

Article

Powering Mode-Integrated Energy Management Strategy for a Plug-In Hybrid Electric Truck with an Automatic Mechanical Transmission Based on Pontryagin's Minimum Principle

Shaobo Xie ^{1,†}, Xiaosong Hu ^{2,*}, Kun Lang ¹, Shanwei Qi ¹ and Tong Liu ¹

¹ School of Automotive Engineering, Chang'an University, Xi'an 710064, China; xieshaobo@chd.edu.cn (S.X.); 2017222079@chd.edu.cn (K.L.); 2017122004@chd.edu.cn (S.Q.); 2014222050@chd.edu.cn (T.L.)

² State Key Laboratory of Mechanical Transmissions, Chongqing University, Chongqing 400044, China

* Correspondence: xiaosonghu@ieee.org; Tel.: +86-136-1820-9956

† The authors equally contributed to this research work.

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Abstract: Pontryagin's Minimum Principle (PMP) has a significant computational advantage over dynamic programming for energy management issues of hybrid electric vehicles. However, minimizing the total energy consumption for a plug-in hybrid electric vehicle based on PMP is not always a two-point boundary value problem (TPBVP), as the optimal solution of a powering mode will be either a pure-electric driving mode or a hybrid discharging mode, depending on the trip distance. In this paper, based on a plug-in hybrid electric truck (PHET) equipped with an automatic mechanical transmission (AMT), we propose an integrated control strategy to flexibly identify the optimal powering mode in accordance with different trip lengths, where an electric-only-mode decision module is incorporated into the TPBVP by judging the auxiliary power unit state and the final battery state-of-charge (SOC) level. For the hybrid mode, the PMP-based energy management problem is converted to a normal TPBVP and solved by using a shooting method. Moreover, the energy management for the plug-in hybrid electric truck with an AMT involves simultaneously optimizing the power distribution between the auxiliary power unit (APU) and the battery, as well as the gear-shifting choice. The simulation results with long- and short-distance scenarios indicate the flexibility of the PMP-based strategy. Furthermore, the proposed control strategy is compared with dynamic programming (DP) and a rule-based charge-depleting and charge-sustaining (CD-CS) strategy to evaluate its performance in terms of computational accuracy and time efficiency.

Keywords: plug-in hybrid electric vehicle; energy management strategy; automatic mechanical transmission; Pontryagin's minimum principle

1. Introduction

Due to increasingly severe air pollution and energy crisis, as well as growingly stringent emission regulations, automotive manufactures over the world have been paying enormous attention to electrified vehicles [1–3]. Among them, plug-in hybrid electric vehicles (PHEVs), which are capable of absorbing electricity from the power grid, have been considered as a promising, sustainable mobility solution, because they not only exhibit excellent fuel economy but also are immune from range anxiety during long-distance trips [4].

The presence of two or more power sources such as the auxiliary power unit and the battery in a PHEV makes it necessary to develop a supervisory control strategy where such hybrid power sources are expected to be smartly blended to minimize the energy consumption or the total consumed

energy cost during a driving cycle [5]. A common on-board control strategy is the charge-depleting and charge-sustaining strategy [6]. Such a rule-based method involves driving the vehicle in an electric-only mode until a pre-set lower state-of-charge (SOC) threshold value is reached, and a charging-sustaining mode is then launched. Fuzzy rules were also introduced to further optimize the rule-based methods [7]. The energy distribution policy in these rule-based strategies, however, often depends on engineering experiences/heuristics and lacks design flexibility, largely weakening the potential performance of PHEVs.

To take full advantage of a multiple-source powertrain for optimal fuel economy, the blended discharging strategy that carefully uses the battery and other energy sources simultaneously was developed, including global optimization methods [8], instantaneous minimum approaches [9], and even data-driven algorithms [10,11]. Dynamic programming, as a typically global optimization method, has been extensively applied for energy management problems in HEVs and PHEVs, making it a benchmark to assess other strategies [12,13] or develop real-time power management strategies e.g., in a predictive manner [14,15]. The stochastic dynamic programming also has been proposed to optimize the energy management issue with a Markov chain model [16]. Moreover, the DP algorithm can be employed in a receding horizon to form a model predictive control strategy [17,18]. However, the challenge for DP methods lies in a heavy computational burden, because of the interpolation calculation with massive discrete points of state variables to evaluate the cost-to-go.

Pontryagin's Minimum Principle (PMP) is another extensively utilized global optimization algorithm based on the Hamilton–Jacobi–Bellman equation [19–21], and its solution is capable of inspiring the Equivalent Consumption Minimum Strategy for practical applications [9,22]. Given that a final SOC value equals its initial level, the optimal energy consumption problem for HEVs can form a regular two-point boundary value problem [22], which can be solved directly by numerical methods, i.e., a shooting method. For a PHEV, however, the PMP-based energy management issue over a known driving cycle will exhibit a completely different feature, because the lower SOC boundary value highly related to the driving distance cannot easily be set in advance [23,24]. Specifically, if the lower boundary value is unreasonably preset, the final SOC at the end of the trip may never approach the preassigned value. As a result, the PMP-based energy management problem for plug-in hybrid electric vehicles is not necessarily a two-point boundary value problem. Generally speaking, the optimal solution of the PMP-based strategy induces two possible operating modes: (1) an electric-only mode corresponding to short-distance trips and (2) a hybrid mode corresponding to long-distance trips. The hybrid mode, where the battery charge is depleted such that its final SOC arrives at the preset lower boundary value, can merely result in a normal TPBVP.

So far, various PMP-based control strategies that focus on the hybrid discharging mode and are examined by running cycles with definitive upper and lower SOC boundaries have been proposed. However, the electric-only mode, as a possible optimal choice for short-distance driving scenarios, has not been addressed [25–28]. It is impossible to recognize the optimal powering mode for all scenarios, based on the initial SOC and the driving distance for only a specific driving cycle. Thus, to be a robust controller, the proposed control strategy should be able to deal with the energy management problem for cases with different driving distances. Moreover, for a plug-in hybrid electric truck with an AMT, the choice of gear ratio will change working points of the tractor motor and affect the driveline efficiency and thus the total cost of the consumed energy. As a result, the control strategy should simultaneously optimize the power distribution between the auxiliary power unit (APU) and the battery, as well as the gear ratio choice, inducing a two-dimensional optimization problem.

The main contributions of this paper lie in two aspects. First, we propose a flexible PMP-based energy management controller by integrating electric-only and hybrid powering modes, so as to adapt to any given testing cycle. The power split and the gear shifting are co-optimized via the PMP algorithm, where the gear ratio is regarded as an input variable rather than a state variable in the dynamic equation, which greatly alleviates the numerical difficulty of the two-dimensional optimization issue. Second, the dynamic programming and charge-depletion and charge-sustaining

counterparts are contrasted with the proposed PMP-based method in terms of the total cost and algorithmic efficiency.

The remainder of this paper is organized as follows: Section 2 describes the powertrain configuration and its modeling, and we then formulate the energy management problem based on PMP in Section 3. An integrated control strategy and its solution are detailed in Section 4. Section 5 examines the proposed method with different scenarios. A comprehensive comparison with existing methods is performed in Section 6, followed by main conclusions in Section 7.

2. Powertrain Modeling

2.1. Powertrain Description

The prototype of the considered PHEV is a medium-duty truck, and the architecture of its series powertrain is illustrated in Figure 1. The APU consisting of an internal combustion engine and an integrated-starter-generator (ISG) is mechanically decoupled from the driveline. The engine is a 2.78-L diesel engine, and the ISG is a permanent magnet synchronous motor. The voltage of the energy storage system (an Li-ion battery pack) totals 537.6 V, and its normal capacity is 180 Ah. The electric motor coupled to a three-speed automatic mechanical transmission can operate in either a driving or a regenerating mode.

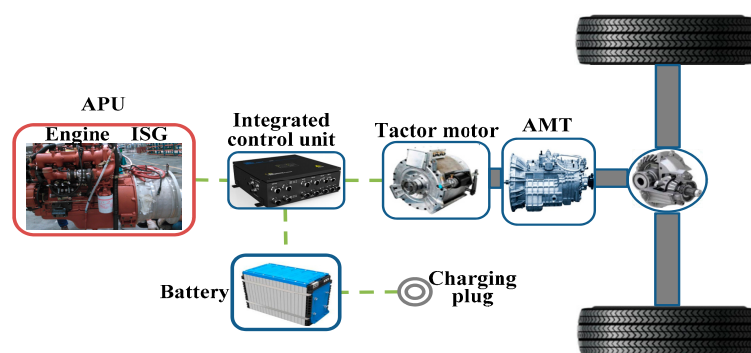


Figure 1. Powertrain architecture of the plug-in hybrid electric truck.

The main parameters of the powertrain are listed in Table 1, wherein the three gear ratios of the automatic mechanical transmission and the maximum dynamic parameters such as the power, speed, and torque for the engine, ISG, and tractor motor are detailed.

Table 1. Main specifications of the plug-in hybrid electric truck.

Item	Parameter	Value
Vehicle	Curb mass (kg)	10,800
	Final gear ratio	5.286
AMT	Gear 1 ratio	4.406
	Gear 2 ratio	2.446
	Gear 3 ratio	1.481
Engine	Displacement (L)	2.78
	Max power (kW)	110
	Max speed (rpm)	3200
ISG	Max power (kW)	90
	Max torque (Nm)	330
	Max speed (rpm)	3000
Tractor motor	Max power (kW)	150
	Max torque (Nm)	850
	Max speed (rpm)	3000
Battery	Capacity (Ah)	180
	Battery total voltage (V)	537.6

2.2. System Model

A quasi-static approach is used to model the powertrain of the PHET.

2.2.1. APU Model

The brake-specific fuel consumption (BSFC) of the engine is mapped from its rotational speed and torque with a look-up table, as shown in Figure 2. The ISG motor efficiency is described as functions of its speed and torque, as depicted in Figure 3.

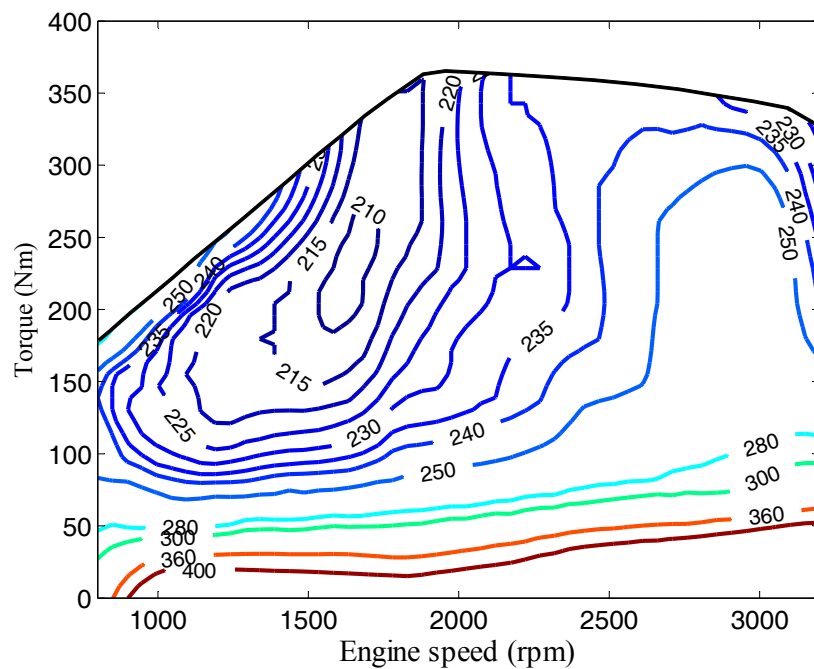


Figure 2. Engine brake-specific fuel consumption (BSFC) map.

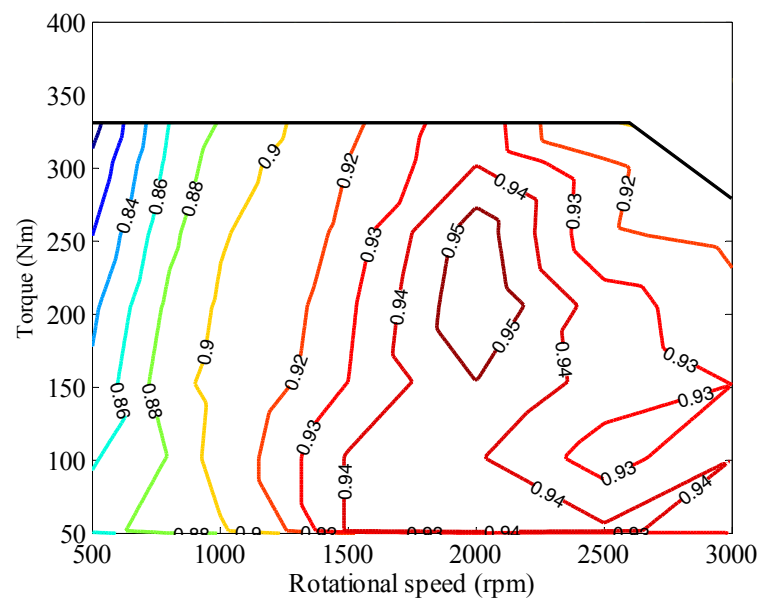


Figure 3. Integrated-starter-generator (ISG) efficiency map.

2.2.2. Electric Motor

The electric motor efficiency is also described as functions of its speed and torque, as plotted in Figure 4.

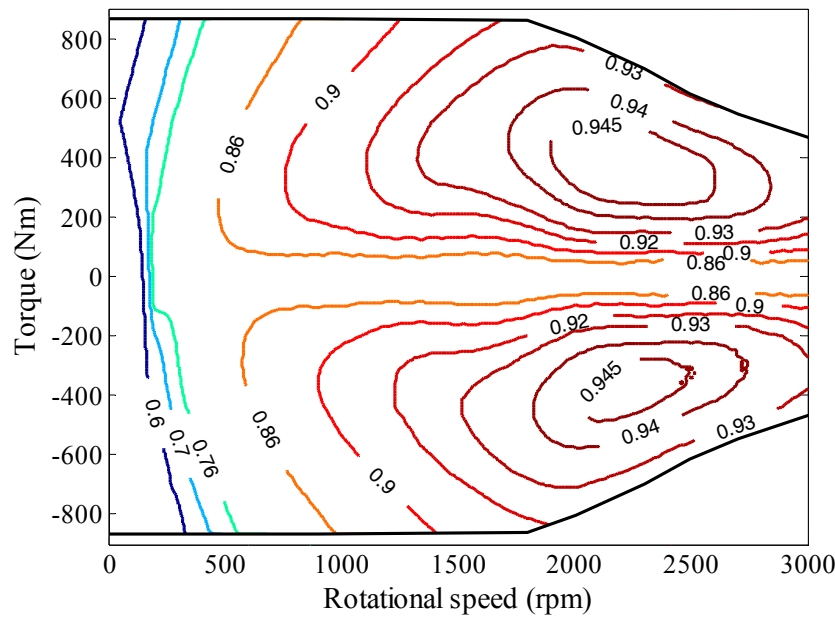


Figure 4. Electric motor efficiency map.

2.2.3. Battery Model

The battery system, which ignores thermal effects and transient influences, is modeled by an equivalent electric circuit, and the simplified model includes an open-circuit voltage in series with an internal resistance, both of which are a function of the battery SOC [18].

The power balance relationship for the battery system can be expressed as

$$P_{bat} = P_b + P_{loss} = P_b + I_b^2 R_b \quad (1)$$

where P_{bat} is the internal power, P_b is the terminal power, and P_{loss} is the internal power loss, and I_b and R_b are the battery electric current and equivalent resistance, respectively.

Accordingly, the voltage balance has the form of

$$V_L = V_{oc} - I_b R_b \quad (2)$$

where V_L is the load voltage, and V_{oc} is the open-circuit voltage.

The dynamic equation for the state of charge is described by [29]

$$\dot{SOC} = -\frac{I_b}{Q_b} \quad (3)$$

where Q_b is the nominal battery capacity.

2.3. Vehicle Dynamics

The power absorbed by the tractor motor is used to overcome the rolling resistance force, the air resistance force, and the accelerating resistance, so the longitudinal dynamics can be given by [30]

$$\begin{cases} T_r = \left(\delta m \frac{dv}{dt} + mgf + 0.5C_d A v^2 \right) r \\ P_r = \frac{1}{\eta_m^{sgn(T_r)} \eta_d^{sgn(T_r)}} \frac{i_0 i_g T_r n_m}{9550} \end{cases} \quad (4)$$

where T_r is the equivalent torque request on the wheel; P_r is the power demand on the output side of the electric machine; η_m denotes the tractor motor efficiency; η_d presents the mechanical efficiency of the driveline; m is the total vehicle mass; v is the speed; δ is the equivalent rotational inertia; i_0 and

i_g denote the final drive ratio and gear ratio, respectively; r is the tire radius; C_d is the air resistance coefficient; A is the front face area; n_m is the rotational speed of the tractor motor.

The electric power balance equation between the power sources is

$$P_{bat} + P_{APU} = P_m + P_{aux} \quad (5)$$

where P_{APU} is the APU output power, P_m is the electric machine power, and P_{aux} denotes the auxiliary component power, e.g., the electric steering system.

3. PMP-Based Optimization Problem

3.1. Optimization Problem Formation

As the optimization problem discussed in this paper is to minimize the total energy cost over a known driving event, the objective function can take the mathematical form as follows [9]:

$$J = \min \int_0^{t_f} (c_f \dot{m}_f + c_e P_{bat}) dt \quad (6)$$

where J is the total cost of the consumed energy, and $\dot{m}_f$ is the fuel consumption rate of the APU; c_f and c_e denote the market prices for oil and electricity, respectively; t_f presents the terminal time of the driving cycle.

In terms of powering mode, the optimal discharging policy for a PHEV can be divided into two modes according to different trip lengths: the electric-only mode for short-distance trips and the hybrid discharging mode for long-distance trips. Unfortunately, it is impossible to initially distinguish which mode will be the optimum for any given driving cycle, because the relationship between the energy requirement of the whole trip and the battery energy available is not clear. As a consequence, to guarantee the algorithmic flexibility, the controller should be designed to smoothly switch between both modes, according to different driving distances.

During the electric-only mode for the PHET, its APU will be off-state, and the battery is responsible for supplying the required power for the driving mission. Therefore, the electric-only mode can be integrated into the TPBVP so that the optimal mode can be filtered by checking the APU output power and the final SOC level. To be more specific, if the APU maintains the off-state during the entire trip, and the final SOC does not exceed the lower boundary value, the electric-only mode is triggered (the charge stored in the battery satisfies the demanded power over the whole trip); otherwise, the hybrid mode is activated. Consequently, the optimal working mode logic of the PHET can be described as

$$\text{optimal mode} = \begin{cases} \text{all electric mode} & P_{APU} = 0 \& SOC_{k_{max}} < SOC_f \\ \text{hybrid mode} & \text{else} \end{cases} \quad (7)$$

where the vector P_{APU} is the sequence of the APU output power over the whole trip, $SOC_{k_{max}}$ is the SOC level at the end of the driving cycle, which is derived by its state equation, and SOC_f is the pre-set lower SOC boundary value.

For a PHET with a 3-speed AMT, minimizing the total energy cost involves the power split between the APU and the battery, as well as the gear-shifting choice. Instead of taking the gear ratio as a state variable that will extend the system state dimensions [13], here it is regarded as an input variable. Hence, the system equation contains two input variables: one represents the power split, and the other represents the gear-shifting. That is, the control vector is given as

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} P_{APU} \\ i_g \end{bmatrix} \quad (8)$$

where u_1 is the output power of the APU, and u_2 is the gear ratio of the AMT.

The battery SOC is selected as the state variable

$$x = SOC. \quad (9)$$

The generic system state equation is then stated as

$$\dot{x} = f(x, u). \quad (10)$$

3.2. Pontryagin's Minimum Principle

As the system dynamics contains two input variables and one state variable, the objective function can be rewritten in the following form:

$$J = \min \int_0^{t_f} (c_f \dot{m}_f(P_{APU}) + c_e P_{bat}(SOC, P_{APU}, i_g)) dt. \quad (11)$$

Based on Equations (2) and (3), the state variable can be described as

$$f(SOC, P_{APU}, i_g) = -\frac{I_b}{Q_b} = -\frac{V_{oc}(SOC) - \sqrt{V_{oc}^2(SOC) - 4R_b(SOC)P_b(P_{APU}, i_g)}}{2Q_b R_b(SOC)} \quad (12)$$

where P_b is the function of the APU power and gear ratio.

The Hamiltonian function is then depicted by

$$H = c_f \dot{m}_f + c_e P_{bat} + \lambda f(SOC, P_{APU}, i_g) \quad (13)$$

where H is the Hamiltonian function, and λ is the co-state variable.

The co-state dynamics and the normal equation can be, respectively, portrayed by

$$\dot{\lambda} = -\frac{\partial H(SOC, \lambda, P_{APU}, i_g)}{\partial SOC} \quad (14)$$

and

$$\dot{SOC} = \frac{\partial H(SOC, \lambda, P_{APU}, i_g)}{\partial \lambda}. \quad (15)$$

The method that is adapted to the electric-only mode and the hybrid mode is an expansion of the normal TPBVP. Therefore, to include both powering modes, the boundary condition for the SOC is described as follows:

$$\begin{cases} SOC(t_0) = SOC_0 \\ SOC(t_f) = \begin{cases} SOC_{k_{max}} & \text{all electric mode} \\ SOC_f & \text{hybrid mode} \end{cases} \end{cases} \quad (16)$$

where SOC_0 and SOC_f are the upper and lower boundary values, respectively, and $SOC_{k_{max}}$ is the final SOC calculated by the dynamic equation.

In addition to the global constraint on the battery SOC, the instantaneous constraints imposed on power components due to the physical limits can be given as

$$\begin{cases} P_{APU, min} \leq P_{APU}(t) \leq P_{APU, max} \\ P_{bat, min} \leq P_{bat}(t) \leq P_{bat, max} \\ P_m, min \leq P_m(t) \leq P_m, max \\ n_m, min \leq n_m(t) \leq n_m, max \\ T_m, min \leq T_m(t) \leq T_m, max \end{cases} \quad (17)$$

where T_m is the torque of the tractor motor, and the subscript *max* and *min* denote the upper and lower bounds for each variable, respectively.

The energy management framed as a constrained optimization problem then takes the following formulation:

$$\begin{aligned} [u_1^*, u_2^*] = \operatorname{argmin} \quad & H(SOC, \lambda, P_{APU}, i_g) \\ & u_1 \in U_1 \\ & u_2 \in U_2 \end{aligned} \quad (18)$$

subject to

$$\begin{cases} \dot{SOC} = -\frac{I_b}{Q_b} \\ SOC(t_0) = SOC_0 \\ SOC(t_f) = \begin{cases} SOC_{kmax} & \text{all electric mode} \\ SOC_f & \text{hybird mode} \end{cases} \\ u_1 \in U_1 \\ u_2 \in U_2 \end{cases} \quad (19)$$

where U_1 and U_2 are defined as the allowable input variable sets.

By minimizing the Hamiltonian function with boundary conditions, we can obtain the optimal Hamiltonian value associated with the optimal control inputs:

$$H(SOC^*(t), \lambda^*(t), P_{APU}^*(t), i_g^*(t)) \leq H(SOC^*(t), \lambda(t), P_{APU}^*(t), i_g^*(t)) \quad (20)$$

where λ^* denotes the optimal co-state value.

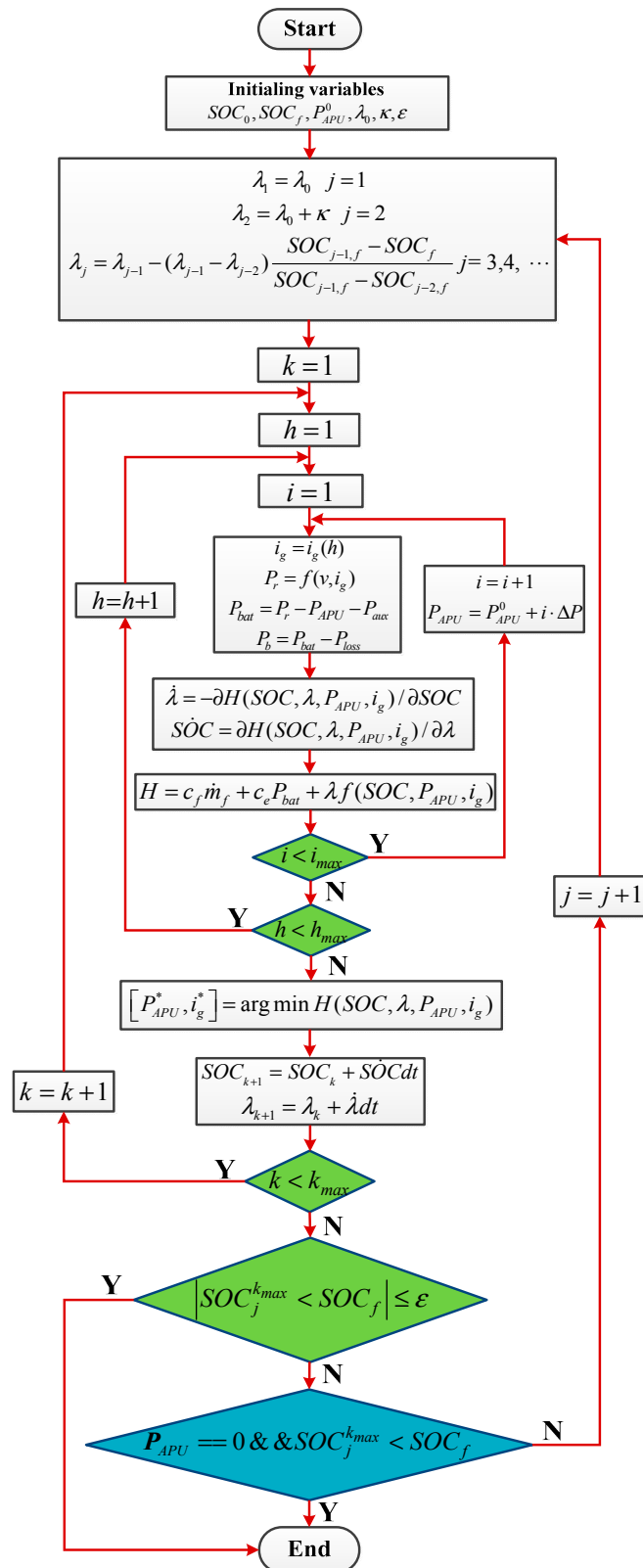
The optimal control inputs can be obtained as

$$[P_{APU}^*, i_g^*] = \operatorname{arg min} H(SOC, \lambda, P_{APU}, i_g). \quad (21)$$

4. Integrated Control Strategy and Its Numerical Solution

4.1. Integrated Control Strategy

As mentioned above, the PMP-based energy management problem for driving missions with different trip distances involves the selection between the electric-only mode and the hybrid mode. By adding the mode decision module that incorporates an appropriate mode into the normal TPBVP, an integrated control strategy is constructed. In the control scheme, the electric-only mode can be triggered by judging the sequence of the APU output power over the whole trip and the final battery SOC level. The detailed flow chart is illustrated in Figure 5.



(a) Flow-chart of the integrated control strategy.

Figure 5. Cont.

Algorithm	
1	Initializing variables $\{SOC_0, SOC_f, P_{APU}^0, \lambda, \varepsilon\}$;
2	$j = 1; k = 1; h = 1; i = 1$;
3	while $\text{abs}(SOC_j^{k_{max}} - SOC_f) > \varepsilon$ do
4	calculate λ_j ;
5	if $k < k_{max}$ then
6	if $h < h_{max}$ then
7	if $I < i_{max}$ then
8	$\{i_g = i_g(h), P_r = f(v, i_g), P_{bat} = P_r - P_{APU} - P_{aux}, P_b = P_{bat} - P_{loss}\}$;
9	calculate $\dot{SOC}, \dot{\lambda}, H$;
10	update $\{P_{APU}\}$;
11	$i = i + 1$;
12	end if
13	$h = h + 1$;
14	end if
15	update $\{P_{APU}^*, i_g^*\}$ and $\{SOC_{k+1}, \lambda_{k+1}\}$;
16	$k = k + 1$;
17	end if
18	if $P_{APU} == 0 \&\& SOC_j^{k_{max}} < SOC_f$ then
19	break;
20	else
21	$j = j + 1$;
22	end if
23	end while

(b) Pseudo code of the algorithm.

Figure 5. (a) Flow-chart and (b) Pseudo code of the proposed control strategy.

(1) A four-loop structure is designed to seek the optimal policy of the power-split and the gear-shifting, where the index j denotes the j th shooting sequence, k denotes the k th time step of the driving cycle, h denotes the h th gear ratio of the AMT, and i denotes an i th increment of the APU power in the allowable set.

(2) The electric-only mode highlighted by the blue color can be activated if the condition $(SOC_j^{max} < SOC_f \text{ and } P_{APU} == 0)$ is satisfied; otherwise, the hybrid mode is triggered, where the normal TPBVP is solved by using the shooting method [31].

4.2. Numerical Solution

For a normal TPBVP solved by the shooting method, to efficiently tune the initial co-state value so as to guide the final SOC to target the desired lower value, the Secant method is used, which takes the following mathematical expressions [9,11]:

$$\begin{cases} \lambda_1 = \lambda_0 & j = 1 \\ \lambda_2 = \lambda_0 + \kappa & j = 2 \\ \lambda_j = \lambda_{j-1} - (\lambda_{j-1} - \lambda_{j-2}) \frac{SOC_{j-1,f} - SOC_f}{SOC_{j-1,f} - SOC_{j-2,f}} & j = 3, 4, \dots \end{cases} \quad (22)$$

where λ_j denotes the initial co-state value in the j th shooting; the first two co-state variables λ_1 and λ_2 are initialized by the given constants λ_0 and κ , which start the iterative process for λ_j ($j = 3, 4, \dots$); $SOC_{j,f}$ denotes the SOC level at the end of the trip in the j th shooting.

From the co-state Equation (14) and the normal Equation (15), we can further formulate the following equations:

$$\begin{cases} \dot{\lambda} = -\lambda \frac{\partial \dot{SOC}}{\partial SOC} \\ \dot{SOC} = f(SOC, P_{APU}, i_g) \end{cases} \quad (23)$$

The numerical dynamics for the state SOC and the co-state are evolved by the Euler methods as follows [11]:

$$\begin{cases} \text{SOC}_{k+1} = \text{SOC}_k + \dot{\text{SOC}}dt \\ \lambda_{k+1} = \lambda_k + \dot{\lambda}dt \end{cases} \quad (24)$$

where the time step dt is set to 1 s.

5. Results and Analysis

To examine the developed integrated control strategy, two scenarios with different instances (long and short distances) based on the Chinese City Bus Driving Cycle (CCBDC) [32] (see Figure 6) are simulated, given different initial SOC levels. The lower SOC boundary value SOC_f is set to 0.3 in both cases. To imitate a realistically reasonable condition to avoid the jumping transition between the first and third gear, a penalty term is added to the Hamiltonian function. The variable λ_0 and κ are set to -55 and 0.4 , respectively, and the converging factor ε is set to 0.00005 . Additionally, the increment of the APU output power ΔP is specified to 1 kW.

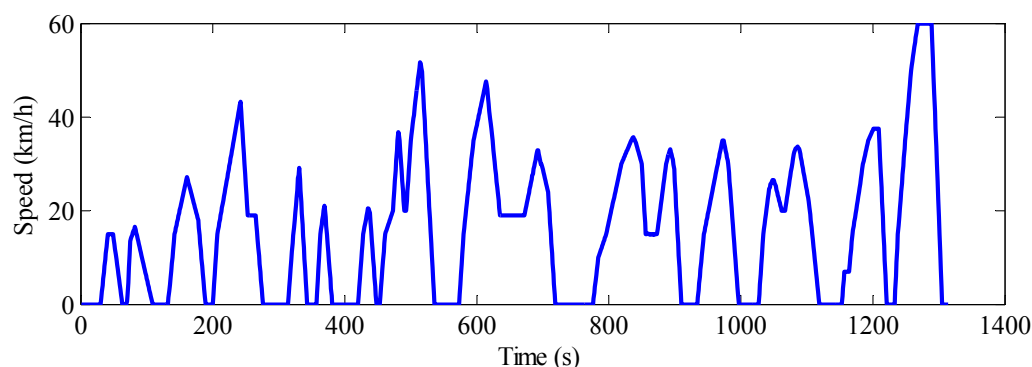


Figure 6. Speed profile of a single Chinese City Bus Driving Cycle (CCBDC) (duration = 1314 s, distance = 5.78 km).

5.1. Long-Distance Driving Cycle

Scenario 1 is composed of eight concatenated CCBDCs ($8 \times \text{CCBDC}$), with a total distance of 47.2 km and a time length of 2.92 h. The initial battery SOC is intentionally set to 0.5 to model a half fully charged battery.

Figure 7 plots the SOC profiles yielded by the shooting method, which indicates that seven shootings were required until the final SOC targeted the preset level (0.3). Moreover, it can be observed that the SOC curves appear in a non-sequential order, indicating an effective tuning of the co-state values by the Secant method. Since the final SOC reaches the preset lower boundary level, the hybrid mode was activated in this case. As a result, the seventh SOC profile can be regarded as the optimal discharging trajectory.

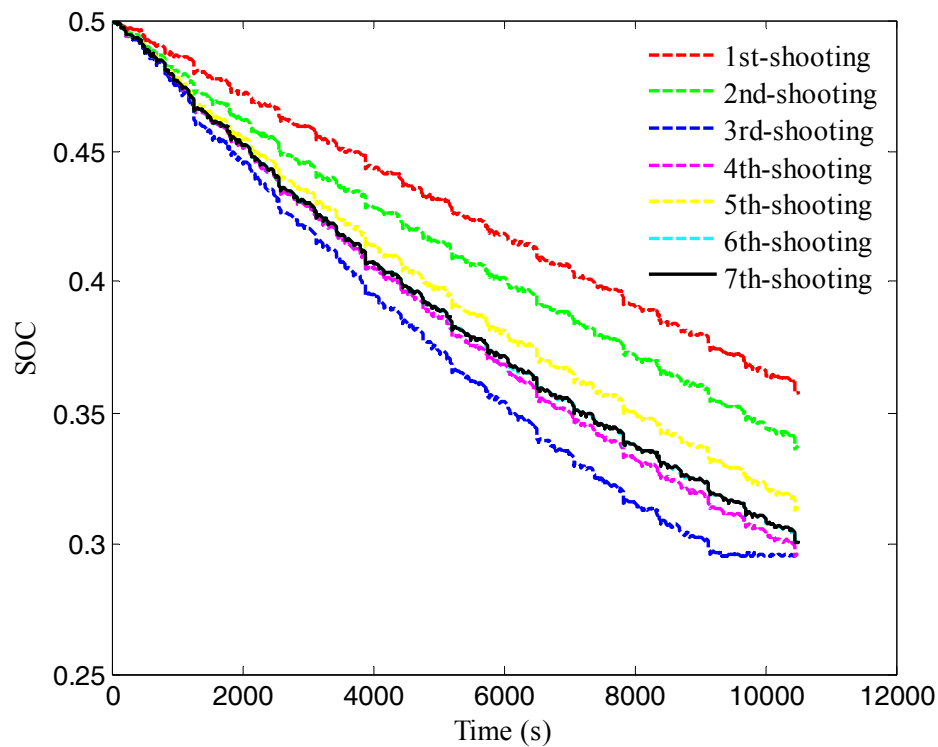


Figure 7. State-of-charge (SOC) trajectories in all shooting operations in Scenario 1.

The initial co-state values corresponding to different shooting processes are depicted in Figure 8. It is apparent that the co-state variable increases in the first stage, drops in the middle stage, and finally tends to a stable value. The optimal co-state variable, as shown in Figure 9, declines in an approximately linear manner ranging from -54.17 to -54.78 CNY (Chinese yuan) during the whole trip. The APU output power and the battery terminal power, as illustrated in Figure 10, reveal that the control algorithm carefully blends both power sources until the SOC approaches the lower boundary value. Overall, the optimal energy management problem in Scenario 1 is essentially cast into a normal TPBVP, which can be solved by the shooting method.

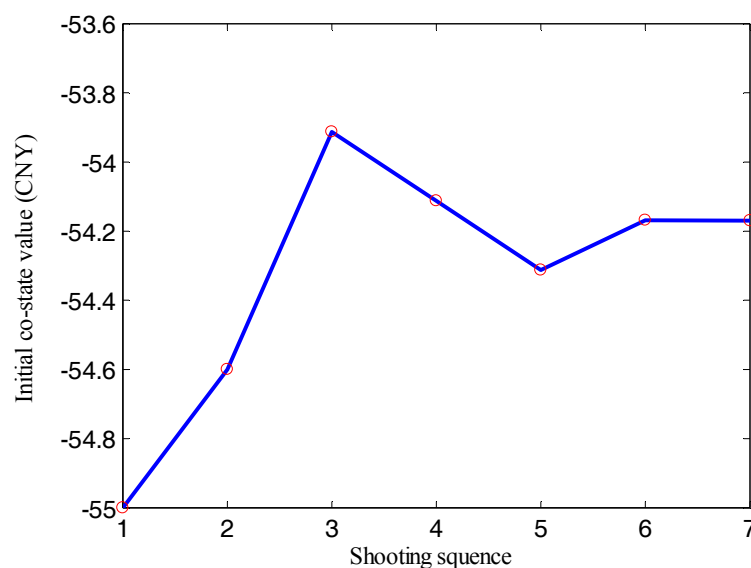


Figure 8. Initial co-state values in all shooting operations in Scenario 1.

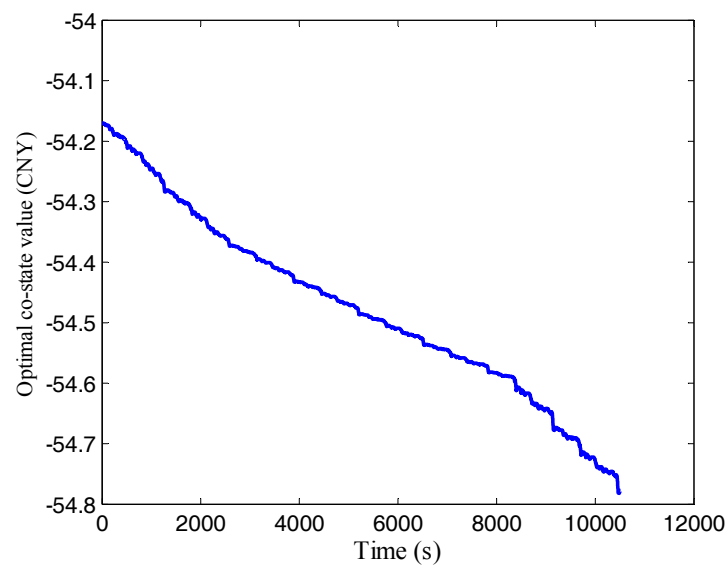


Figure 9. Optimal co-state trajectory in Scenario 1.

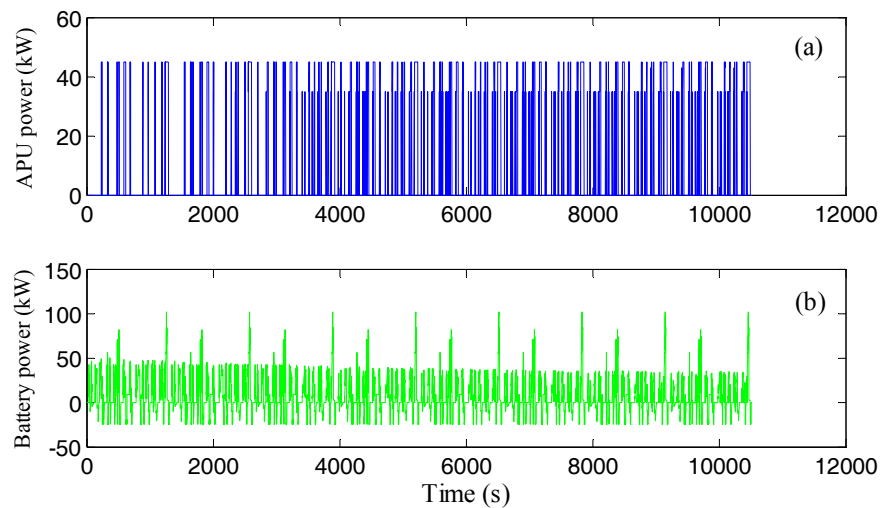


Figure 10. Power output of (a) the auxiliary power unit (APU) and (b) the battery in Scenario 1.

Figure 11 depicts the electric motor operating points (OPs) corresponding to different shifting gears of the AMT. It can be observed that the rotational speed of the tractor motor falls within a wide scope, and the torque mainly varies between ± 200 Nm, which is determined by not only the gear-shifting control strategy but also the feature of the driving cycle. Figure 12 shows the bar graph of the accumulative operating time for each gear. Obviously, the controller chooses the first gear most frequently, followed by the second gear, and then the third gear, with their time percentages accounting for 50.4, 30.4, and 19.2%, respectively.

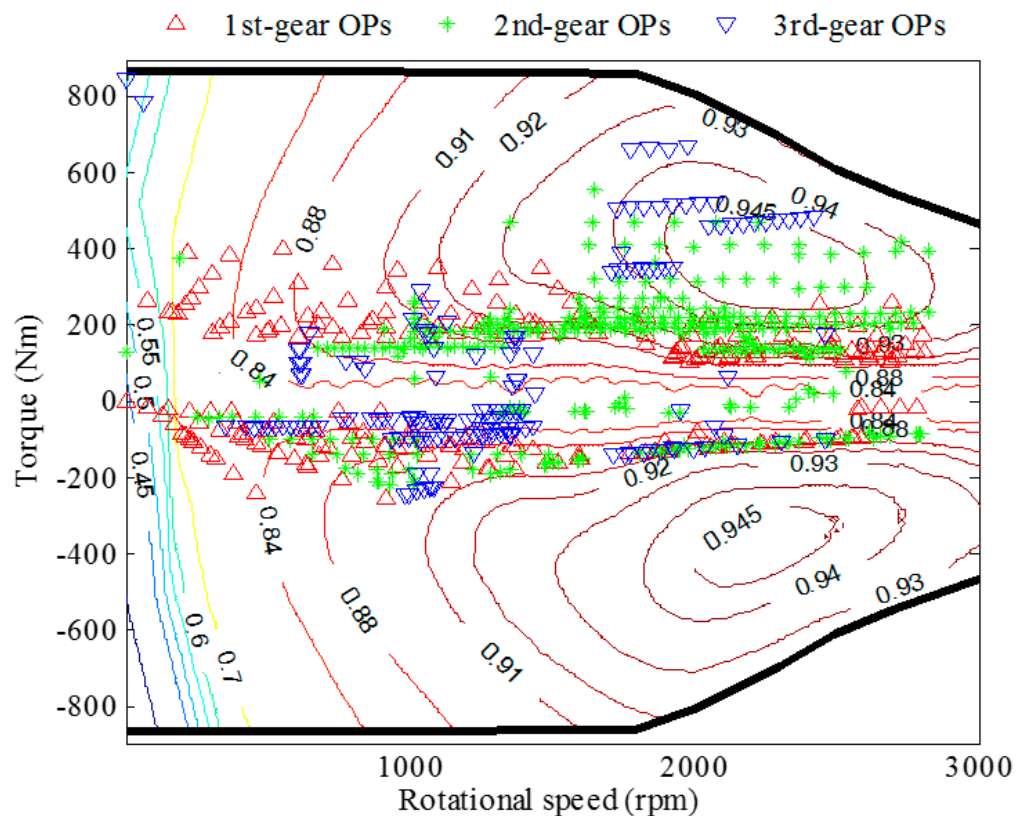


Figure 11. Tractor motor operating points (OPs) with different gears in Scenario 1.

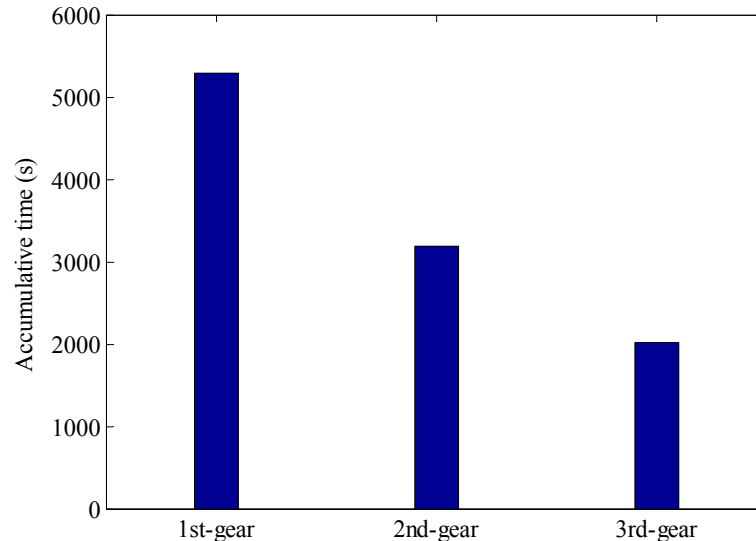


Figure 12. Accumulative time of three gears in Scenario 1.

5.2. Short-Distance Driving Cycle

To further examine the flexibility of the proposed integrated energy management strategy for short-distance cases, Scenario 2 consists of only $2 \times$ CCBDs (about 11.5 km, 0.7 h), and the initial SOC is arranged to 0.6 to simulate a moderate charging level of the battery.

As plotted in Figure 13, the shooting method in this case generates three SOC profiles, indicating that three shootings were required to seek the optimal SOC trajectory. Despite the same lower SOC boundary value (0.3) as in Scenario 1, the final SOC, however, reaches 0.491 without exceeding the preset level, which suggests that the electric-only mode is triggered by checking the APU output power

and the final SOC level. Additionally, the output power profiles in Figure 14 show that the battery is the only energy provider, whereas the APU maintains the off-state at all times. This observation again demonstrates the optimal electric-only driving mode in this case.

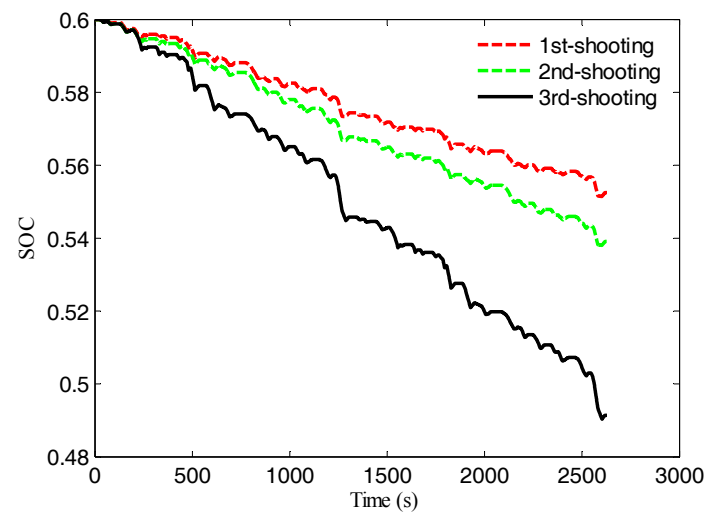


Figure 13. SOC trajectories in all shootings in Scenario 2.

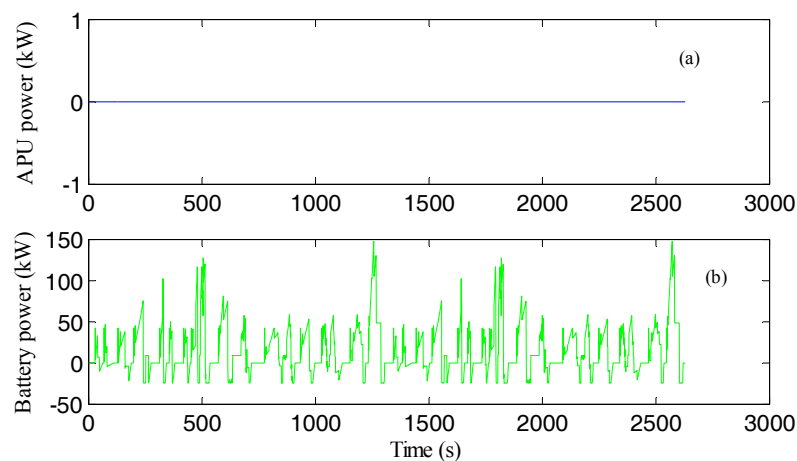


Figure 14. Output power of (a) APU and (b) battery in Scenario 2.

The initial co-state value in Figure 15 presents a sudden increase in the third shooting, because the same setting of the initial co-state values (λ_1 and λ_2) as that in Scenario 1 needs a deep adjustment by the Secant method in this case. The optimal co-state value (see Figure 16) varies from -47.50 to -47.85 CNY during the whole trip.

The gear-shifting OPs are plotted in Figure 17, and the accumulative time of the gear-shifting in this case is given in Figure 18, with the percentages of time distribution for the three gears being 50.3, 30.4, and 19.3%, respectively.

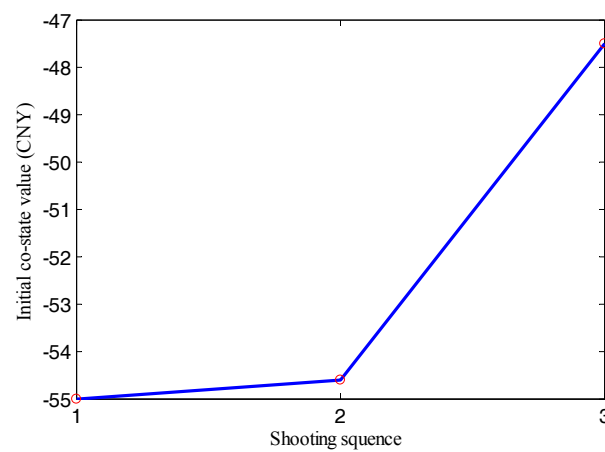


Figure 15. Initial co-state values in all shootings in Scenario 2.

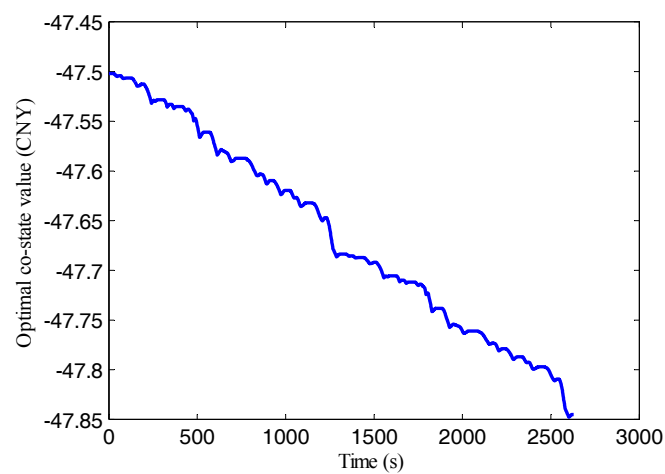


Figure 16. Optimal co-state trajectory in Scenario 2.

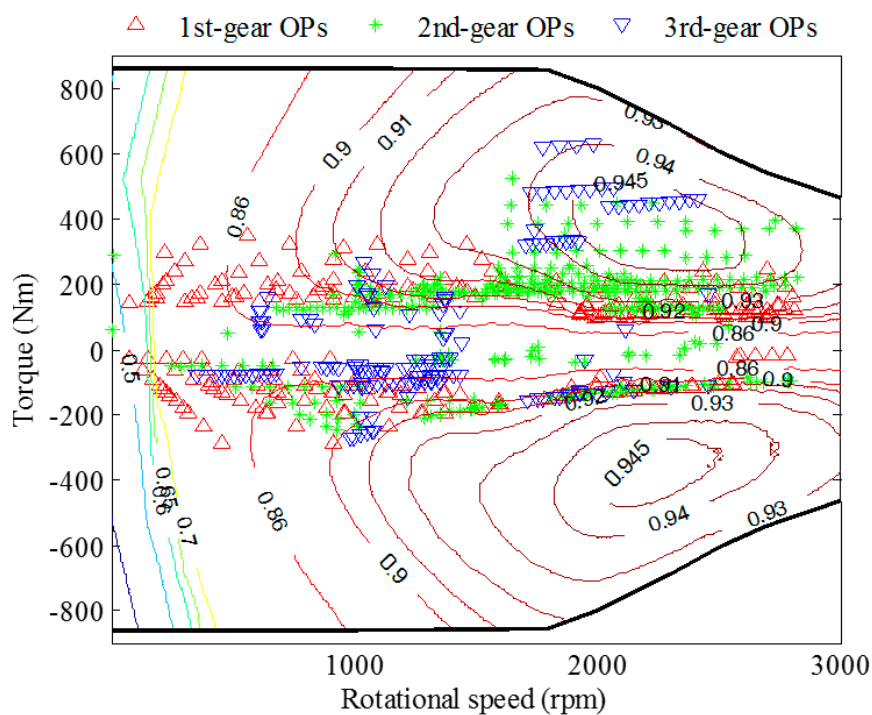


Figure 17. Tractor motor OPs with different gears in Scenario 2.

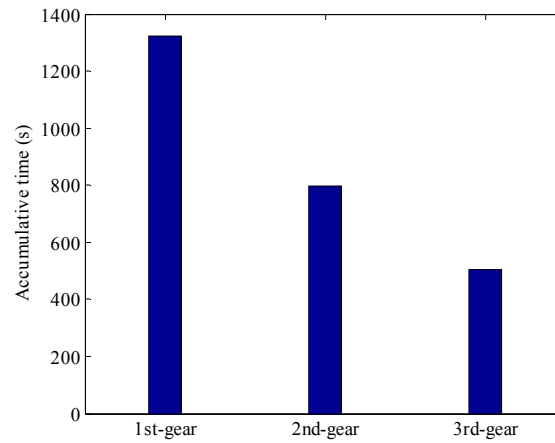


Figure 18. Accumulative time of three gears in Scenario 2.

6. Comparisons with Other Existing Strategies

To evaluate the performance of the presented method regarding the computational efficiency and accuracy, a comparative analysis is carried out with the DP algorithm and the rule-based CD-CS strategy.

6.1. DP-Based Strategy

Based on the Bellman principle [33], the DP-based energy consumption optimization for the PHET with the three-speed AMT can be expressed as follows:

$$\left\{ \begin{array}{l} J_k(x_h) = \min \{ g_k(x_h, u_{1,i}, u_{2,j}) + J_{k+1}(f_{k+1}(x_h, u_{1,i}, u_{2,j})) \} \\ (k = k_{max} - 1, k_{max} - 2, \dots, 1; i = 1, 2, \dots, i_{max}, j = 1, 2, 3) \\ J_{k_{max}}(x_h) = \min_{\substack{u_1 \in U_1 \\ u_2 \in U_2}} \{ g_{k_{max}}(x_h, u_{1,i}, u_{2,j}) \} \\ (k = k_{max}; i = 1, 2, \dots, i_{max}; j = 1, 2, 3) \end{array} \right. \quad (25)$$

where k denotes the time step; g_k represents the current energy cost; x_h is the h th discrete value of the state variable; $u_{1,i}$ represents the i th discrete APU power, and $u_{2,j}$ denotes the j th gear ratio; J_k is the cost-to-go value from the start point of x_k ; f_k describes the SOC dynamic equation, which is used to estimate the SOC in the next time step.

Since the solving accuracy and computational burden of the DP algorithm are affected by the scale of the discretization of the state variable, the number of SOC grid points is set to 1000 after many trials to balance the calculative efficiency and precision.

6.2. CD-CS Strategy

For a CD-CS strategy, the transition from the CD stage to the CS stage is determined by a preset SOC threshold value. Here the rule is regulated such that if the battery SOC is lower than 0.3, the APU outputs a constant power (45 kW) corresponding to the optimal fuel rate; if the battery SOC exceeds 0.35, the APU turns off.

By balancing the drivability and energy economy, a rule-based gear-shifting control policy is used here, on the basis of two supervisory variables including the vehicle speed and the torque request at the output side of the AMT shaft. As shown in Figure 19, the dashed lines denote the down-shift boundary lines, the solid lines represent the up-shift boundary lines, and the intermediate region between each two different lines indicates an overlapping operation to avoid frequent gearshift.

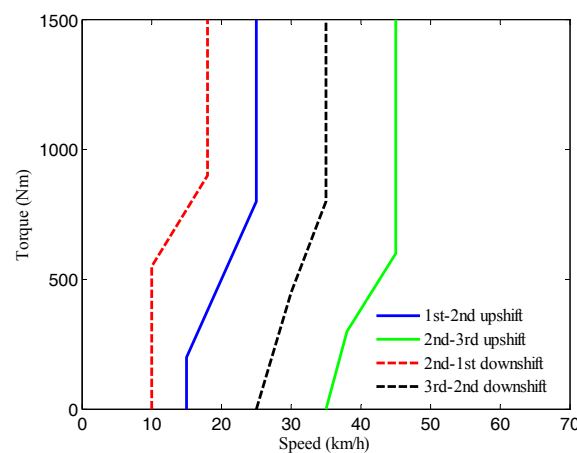


Figure 19. Rule-based gear-shifting control policy.

6.3. Results

The aforementioned DP and CD-CS methods are compared with the devised PMP-based energy management strategy. For a fair, judicious comparison, Scenario 1, which triggers the hybrid powering mode, is used as the driving cycle. The results are summarized in Table 2. The total cost consumed by the CD-CS strategy is 10.03% and 10.04% more than those in the DP and PMP algorithms, respectively. Both global optimal methods (DP and PMP) produce almost the same total cost, and the slight difference between them is due to the fact that the solution accuracy of DP is influenced by the discretization scale of the battery SOC and the interpolation method used to estimate the cost-to-go; for the latter, the converging factor exerts a direct effect on the calculative precision. As for the computational efficiency, the time required by the PMP algorithm is tremendously diminished compared to the DP method, exhibiting the potential to leverage this time-efficient algorithm to develop a real-time energy management strategy. While the CD-CS strategy supports a real-time implementation, it is remarkably more energy-consuming.

Table 2. Results of the three strategies.

Result	Method		
	CD-CS	DP	PMP
Final SOC	0.323	0.301	0.301
Electric consumption (kWh)	17.119	19.236	19.246
Fuel consumption (L)	6.641	5.438	5.436
Total cost (CNY)	51.680	46.493	46.488
Time consumption (s)	12	20689	71

Note: the electricity price is 0.8 CNY/kWh, and the fuel price is 5.72 CNY/L.

The SOC profiles in Figure 20 disclose that the CD-CS strategy undergoes a charge-depleting stage, followed by a charge-sustaining stage. Despite both SOC profiles of the DP and PMP algorithms decline to the identical terminal value, the difference between both curves are visible. Specifically, the DP-based SOC profile decreases almost linearly with time, whereas the PMP-based SOC trajectory features an arc shape, mainly because of their different optimization mechanisms—the PMP algorithm seeks the global optimal solution by instantaneously minimizing the Hamiltonian function with the constrained SOC boundary condition, whereas the DP selects the optimal control by minimizing the cost-to-go value. Quantitative results reveal that, in the first-half period of the trip (around 0–4800 s), the battery output power generated by the PMP method exceeds that of the DP method, so the associated APU output power is smaller than that of the DP solution. However, the opposite happens in the second-half period of the trip, eventually resulting in nearly the same total energy consumption cost.

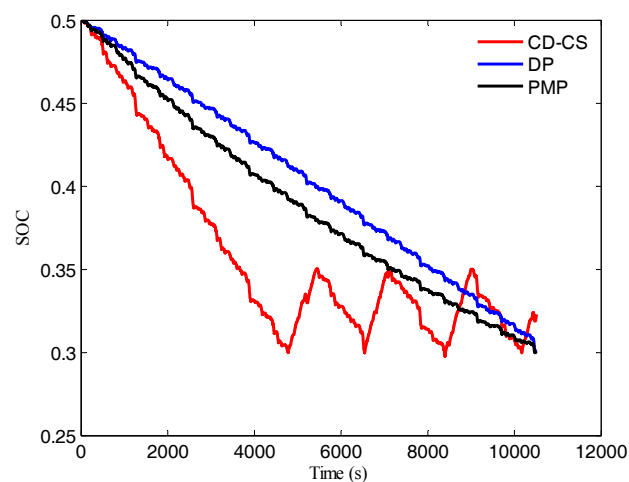


Figure 20. SOC profiles of three strategies.

Moreover, Figure 21 depicts the growing total cost versus the trip distance. As can be seen, the total cost growth of the CD-CS strategy can be divided into two stages. In the first stage or the CD stage, the total cost grows slower than others, because the electric-only mode dissipates less expensive electricity; nevertheless, during the CS stage, the total cost rises rapidly and exceeds those of the optimal strategies, owing to non-optimal heuristics. For the DP and PMP methods, their cost growth curves are decided by the summed amount of the fossil fuel and electricity consumption. The total cost of the PMP is first lower than that of the DP method; however, it increases gradually after about 35 km, because of more fuel use and less use of the battery electricity, compared to the DP method.

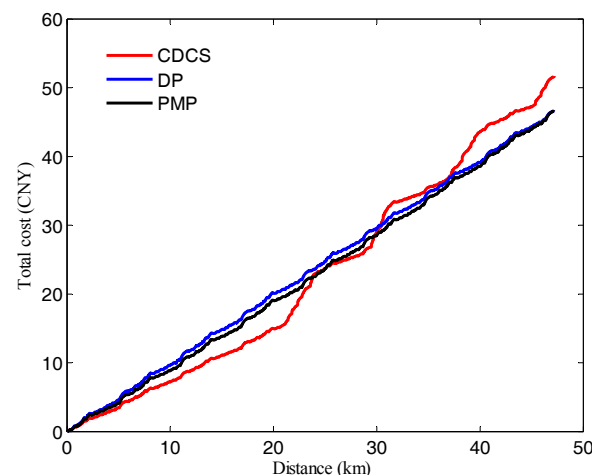


Figure 21. Total energy cost with respect to driving distance.

The accumulative time distribution of the three gears is shown in Figure 22. It can be seen that the DP and PMP methods almost choose the same gear ratio. For the CD-CS strategy, together with a ruled-based gear-shifting control, the first gear occupies more time than the other two gears, being consistent with both optimal methods; nonetheless, the CD-CS strategy selects the second gear more frequently than do the DP and PMP methods, and takes shorter time for the third gear.

Table 3 summaries the time percentage of the same gearshift between each two of the three control strategies. The gearshift similarity between the DP and PMP is up to 99.01%. However, such a level decreases to 73.14% between the CD-CS and DP and to 73.21% between the CD-CS and PMP.

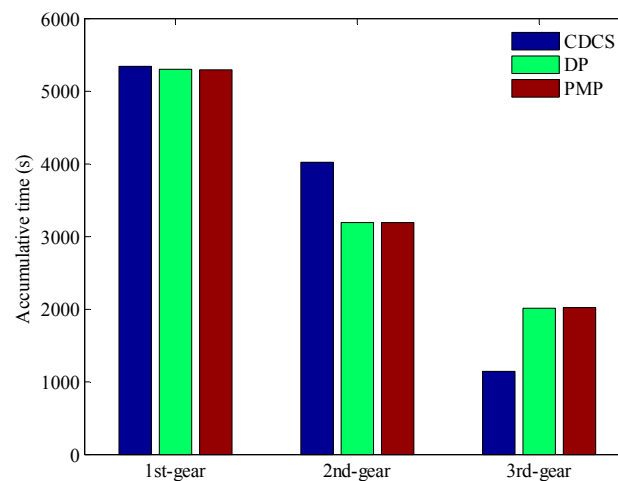


Figure 22. Gear-shifting distributions for the three methods.

Table 3. Time percentage of the same gearshift between each two of the three methods.

Method	Result	
	CD-CS	DP
CD-CS	-	73.14%
DP	73.14%	-
PMP	73.21%	99.01%

For further clarification, local gear-shifting sequences during 5000–6000 s for the three approaches are illustrated in Figure 23. It is evident that the DP and PMP methods have the identical gear-shifting sequence over the whole period. Additionally, both global methods operate the gear-shifting more frequently than does the rule-based strategy, because both optimization algorithms always search for the optimal gear ratio to minimize the total energy cost, from a global perspective.

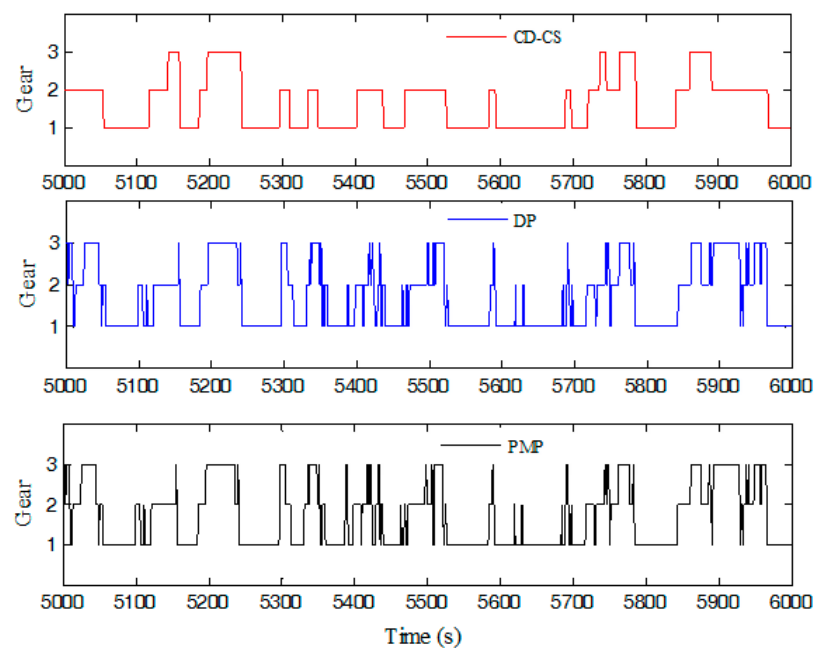


Figure 23. Local gear-shifting sequences for the three methods (5000–6000 s).

7. Conclusions

To make a PMP-based energy management strategy for PHEVs resilient against scenarios with different driving distances, this paper proposes an integrated control strategy by meticulously manipulating the electric-only mode and the hybrid powering mode. Several important conclusions are summarized below:

(1) The PMP-based energy management problem for PHEVs is not necessarily a two-point boundary value problem, because of the uncertain lower SOC boundary related to a specific driving distance. Since the optimal discharging policy can be either an electric-only mode or a hybrid discharging mode, an electric-only driving mode decision module is thereby incorporated into the solving process of the TPBVP to build an integrated control strategy, where the electric-only mode is recognized by checking the APU output power and the final SOC level. As such, the energy management optimization problem for different trip lengths is cast into a TPBVP with the capability of recognizing the electric-only mode.

(2) For a series plug-in hybrid electric truck with an AMT, the optimal control problem involves the power-split between the APU and the battery, as well as the gear-shifting choice, which results in an optimization problem with multiple control variables. Here the gear ratio is treated as an input variable instead of a state variable to avoid complicating the system dynamics as a multi-state issue.

(3) The result in a long-distance scenario shows that, despite that the DP and PMP have the same energy consumption cost and gear-shifting sequence, their SOC profiles and cost growth curves exhibit different shapes, due to their different optimizing mechanisms. The significant difference between the PMP and DP algorithms lies in the computational efficiency—the time consumed by the PMP algorithm is massively reduced compared to the DP method. Moreover, the CD-CS method with a rule-based gear-shifting strategy increases the energy cost by nearly 10% compared with the PMP method.

Author Contributions: S.X. and X.H. conceived the energy management strategy and solved the critical computational issue in the model simulation and wrote the manuscript; K.L., S.Q., and T.L. conducted the numerical simulation; all the authors contributed to discussions and analyses of the results.

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