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Study of Winding Structure to Reduce Harmonic Currents in Dual Three-Phase Motor

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Abstract: Dual three-phase motors have the problem of a large generation of harmonic currents while driving due to small inductance for certain harmonics. The purpose of this paper is to theoretically clarify the effect of the winding structure of dual three-phase motors on harmonic currents. Analytical calculations considering the spatial harmonics of the magnetomotive force are used to formulate the inductance that depends on the winding structure and the harmonic order. The validity of the formulated theoretical expression of the inductance is confirmed by finite element analysis. In addition, drive simulations of the coil pitch 5/6 dual 3-phase motor and the coil pitch 1 dual 3-phase motor are performed, and it is found that the magnitude of the generated harmonic currents can be explained using the formulated inductance values. In conclusion, it is found that dual three-phase motors with large winding factors for fifth and seventh order spatial harmonics can reduce the harmonic currents.

Keywords: dual 3-phase motor; harmonic currents; winding structure



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1. Introduction

In recent years, the realization of a low-carbon society has become an urgent issue. The electrification of automobiles and all other fields is accelerating, and further performance improvement of motors is required.

Against this background, dual inverter drive systems have been attracting attention for improving motor performance. The advantages of this system include a higher torque and power using two inverters of conventional ratings [1–7], and an increased degree of control freedom and reliability by increasing the number of phases [1–8]. The winding arrangement of the motor applied in this system can be broadly classified into 3 types with a phase difference between 2 sets of 3-phase windings of 0 degrees, 30 degrees, and 60 degrees in electrical angle [5]. In particular, the dual 3-phase motor with a phase difference of 30 degrees in the electrical angle shown in Figure 1 has attracted more attention because it completely eliminates the 6th harmonic component of the torque ripple, in addition to the above advantages of the dual inverter drive system [3–5,8].

However, dual three-phase motors have the problem of generating more harmonic currents during driving than conventional three-phase motors [1–4,6,7]. The cause of this problem can be explained using vector space decomposition (VSD), as proposed in [1]. As described in detail below, the harmonic components of phase voltage and phase current are decomposed into three linearly independent two-dimensional Cartesian coordinate systems (α - β subspace, z_1 - z_2 subspace, o_1 - o_2 subspace) by a Cartesian coordinate transformation based on VSD. Assuming a sinusoidal distribution winding, the magnetizing inductance component is decomposed into only the α - β subspace, and the inductance in the z_1 - z_2 subspace is only the leakage component. Therefore, the inductance in the z_1 - z_2 subspace is small, and large harmonic currents are generated even if the harmonic voltage in the z_1 - z_2 subspace is small [2–4]. The harmonic currents in the z_1 - z_2 subspace do not contribute to the generation of torque ripple but must be suppressed because they cause increased losses

and inverter current ratings [2–4,6]. Since the o1 and o2 phases are the zero sequence of the first and second windings, respectively, separating the neutral points of the first and second windings, as shown in Figure 1, can prevent the generation of harmonic currents in the o1-o2 subspace, such as 3rd order harmonic currents [1,7].

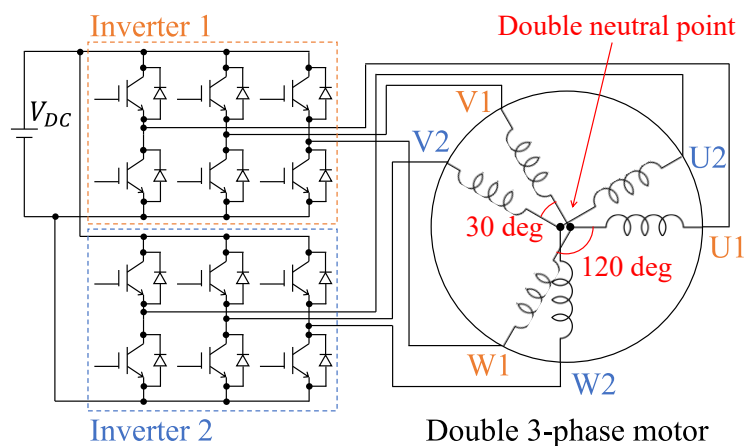


Figure 1. Dual 3-phase motor.

The harmonic currents that are the problem can be broadly classified into low-order harmonic currents (mainly fifth and seventh harmonics) caused by harmonic components of the back electromotive force (back-EMF) and nonlinearity of the inverter, and high-order carrier harmonic currents caused by the pulse width modulation (PWM) of the inverter [4,7]. Many studies have focused on control approaches, proposing low-order harmonic current suppression methods using proportional-integral control with resonant control (PI-R) and multiple synchronous reference frames (MSRFs) [3,4], and carrier harmonic current suppression methods using space vector pulse width modulation (SVPWM) [1,2,7].

On the other hand, the cause of large harmonic currents is the small inductance in the z1-z2 subspace as described above, and an approach to reduce harmonic currents by designing the windings with a large inductance in the z1-z2 subspace is also possible. However, as far as the author has been able to find, there are few studies that clarify how the inductance in the z1-z2 subspace changes depending on the winding structure.

Only in [6], the characteristics of the slot leakage inductance in the α - β and z1-z2 subspaces with respect to the coil pitch are clarified analytically. However, these inductances have not been formulated using a more basic parameter, the winding factor. There is also room for further study of the relationship between the inductance in each subspace and the magnitude of harmonic currents. In particular, the short-pitch winding with the coil pitch of 5/6 is known to have small winding factors for 5th and 7th spatial harmonics. This reduces the fifth and seventh harmonic components of the back-EMF, which is an advantage compared to conventional three-phase motors because the fifth and seventh harmonic currents can be reduced. However, it has been reported that the fifth and seventh harmonic currents conversely increase in the case of dual three-phase motors [6]. To explain the cause of this, it is necessary to clarify the inductance analytically using the winding factor and the relationship with the back-EMF.

In this paper, the inductance of dual three-phase motors, which varies depending on the winding structure and subspaces, is analytically clarified. The validity of the formulated inductances is confirmed by finite element analysis (FEA) and the magnitude of the harmonic currents generated during drive is quantitatively evaluated using these inductance values. The model used in the FEA and simulation is an interior permanent magnet synchronous motor (IPMSM) for electric vehicles.

2. Coordinate System of Dual Three-Phase Motors

2.1. Stationary Orthogonal Subspaces

The original transformation matrix based on VSD is a 6×6 matrix, which maps the current, voltage, and flux linkage to three two-dimensional stationary subspaces: α - β subspace, z1-z2 subspace, and o1-o2 subspace. On the other hand, as mentioned above, separating the neutral points of the 1st and 2nd windings prevents the generation of harmonic currents in the o1-o2 subspace, such as 3rd order harmonic currents, so this paper considers a winding structure with the separated neutral points of the 1st and 2nd windings. In this case, the mapping to the o1-o2 subspace can be omitted, and the orthogonal coordinate transformation in this paper uses the 4×6 matrix shown in the following Equation (1).

$$T_6 = \frac{1}{3} \begin{bmatrix} \cos 0 & \cos(2\pi/3) & \cos(4\pi/3) & \cos(\pi/6) & \cos(5\pi/6) & \cos(3\pi/2) \\ \sin 0 & \sin(2\pi/3) & \sin(4\pi/3) & \sin(\pi/6) & \sin(5\pi/6) & \sin(3\pi/2) \\ \cos 0 & \cos(4\pi/3) & \cos(2\pi/3) & \cos(5\pi/6) & \cos(\pi/6) & \cos(3\pi/2) \\ \sin 0 & \sin(4\pi/3) & \sin(2\pi/3) & \sin(5\pi/6) & \sin(\pi/6) & \sin(3\pi/2) \end{bmatrix} \quad (1)$$

The orthogonal subspaces used in this paper are shown in Figure 2. As an example, the currents in each phase are mapped to the α - β and z1-z2 subspaces as follow (Equation (2)).

$$\begin{bmatrix} i_{\alpha\beta} \\ i_{z1z2} \end{bmatrix} = T_6 i_{ph} \quad (2)$$

i_{ph} is the phase current vector, $i_{\alpha\beta}$ and i_{z1z2} are the current vectors in the α - β and z1-z2 subspaces. Where i_{U1} , i_{V1} , i_{W1} , i_{U2} , i_{V2} , and i_{W2} are currents in each phase; i_{α} , and i_{β} are currents in the α - β subspace; i_{z1} and i_{z2} are currents in the z1-z2 subspace; and i_{ph} , $i_{\alpha\beta}$, and i_{z1z2} are expressed as (Equations (3) and (4)).

$$i_{ph} = [i_{U1} \ i_{V1} \ i_{W1} \ i_{U2} \ i_{V2} \ i_{W2}]^T \quad (3)$$

$$\begin{cases} i_{\alpha\beta} = [i_{\alpha} \ i_{\beta}]^T \\ i_{z1z2} = [i_{z1} \ i_{z2}]^T \end{cases} \quad (4)$$

Similarly, the voltage and the flux linkage can be transformed to the α - β subspace and the z1-z2 subspace.

The harmonic components in the currents, voltage, and flux linkage of each phase are mapped to either the α - β subspace or the z1-z2 subspace, depending on their orders. As an example, the orthogonal coordinate transformation is performed for the k th order harmonic currents, as shown in the following equation:

$$i_{ph} = I \begin{bmatrix} \cos(k\omega_e t) \\ \cos k(\omega_e t - \frac{2}{3}\pi) \\ \cos k(\omega_e t - \frac{4}{3}\pi) \\ \cos k(\omega_e t - \frac{\pi}{6}) \\ \cos k(\omega_e t - \frac{5}{6}\pi) \\ \cos k(\omega_e t - \frac{3}{2}\pi) \end{bmatrix} \quad (5)$$

where I is the amplitude of currents and ω_e is the fundamental electric angular frequency. Furthermore, k is an odd number. The expression for the k th harmonic currents in the stationary orthogonal subspaces, obtained by substituting (5) into (2), can be divided into the following four cases depending on the value of k .

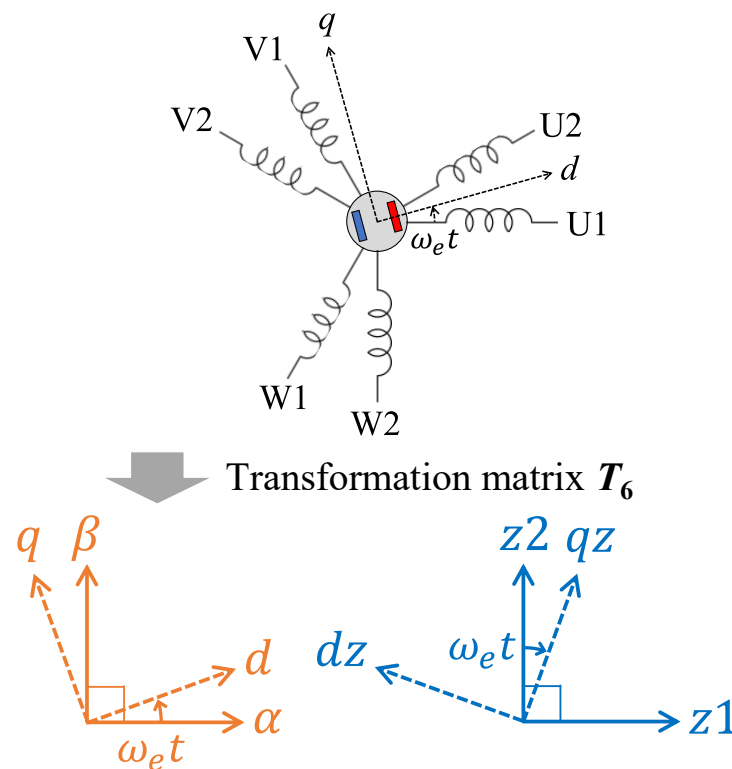


Figure 2. Orthogonal subspaces.

1. $k = 1, 13, 25, \dots, 12m + 1$ ($m = 0, 1, 2, \dots$):

$$\begin{bmatrix} i_{\alpha\beta} \\ i_{z1z2} \end{bmatrix} = I \begin{bmatrix} \cos(k\omega_e t) & \sin(k\omega_e t) & 0 & 0 \end{bmatrix}^T \quad (6a)$$

From (6a), it can be seen that the above harmonic orders, including the fundamental and 13th order, represent the positive sequence components of the α - β subspace.

2. $k = 11, 23, 35, \dots, 12m + 11$ ($m = 0, 1, 2, \dots$):

$$\begin{bmatrix} i_{\alpha\beta} \\ i_{z1z2} \end{bmatrix} = I \begin{bmatrix} \cos(-k\omega_e t) & \sin(-k\omega_e t) & 0 & 0 \end{bmatrix}^T \quad (6b)$$

From (6b), it can be seen that the above harmonic orders, including 11th and 23rd order, represent the negative sequence components of the α - β subspace.

3. $k = 5, 17, 29, \dots, 12m + 5$ ($m = 0, 1, 2, \dots$):

$$\begin{bmatrix} i_{\alpha\beta} \\ i_{z1z2} \end{bmatrix} = I \begin{bmatrix} 0 & 0 & \cos(k\omega_e t) & \sin(k\omega_e t) \end{bmatrix}^T \quad (6c)$$

From (6c), it can be seen that the above harmonic orders, including the 5th and 17th order, represent the positive sequence components of the $z1$ - $z2$ subspace.

4. $k = 7, 19, 31, \dots, 12m + 7$ ($m = 0, 1, 2, \dots$):

$$\begin{bmatrix} i_{\alpha\beta} \\ i_{z1z2} \end{bmatrix} = I \begin{bmatrix} 0 & 0 & \cos(-k\omega_e t) & \sin(-k\omega_e t) \end{bmatrix}^T \quad (6d)$$

From (6d), it can be seen that the above harmonic orders, including the 7th and 19th order, represent the negative components of the $z1$ - $z2$ subspace.

Note that $k = 3, 9, 15, \dots, 6m + 3$ ($m = 0, 1, 2, \dots$) are not included above because they are components that are mapped to the $o1$ - $o2$ subspace. Table 1 shows a comparison of

the harmonic distributions in conventional 3-phase motors and dual 3-phase motors. As shown in Table 1, in conventional 3-phase motors, harmonic components of orders such as $k = 5, 7, \dots$ are superimposed in the α - β subspace which is the fundamental axes. On the other hand, in dual three-phase motors, these harmonic components are superimposed in the $z1$ - $z2$ subspace, which is the coordinate system unique to dual 3-phase motors. As described above, it should be noted that the harmonics of dual three-phase motors show a different distribution from those of conventional three-phase motors.

Table 1. Comparison of harmonic distributions.

Harmonic Order k	1	3	5	7	9	11	13	...
Conventional 3-phase motor	α - β Positive sequence	Zero sequence	α - β Negative sequence	α - β Positive sequence	Zero sequence	α - β Negative sequence	α - β Positive sequence	...
Dual 3-phase motor	α - β Positive sequence	o1-o2 Zero sequence	$z1$ - $z2$ Positive sequence	$z1$ - $z2$ Negative sequence	o1-o2 Zero sequence	α - β Negative sequence	α - β Positive sequence	...

2.2. Rotational Orthogonal Subspaces

The transformation from the stationary orthogonal subspaces, α - β and $z1$ - $z2$ subspaces, to the rotational orthogonal subspaces uses the following matrix [4]:

$$T_{dq} = \begin{bmatrix} \cos(\omega_e t) & \sin(\omega_e t) & 0 & 0 \\ -\sin(\omega_e t) & \cos(\omega_e t) & 0 & 0 \\ 0 & 0 & -\cos(\omega_e t) & \sin(\omega_e t) \\ 0 & 0 & \sin(\omega_e t) & \cos(\omega_e t) \end{bmatrix} \quad (7)$$

As an example, currents in the α - β and $z1$ - $z2$ subspaces are transformed to the d - q and dz - qz subspaces, respectively, as follows:

$$\begin{bmatrix} i_d & i_q & i_{dz} & i_{qz} \end{bmatrix}^T = T_{dq} \begin{bmatrix} i_\alpha & i_\beta & i_{z1} & i_{z2} \end{bmatrix}^T \quad (8)$$

where i_d and i_q are currents in the d - q subspace, and i_{dz} and i_{qz} are currents in the dz - qz subspace. As is clear from (8), the components in the α - β subspace are mapped to the d - q subspace only, and the components in the $z1$ - $z2$ subspace are mapped to the dz - qz subspace only. Therefore, the transformation matrix (7) does not change the harmonic distribution discussed in the previous section.

3. Mathematical Model of Dual Three-Phase IPMSM

IPMSMs have saliency in inductance, and it is common to consider inductance saliency in mathematical models. However, the inductance saliency causes interference between harmonic orders [9], which complicates the discussion when considering the relationship between harmonic current amplitude and inductance value for each harmonic order. Therefore, in this chapter, we first define a model that takes into account inductance saliency, and then present a model that ignores inductance saliency to simplify the discussion in the following sections.

3.1. Model with Inductance Saliency

The voltage equation for a dual 3-phase IPMSM is shown below in Equation (9):

$$v_{ph} = R i_{ph} + p(\phi_{ph} + \psi_{ph}) \quad (9)$$

where R is the phase resistance and p is the differential operator. Furthermore, where v_{ph} is the phase voltage vector, ϕ_{ph} is the flux linkage vector by an armature current, ψ_{ph} is the flux linkage vector by a permanent magnet, v_{ph} , ϕ_{ph} , and ψ_{ph} are expressed as:

$$v_{ph} = [v_{U1} \ v_{V1} \ v_{W1} \ v_{U2} \ v_{V2} \ v_{W2}]^T \quad (10)$$

$$\phi_{ph} = [\phi_{U1} \ \phi_{V1} \ \phi_{W1} \ \phi_{U2} \ \phi_{V2} \ \phi_{W2}]^T \quad (11)$$

$$\psi_{ph} = [\psi_{U1} \ \psi_{V1} \ \psi_{W1} \ \psi_{U2} \ \psi_{V2} \ \psi_{W2}]^T \quad (12)$$

The flux linkage vector by armature current ϕ_{ph} is:

$$\phi_{ph} = L_6 i_{ph} \quad (13)$$

$$L_6 = \begin{bmatrix} L_1 & M \\ M^T & L_2 \end{bmatrix} \quad (14)$$

$$L_1 = \begin{bmatrix} L_{U1} & M_{U1V1} & M_{U1W1} \\ M_{V1U1} & L_{V1} & M_{V1W1} \\ M_{W1U1} & M_{W1V1} & L_{W1} \end{bmatrix} \quad (15a)$$

$$L_2 = \begin{bmatrix} L_{U2} & M_{U2V2} & M_{U2W2} \\ M_{V2U2} & L_{V2} & M_{V2W2} \\ M_{W2U2} & M_{W2V2} & L_{W2} \end{bmatrix} \quad (15b)$$

$$M = \begin{bmatrix} M_{U1U2} & M_{U1V2} & M_{U1W2} \\ M_{V1U2} & M_{V1V2} & M_{V1W2} \\ M_{W1U2} & M_{W1V2} & M_{W1W2} \end{bmatrix} \quad (15c)$$

where L_6 is the overall inductance matrix, L_1 is the inductance matrix of the first winding, L_2 is the inductance matrix of the second winding, and M is the mutual inductance matrix of the first and second windings, respectively. For each inductance component, a model that takes into account the mutual inductance between the first and second windings is used [5]. For the inductance matrices in (15a,b),

$$L_i = L_{sl} + L_0 + L_2 \cos(2\varphi_{di}) \quad (16a)$$

$$M_{ij} = M_0 \cos(\varphi_{di} - \varphi_{dj}) + M_2 \cos(\varphi_{di} + \varphi_{dj}) \quad (16b)$$

For the inductance matrix in (15c),

$$M_{ij} = M'_0 \cos(\varphi_{di} - \varphi_{dj}) + M'_2 \cos(\varphi_{di} + \varphi_{dj}) \quad (16c)$$

where L_{sl} is the leakage inductance, L_0 and L_2 are the DC and AC components of the magnetizing inductance, M_0 and M_2 are the DC and AC components of mutual inductance between different phases in the same winding set, M'_0 and M'_2 are the DC and AC components of mutual inductance among different phases in different winding sets, φ_{di} and φ_{dj} are the phase differences between the d -axis and phases i and j (i and j are U1, V1, W1, U2, V2, or W2, and $i \neq j$).

The flux linkage in phase i by the permanent magnet ψ_i is expressed by the following equation when harmonic components are taken into account:

$$\psi_i = \sum_{k=1,3,5,\dots}^{\infty} \psi_k \cos k(\omega_e t - \varphi_i) \quad (17)$$

where ψ_k is the amplitude of the k th harmonic component of the flux linkage by the permanent magnet and φ_i is the phase of phase i . In this paper, we consider up to $k = 13$.

Transforming (9) into the rotational orthogonal subspaces using (1) and (7) yields

$$\begin{bmatrix} v_d \\ v_q \\ v_{dz} \\ v_{qz} \end{bmatrix} = \begin{bmatrix} R + pL_d & -\omega_e L_q & 0 & 0 \\ \omega_e L_d & R + pL_q & 0 & 0 \\ 0 & 0 & R + pL_{dz} & -\omega_e L_{qz} \\ 0 & 0 & \omega_e L_{dz} & R + pL_{qz} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_{dz} \\ i_{qz} \end{bmatrix} + \omega_e \begin{bmatrix} -11\psi_{11}\sin(12\omega_e t) - 13\psi_{13}\sin(12\omega_e t) \\ \psi_1 - 11\psi_{11}\cos(12\omega_e t) + 13\psi_{13}\cos(12\omega_e t) \\ 5\psi_5\sin(6\omega_e t) + 7\psi_7\sin(6\omega_e t) \\ 5\psi_5\cos(6\omega_e t) - 7\psi_7\cos(6\omega_e t) \end{bmatrix} \quad (18)$$

where v_d and v_q are the voltages in the d - q subspace and v_{dz} and v_{qz} are the voltages in the dz - qz subspace. The inductances L_d and L_q in the d - q subspace, and L_{dz} and L_{qz} in the dz - qz subspace are:

$$\begin{cases} L_d = L_{\alpha\beta} + \frac{L_2}{2} + M_2 + \frac{3}{2}M_2' \\ L_q = L_{\alpha\beta} - \frac{L_2}{2} - M_2 - \frac{3}{2}M_2' \\ L_{dz} = L_{z1z2} + \frac{L_2}{2} + M_2 - \frac{3}{2}M_2' \\ L_{qz} = L_{z1z2} - \frac{L_2}{2} - M_2 + \frac{3}{2}M_2' \end{cases} \quad (19)$$

where $L_{\alpha\beta}$ is the inductance in the α - β subspace, L_{z1z2} is the inductance in the $z1$ - $z2$ subspace, $L_{\alpha\beta}$ and L_{z1z2} are:

$$\begin{cases} L_{\alpha\beta} = L_{sl} + L_0 + \frac{1}{2}M_0 + \frac{3}{2}M_0' \\ L_{z1z2} = L_{sl} + L_0 + \frac{1}{2}M_0 - \frac{3}{2}M_0' \end{cases} \quad (20)$$

In the case of sinusoidal distribution winding, $L_0 = M_0 = M_0'$ and $L_2 = M_2 = M_2'$ in (16a–c). In this case, (19) and (20) can be expressed as:

$$L_{z1z2} = L_{dz} = L_{qz} = L_{sl} \quad (21)$$

From the above equation, the inductance in the $z1$ - $z2$ or dz - qz subspaces is only the leakage component in the case of sinusoidal distribution winding, as described in Chapter 1.

3.2. Model Ignores Inductance Saliency

For a model that ignores the inductance saliency, it is sufficient to set $L_2 = 0$, $M_2 = 0$, and $M_2' = 0$ in (16a–c). In this case, transforming (9) to the stationary orthogonal subspaces using (1) yields:

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ v_{z1} \\ v_{z2} \end{bmatrix} = \begin{bmatrix} R + pL_{\alpha\beta} & 0 & 0 & 0 \\ 0 & R + pL_{\alpha\beta} & 0 & 0 \\ 0 & 0 & R + pL_{z1z2} & 0 \\ 0 & 0 & 0 & R + pL_{z1z2} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_{z1} \\ i_{z2} \end{bmatrix} + \omega_e \begin{bmatrix} -\psi_1\sin(\omega_e t) + 11\psi_{11}\sin(-11\omega_e t) - 13\psi_{13}\sin(13\omega_e t) \\ \psi_1\cos(\omega_e t) - 11\psi_{11}\cos(-11\omega_e t) + 13\psi_{13}\cos(13\omega_e t) \\ -5\psi_5\sin(5\omega_e t) + 7\psi_7\sin(-7\omega_e t) \\ 5\psi_5\cos(5\omega_e t) - 7\psi_7\cos(-7\omega_e t) \end{bmatrix} \quad (22)$$

where v_α and v_β are the voltages in the α - β subspace, and v_{z1} and v_{z2} are the voltages in the $z1$ - $z2$ subspace. From (22), it can be confirmed that the voltage equation in the stationary orthogonal subspaces when ignoring the inductance saliency is not only free from interference between harmonic orders (the frequency of the output current matches the frequency of the input voltage), but also free from interference between axes. In Chapter 6, when analyzing the magnitude of the harmonic currents generated using inductance values, Equation (22) is used for simplifying.

4. Design Method for Inductance of Dual Three-Phase Motors

This chapter describes a method for formulating the inductance $L_{\alpha\beta}$ and L_{z1z2} in the α - β and $z1$ - $z2$ subspaces.

4.1. Definition of Winding Distribution Function

Let the winding distribution function $R(\theta_e)$ be the magnetomotive force (MMF) distribution when DC 1 [A] is energized in the U1 phase winding, where θ_e is the position

(electric angle) on the stator. Assuming that the MMF distribution generated by the coil is a square wave, $R(\theta_e)$ for the winding with the number of phase a , the coil pitch γ , and the number of slots per pole per phase q can be expressed as follows by Fourier series expansion:

$$R(\theta_e) = \frac{4N}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{k_{wn}}{n} \cos n\theta_e \quad (23)$$

where N is the number of turns and k_{wn} is the winding factor for n th spatial harmonics. Furthermore, the winding factor k_{wn} is:

$$k_{wn} = k_{pn} \times k_{dn} \quad (24a)$$

$$k_{pn} = \sin\left(\frac{n\gamma\pi}{2}\right) \quad (24b)$$

$$k_{dn} = \frac{\sin\left(\frac{n\pi}{2a}\right)}{q \sin\left(\frac{n\pi}{2aq}\right)} \quad (24c)$$

where k_{pn} is the pitch factor for n th spatial harmonics and k_{dn} is the distribution factor for n th spatial harmonics. In this paper, the number of slots per pole per phase q is assumed to be an integer. Figure 3 shows the waveform of the winding distribution function $R(\theta_e)$ when $k_{dn} = 1$ and only the pitch factor k_{pn} is considered. From Figure 3, the MMF distribution of the winding with the coil pitch γ can be seen.

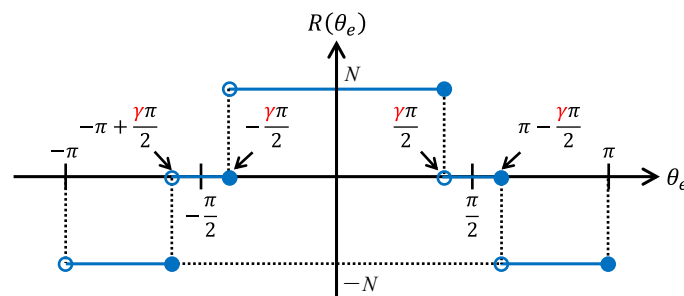


Figure 3. Winding distribution function $R(\theta_e)$ when $k_{dn} = 1$.

4.2. Magnetomotive Force Distribution for Harmonic Currents

The synthetic MMF distribution $F(\theta_e, t)$ by the armature currents of a dual three-phase motor is:

$$F(\theta_e, t) = i_{U1}R(\theta_e) + i_{V1}R\left(\theta_e - \frac{2}{3}\pi\right) + i_{W1}R\left(\theta_e - \frac{4}{3}\pi\right) + i_{U2}R\left(\theta_e - \frac{\pi}{6}\right) + i_{V2}R\left(\theta_e - \frac{5}{6}\pi\right) + i_{W2}R\left(\theta_e - \frac{3}{2}\pi\right) \quad (25)$$

The synthetic MMF distribution $F(\theta_e, t)$ when the k th harmonic current is energized can be obtained by substituting (5) into (25), and, as in (6a–d), it can be divided into the following 4 cases depending on the value of k .

1. $k = 1, 13, 25, \dots, 12m + 1$ ($m = 0, 1, 2, \dots$):

The energizing currents are the positive sequence of the α - β subspace, and $F(\theta_e, t)$ is:

$$F(\theta_e, t) = \frac{12NI}{\pi} \left\{ \sum_{n=1,13,25,\dots}^{\infty} \frac{k_{wn}}{n} \cos(n\theta_e - k\omega_e t) + \sum_{n=11,23,35,\dots}^{\infty} \frac{k_{wn}}{n} \cos(n\theta_e + k\omega_e t) \right\} \quad (26a)$$

2. $k = 11, 23, 35, \dots, 12m + 11$ ($m = 0, 1, 2, \dots$):

The energizing currents are the negative sequence of the α - β subspace, and $F(\theta_e, t)$ is:

$$F(\theta_e, t) = \frac{12NI}{\pi} \left\{ \sum_{n=1,13,25,\dots}^{\infty} \frac{k_{wn}}{n} \cos(n\theta_e + k\omega_e t) + \sum_{n=11,23,35,\dots}^{\infty} \frac{k_{wn}}{n} \cos(n\theta_e - k\omega_e t) \right\} \quad (26b)$$

$$3. \quad k = 5, 17, 29, \dots, 12m + 5 \quad (m = 0, 1, 2, \dots):$$

The energizing currents are the positive sequence of the z1-z2 subspace, and $F(\theta_e, t)$ is:

$$F(\theta_e, t) = \frac{12NI}{\pi} \left\{ \sum_{n=5,17,29,\dots}^{\infty} \frac{k_{wn}}{n} \cos(n\theta_e - k\omega_e t) + \sum_{n=7,19,31,\dots}^{\infty} \frac{k_{wn}}{n} \cos(n\theta_e + k\omega_e t) \right\} \quad (26c)$$

$$4. \quad k = 7, 19, 31, \dots, 12m + 7 \quad (m = 0, 1, 2, \dots):$$

The energizing currents are the negative sequence of the z1-z2 subspace, and $F(\theta_e, t)$ is:

$$F(\theta_e, t) = \frac{12NI}{\pi} \left\{ \sum_{n=5,17,29,\dots}^{\infty} \frac{k_{wn}}{n} \cos(n\theta_e + k\omega_e t) + \sum_{n=7,19,31,\dots}^{\infty} \frac{k_{wn}}{n} \cos(n\theta_e - k\omega_e t) \right\} \quad (26d)$$

In (26a–d), $\cos(n\theta_e - k\omega_e t)$ and $\cos(n\theta_e + k\omega_e t)$ represent n th spatial harmonic components of the MMF, the former being a magnetic field rotating with speed $k\omega_e$ in the positive direction and the latter with speed $k\omega_e$ in the negative direction.

As an example, Figure 4 shows the waveforms of (26a–d) when $\gamma = 1$ and $q = 1$. In Figure 4, the waveform at $t = 0$ is shown as a solid line and the waveform at $t = T/4k$ (where $T = 2\pi/\omega_e$) as a dotted line. From Figure 4a,b, it can be seen that the amplitude of the MMF is large when the energizing currents are in the α - β subspace. On the other hand, Figure 4c,d shows that the amplitude of the MMF is small when the energizing currents are in the z1-z2 subspace. The reason for this can be explained by focusing on the coefficient k_{wn}/n in (26a–d). Since $|k_{wn}|$ is always less than one, the larger n is, the smaller the n th spatial harmonic component of the MMF becomes. Here, the minimum n in (26a,b) is 1, while the minimum n in (26c,d) is 5. Therefore, the amplitude of the synthetic MMF is smaller in the latter case.

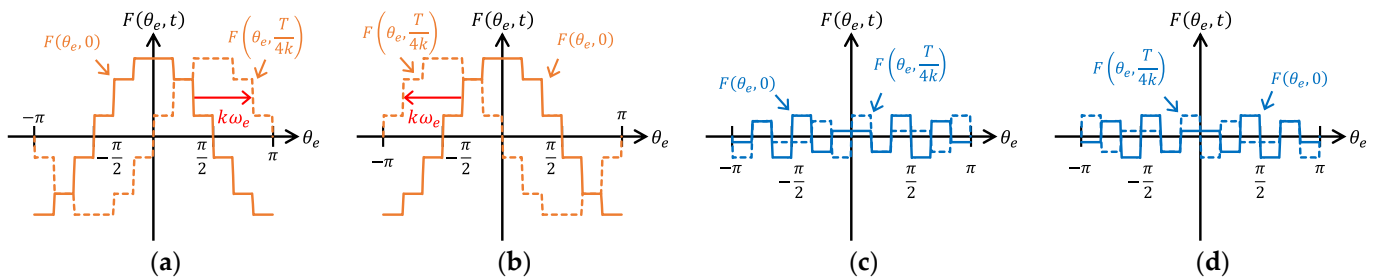


Figure 4. MMF distribution for k th harmonic currents: (a) $k = 12m + 1$ (α - β /positive sequence); (b) $k = 12m + 11$ (α - β /negative sequence); (c) $k = 12m + 5$ (z1-z2/positive sequence); (d) $k = 12m + 7$ (z1-z2/negative sequence).

Equation (26a,b) are similar to the expressions for the synthetic MMF distribution in conventional 3-phase motors. The difference is that when the currents in the α - β subspace (see Table 1) are energized in a conventional 3-phase motor, the spatial orders included in the synthetic MMF are $n = 1, 5, 7, 11, 13, \dots$, whereas in a dual 3-phase motor $n = 1, 11, 13, \dots$. In the case of a dual 3-phase motor, $n = 5, 7, \dots$ are included in the MMF when the current in the z1-z2 subspace are energized (see (26c,d)).

4.3. Calculation of Inductance

The air gap magnetic flux density distribution $B(\theta_e, t)$ is obtained by the following equation, where the permeance distribution density is $p(\theta_e, t)$:

$$B(\theta_e, t) = p(\theta_e, t) \times F(\theta_e, t) \quad (27)$$

Essentially, the permeance distribution density, $p(\theta_e, t)$, depends on the shape of the slot and rotor. However, taking these into account complicates the calculation of $B(\theta_e, t)$. Since the purpose of this chapter is to formulate the inductance $L_{\alpha\beta}$ and L_{z1z2} , ignoring the inductance saliency, a simple model shown in Figure 5 is used to model $p(\theta_e, t)$. As shown in Figure 5, the shape of the rotor is assumed to be a disk without the inductance saliency and the air gap width δ is uniform regardless of the position. In this model, if the magnetic resistance and magnetic saturation of the steel plate are neglected, $p(\theta_e, t)$ can be approximated as only the permeance distribution density in the gap area.

$$p(\theta_e, t) = \frac{\mu_0}{\delta} \quad (28)$$

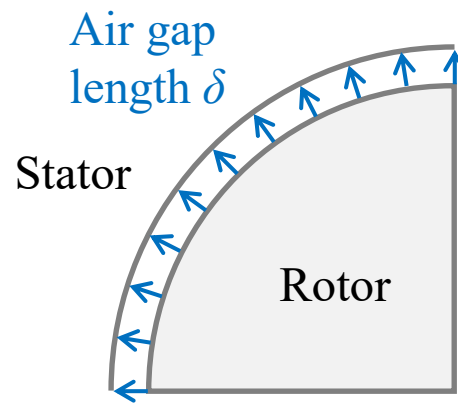


Figure 5. Simplified model of permeance distribution.

The magnetic flux linked to the coil of one pole pair in phase i (i is one of U1, V1, W1, U2, V2, or W2), ϕ_i , is obtained by the following equation, noting that the coil pitch γ and the number of slots per pole per phase q are considered:

$$\phi_i = \sum_{x=0}^{q-1} \frac{2Nl}{q} \int_{-\frac{\gamma\pi}{2} + \varphi_x + \varphi_i}^{\frac{\gamma\pi}{2} + \varphi_x + \varphi_i} B(\theta_e, t) \frac{r}{P} d\theta_e \quad (29)$$

where l is the stack length, r is the stator inner diameter, and P is the number of pole pairs. Furthermore, φ_i is the phase of phase i and φ_x is:

$$\varphi_x = \frac{\pi}{2aq} (2x - q + 1) \quad (30)$$

where $\varphi_x + \varphi_i$ represents the phase of the x th coil of phase i .

In the following, P -pole pairs of coils are assumed to be connected in parallel number b . We substitute $F(\theta_e, t)$ in (26a–d) and $p(\theta_e, t)$ in (28) into (27) to obtain $B(\theta_e, t)$. Furthermore, we substitute $B(\theta_e, t)$ into (29) to obtain the flux linkage in each phase. Furthermore, we use the orthogonal coordinate transformation matrix in (1) to transform the flux linkage in the α - β subspace and the $z1$ - $z2$ subspace. The calculation results are shown in (31a–d).

1. $k = 1, 13, 25, \dots, 12m + 1$ ($m = 0, 1, 2, \dots$):

$$\begin{bmatrix} \phi_\alpha \\ \phi_\beta \\ \phi_{z1} \\ \phi_{z2} \end{bmatrix} = L \sum_{n=1,11,13,23,25,\dots}^{\infty} \left(\frac{k_{wn}}{n} \right)^2 \times I \begin{bmatrix} \cos(k\omega_e t) \\ \sin(k\omega_e t) \\ 0 \\ 0 \end{bmatrix} \quad (31a)$$

2. $k = 11, 23, 35, \dots, 12m + 11$ ($m = 0, 1, 2, \dots$):

$$\begin{bmatrix} \phi_\alpha \\ \phi_\beta \\ \phi_{z1} \\ \phi_{z2} \end{bmatrix} = L \sum_{n=11,13,23,25,\dots}^{\infty} \left(\frac{k_{wn}}{n} \right)^2 \times I \begin{bmatrix} \cos(-k\omega_e t) \\ \sin(-k\omega_e t) \\ 0 \\ 0 \end{bmatrix} \quad (31b)$$

3. $k = 5, 17, 29, \dots, 12m + 5$ ($m = 0, 1, 2, \dots$):

$$\begin{bmatrix} \phi_\alpha \\ \phi_\beta \\ \phi_{z1} \\ \phi_{z2} \end{bmatrix} = L \sum_{n=5,7,17,19,29,31,\dots}^{\infty} \left(\frac{k_{wn}}{n} \right)^2 \times I \begin{bmatrix} 0 \\ 0 \\ \cos(k\omega_e t) \\ \sin(k\omega_e t) \end{bmatrix} \quad (31c)$$

4. $k = 7, 19, 31, \dots, 12m + 7$ ($m = 0, 1, 2, \dots$):

$$\begin{bmatrix} \phi_\alpha \\ \phi_\beta \\ \phi_{z1} \\ \phi_{z2} \end{bmatrix} = L \sum_{n=5,7,17,19,29,31,\dots}^{\infty} \left(\frac{k_{wn}}{n} \right)^2 \times I \begin{bmatrix} 0 \\ 0 \\ \cos(-k\omega_e t) \\ \sin(-k\omega_e t) \end{bmatrix} \quad (31d)$$

where ϕ_α and ϕ_β are the flux linkage in the α - β subspace and ϕ_{z1} and ϕ_{z2} are the flux linkage in the z1-z2 subspace. In addition, L is the inductance constant expressed by the following equation:

$$L = \frac{24\mu_0 N^2 r l}{\pi \delta b^2} \quad (32)$$

Here, in the mathematical model neglecting the inductance saliency, the flux linkage by the armature currents can be expressed from (22) as:

$$\begin{bmatrix} \phi_\alpha \\ \phi_\beta \\ \phi_{z1} \\ \phi_{z2} \end{bmatrix} = \begin{bmatrix} L_{\alpha\beta} & 0 & 0 & 0 \\ 0 & L_{\alpha\beta} & 0 & 0 \\ 0 & 0 & L_{z1z2} & 0 \\ 0 & 0 & 0 & L_{z1z2} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_{z1} \\ i_{z2} \end{bmatrix} \quad (33)$$

Therefore, by substituting the expressions for the k th harmonic currents shown in (6a–d) and the expressions for the flux linkage when the k th harmonic currents are energized, as shown in (31a–d) into (33), the inductances $L_{\alpha\beta}$ and L_{z1z2} in the α - β and z1-z2 subspaces are obtained as follows:

$$L_{\alpha\beta} = L \sum_{n=11,13,23,25,\dots}^{\infty} \left(\frac{k_{wn}}{n} \right)^2 \quad (34a)$$

$$L_{z1z2} = L \sum_{n=5,7,17,19,29,31,\dots}^{\infty} \left(\frac{k_{wn}}{n} \right)^2 \quad (34b)$$

$L_{\alpha\beta}$ and L_{z1z2} for the coil pitch γ and the number of slots per pole per phase q are normalized by L and shown in Figure 6. From Figure 6, it can be confirmed that L_{z1z2} is basically smaller than $L_{\alpha\beta}$ regardless of the values of γ and q . The reason for this is that Equation (34a,b) are heavily influenced by the inverse of n^2 , with the former dominated by the term at $n = 1$ and the latter by the terms at $n = 5$ and 7 .

The magnitude of $L_{\alpha\beta}$ depends on the winding factor k_{w1} for the first spatial order, and the magnitude of L_{z1z2} depends on the winding factors k_{w5} and k_{w7} for the fifth and seventh spatial order. Figure 7 shows the pitch factor k_{pn} for the coil pitch γ and Figure 8 shows the distribution factor k_{dn} for the number of slots per pole per phase q . From Figure 6a, we can see that $L_{\alpha\beta}$ is larger when the coil pitch γ is larger. From Figure 7, it can be seen that the trend of $L_{\alpha\beta}$ is similar to that of the pitch factor k_{p1} for the 1st spatial order. The fact that $L_{\alpha\beta}$ does not show much dependence on the number of slots per pole per phase q is

also similar to that of the distribution factor k_{d1} for the 1st spatial order in Figure 8. On the other hand, Figure 6b shows that the magnitude of L_{z1z2} has valleys around the coil pitch γ of $2/6$ ($=0.333$) and $5/6$ ($=0.833$) and peaks around $1/6$ ($=0.167$), $4/6$ ($=0.667$), and 1 . From Figure 7, it can be seen that this trend is influenced by the pitch factors k_{p5} and k_{p7} for the 5th and 7th spatial order. Furthermore, the larger the number of slots per pole per phase q , the smaller L_{z1z2} is, and this tendency is also similar to the distribution factors k_{d5} and k_{d7} for the 5th and 7th spatial order in Figure 8.

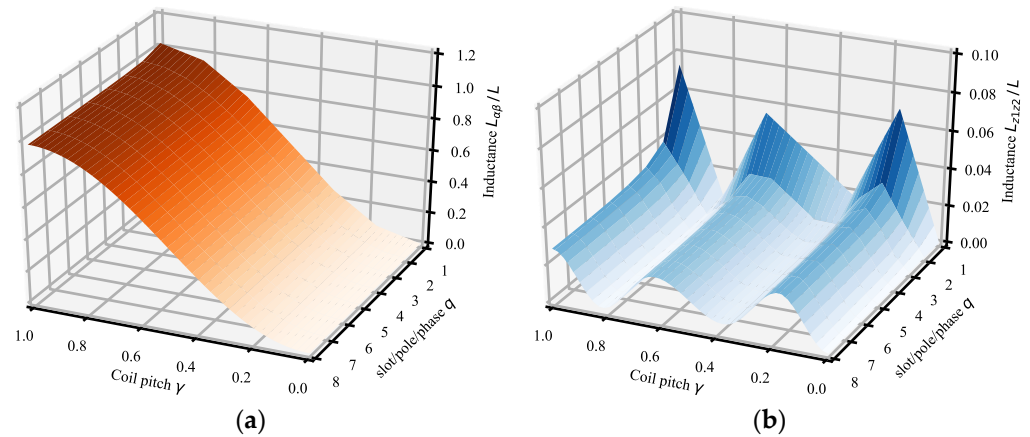


Figure 6. Theoretical value of inductance for coil pitch γ , number of slots per pole per phase q : (a) in α - β subspace; (b) in $z1$ - $z2$ subspace.

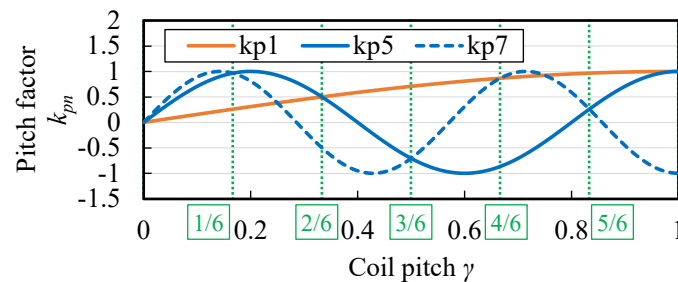


Figure 7. Pitch factor k_{pn} versus coil pitch γ .

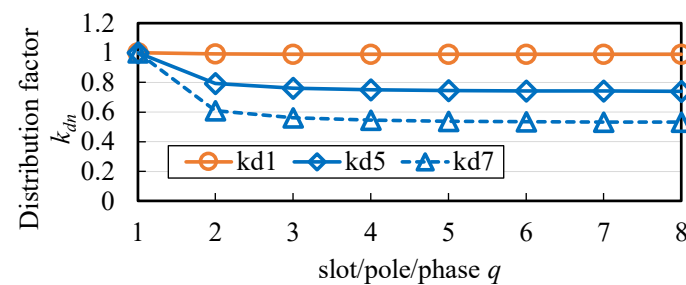


Figure 8. Distribution factor k_{dn} versus number of slots per pole per phase q .

As mentioned in Chapter 1, in order to reduce the harmonic currents during driving in a dual 3-phase motor, the inductance L_{z1z2} in the $z1$ - $z2$ subspace must be designed to be large. Figure 6b shows that L_{z1z2} can be maximized when the coil pitch $\gamma = 1$ and the number of slots per pole per phase $q = 1$. On the other hand, the short-pitch winding with the coil pitch $\gamma = 5/6$, which is often used in conventional 3-phase motors, has small pitch factors k_{p5} and k_{p7} , which means that the 5th and 7th order components of the back-EMF can be reduced when applied to a dual 3-phase motor, but at the same time L_{z1z2} is also reduced.

5. Verification by Finite Element Analysis

To verify the validity of the theoretical inductance equations presented in (34a,b), comparisons were made between the theoretical and FEA values. The parameters of the motor for evaluation are shown in Table 2. Since the motor for evaluation is a dual 3-phase motor with 48-slots/8-poles, the number of slots per pole per phase is $q = 1$. The model used for FEA is shown in Figure 9. For the shape of the rotor, L_d , L_q , L_{dz} , and L_{qz} are obtained for the interior magnet model with the saliency in Figure 9a, and $L_{\alpha\beta}$ and L_{z1z2} are obtained for the disk model without the saliency in Figure 9b. The purpose of this is to confirm that the parameters L_d , L_q , L_{dz} , and L_{qz} , which take into account the inductance saliency, show the same trend as $L_{\alpha\beta}$ and L_{z1z2} , which ignore the saliency. The analysis was performed for the cases of the coil pitch $\gamma = 1/6, 2/6, 3/6, 4/6, 5/6$, and 1, with a two-layer structure where two coils fit in one slot, as shown in Figure 9.

Table 2. Parameters of motor for evaluation.

Parameter	Value
Number of poles	8
Number of slots	48
Stator inner diameter r [mm]	131
Stack length l [mm]	141
Air gap length δ [mm]	0.5
Number of turns N	4
Number of parallels in circuit b	2

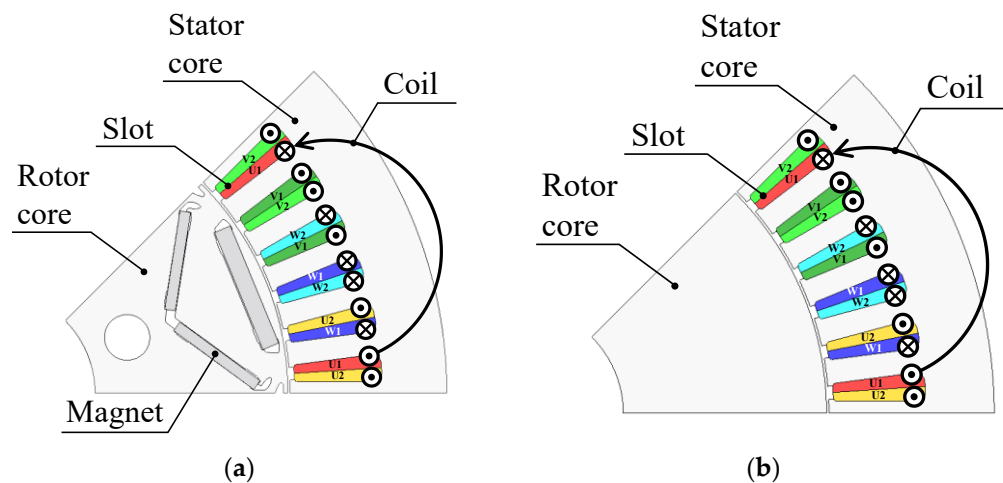


Figure 9. Model used for FEA: (a) interior magnet rotor with the saliency; (b) disk rotor without the saliency.

Figure 10 shows a comparison of the theoretical and FEA values. Note that in the calculation of the theoretical values, the permeance distribution is given approximately as in (28) and the errors due to differences from the exact permeance of the FEA model are introduced. Therefore, the theoretical values of $L_{\alpha\beta}$ and L_{z1z2} in Figure 10 are multiplied by the correction factors $K_{\alpha\beta}$ and K_{z1z2} , respectively, to the values calculated by (34a,b). From Figure 10, the theoretical values of $L_{\alpha\beta}$ and L_{z1z2} are in good agreement with the FEA values. Figure 10a also shows that $L_{\alpha\beta}$ and L_q are close to each other. This is because $L_{\alpha\beta}$ obtained using the model in Figure 9b is the ideal q -axis inductance. On the other hand, L_d is smaller than $L_{\alpha\beta}$ because the magnetic resistance of the d -axis magnetic path is larger due to the presence of the magnet region. Figure 10b also shows that L_{dz} and L_{qz} are not affected by the saliency and are consistent with L_{z1z2} . These results confirm the validity of the theoretical inductance equations presented in (34a,b). It was also confirmed that the

inductance values L_d , L_q , L_{dz} , and L_{qz} , which take into account the saliency, show the same tendency as the inductance values $L_{\alpha\beta}$ and L_{z1z2} , which ignore the saliency.

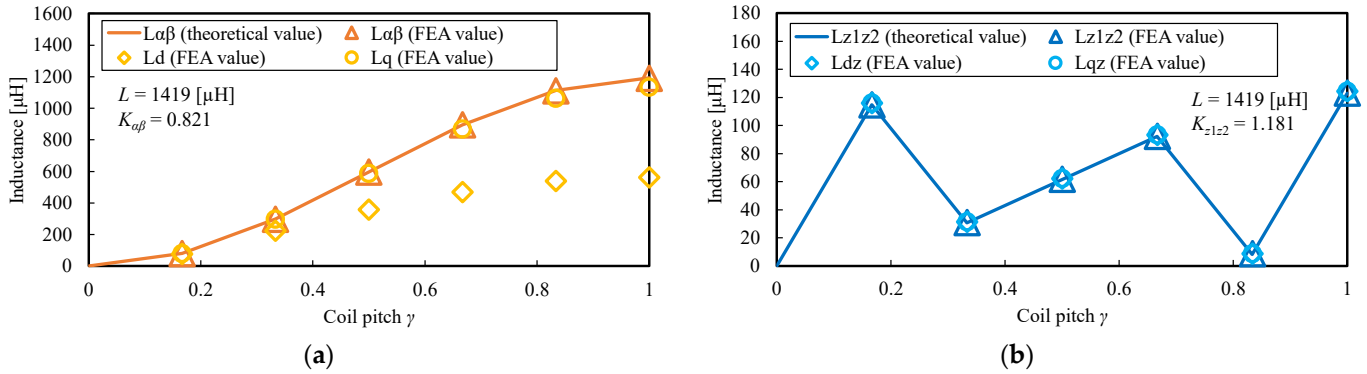


Figure 10. Comparison of theoretical and FEA values of inductance: (a) in α - β subspace; (b) in $z1$ - $z2$ subspace.

6. Verification by Circuit Simulation

In this chapter, the magnitude of the harmonic currents is studied by a simulation of the IPMSM drive. The magnitude of the harmonic component superimposed on the current obtained from this simulation is quantitatively evaluated using the inductance $L_{\alpha\beta}$ and L_{z1z2} in the α - β and $z1$ - $z2$ subspaces.

6.1. Simulation Method

Figure 11 shows the simulation configuration. In this simulation, the controller and ideal voltage-source inverter are implemented in the circuit simulator (MATLAB/Simulink), and the voltage and currents are inputted and outputted to the FEA model of the IPMSM (JMAG). The parameters of the IPMSM model are the same as in Table 2, and the shape of the rotor is the same as in Figure 9a. In this chapter, simulations are performed for coil pitch $\gamma = 5/6$ and 1.

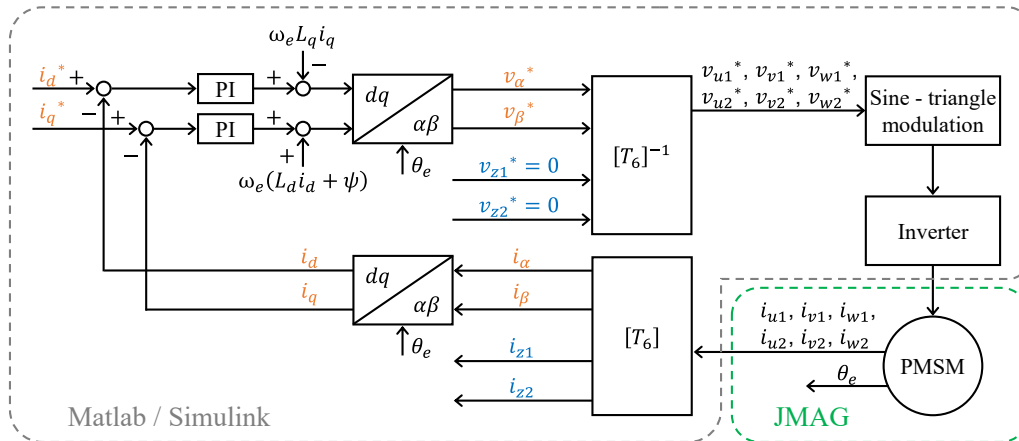


Figure 11. Simulation configuration.

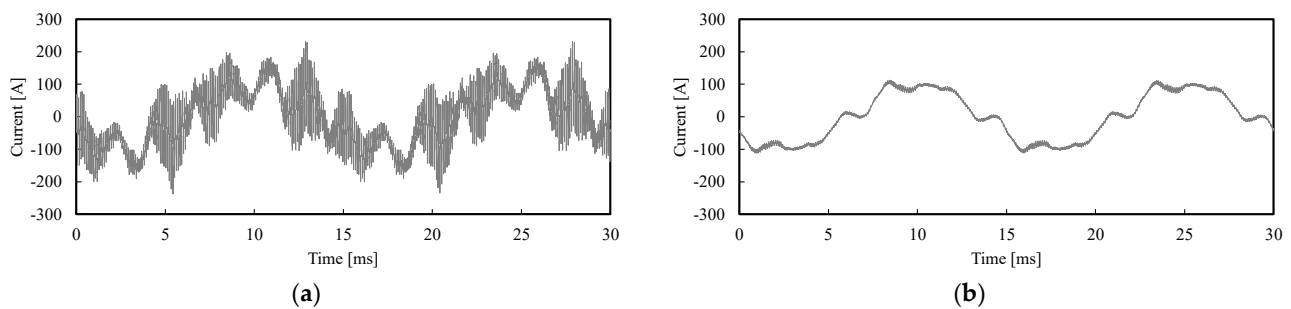
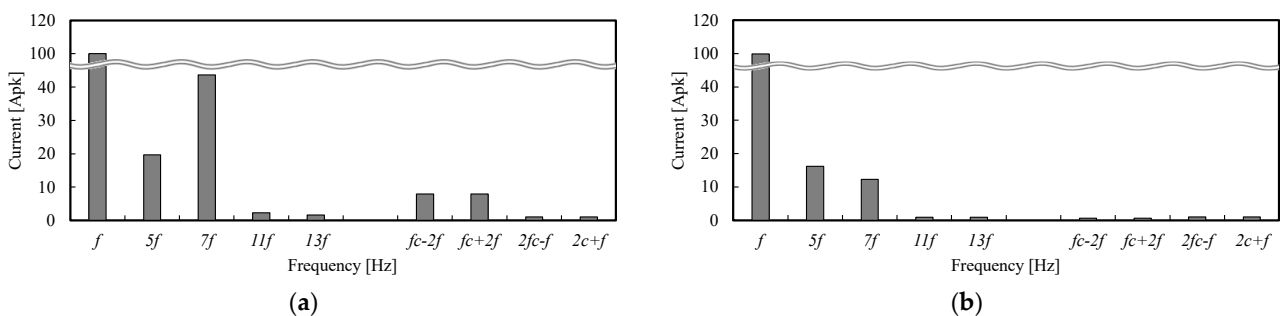
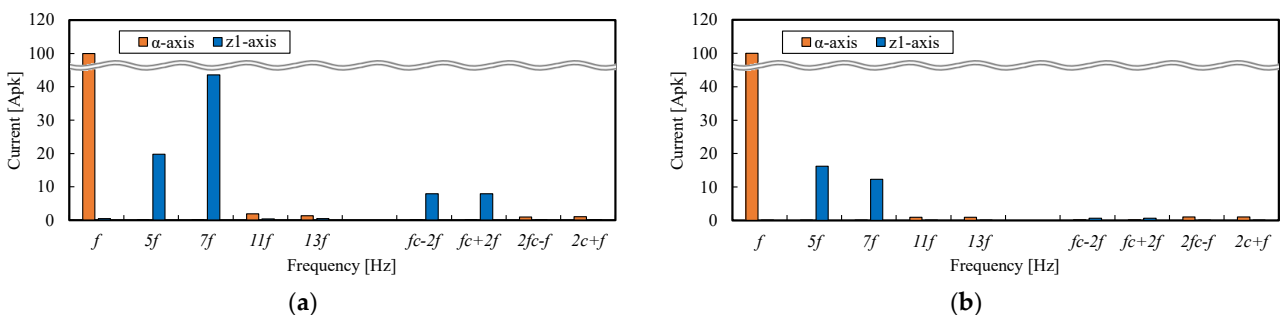
Next, the control configuration is described. As shown in Figure 11, PI control was applied to the fundamental superimposed on the α - β subspace. In this simulation, the harmonic currents are not controlled for the purpose of obtaining the magnitude of the generated harmonic currents. Therefore, the cutoff frequency of the controller is determined so that the voltage applied to the motor is sinusoidal. For the $z1$ - $z2$ subspace, where the 5th and 7th order harmonics are superimposed, open-loop control was used with voltage reference values $v_{z1}^* = 0$ and $v_{z2}^* = 0$. The drive conditions are shown in Table 3.

Table 3. Drive conditions.

Parameter	Value
Reference current [Apk]	100
Fundamental electrical frequency f [Hz]	66.67
Carrier frequency f_c [Hz]	10,000
DC voltage [V]	365

6.2. Simulation Results

As simulation results, the waveforms of the phase U1 currents and the FFT results are shown in Figures 12 and 13, respectively. The α -axis and z1-axis currents were calculated using (2), and the FFT results are shown in Figure 14. In Figures 13 and 14, the kf [Hz] component on the horizontal axis indicates the k th harmonic component, and the $f_c \pm 2f$ and $2f_c \pm f$ [Hz] components indicate the carrier harmonic sideband components. From Figures 12 and 13, it can be seen that the phase currents contain large harmonic components for both coil pitch $\gamma = 5/6$ and 1. For the coil pitch $\gamma = 1$, the phase currents contain mainly 5th and 7th harmonics. At coil pitch $\gamma = 5/6$, in addition to the 5th and 7th harmonics, the phase currents also contain large $f_c \pm 2f$ [Hz] components. Figure 14 shows that these large harmonic components are decomposed into the z1-z2 subspace by the orthogonal coordinate transformation.

**Figure 12.** Waveforms of phase U1 currents: (a) Coil pitch $\gamma = 5/6$; (b) Coil pitch $\gamma = 1$.**Figure 13.** FFT results of phase U1 currents: (a) Coil pitch $\gamma = 5/6$; (b) Coil pitch $\gamma = 1$.**Figure 14.** FFT results of α -axis and z1-axis currents: (a) Coil pitch $\gamma = 5/6$; (b) Coil pitch $\gamma = 1$.

6.3. Consideration of the Relationship between the Magnitude of Harmonic Currents and Inductance Value

The voltage equation in the stationary orthogonal subspace shown in (22), which ignores the inductance saliency, is used. In (22), the left side corresponds to the voltage applied by the inverter and the second term on the right side corresponds to the back-EMF by the permanent magnet. Therefore, in the α - β and $z1$ - $z2$ subspaces, respectively, Equation (22) can be regarded as an RL circuit with the applied voltage by the inverter and the back-EMF by the permanent magnet as the voltage input, and $i_{\alpha\beta}$ and i_{z1z2} as the current output. Here, the impedance, which is the ratio of the voltage and currents, is expressed by the following equation:

$$Z_{\alpha\beta}(\omega) = \sqrt{R^2 + (\omega L_{\alpha\beta})^2} \quad (35a)$$

$$Z_{z1z2}(\omega) = \sqrt{R^2 + (\omega L_{z1z2})^2} \quad (35b)$$

where ω is an arbitrary angular frequency. Figure 15 shows the frequency characteristics of $Z_{\alpha\beta}(\omega)$ and $Z_{z1z2}(\omega)$. From Figure 15, it can be seen that the impedance $Z_{z1z2}(\omega)$ is small even in the high frequency band, basically due to the inductance L_{z1z2} being smaller than $L_{\alpha\beta}$, regardless of the coil pitch γ . This results in large harmonic currents in the $z1$ - $z2$ subspace even when the harmonic components in the voltage input of the $z1$ - $z2$ subspace are small.

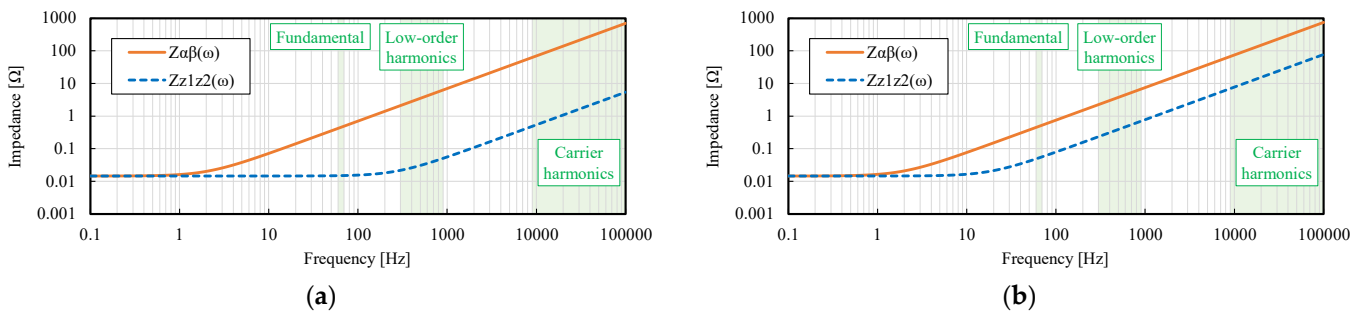


Figure 15. Frequency characteristic of impedance in α - β and $z1$ - $z2$ subspaces: (a) Coil pitch $\gamma = 5/6$; (b) Coil pitch $\gamma = 1$.

First, we discussed the cause of the excessive carrier harmonic currents ($f_c \pm 2f$ [Hz] component) in the $z1$ - $z2$ subspace for $\gamma = 5/6$ in Figure 14a. The FFT results of the applied voltage by the inverter are shown in Figure 16. From Figure 16, we see that for both $\gamma = 5/6$ and 1, the $f_c \pm 2f$ [Hz] component is included in the $z1$ - $z2$ subspace. On the other hand, Figure 15a shows that the impedance $Z_{z1z2}(\omega)$ around $f_c \pm 2f$ [Hz] for $\gamma = 5/6$ is very small, about 1 [Ω], despite the high frequency. This is thought to cause large harmonic currents at the corresponding frequencies.

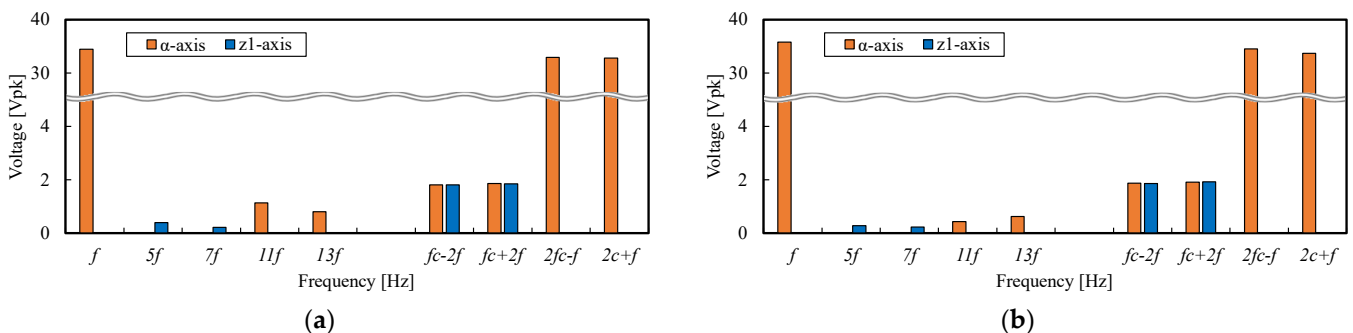


Figure 16. FFT results of applied voltage by the inverter: (a) Coil pitch $\gamma = 5/6$; (b) Coil pitch $\gamma = 1$.

Next, we discussed the reason why the 5th and 7th harmonic currents are larger for the coil pitch $\gamma = 5/6$ than for the coil pitch $\gamma = 1$. The FFT results of the no-load back-EMF are shown in Figure 17. Figure 17 shows that for coil pitch $\gamma = 5/6$, the 5th and 7th harmonic components of the back-EMF are smaller than for $\gamma = 1$. This is because, in general, the 5th and 7th harmonic components of the back-EMF can be reduced by a factor of $|k_{p5}/5|$ and $|k_{p7}/7|$ for the winding with the coil pitch γ , and, as shown in Figure 7, the pitch factors k_{p5} and k_{p7} are particularly small when $\gamma = 5/6$. On the other hand, from Figure 15, the impedance $Z_{z1z2}(\omega)$ for $\gamma = 5/6$ is about 1/10 of that for $\gamma = 1$ in the 5th and 7th harmonic frequency bands. This is because, from (34b), the magnitude of the inductance L_{z1z2} is strongly influenced by $(k_{p5}/5)^2$ and $(k_{p7}/7)^2$. Therefore, for the coil pitch $\gamma = 5/6$, even though the 5th and 7th harmonic components of the no-load back-EMF can be smaller than for $\gamma = 1$, the inductance L_{z1z2} decrease is more dominant, and large 5th and 7th harmonic currents are generated.

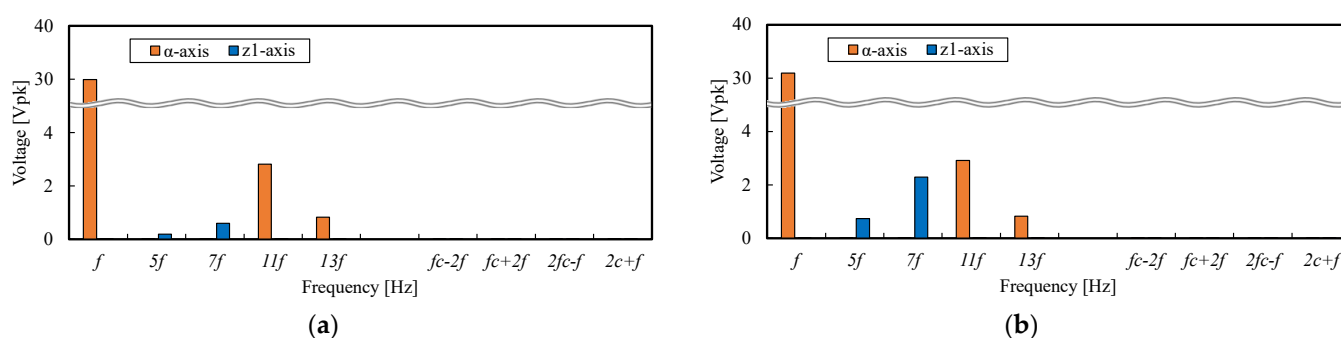


Figure 17. FFT results of the no-load back-EMF: (a) Coil pitch $\gamma = 5/6$; (b) Coil pitch $\gamma = 1$.

7. Conclusions

In this paper, the inductance in the α - β and $z1$ - $z2$ subspaces for dual 3-phase motors, which varies depending on the winding structure, was formulated. The validity of the formulated inductance was confirmed by FEA. Furthermore, using these inductance values, it was clarified that the magnitude of the generated harmonic currents could be quantitatively explained. In conclusion, it was theoretically shown that dual 3-phase motors with large winding factors for 5th and 7th spatial harmonics have a relatively large inductance in the $z1$ - $z2$ subspace, which can reduce the harmonic currents.

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