



Article

# **Optimization Method of Customized Shuttle Bus Lines under Random Condition**

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Abstract: Transit network optimization can effectively improve transit efficiency, improve traffic conditions, and reduce the pollution of the environment. In order to better meet the travel demands of passengers, the factors influencing passengers' satisfaction with a customized bus are fully analyzed. Taking the minimum operating cost of the enterprise as the objective and considering the random travel time constraints of passengers, the customized bus routes are optimized. The K-means clustering analysis is used to classify the passengers' needs based on the analysis of the passenger travel demand of the customized shuttle bus, and the time stochastic uncertainty under the operating environment of the customized shuttle bus line is fully considered. On the basis of meeting the passenger travel time requirements and minimizing the cost of service operation, an optimization model that maximizes the overall satisfaction of passengers and public transit enterprises is structured. The smaller the value of the objective function is, the lower the operating cost. When the value is negative, it means there is profit. The model is processed by the deterministic processing method of random constraints, and then the hybrid intelligent algorithm is used to solve the model. A stochastic simulation technique is used to train stochastic constraints to approximate uncertain functions. Then, the improved immune clonal algorithm is used to solve the vehicle routing problem. Finally, it is proved by a case that the method can reasonably and efficiently realize the optimization of the customized shuttle bus lines in the region.

**Keywords:** urban transport; lines optimization; customized shuttle bus; hybrid intelligent algorithm; random constraints

# 1. Introduction

In recent years, as a special mode of public transport, a customized shuttle bus has been increasingly popular. Taking Beijing as an example, the daily average passenger volume of customized buses (business buses, high-speed rail lines, leisure tourism lines, etc.) has reached 37,000. A customized shuttle bus is a public transit mode, serving the individual demands for the passengers whose origins and destinations, travel time, and service level requirement is similar. A customized shuttle bus is an important part of urban public transit. The stops (including transfer stops) and routes of customized shuttle bus are set up according to the individual demands. The ticket fare of customized shuttle bus is higher than one of the bus and subway, but is lower than one of the self-driving and taxi. Passengers usually need to pay a ticket fare in advance in months or weeks. A customized shuttle bus can provide door-to-door service and seat for each passenger. Moreover, WIFI, travel information search, and multimedia are available to passengers. Locations of stops are determined according to demands. The stops can be specialized or bus stops. The routes of the customized shuttle bus can be fixed or adjusted according to the traffic conditions. However, the stop coverage must meet the demands. Related research results showed that four factors can influence passengers' willingness to choose



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a customized shuttle bus: Private car, distance between the home and work place, travel satisfaction level, and work overtime [1].

To reflect the advantages of customized shuttle bus, especially the accuracy of travel time, the routes of vehicles should be optimized reasonably. Route optimization or scheduling optimization of the customized shuttle bus or a similar service have been studied by a few papers. Lei et al. (2017) used the K-means clustering analysis to cluster real-time demands of residents, and a dynamic scheduling model was developed with the maximal demand service rate and minimum cost as the objectives and the maximal capacity and passenger time threshold factors as the constraints. Then, a parallel ant colony algorithm based on the Hadoop platform was proposed [2]. Cheng (2014) studied the layout of the customized shuttle bus with the level analysis method of point, line, and plane [3]. To design the service area and routing for a flexible feeder transit system, Pan et al. (2014) presented a mathematical model with objectives to maximize the number of served passengers and minimize the operational cost [4]. A route planning and coordinated scheduling model was developed by Pan et al. (2016) to minimize passengers' cost and operator cost [5].

For similarities between the customized shuttle bus and common public transit, the line optimization method for a common public transit can be used for reference when the characteristics of the customized shuttle bus are fully considered. Lin et al. (1999) presented a nonlinear 0-1 programming model for the bus network design problem from the viewpoint of a combination of optimization and minimizing of passengers travel time and operation cost of the bus network. The stop capacity is taken into account and the simulated annealing algorithm was adopted to solve the model [6]. Zhou and Luo (2013) established a layout bi-level optimization model of a comprehensive transit system and an algorithm based on improved IOA was proposed, which overcame the defects of a lack of significant hierarchy and improved solve efficiency [7]. He and Fan (2007) considered the impacts of transfer times, transfer point choice, and travel cost on solving the optimal path. The transfer walk-time matrix was constructed and a multiplication of the transfer matrix was transformed to the addition of the transfer walk-time matrix and public traffic travel-time matrix [8]. Mireia R.R. et al. (2012) proposed a bilevel formulation for solving the Bus Network Design Problem (BNDP) of interurban services entering a major city. The objective function of the first level was defined with the aim of reducing the user and agency costs. In the second level, the performance of users was addressed. Furthermore, a local search method based on the Tabu search algorithm was carried out to guide the exploration in the solution domain [9]. Perugia A. et al. (2012) presented a model and an algorithm for the design of a home-to-work bus service in a metropolitan area. A multiobjective model was introduced, among other aspects, equity was considered by time windows on the arrival time of a bus at a stop. In addition, a cluster routing approach was proposed to model both the bus stop location and routing in urban road networks where turn restrictions exist. The resulting multi-objective location-routing model was solved by a Tabu search algorithm [10]. Amiripour S.M.M. et al. (2014) based on the analysis of seasonal transit demand variations, studied the design of the bus network. In order to solve this NP hard problem, a genetic algorithm-based method was proposed. The optimization was realized by a defined objective function [11]. Leksakul et al. (2017) compared solution methods for different location-routing problems and the optimal route between 300-700 bus stops allocated by the K-means clustering analysis was determined with the ant colony optimization [12]. A methodological framework was formulated by Ouyang et al. (2014), and heterogeneous route configurations generated by this framework can reduce the cost of bus users and operating agency [13]. Chang and Hu (2005) proposed a linear mathematical model on the bus line network, and passengers' travel time, public transit network density, as well as operator's benefit were considered [14]. Tom and Mohan (2003) designed the transit route network in a two-phase solution process: A large set of the candidate route is generated in the first phase and a solution route set is selected from the candidate route set in the second phase, while the bus operating cost and passenger total travel time were minimized [15]. Wei and Machemehl (2006, 2012) considered the user cost, Algorithms **2021**, 14, 52 3 of 13

operator cost, and unsatisfied demand cost [16,17]. Xu et al. (2015) considered the interests of passengers and bus companies and established a bi-level programming model of public transit network planning, while the equilibrium analysis of the passenger travelling choice behavior was made based on the strategy equilibrium transit assignment [18]. Yu and Liang (2016) established an integer nonlinear programming model for designing the optimal bus transit network to minimize the total cost of the bus operation and passenger travel time [19]. A network optimization model was developed by Hu et al. (2008), and benefits maximization, costs minimization, and sustainable development were considered [20]. Yu et al. (2012) built a direct traveler density model and extended it to design the transit network, aiming to minimize the number of transfer times and maximize the demand density of the route [21]. Zhao and Zeng (2007) presented a methodology for optimizing transit networks based on both passenger and operator costs [22]. Szeto and Jiang (2014) proposed a bi-level transit network design problem and the objective of upper-level was to minimize the number of passenger transfers, while the lower-level problem was a transit assignment problem [23].

We can conclude that the objectives of works on the bus network optimization are mainly minimal travel time, cost or transfer times, while residents' travel behavior is less considered. Among the factors affecting the optimization of customized bus routes, the most important are the travel cost and time, in which travel time often has a certain randomness. Given that stops of the customized shuttle bus are set up according to demands, this paper optimizes the customized shuttle bus network from the point of residents' travel behavior. The remainder of this paper is organized as follows: The next section analyzes passengers' travel demands along with the model formulation. A heuristic algorithm is proposed and illustrated in Section 3. Section 4 gives an illustrative example to demonstrate the validity of the model and effectiveness of the heuristic algorithm. The concluding remarks are made in the last section.

#### 2. Problem Formulation

In a certain region, the customized shuttle bus travel demand data is obtained through the OD demand survey, and the alternative bus stops are determined based on the K-means clustering analysis. The objective is to minimize the difference between the operating costs and ticket revenue, and the constraint conditions are vehicle stops, passenger capacity, and operation time. The customized shuttle bus route optimization model is constructed, focusing on the time uncertainty in the process of vehicle operation, the stochastic uncertainty constraints of time are given.

## 2.1. Travel Demand Analysis

Passengers often choose the shortest route. In medium or small cities where the traffic condition is better, the shortest route is usually the most convenient, economical, and efficient route. However, in large cities, especially mega-cities, the shortest route often means more travel time and cost for traffic jam.

Different from the common public transit, customized shuttle bus stops are set up based on travel demand and the travel time of each route is fixed. Routes can be adjusted according to the traffic condition. Therefore, customized shuttle bus routes can be optimized by the analysis of the situation of land utilization along the route and detouring around the traffic jam.

Based on big data analytics, we adopt the K-means clustering analysis to classify passengers [24]. There is a positive integer set  $X = \{x_1, x_2, \dots, x_n\}$  and the data in this set are divided into k subsets,  $C = \{c_k, k = 1, 2, \dots, N\}$ . There is a data center  $\mu_k$  in each subset which represents a kind of data. Euclidean distance is used as the criterion. The quadratic sum of distances between the data points and data center  $\mu_k$  will be calculated and minimized.

$$\min f(x_i) = \sum_{x_i \in c_k} \|x_i - \mu_k\|^2 = \sum_{i=1}^n d_{ki} \|x_i - \mu_k\|^2$$
 (1)

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$$d_{ki} = \begin{cases} 1, x_i \in c_k \\ 0, x_i \notin c_k \end{cases} \tag{2}$$

Whether travel demand points will be divided into the same subset depends on whether the distances between the cluster center and origins or destinations meet passengers' expectation which is 500 m in this paper. The cluster center of origins or destinations will be the customized shuttle bus stop.

$$||x_i - \mu_k|| \le r, r = 500 \tag{3}$$

The principle of the K-means algorithm is as follows:

Step 1: Data normalization and outlier processing;

Step 2: Randomly select k-clustering centroids (alternative bus stops);

Step 3: All data points are related and divided to the nearest centroid, and clustering is based on it;

Step 4: Move the particle to the center (means) of the current partition cluster containing all the data points;

Repeat step 2 and step 3 N times until the sum of squares of the distances from all points to their cluster centroid is the smallest.

Through the K-means clustering algorithm, the group of  $\mu_k$  covering the most OD is selected as the candidate bus stops.

## 2.2. Model Establishment

Based on the candidate stations calculated by the above clustering analysis method, the following assumptions need to be considered before establishing the route optimization model and the corresponding solution method.

# 2.2.1. Modeling Assumption

The operation cost and travel time requirement will be considered based on a reasonable ticket fare. The optimization objective is to minimize the operation cost. To simplify the problem, the following conditions will be satisfied.

- Each depot will be analyzed alone. The vehicle type for one depot will be the same. The unit operation cost is fixed and proportional to the operation mileages.
- Each vehicle can only travel along one route while serving several stops. The trip volume of each OD at each stop will not exceed the vehicle capacity.
- The number of passengers getting on or off the bus at each stop is known in the decision period. Passengers without an appointment are not allowed to take the bus.
- The timetable will be observed strictly. A delay caused by emergencies such as accidents or disasters will be neglected.
- A single ticket fare will be adopted.

# 2.2.2. Notations

The following terms have been defined and commonly used in our model.

Sets	
A	set of depots (departure point of vehicles), $A = \{i \mid i=1, 2,, n\}$
V	set of demand points (stops), $V = \{v \mid v = 1, 2,, m\}$
B	set of vehicles, $B = \{b \mid b = 1, 2, \dots, k\}$
S	set of the number of stops served by vehicles, $S = \{s \mid s = 1, 2,, m\}$
Parameters	3
$m_b$	the number of stops served by vehicle $b$ , $\sum_{b=1}^{k} m_b = m$
$Q_v$	the number of passengers getting on the bus at stop $v$
$P_v$	the number of passengers getting off the bus at stop $v$
$d_s$	the number of passengers in the area served by stop s

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$f_s$	ticket fare for one passenger served by stop s
$r_h$	capacity of vehicle <i>b</i>
$l_v$	distance between the depot and demand point $v$
$l_{u,v}$	length of the shortest route between stop $u$ and stop $v$
$c_b$	unit operation cost of a vehicle
$c_h'$	fixed cost of a vehicle for one trip
$W_s^b$	the number of passengers in vehicle <i>b</i> after the <i>s</i> th stop served
$t_0^b$	the time when the vehicle <i>b</i> starts from the depot
$c_b^\prime \ W_s^b \ t_{u,v}^b$	vehicle $b$ running time from stop $u$ to stop $v$
$a_v$	bus arrival time required by stop $v$
	distance between the depot and first stop served by vehicle $b$ ,
$L_b$	$L_b = \sum\limits_{v=1}^m l_v  imes X_{1,v}^b, orall b \in B$
	distance between the first stop and last stop served by vehicle $b$ ,
$L'_b$	$L'_{b} = \sum_{s=1}^{m_{b}-1} \sum_{u=1}^{m} \sum_{v=1}^{m} l_{u,v} \times X_{s,u}^{b} \times X_{s+1,v}^{b}, \forall b \in B$
$T_v^b$	The time when vehicle $b$ reaches stop $v$
Decision va	ariables
$X^b_{s,v}$	0– $1$ if or not the sth stop served by vehicle $b$ is stop $v$

# 2.2.3. Mathematical Formulation

We aim at deciding routes of vehicles reasonably to meet demands while the number of stops served by each vehicle is unknown. The optimization objective is to minimize the operation cost of all vehicles under a vehicle capacity constraint.

Based on the above, we now introduce the objectives and constraints.

$$minZ = \sum_{b=1}^{k} \left\{ \left[ (L_b + L'_b) \times c_b + c'_b \right] - \sum_{s=1}^{m_b} f_s \times d_s \right\}$$
 (4)

s.t.

$$\sum_{v=1}^{m} X_{s,v}^{b} = 1, \forall b \in B, s \in S$$

$$\tag{5}$$

$$\sum_{s=1}^{m_b} X_{s,v}^b = 1, \forall b \in B, v \in V$$

$$\tag{6}$$

$$W_1^b = \sum_{v=1}^m \left( Q_v \times X_{1,v}^b \right), \forall b \in B$$
 (7)

$$W_2^b = W_1^b - \sum_{v=1}^m (P_v - Q_v) \times X_{2,v}^b, \forall b \in B$$
 (8)

$$W_{s}^{b} = W_{s-1}^{b} - \sum_{v=1}^{m} \left[ (P_{v} - Q_{v}) \times X_{s,v}^{b} \right], \forall b \in B, s \in S$$
 (9)

$$W_s^b \le r_b, \forall b \in B, s \in S \tag{10}$$

$$T_0^b = t_{0, i}^b \forall b \in B \tag{11}$$

$$T_1^b = t_0^b + \sum_{v=1}^m t_{0,v}^b \times X_{1,v}^b, \forall b \in B$$
 (12)

$$T_{v}^{b} = t_{0}^{b} + \sum_{v=1}^{m} \left( t_{v-1,v}^{b} \times X_{s,v}^{b} \right) + \sum_{v=1}^{m} \left( t_{v-1}^{b} \times X_{s-1,v-1}^{b} \right), \forall b \in B, s \in S$$
 (13)

$$T_v^b \in [a_v - d_v^-, a_v + d_v^+], \forall b \in B, v \in V \text{ or } a_v - d_v^- \le T_v^b \le a_v + d_v^+, \forall b \in B, v \in V$$
 (14)

The objective function (4) minimizes the cost of all the vehicles. Constraint (5) means that each vehicle cannot serve more than one stop simultaneously. Constraint (6) ensures

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that each stop can only be served once for each vehicle in one trip. Constraint (7)–(10) are the vehicle capacity constraints. Constraint (7) represents the number of passengers at the first stop in each vehicle. Constraint (8) indicates the number of passengers aboard when the vehicles departure from the second stop. Constraint (9) is the recursion formula to calculate the number of passengers in each vehicle at each stop. Constraint (10) sets the limit of the number of passengers in each vehicle. Constraint (11)–(14) are time constraints. Constraint (11) states the departure time of each vehicle. Constraint (12) represents when each vehicle arrives at the first stop served. Constraint (13) indicates when each vehicle arrives at other stops served. Constraint (14) states the uncertainty for vehicles' arrival time at stops.  $d_v^+$  and  $d_v^-$  are the definitions of positive and negative deviation of arrival time at stop v for vehicle b, respectively.

# 3. Solution Algorithm

We will try to guarantee that the probability that the vehicle's arrive stop v within the specified time is not less than  $\beta_v$ . Constraint (14) can be transferred to a chance constraint (Liu et al. 2003) [25].

$$Pr\left\{T_v^b - a_v \le d_v^+\right\} \ge \beta_v, \forall b \in B, v \in V, Pr\left\{a_v - T_v^b \le d_v^-\right\} \ge \beta_v, \forall b \in B, v \in V$$
 (15)

The prominent feature of the programming model with a stochastic chance constraint is that the decision-maker is allowed to make a decision that does not meet the constraint conditions to a certain extent, but the probability of the constraint conditions is not less than a certain confidence level. The neural network has a strong nonlinear approximation function, and its approximation ability for complex objects with uncertainty is better than any previous method based on the accurate mathematical model. It has been proven that the neural network with a deviation and at least one S-type hidden layer plus a linear output layer can approximate any rational function. Therefore, this manuscript uses the neural network to realize the approximation of the random function. In the process of algorithm implementation, the neural network can be used to approximate the random function to reduce the amount of calculation and speed up the solution process. It is feasible to use the trained neural network to calculate the random function. The trained neural network is nested in the immune clonal algorithm, and the optimal solution is obtained by using the search performance of the immune clonal algorithm.

We propose a hybrid intelligent optimization algorithm based on the immune clone algorithm, stochastic simulation technique, and neural network. The algorithm works as follows:

Step 1

Generate input and output data of the uncertain function (15) with the stochastic simulation technique.

$$U_1: T_v^b \to Pr \left\{ T_v^b - a_v \le d_v^+ \right\} \ge \beta_v, \forall b \in B, \ v \in V$$

$$U_2: T_v^b \to Pr \left\{ a_v - T_v^b \le d_v^- \right\} \ge \beta_v, \forall b \in B, \ v \in V$$

Step 2

A neural network is trained to approximate the uncertain function from the input and output data generated in Step 1.

Step 3

Create the initial population and the feasibility of antibodies will be checked with the neural network.

Step 4

Update antibodies by a clone operator and immune genic operator and the feasibility of child antibodies will be checked with the neural network.

Step 5

Calculate all antibodies' objective values with the neural network.

Step 6

Calculate all antibodies' affinity based on the objective values.

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Step 7

Select antibodies by the clone selection.

Step 8

Iterate from Step 4 to 7 until the end criterion is met.

Step 9

Output the optimal solutions.

## 3.1. Antibody Coding

Antibody v = (x, y, t) represents a decision process and gene  $x = (x_1, x_2, x_3, \ldots, x_n)$  represents the n stops;  $y = (y_1, y_2, y_3, \ldots, y_{m-1})$  is the number of stops served by the vehicle,  $y_0 \equiv 0 \le y_1 \le y_2 \le y_3 \cdots \le y_{m-1} \le n \equiv y_m$  and "0" is the depot;  $t = (t_1, t_2, t_3, \ldots, t_b)$  and  $t_b$  represent the departure time from the depot for vehicle b; and  $y_m = y_{m-1}$  means that vehicle b has not departed from the depot, while  $y_m > y_{m-1}$  means that vehicle b has departed from the depot and the departure time is  $t_b$ . Schematic plot of antibody coding presented in Figure 1.

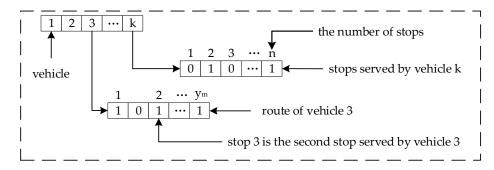


Figure 1. Schematic plot of antibody coding.

# 3.2. Initial Population

For gene x, we define  $x_i = i$  and a series  $\{x_1, x_2, x_3, \dots, x_n\}$  is defined,  $I = 1, 2, 3, \dots, n$ . The following process will be repeated: Select a position n' randomly between j and n, then exchange the values of  $x_j$  and  $x_{n'}$  to ensure that  $\{x_1, x_2, x_3, \dots, x_n\}$  is a reset of  $\{1, 2, 3, \dots, n\}$ . Gene  $x = \{x_1, x_2, x_3, \dots, x_n\}$  will be generated.

If  $1 \le i \le m-1$ , generate random number  $y_i$  between 0 and n, then the series will be reset from small to large and we will obtain gene  $y = (y_1, y_2, y_3, \dots, y_{m-1})$ .

For i = 1, 2, 3, ..., k, we define  $t_i$  is a random number for departure time a, then gene  $t = (t_1, t_2, t_3, ..., t_k)$ .

If antibody v = (x, y, t) is feasible, it will be accepted. Otherwise repeat the process above until a feasible antibody is generated.

# 3.3. Clone Operator

For the clone operator, the population V is updated to V' as follows:

$$V' = T_c(V) = (v'_1(x, y, t), v'_2(x, y, t), \cdots, v'_k(x, y, t))^T$$
(16)

 $v'_i(x,y,t) = T_c(v_i(x,y,t)) = I_i \times v_i(x,y,t), i = 1,2,\cdots,k$ ,  $I_i$  is a  $q_i$ -dimension row vector of where all elements are "1". The clone scale of antibody  $v_i$  (x,y,t) should be adjusted adaptively according to its affinity to antigens and avidities to other antibodies. The clone scale should be calculated as follows:

$$q_{i}(\mathbf{x}, \mathbf{y}, \mathbf{t}) = ceil \left( n_{c} \cdot \frac{f(\mathbf{v}'_{i}(\mathbf{x}, \mathbf{y}, \mathbf{t}))}{\sum\limits_{j=1}^{k} f(\mathbf{v}_{i}(\mathbf{x}, \mathbf{y}, \mathbf{t}))} \cdot \varphi_{i} \right), i = 1, 2, \dots, k$$
(17)

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where  $ceil(\cdot)$  is a function to round up the input to an integer.  $n_c$  is a set value in connection with the clone scale and  $n_c > k$ .  $f(v_i(x, y, t))$  is the affinity of antibody  $v_i(x, y, t)$  to the antigens.  $\varphi_i$  is the avidity of antibody  $v_i(x, y, t)$  to other antibodies and can be calculated by Formula (18).

$$\varphi_i = min\{exp(\|v_i - v_j\|)\}, i \neq j; \quad i, j = 1, 2, \dots, k$$
 (18)

where  $\|\cdot\|$  is the Euclidean distance which means that  $0 \le \|\cdot\| \le 1$ . Obviously the larger the antibody's avidities, the more the antibody is similar to other antibodies and inhibition among the antibodies is stronger.

## 3.4. Immune Genic Operator

#### Crossover

For antibodies  $v_1$  and  $v_2$ , where  $v_1 = (x_1, y_1, t_1)$  and  $v_2 = (x_2, y_2, t_2)$ , the crossover operator can be executed as follows: Generate a random number c in the open interval (0,1), then we define  $t'_1 = c \cdot t_1 + (1-c) \cdot t_2$ ,  $t'_2 = (1-c) \cdot t_1 + c \cdot t_2$ . Child antibodies  $v'_1$  and  $v'_2$  can be obtained by the crossover operator, where  $v'_1 = (x_1, y_2, t'_1), v'_2 = (x_2, y_1, t'_2)$ .

#### Mutation

For gene x of antibody v=(x,y,t), we generate two positions  $n_1$  and  $n_2$  between 1 and n randomly for the mutation operator. Then, the series  $\{x_{n_1}, x_{n_1+1}, x_{n_1+2}, \cdots, x_{n_2}\}$  will be reset and a new series  $\{x'_{n_1}, x'_{n_1+1}, x'_{n_1+2}, \cdots, x'_{n_2}\}$  will be obtained. A new gene will be as follows:

$$\mathbf{x}' = \left\{ x_1, x_2, \cdots, x_{n_1-1}, x'_{n_1}, x'_{n_1+1}, x'_{n_1+2}, \cdots, x'_{n_2}, x_{n_2+1}, x_{n_2+2}, \cdots, x_n \right\}$$
 (19)

For gene y, two positions  $n_1$  and  $n_2$  between 1 and m-1 will be generated randomly. For  $I = n_1, n_{1+1}, n_{1+2}, \ldots, n_2$ , we define  $y_i$  as a random number between 0 and n. Then, we reset the series  $\left\{y_1, y_2, \cdots, y_{n_1-1}, y'_{n_1}, y'_{n_1+1}, y'_{n_1+2}, \cdots, y'_{n_2}, y_{n_2+1}, y_{n_2+2}, \cdots, y_{m-1}\right\}$  from small to large in order to obtain a new gene y'.

For gene t, we generate a mutation direction d in  $R_k$ . If  $t + M \cdot d$  is out of the time window  $[a_v - d_v^-, a_v + d_v^+]^k$  for a predetermined step size M, M will be set as a random number between 0 and M until the time window  $[a_v - d_v^-, a_v + d_v^+]^k$  is met. When gene t in the time window cannot be obtained by the above process within iterations, we set M = 0. Finally, t will be replaced by child gene  $t' = t + M \cdot d$ .

# Selection operator

Combine the parent population and children as one, then select an antibody with the highest affinity to be kept to the children population, which avoids the degradation of the algorithm [26].

#### 4. Numerical Example

In this section, we analyze travel demands for the customized shuttle bus in an area served by a depot. We optimize the customized shuttle bus route in the decision period. The area is presented in Figures 2 and 3. The K-means clustering analysis is adopted to analyze the travel demand and the result is shown in Figure 4. Passengers get on customized shuttle bus vehicles mainly in residential areas, while their destinations are mainly government agencies, hospitals, the central business district, enterprises, and creative industry parks. The number of pairs of origin-destination is 200 and this paper has not listed them all due to the limited space. There is a line between any two stops. The distances matrices are given in Tables 1 and 2. The travel time depends on the traffic condition. The arrival time and dwell time are given by Table 3.  $d_v^+$  and  $d_v^-$  are normally distributed, while the

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expectation is 3 and the variance is 1. Under 90% confidence, all stops can be served in their time windows, which means that the chance constraint is as follows:

$$Pr\left\{T_{v}^{b} \in \left[a_{v} - d_{v}^{-}, a_{v} + d_{v}^{+}\right]\right\} \ge 0.9, \forall b \in B, v \in V$$
 (20)

The capacity of a vehicle is 45 passengers. The fixed cost of a trip is USD 200 and unit operation cost is USD 2. The ticket fare is USD 10.

**Table 1.** Distance between the bus station and each stop (km).

Stop	$v_1$	$v_2$	$v_3$	$v_4$	$v_5$	$v_6$	$v_7$	$v_8$	$v_9$	$v_{10}$	$v_{11}$	$v_{12}$
Distance	4	1	1.5	6.5	11.5	14.5	17.5	5.5	8.5	10.5	5	4

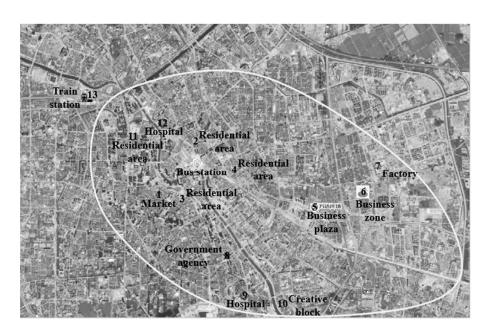


Figure 2. Layout of the customized shuttle bus service demand points.

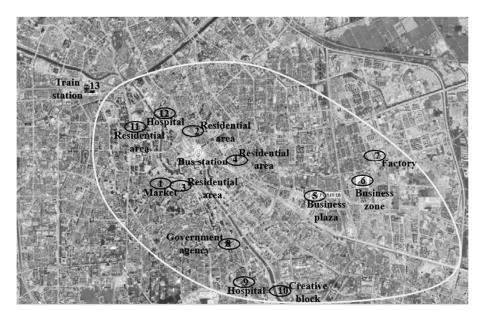


Figure 3. Results of the cluster analysis.

We adopt the following parameter values: The number of iterations is 1000, the number of clone operator times is 500, and the population size is 1200. The time for the calculation is 1.5 s. The optimized customized shuttle bus lines are shown in Figure 4. Two customized shuttle bus lines are obtained (2-4-5-6-7, 2-1-3-8-9-10) and 90% demands can be satisfied. To verify the practical effect of the model and algorithm, we compare the results for different travel demands, as shown in Tables 4 and 5.

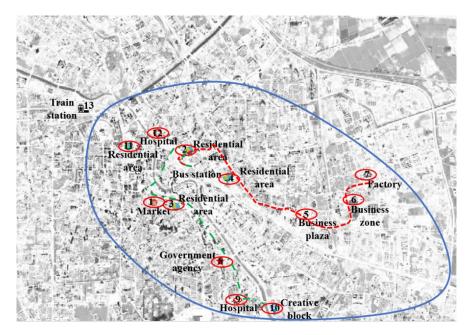


Figure 4. Schematic diagram of the customized shuttle bus routes planning.

<b>Table 2.</b> Distance matrix among stops (	km).	
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$v_1$	$v_2$	$v_3$	$v_4$	$v_5$	$v_6$	$v_7$	$v_8$	$v_9$	$v_{10}$	$v_{11}$	$v_{12}$
0	4	1	6	11	15	18	5	8	10	4	6
4	0	5	2	7	10	13	9	12	14	4	3
1	5	0	7	12	15	18	4	7	9	5	7
6	2	7	0	5	8	11	11	14	16	6	5
11	7	12	5	0	3	6	16	19	21	11	10
15	10	15	8	3	0	3	19	22	24	14	13
18	13	18	11	6	3	0	22	25	27	17	16
5	9	4	11	16	19	22	0	3	5	9	11
8	12	7	14	19	22	25	3	0	2	12	14
10	14	9	16	21	24	27	5	2	0	14	16
4	4	5	6	11	14	17	9	12	14	0	3
6	6	7	5	10	13	16	11	14	16	3	0
	0 4 1 6 11 15 18 5 8 10 4	0 4 4 0 1 5 6 2 11 7 15 10 18 13 5 9 8 12 10 14 4 4	0     4     1       4     0     5       1     5     0       6     2     7       11     7     12       15     10     15       18     13     18       5     9     4       8     12     7       10     14     9       4     4     5	0     4     1     6       4     0     5     2       1     5     0     7       6     2     7     0       11     7     12     5       15     10     15     8       18     13     18     11       5     9     4     11       8     12     7     14       10     14     9     16       4     4     5     6	0     4     1     6     11       4     0     5     2     7       1     5     0     7     12       6     2     7     0     5       11     7     12     5     0       15     10     15     8     3       18     13     18     11     6       5     9     4     11     16       8     12     7     14     19       10     14     9     16     21       4     4     5     6     11	0     4     1     6     11     15       4     0     5     2     7     10       1     5     0     7     12     15       6     2     7     0     5     8       11     7     12     5     0     3       15     10     15     8     3     0       18     13     18     11     6     3       5     9     4     11     16     19       8     12     7     14     19     22       10     14     9     16     21     24       4     4     5     6     11     14	0     4     1     6     11     15     18       4     0     5     2     7     10     13       1     5     0     7     12     15     18       6     2     7     0     5     8     11       11     7     12     5     0     3     6       15     10     15     8     3     0     3       18     13     18     11     6     3     0       5     9     4     11     16     19     22       8     12     7     14     19     22     25       10     14     9     16     21     24     27       4     4     5     6     11     14     17	0     4     1     6     11     15     18     5       4     0     5     2     7     10     13     9       1     5     0     7     12     15     18     4       6     2     7     0     5     8     11     11       11     7     12     5     0     3     6     16       15     10     15     8     3     0     3     19       18     13     18     11     6     3     0     22       5     9     4     11     16     19     22     0       8     12     7     14     19     22     25     3       10     14     9     16     21     24     27     5       4     4     5     6     11     14     17     9	0       4       1       6       11       15       18       5       8         4       0       5       2       7       10       13       9       12         1       5       0       7       12       15       18       4       7         6       2       7       0       5       8       11       11       14         11       7       12       5       0       3       6       16       19         15       10       15       8       3       0       3       19       22         18       13       18       11       6       3       0       22       25         5       9       4       11       16       19       22       0       3         8       12       7       14       19       22       25       3       0         10       14       9       16       21       24       27       5       2         4       4       5       6       11       14       17       9       12	0       4       1       6       11       15       18       5       8       10         4       0       5       2       7       10       13       9       12       14         1       5       0       7       12       15       18       4       7       9         6       2       7       0       5       8       11       11       14       16         11       7       12       5       0       3       6       16       19       21         15       10       15       8       3       0       3       19       22       24         18       13       18       11       6       3       0       22       25       27         5       9       4       11       16       19       22       0       3       5         8       12       7       14       19       22       25       3       0       2         10       14       9       16       21       24       27       5       2       0         4       4       5       6       11	0       4       1       6       11       15       18       5       8       10       4         4       0       5       2       7       10       13       9       12       14       4         1       5       0       7       12       15       18       4       7       9       5         6       2       7       0       5       8       11       11       14       16       6         11       7       12       5       0       3       6       16       19       21       11         15       10       15       8       3       0       3       19       22       24       14         18       13       18       11       6       3       0       22       25       27       17         5       9       4       11       16       19       22       0       3       5       9         8       12       7       14       19       22       25       3       0       2       12         10       14       9       16       21       24       27

In the case of the same random time, compared with the traditional simulated annealing algorithm (the cooling rate is 0.98, the initial temperature is 1000, the end temperature is 0.001, and the chain length L of the Metropolis is 500), the proposed hybrid intelligent optimization algorithm can improve the solving efficiency of the model, especially when there is a large travel demand. The advantage of the proposed hybrid intelligent optimization algorithm is obvious in terms of the solving speed. The proportion of passengers served will be improved with the increasing of the travel demand. The simulated annealing algorithm works as follows:

## Step 1

Initialization. Set a large enough initial temperature  $T_0$ ,  $T = T_0$ , and determine the number of iterations of each T (that is, the chain length L of the Metropolis) by the randomly generated initial solution S.

Step 2

For the current temperature *T* and k = 1, 2, ..., L, repeat Step  $3 \sim 6$ .

Step 3

A new solution *S'* is generated by randomly perturbing the current solution *S*.

Step 4

The increment d of S' is calculated as f(S') - f(S), where f(S) is the cost function of S. Step 5

If d < 0, then accept S' as the new current solution; otherwise, calculate the acceptance  $\exp(-d/T)$  of S', and determine whether to retain S' by comparing the value of the random number rand generated in (0,1) interval with the acceptance; if  $\exp(-d/T) > rand$ , then accept S' as the current solution, otherwise, retain S.

Step 6

The termination condition of the algorithm is usually set as: In several continuous metropolis chains, the new solution S' cannot be accepted, or reaches the set end temperature. If the termination condition of the algorithm is satisfied, the current solution will be the output as the optimal solution. Otherwise, the attenuation function T = qT (q is the cooling rate) attenuates T and returns to Step 2.

**Table 3.** Arrival time and waiting time of vehicles required by each stop.

Stop	Arrival Time	Waiting Time (min)	Stop	Arrival Time	Waiting Time (min)
$v_1$	8:00	2	$v_7$	8:50	2
$v_2$	8:10	2	$v_8$	8:20	2
$v_3$	8:05	2	$v_9$	8:30	2
$v_4$	8:15	3	$v_{10}$	8:35	1
$v_5$	8:30	2	$v_{11}$	8:05	2
$v_6$	8:40	2	$v_{12}$	8:10	2

Table 4. Comparison of operation parameters of different travel demand.

Travel Demand/Trip	Operation Time/s	Number of Vehicles	Service Rate/%		
200	1.5	4	85		
500	1.7	10	88		
1000	2.2	22	90		

**Table 5.** Comparison of operation time comparison (s).

	Travel Demand/Trip			
Solution Method	200	500	1000	
Hybrid intelligent optimization algorithm	1.5	1.7	2.2	
Simulated annealing algorithm	5	11	21	

# 5. Concluding Remarks

This work classifies passengers' travel demand for a customized shuttle bus with the K-means clustering analysis. A customized shuttle bus line optimization model is established considering the operation cost and travel time requirement of passengers. Considering the delay of buses caused by the traffic jam, etc., a stochastic travel time constraint is included in the model. On the basis of the immune clonal algorithm, according to the characteristics of the model, the hybrid intelligent optimization algorithm is used to solve the model. Aiming at the chance constraint, a neural network is trained by the stochastic simulation technique to approximate the uncertain function. On this basis, the improved immune clonal algorithm is used to solve the problem. The effect of the model

and algorithm is verified by the numerical example with different travel demand sizes. On the basis of the minimum cost, it can maximize the demand of the customized shuttle bus (90%), and ensure the travel time demand of passengers. This method can improve the efficiency of the customized shuttle bus line planning effectively, which is nearly 10 times faster than the traditional simulated annealing algorithm. By optimizing the customized bus routes, it can continuously improve the service quality and efficiency, promote the diversification of urban public transit, and improve the sharing of urban public transit. However, how to schedule vehicles after they arrive at the terminals is neglected and could be a path for future work.

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