta)c_e\overline{G}(c_e)\left(\mu - Z_w + \int_0^{Z_w} F(x)dx\right) + (1-\beta)s\int_{Q_w}^{Z_w} F(x)dx.
\end{aligned}
$$

Take the first-order and second-order derivatives of $\Pi_S^W(Z_w|Q_w)$:

$$\frac{d\Pi_S^W(Z_w|Q_w)}{dZ_w} = -c + (1-\beta)\Big(\underline{p}(c_e) + c_e\overline{G}(c_e)\Big)(1 - F(Z_w)) + (1-\beta)sF(Z_w),$$

$$\frac{d^2\Pi_S^W(Z_w|Q_w)}{d(Z_w)^2} = -(1-\beta)\Big(\underline{p}(c_e) + c_e\overline{G}(c_e) - s\Big)f(Z_w).$$

Then, $\Pi_S^W(Z_w|Q_w)$ is concave with respect to Z_w. By the first-order condition, then $F(Z_w^*) = \frac{(1-\beta)\hat{p}-c}{(1-\beta)(\hat{p}-s)}$. Due to $Z_w^* \geq Q_w$, then $F(Z_w^*) = \max\{\frac{(1-\beta)\hat{p}-c}{(1-\beta)(\hat{p}-s)}, Q_w\}$. Define $F(Z_w^l) = \frac{(1-\beta)\hat{p}-c}{(1-\beta)(\hat{p}-s)}$, it can be derived by the Theorem 3. $\square$

Proof of Theorem 4. The firm's expected profit is:

$$
\begin{aligned}
\pi_R^W(Q_w) = \ & -(1-\beta)wQ_w + (1-\beta)rG(c_e)\Big(Z_w - \int_0^{Z_w} F(x)dx\Big) + (1-\beta)r\overline{G}(c_e)\mu \\
& -(1-\beta)\underline{p}(c_e)\Big(Z_w - Q_w - \int_{Q_w}^{Z_w} F(x)dx\Big) \\
& -(1-\beta)\overline{p}(c_e)\Big(\mu - Q_w + \int_0^{Q_w} F(x)dx\Big) + (1-\beta)s\int_0^{Q_w} F(x)dx.
\end{aligned}
$$

Take the first-order and second-order derivatives of $\pi_R^W(Q_w)$ under two cases.
Case 1: when $Z_w^* > Q_w$,

$$\frac{d\pi_R^W(Q_w)}{dQ_w} = -(1-\beta)w + (1-\beta)\Big(\underline{p}(c_e) + \overline{p}(c_e)\Big)(1 - F(Q_w)) + (1-\beta)sF(Q_w),$$

$$\frac{d^2\pi_R^W(Q_w)}{d(Q_w)^2} = -(1-\beta)\Big(\underline{p}(c_e) + \overline{p}(c_e)\Big)f(Q_w) + (1-\beta)sf(Q_w) < 0.$$

Hence, $\pi_R^W(Q_w)$ is concave in Q_w and $F(Q_w^*) = \frac{\tilde{p}-w}{\tilde{p}-s}$.
Case 2: when $Z_w^* = Q_w$,

$$\frac{d\pi_R^W(Q_w)}{dQ_w} = -(1-\beta)w + (1-\beta)(rG(c_e) + \overline{p}(c_e))(1 - F(Q_w)) + (1-\beta)sF(Q_w),$$

$$\frac{d^2\pi_R^W(Q_w)}{d(Q_w)^2} = -(1-\beta)(rG(c_e) + \overline{p}(c_e))f(Q_w) + (1-\beta)sf(Q_w) < 0.$$

Hence, $\pi_R^W(Q_w)$ is concave in Q_w and $F(Q_w^*) = \frac{p_2-w}{p_2-s}$.
Define $F(Q_w^h) = \frac{p_2-w}{p_2-s}$, $F(Q_w^l) = \frac{\tilde{p}-w}{\tilde{p}-s}$ and $F(Z_w^l) = \frac{(1-\beta)\hat{p}-c}{(1-\beta)(\hat{p}-s)}$. Based on the supplier's best response, the firm's optimal decision satisfies:

$$F(Q_w^*) = \begin{cases} \frac{p_2-w}{p_2-s}, & if \quad Z_w^* = Q_w, \\ \frac{\tilde{p}-w}{\tilde{p}-s}, & if \quad Z_w^* > Q_w. \end{cases}$$

Now, analyze the optimal decisions of the supplier and firm as follows:
(1) If $Z_w^l < Q_w^l$ (that is also $\beta > \frac{(\hat{p}-c)(\tilde{p}-s)-(\hat{p}-s)(\tilde{p}-w)}{\hat{p}(\tilde{p}-s)-(\hat{p}-s)(\tilde{p}-w)}$), then $Q_w^* = Z_w^* = Q_w^h$.

(2) If $Q_w^l \leq Z_w^l \leq Q_w^h$ (that is also $\frac{(\hat{p}-c)(p_2-s)-(\hat{p}-s)(p_2-w)}{\hat{p}(p_2-s)-(\hat{p}-s)(p_2-w)} \leq \beta \leq \frac{(\hat{p}-c)(\hat{p}-s)-(\hat{p}-s)(\tilde{p}-w)}{\hat{p}(\tilde{p}-s)-(\hat{p}-s)(\tilde{p}-w)}$), let $Q^H = Z^H = Q_w^h$, $Q^L = Q_w^l$ and $Z^L = Z_w^l$, then Q^H and Z^H, Q^L and Z^L are two pairs optimal solutions of the supplier and firm. Next, compare the optimal profits under the two cases. Then:

$$\pi_{R2}^W(Q^H) = -(1-\beta)wQ^H + (1-\beta)rG(c_e)\left(Q^H - \int_0^{Q^H} F(x)dx\right) + (1-\beta)r\overline{G}(c_e)\mu -$$
$$(1-\beta)\overline{p}(c_e)\left(\mu - Q^H + \int_0^{Q^H} F(x)dx\right) + (1-\beta)s\int_0^{Q^H} F(x)dx,$$

$$\pi_{R1}^W(Q^L) = -(1-\beta)wQ^L + (1-\beta)rG(c_e)\left(Z^L - \int_0^{Z^L} F(x)dx\right) + (1-\beta)r\overline{G}(c_e)\mu -$$
$$(1-\beta)\underline{p}(c_e)\left(Z^L - Q^L - \int_{Q^L}^{Z^L} F(x)dx\right) - (1-\beta)\overline{p}(c_e)\left(\mu - Q^L + \int_0^{Q^L} F(x)dx\right) + (1-\beta)s\int_0^{Q^L} F(x)dx,$$

$$\pi_{R2}^W(Q^H) - \pi_{R1}^W(Q^L) = -(1-\beta)w(Q^H - Q^L) + (1-\beta)(rG(c_e) + \overline{p}(c_e))(Q^H - Z^L -$$
$$\int_{Z^L}^{Q^H} F(x)dx\right) + (1-\beta)\underline{p}(c_e)\left(Z^L - Q^L - \int_{Q^L}^{Z^L} F(x)dx\right) + (1-\beta)s\int_{Q^L}^{Q^H} F(x)dx.$$

Assume that:

$$H_2(t) = -(1-\beta)w(Q^H - Q^L) + (1-\beta)(rG(c_e) + \overline{p}(c_e))(Q^H - t - \int_t^{Q^H} F(x)dx) +$$
$$(1-\beta)\underline{p}(c_e)\left(t - Q^L - \int_{Q^L}^t F(x)dx\right) + (1-\beta)s\int_{Q^L}^{Q^H} F(x)dx,$$

then, $Q^L \le t \le Q^H$. Taking the first-order derivate of $H_2(t)$ with respect to t,

$$\frac{dH_2(t)}{dt} = -(1-\beta)\left(rG(c_e) + \overline{p}(c_e) - \underline{p}(c_e)\right)(1 - F(t)) < 0.$$

Therefore, $H_2(t)$ is decreasing in t for $t \in [Q^L, Q^H]$.
If $t = Q^L$, then:

$$H_2(Q^L) = \quad -(1-\beta)w(Q^H - Q^L)$$
$$+ (1-\beta)(rG(c_e) + \overline{p}(c_e))(Q^H - Q^L - Q^H F(Q^H) + Q^L F(Q^L)$$
$$+ \int_{Q^L}^{Q^H} xf(x)dx\right) + (1-\beta)s\left(Q^H F(Q^H) - Q^L F(Q^L) - \int_{Q^L}^{Q^H} xf(x)dx\right)$$
$$> -(1-\beta)w(Q^H - Q^L) + (1-\beta)(rG(c_e) + \overline{p}(c_e))(Q^H - Q^L)(1 - F(Q^H)) - (1-\beta)s(Q^H - Q^L)F(Q^H)$$
$$= (1-\beta)(Q^H - Q^L)\left((rG(c_e) + \overline{p}(c_e))(1 - F(Q^H)) - w - sF(Q^H)\right)$$
$$= (1-\beta)(Q^H - Q^L)\frac{(w-s)(rG(c_e) + \overline{p}(c_e)) - (w-s)(rG(c_e) + \overline{p}(c_e))}{rG(c_e) + \overline{p}(c_e) - s} = 0.$$

If $t = Q^H$, then:

$$H_2(Q^H) = \quad -(1-\beta)w(Q^H - Q^L)$$
$$+ (1-\beta)\underline{p}(c_e)\left(Q^H - Q^L - Q^H F(Q^H) + Q^L F(Q^L) + \int_{Q^L}^{Q^H} xf(x)dx\right)$$
$$+ (1-\beta)s\left(Q^H F(Q^H) - Q^L F(Q^L) - \int_{Q^L}^{Q^H} xf(x)dx\right)$$
$$< -(1-\beta)w(Q^H - Q^L) + (1-\beta)\underline{p}(c_e)(Q^H - Q^L)(1 - F(Q^L))$$
$$+ (1-\beta)s(Q^H - Q^L)(1 - F(Q^L))$$
$$= (1-\beta)(Q^H - Q^L)\frac{(w-s)\underline{p}(c_e) - (w-s)\widetilde{p}}{\widetilde{p} - s} < 0.$$

Due to $H_2(Q^L) > 0$ and $H_2(Q^H) < 0$, therefore, there exists a unique $t_1 \in [Q^L, Q^H]$ which satisfies $H_2(t_1) = 0$. Because Z^L is decreasing in β, then there also exists a unique $\hat{\beta}_w$ which satisfies $F(t_1) = \frac{(1-\hat{\beta}_w)\hat{p} - c}{(1-\hat{\beta}_w)(\hat{p}-s)}$. Therefore, when $\beta \le \hat{\beta}_w$, $Q_w^* = Q_w^l$ and $Z_w^* = Z_w^l$; otherwise, $Q_w^* = Z_w^* = Q_w^h$.

(3) If $Z_w^l > Q_w^h$ (that is also $\beta < \frac{(\hat{p}-c)(p_2-s) - (\hat{p}-s)(p_2-w)}{\hat{p}(p_2-s) - (\hat{p}-s)(p_2-w)}$), $Q_w^* = Q_w^l$ and $Z_w^* = Z_w^l$.

To summarize, (1) when $0 \le \beta \le \hat{\beta}_w$, $Q_w^* = Q_w^l$ and $Z_w^* = Z_w^l$; (2) when $\hat{\beta}_w < \beta \le 1 - \frac{c}{\hat{p}}$, $Q_w^* = Z_w^* = Q_w^h$.

Therefore, the Theorem 4 is derived. □

Proof of Corollaries 1–3. Due to the simplicity, the authors omit the proof of Corollaries 1–3. □

References

1. Hu, X.; Gurnani, H.; Wang, L. Managing risk of supply disruptions: Incentives for capacity restoration. *Prod. Oper. Manag.* **2013**, *22*, 137–150. [CrossRef]
2. Hu, B.; Kostamis, D. Managing supply disruptions when sourcing from reliable and unreliable suppliers. *Prod. Oper. Manag.* **2015**, *24*, 808–820. [CrossRef]
3. Hendricks, K.B.; Singhal, V.R. Association between supply chain glitches and operating performance. *Manag. Sci.* **2005**, *51*, 695–711. [CrossRef]
4. Barbosa-Póvoa, A.P.; da Silva, C.; Carvalho, A. Opportunities and challenges in sustainable supply chain: An operations research perspective. *Eur. J. Oper. Res.* **2018**, *268*, 399–431. [CrossRef]
5. Tokui, J.; Kawasaki, K.; Miyagawa, T. The economic impact of supply chain disruptions from the Great East-Japan earthquake. *Jpn. World Econ.* **2017**, *41*, 59–70. [CrossRef]
6. Tomlin, B. On the value of mitigation and contingency strategies for managing supply chain disruption risks. *Manag. Sci.* **2006**, *52*, 639–657. [CrossRef]
7. Wang, Y.; Gilland, W.; Tomlin, B. Mitigating supply risk: Dual sourcing or process improvement? *Manuf. Serv. Oper. Manag.* **2010**, *12*, 489–510. [CrossRef]
8. Dong, L.; Tomlin, B. Managing disruption risk: The interplay between operations and insurance. *Manag. Sci.* **2012**, *58*, 1898–1915. [CrossRef]
9. Li, Y.; Zhen, X.; Qi, X.; Cai, G. Penalty and financial assistance in a supply chain with supply disruption. *Omega* **2016**, *61*, 167–181. [CrossRef]
10. Zhen, X.; Li, Y.; Cai, G.G.; Shi, D. Transportation disruption risk management: Business interruption insurance and backup transportation. *Trans. Res. Part E* **2016**, *90*, 51–68. [CrossRef]
11. Feng, C.; Wang, Z.; Jiang, Z. Retailer's procurement strategy under endogenous supply stability. *Sustainability* **2017**, *9*, 2261. [CrossRef]
12. Kumar, M.; Basu, P.; Avittathur, B. Pricing and sourcing strategies for competing retailers in supply chains under disruption risk. *Eur. J. Oper. Res.* **2018**, *265*, 533–543. [CrossRef]
13. Nagali, V.; Hwang, J.; Sanghera, D.; Gaskins, M.; Pridgen, M.; Thurston, T.; Shoemaker, G. Procurement risk management (PRM) at Hewlett-Packard company. *Interfaces* **2008**, *38*, 51–60. [CrossRef]
14. Chen, J.Y.; Dada, M.; Hu, Q. Designing supply contracts: Buy-now, reserve, and wait-and-see. *IIE Trans.* **2016**, *48*, 881–900. [CrossRef]
15. Ma, L.; Zhao, Y.; Xue, W.; Cheng, T.C.E.; Yan, H. Loss-averse newsvendor model with two ordering opportunities and market information updating. *Int. J. Prod. Econ.* **2012**, *140*, 912–921. [CrossRef]
16. Chen, X.; Shen, Z.J. An analysis of a supply chain with options contracts and service requirements. *IIE Trans.* **2012**, *44*, 805–819. [CrossRef]
17. Yu, H.; Zeng, A.Z.; Zhao, L. Single or dual sourcing: Decision-making in the presence of supply chain disruption risks. *Omega* **2009**, *37*, 788–800. [CrossRef]
18. Kim, M.; Jim, C. Global Supply Chain Rattled by Japan Quake, Tsunami. 2011. Available online: http://www.reuters.com/article/2011/03/14/japan-quake-supplychain-idUSL3E7EE05V20110314 (accessed on 20 June 2018).
19. Fang, Y.; Shou, B. Managing supply uncertainty under supply chain Cournot competition. *Eur. J. Oper. Res.* **2015**, *243*, 156–176. [CrossRef]
20. Tomlin, B. Disruption-management strategies for short life-cycle products. *Nav. Logist. Res.* **2009**, *56*, 318–347. [CrossRef]
21. Tang, C.S. Perspectives in supply chain risk management. *Int. J. Prod. Econ.* **2006**, *103*, 451–488. [CrossRef]
22. Snyder, L.V.; Atan, Z.; Peng, P.; Rong, Y.; Schmitt, A.J.; Sinsoysal, B. OR/MS models for supply chain disruptions: A review. *IIE Trans.* **2016**, *48*, 89–109. [CrossRef]
23. Yan, R.; Lu, B.; Wu, J. Contract coordination strategy of supply chain with substitution under supply disruption and stochastic demand. *Sustainability* **2016**, *8*, 676. [CrossRef]
24. Zhao, M.; Freeman, N.K. Robust sourcing from suppliers under ambiguously correlated major disruption risks. *Prod. Oper. Manag.* **2018**. [CrossRef]
25. Yin, Z.; Wang, C. Strategic cooperation with a backup supplier for the mitigation of supply disruptions. *Int. J. Prod. Res.* **2017**. [CrossRef]

26. Wang, C.; Yin, Z. Using backup supply with responsive pricing to mitigate disruption risk for a risk-averse firm. *Int. J. Prod. Res.* **2018**. [CrossRef]

27. He, B.; Huang, H.; Yuan, K. The comparison of two procurement strategies in the presence of supply disruption. *Comput. Ind. Eng.* **2015**, *85*, 296–305. [CrossRef]

28. Bolandifar, E.; Feng, T.; Zhang, F. Simple contracts to assure supply under noncontractible capacity and asymmetric cost information. *Manuf. Serv. Oper. Manag.* **2018**, *20*, 217–231. [CrossRef]

29. Hwang, W.; Bakshi, N.; DeMiguel, V. Wholesale price contracts for reliable supply. *Prod. Oper. Manag.* **2018**. [CrossRef]

30. Xu, H. Managing production and procurement through option contracts in supply chains with random yield. *Int. J. Prod. Econ.* **2010**, *126*, 306–313. [CrossRef]

31. Li, J.C.; Zhou, Y.W.; Huang, W. Production and procurement strategies for seasonal product supply chain under yield uncertainty with commitment-option contracts. *Int. J. Prod. Econ.* **2017**, *183*, 208–222. [CrossRef]

32. Köle, H.; Bakal, I.S. Value of information through options contract under disruption risk. *Comput. Ind. Eng.* **2017**, *103*, 85–97. [CrossRef]

33. Xia, Y.; Ramachandran, K.; Gurnani, H. Sharing demand and supply risk in a supply chain. *IIE Trans.* **2011**, *43*, 451–469. [CrossRef]

34. Xue, K.; Li, Y.; Zhen, X.; Wang, W. Managing the disruption risk: Option contract or order commitment contract? *Ann. Oper. Res.* **2018**. [CrossRef]

35. Li, X.; Li, Y. On the loss-averse dual-sourcing problem under supply disruption. *Comput. Oper. Res.* **2016**. [CrossRef]

36. Mizgier, K.J.; Wagner, S.M.; Holyst, J.A. Modeling defaults of companies in multi-stage supply chain networks. *Int. J. Prod. Econ.* **2012**, *135*, 14–23. [CrossRef]

37. Mizgier, K.J.; Wagner, S.M.; Jüttner, M.P. Disentangling diversification in supply chain networks. *Int. J. Prod. Econ.* **2015**, *162*, 115–124. [CrossRef]

38. Wagner, S.M.; Mizgier, K.J.; Papageorgiou, S. Operational disruptions and business cycles. *Int. J. Prod. Econ.* **2017**, *183*, 66–78. [CrossRef]

39. Tang, S.Y.; Gurnani, H.; Gupta, D. Managing disruptions in decentralized supply chains with endogenous supply process reliability. *Prod. Oper. Manag.* **2014**, *23*, 1198–1211. [CrossRef]

40. Latour, A. Trial by Fire: A Blaze in Albuquerque Sets Off Major Crisis for Cell-Phone Giants. 2001. Available online: https://www.wsj.com/articles/SB980720939804883010 (accessed on 20 June 2018).

41. He, Y. Supply risk sharing in a closed-loop supply chain. *Int. J. Prod. Econ.* **2017**, *183*, 39–52. [CrossRef]