

## Proof for the Cardan-conversion-formulas

The formulas (1) to (3) are showed without any proof in the publication. Within this document the proof will be given for

$$\varphi_i = \arctan2(R_{kk}^*, -sgn_{ijk} R_{jk}^*) \quad , \quad (1)$$

$$\varphi_j = \sin^{-1}(sgn_{ijk} R_{ik}^*) \quad \text{and} \quad (2)$$

$$\varphi_k = \arctan2(R_{ii}^*, -sgn_{ijk} R_{ij}^*) \quad . \quad (3)$$

*Proof:* By executing the multiplication of the uniaxial rotation matrices in every possible combination, it can be shown that the general rotation matrix  $R$ , expressed as a function of the angles, with the associated sequence tuple  $(i, j, k)$  contains the following entries

$$R_{ii} = \cos\varphi_j \cos\varphi_k \quad , \quad (4)$$

$$R_{ij} = -sgn_{ijk} \cos\varphi_j \sin\varphi_k \quad , \quad (5)$$

$$R_{ik} = sgn_{ijk} \sin\varphi_j \quad , \quad (6)$$

$$R_{jk} = -sgn_{ijk} \sin\varphi_i \cos\varphi_j \quad \text{and} \quad (7)$$

$$R_{kk} = \cos\varphi_i \cos\varphi_j \quad . \quad (8)$$

By letting  $R$  equal the given rotation matrix  $R^*$ , it is possible to solve for the values of these angles. That means that angles  $\varphi_i$ ,  $\varphi_j$  and  $\varphi_k$  are needed, for which equations (4) to (8) hold, given that  $R(\varphi_i, \varphi_j, \varphi_k) = R^*$ . Rearranging equations (7) and (8) to solve for  $\sin\varphi_i$  and  $\cos\varphi_i$  and then dividing one by the other leads to

$$\tan\varphi_i = \frac{-sgn_{ijk} R_{jk}}{R_{kk}} = \frac{-sgn_{ijk} R_{jk}^*}{R_{kk}^*} \quad (9)$$

$$\Leftrightarrow \quad \varphi_i = \arctan2(R_{kk}^*, -sgn_{ijk} R_{jk}^*) \quad (10)$$

which is exactly equation (1).

Equation (3) follows analogously from (4) and (5).

Equation (2) is equivalent to equation (6) after replacing  $R$  by  $R^*$ .  $\square$