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Article

An Improved Quantum-Behaved Particle Swarm Optimization Method for Economic Dispatch Problems with Multiple Fuel Options and Valve-Points Effects

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Abstract: Quantum-behaved particle swarm optimization (QPSO) is an efficient and powerful population-based optimization technique, which is inspired by the conventional particle swarm optimization (PSO) and quantum mechanics theories. In this paper, an improved QPSO named SQPSO is proposed, which combines QPSO with a selective probability operator to solve the economic dispatch (ED) problems with valve-point effects and multiple fuel options. To show the performance of the proposed SQPSO, it is tested on five standard benchmark functions and two ED benchmark problems, including a 40-unit ED problem with valve-point effects and a 10-unit ED problem with multiple fuel options. The results are compared with differential evolution (DE), particle swarm optimization (PSO) and basic QPSO, as well as a number of other methods reported in the literature in terms of solution quality, convergence speed and robustness. The simulation results confirm that the proposed SQPSO is effective and reliable for both function optimization and ED problems.

Keywords: economic dispatch; quantum-behaved particle swarm optimization; valve-point effects; multiple fuel options

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1. Introduction

Economic dispatch (ED) is considered to be one of the key functions in electric power system operation. The main objective of ED is to determine the optimal scheduling of power outputs for all generating units that minimizes the total fuel cost while satisfying all the equality and inequality constraints of units and system. Due to valve-point effects, prohibited operating zones and multiple fuel effects, the characteristics of power generating units are inherently highly nonlinear [1].

Multiple fuel options problem (coal, nature gas or oil) is one of the important kinds of ED problems and each part of the hybrid cost function implies some information about the fuel being burned or the operation cost of units. Taking valve-point effects and multiple fuel options into consideration, the ED problem can be represented as a non-smooth optimization problem, which causes difficulties in finding the global or near global optimization solution using conventional approaches.

Over the past two decades, many modern meta-heuristic methods have been applied to ED problems, such as genetic algorithm (GA) [2], particle swarm optimization (PSO) [3], differential evolution (DE) [4], ant colony optimization (ACO) [5] and simulated annealing (SA) [6]. Among these methods, PSO has recently attracted more attention due to its rapid convergence and algorithmic accuracy compared with other optimization methods.

PSO is a population based optimization algorithm, which was introduced by Kennedy and Eberhart in 1995 [7]. PSO is motivated by the simulation of social behaviour of animals such as fish schooling and bird flocking. In the conventional PSO mechanism, a swarm of individuals (called particles) fly within the search space. Each particle represents a potential solution to the optimization problem. The position of a particle is influenced by the best position (*pbest*) found by itself (*i.e.*, its own experience) and the position of the best particle in the whole swarm (*gbest*) (*i.e.*, the experience of neighbouring particles).

Although PSO can converge quickly towards the optimal solution, it has difficulties in reaching a global optimum and suffers from premature convergence. Moreover, PSO has several control parameters. The convergence of the algorithm depends heavily on the value of its control parameters.

Taking advantage of both PSO mechanism and quantum mechanics, in 2004, a new version of PSO, quantum-behaved particle swarm optimization, named QPSO, was proposed by Sun, Xu and Feng [8], which is inspired by quantum mechanics and trajectory analysis of PSO. As a quantum system is an uncertain system that is different from classical stochastic system in which every particle can appear at any position with a certain probability, the swarm can search in the whole feasible region [9]. Besides, unlike PSO, there are no velocity vectors for particles in QPSO, and it has fewer parameters to be adjusted, which makes it easier to implement. In [10–12], convergence analysis and other varients of QPSO have been presented. As an efficient algorithm, QPSO has been applied to many optimization problems, such as system identification [13], non-linear programming problems [14], power system [15], *etc*. Although Coelho *et al.* proposed a quantum-inspired HQPSO using the harmonic oscillator potential well to solve economic dispatch problems [16], Sun and Lu applied QPSO to ED problems [15], and Chakraborty *et al.* presented a hybrid QPSO to solve the ED problems [17], to the best of our knowledge, it has not been used yet to solve ED problems with multiple fuel options.

In this paper, an improved QPSO namely SQPSO is proposed to solve ED problems with multiple fuel options and valve-points effects. In the proposed SQPSO, a new selective probability operator is introduced into the updating mechanism of QPSO, which can balance the global and local searching abilities and enhance the diversity of QPSO. In particular, based on the selective probability operator, *pbest* and *gbest* are used to generate the local attractor of QPSO, with user defined selective probability, to enhance the local search performance. This modification on the original QPSO together with a recombination operator will maintain the best information of the swarm and, in the same time, exchange information between individuals to increase the population diversity.

To show the performance of the proposed SQPSO, five popular benchmark functions and two ED problems with valve-point effects and multi-fuel options are tested. The results obtained by SQPSO are analyzed and compared with PSO, DE and QPSO, as well as some other optimization methods reported in recent literature. The remainder of this paper is organized as follows: Section 2 is the formulation of the ED problem and Section 3 presents the conventional PSO, QPSO and proposed SQPSO, respectively. Section 4 gives the experimental results. Finally, Section 5 concludes the paper.

2. Formulation of the ED Problem

The main objective of solving the ED problem is to minimize the total fuel cost of each thermal generating unit in electric power system while satisfying a variety of equality and inequality constraints. The total fuel cost function of ED problem is described as:

$$\min F_T = \sum_{i=1}^{n} F_i(P_i) \tag{1}$$

where F_T is the total generation cost, n is the total number of generating unit, P_i is the power of the ith generator and F_i is its corresponding fuel cost, which is defined by the following equation as:

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 \tag{2}$$

where a_i, b_i and c_i are the cost coefficients and subject to:

$$\sum_{i=1}^{n} P_i = P_D, i = 1, 2, ..., n \tag{3}$$

$$P_i^{\min} < P_i < P_i^{\max} \tag{4}$$

where P_D is the total demand of the power system, P_i^{min} and P_i^{max} are the minimum and maximum output of the ith generation unit, respectively.

2.1. The ED Problem with Valve-Point Effects

A valve-point is the rippling effect added to the generation unit curve when each steam admission valve in a turbine starts to open [2]. This curve poses higher order non-linearity and discontinuity, which makes the problem of finding the optimum more difficult and increases the number of local minima in the fuel cost function. Considering the valve-point effects, sinusoidal functions are added to the quadratic cost function, which is defined by the following equation:

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 + \left| e_i \sin(f_i(P_i^{\min} - P_i)) \right| \tag{5}$$

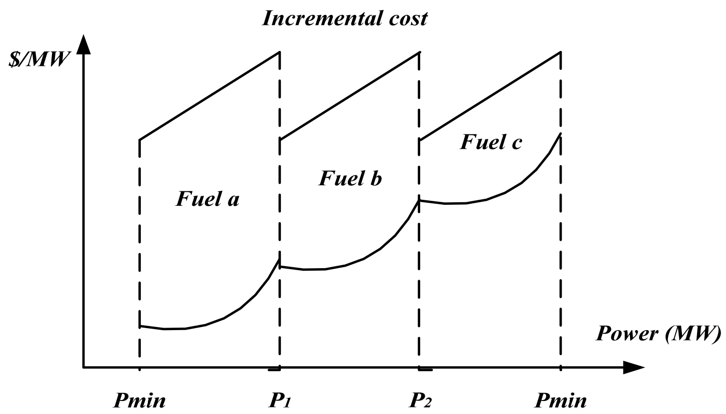
where e_i, f_i are the coefficients of generator i, reflecting the valve-point.

2.2. ED Problem with Multiple Fuels and Valve-Point Effects

To give a more accurate description of the ED problem, the effects of multiple fuels resources (coal, nature gas or oil) should also be considered. Each segment of the hybrid cost function implies some information about the fuel being burned or the unit's operation. Since the dispatching units are practically supplied with multi-fuel sources, each unit should be represented with several piecewise quadratic functions reflecting the effects of fuel type changes, and the generator must identify the most economic fuel to burn [2]. The number of non-differentiable points in the objective function increases when multiple fuels are taken into consideration. The incremental cost functions of a generator with multi-fuel options are illustrated in Figure 1. The ED problems with both multiple and fuels valve-point effects can be represented as follows:

$$F_i(P_i) = \begin{cases} a_{i1} + b_{i1}P_i + c_{i1}P_i^2 + \left|e_{i1}\sin(f_{i1})(P_{i1}^{\min} - P_{i1})\right|, & fuel\ 1,\ P_i^{\min} < P_i < P_{i1} \\ a_{i2} + b_{i2}P_i + c_{i2}P_i^2 + \left|e_{i2}\sin(f_{i2})(P_{i2}^{\min} - P_{i2})\right|, & fuel\ 2,\ P_{i1} < P_i < P_{i2} \\ a_{ik} + b_{ik}P_i + c_{ik}P_i^2 + \left|e_{i2}\sin(f_{ik})(P_{ik}^{\min} - P_{ik})\right|, & fuel\ k,\ P_{ik-1} < P_i < P_i^{\max} \end{cases} \quad (6)$$

Figure 1. Incremental cost function of a generator with multi-fuel options.



3. The Proposed SQPSO Algorithm

3.1. Conventional Particle Swarm Optimization

PSO is a population-based stochastic optimization algorithm, which is inspired by the social intelligence and movements of fishes or birds in the swarm. In PSO, each potential solution is a point in the search space and is called as 'particle'. Each particle is assumed to have two characteristics: a position and a velocity. The target of the particles is to find the best result of the objective function. Initially, a population of particles is randomly generated within the search space. At each iteration, it stores memory of best position of each individual and best position of the whole population. By taking advantages of the particles' own experience and experience of its neighbours, the particles could fly towards the optimal solution.

For example, in a *n*-dimensional search space, the position and velocity of an individual *i* are represented as the vectors: $X_i = (X_{i1}, X_{i1},..., X_{in})$ and $V_i = (V_{i1}, V_{i2},..., V_{in})$. The best position for each particle is denoted as: $pbest_i = (pbest_{1i}, pbest_{2i},…, pbest_{ni})$ and $gbest_i$ is the best solution found in the whole swarm. In standard PSO, the position and velocity of particles are updated by the following equations:

$$V_i^{(t+1)} = w \times V_i^{(t)} + c_1 \times rand() \times \left(pbest_i - x_i^{(t)} \right) + c_2 \times rand() \times \left(gbest_i - x_i^{(t)} \right) \tag{7}$$

$$x_i^{(t+1)} = x_i^t + v_i^{(t+1)} \tag{8}$$

where:

x_i^t and v_i^t represent the position and velocity of individual i at generation t;

w is the inertia weight parameter that controls the momentum of particles;

c_1 and c_2 are positive constants, which balance the need for local and global search;

rand() is a random number between 0 and 1.

3.2. Quantum-Behaved Particle Swarm Optimization

In the conventional PSO, a particle moves in the search space by the moments of its position and velocity. In the quantum model of a PSO, the state of a particle is depicted by wave function $\Psi(x,t)$ [8], instead of position and velocity. QPSO introduces the mean best position into the algorithm and uses a strategy based on a quantum delta potential well model to sample around the previous best points Furthermore, QPSO has only one parameter, which is easier to control than PSO algorithm. Employing the Monte Carlo method, particles are updated according to the following equations:

$$\begin{cases} x_{ij}(t+1) = p_{ij}(t) + \beta * \left| Mbest_{ij}(t) - x_{ij}(t) \right| * In(1/u), & \text{if } k \geq 0.5 \\ x_{ij}(t+1) = p_{ij}(t) - \beta * \left| Mbest_{ij}(t) - x_{ij}(t) \right| * In(1/u), & \text{if } k < 0.5 \end{cases} \tag{9}$$

The following gives the explication of the update Equation (9):

(1) x_{ij} $(t + 1)$ is denoted as the position of the jth dimension of the ith particle for the next generation $t + 1$.

(2) P_{ij} (t) is the local attractor to make sure SQPSO can converge, which is defined as follows:

$$p_{ij} = \phi * Pbest_{ij} + (1 - \phi) * gbest_j \tag{10}$$

where ϕ is a random number uniformly distributed in (0,1); $Mbest_{ij}$ is a global point, which can be calculated by the mean of the *Pbest* of all particles in the population. The definition is given is as follows:

$$Mbest_{ij}(t) = \left(\frac{1}{N}\sum_{i=1}^{N} Pbest_{i1}(t-1), \frac{1}{N}\sum_{i=1}^{N} Pbest_{i2}(t-1), \ldots, \frac{1}{N}\sum_{i=1}^{N} Pbest_{in}(t-1)\right) \tag{11}$$

where N represents the population size and $Pbest_i$ is the best position of the ith particle.

(3) In this paper, β is called the constriction-expansion coefficient, and it is linearly decreasing when the iteration increases:

$$\beta^t = \beta_{\max} - \frac{\beta_{\max} - \beta_{\min}}{itNum} * t \tag{12}$$

where *itNum* is the maximum iteration number, t is the current iteration number $\beta_{\max} = 1.0$ and $\beta_{\min} = 0.5$.

(4) u and k are two random numbers uniformly distributed in (0,1).

3.3. The Proposed Quantum-Behaved Particle Swarm Optimization

In the original QPSO, the local attractor is calculated by Equation (10), which means that the $P_{ij}(t)$ is a random position between the individual best position and the group best position. However, the drawback is the difficulty in maintaining the best information of the swarm, especially when the optimal solution is at the boundary of the problem. In [18], Jong-Bae Park proposed an improved PSO, which introduced a kind of crossover operation. In this operation, particles update the position with the exchange information of previous generation particle position and the individual best position of itself. In this paper, a modified QPSO is proposed, called SQPSO, which introduces a selective probability operator into the update mechanism when calculating the local attractor $P_{ij}(t)$. In SQPSO, the information of global best position and the whole swarm's individual best position are used to update the position for the next generation. The reason behind the inclusion of the selective probability operator is to enable the use of recombination operator into the original QPSO which will help to maintain the best solution and, at the same time, exchange information between individuals in the whole swarm. The pseudo code for the proposed selective probability operator is given in Figure 2.

Figure 2. The pseudo code for the proposed crossover operator of SQPSO.

```
For i=1:PopNum
  For j=1:Dim
     IF rand ≤ SP
        RandPop=floor(rand*popNum)+1;
        P(i,j)=pbest(RandPop,j);
     Else
        P(i,j)=gbest(1,j);
   End
End
```

In Figure 2, *PopNum* is the number of population and *Dim* is the Dimensionality for each individual. *RandPop* is an individual randomly selected from the swarm. *SP* is the selective probability, which can control whether the local attractor $P(i,j)$ is generated from individual best position or global best position. If $rand \le SP$, then the local attractor $P(i,j)$ will select its value from the *Pbest* of the individual *RandPop* and if $rand > SP$, then the value of $P(i,j)$ will select the point of global best position. Using the *SP*, $P(i,j)$ can not only make use of the previous best swarm information but also increase the population diversity and consequently enhance the global search ability. The principle of the modification is illustrated in Figure 3 and the procedure of the proposed SQPSO is described as follows:

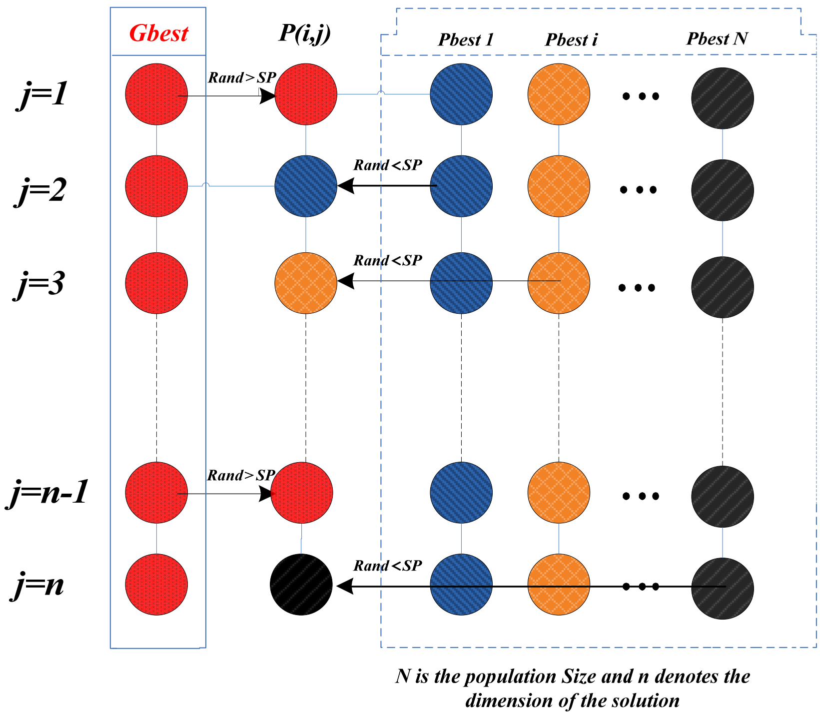
(1) *Initialize the population*, which are generated randomly within the minimum and maximum output of each generator, using the following equations:

$$\text{population} = \begin{bmatrix} X_1 \\ X_2 \\ \cdots \\ X_n \end{bmatrix} \tag{13}$$

$$X_i = [x_{i,1}, x_{i,2}, \ldots, x_{i,n}], \quad x_{ij} = P_{ij}^{\min} + rand * (P_{ij}^{\max} - P_{ij}^{\min})$$

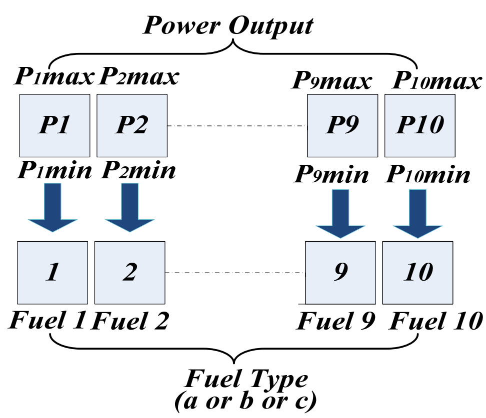
where X_i is the *ith* individual of the population,(x_{ij} is the *jth* data vector of *ith* individual; P_{ij}^{min} and P_{ij}^{max} are the maximum and minimum output limit values of the *jth* control variable.

Figure 3. Principle of the modified of SQPSO.



For the multi-fuel ED problem, the relationship between unit output and fuel type is shown in Figure 4, taking a 10-generator problem as an example, each unit has its minimum and maximum output of generation and the sum of the whole power output should satisfy the total output demand, and as shown in Figure 4, different range of unit output corresponds to different type of fuel.

Figure 4. Relationship between unit output and fuel type.



(2) *Constraint handling for real power balance.* Since the individuals of the population are created randomly and with the evolution of particles, newly generated individual may violate the constraints. Therefore, it is important to keep all the individual variables within their feasible ranges. Hence, the following procedure is adopted by the SQPSO to modify the value of new generated variables to satisfy the power balance constraint.

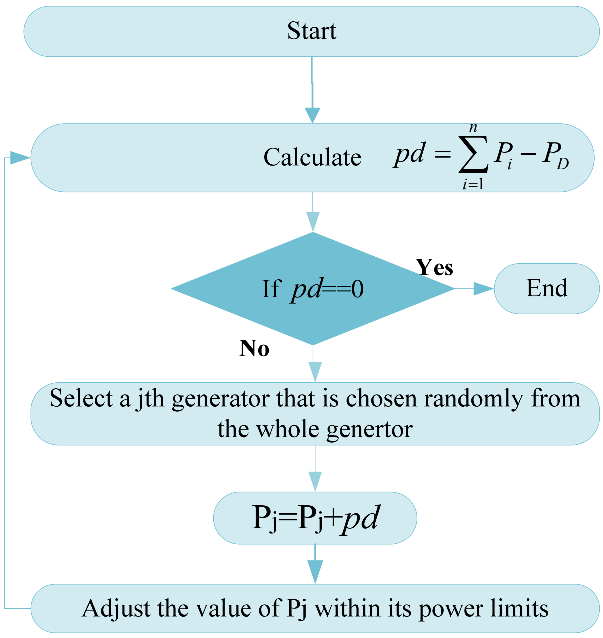
$$x_{ij} = \begin{cases} P_{ij}^{\min} & \text{if } x_{ij} \le P_{ij}^{\min} \\ P_{ij}^{\max} & \text{if } x_{ij} > P_{ij}^{\min} \\ x_{ij} & \text{otherwise} \end{cases} \tag{14}$$

The amount of power balance violation is calculated by:

$$pd = \sum_{i=1}^{n} P_i - P_D \tag{15}$$

if $pd = 0$, go to step 3; if $pd \neq 0$, the value of pd will be adjusted by allocating it to the output of a unit, which is chosen randomly from the whole set of generating units, so that the generating constraints can be satisfied. If the output of the chosen unit goes outside the feasible boundaries, its value should be modified using Equation (14). The constraints handling procedure is illustrated in Figure 5.

Figure 5. Procedure of constraint handling of the SQPSO algorithm.



(3) *Parameter setting*. There are two parameters in SQPSO, one is the constriction-expansion coefficient which decreases from 1.0 to 0.5 linearly. Another parameter is the introduced selective probability (*SP*). In this paper, the SP for SQPSO increases from 0.5 to 0.8 linearly using the following equation:

$$SP^t = SP_{\max} - \frac{SP_{\max} - SP_{\min}}{itNum} * t \tag{16}$$

where SP^t is the value of SP at iteration t. SP_{max} and SP_{min} are maximum and minimum selective probability. At the early stage, the population will select more vectors from the group best position, which can accelerate the convergence speed. As the iteration number increases, the population will draw more vectors from the individual best positions to enhance the diversity of the whole swarm.

(4) *Evaluate the objective function value of each particle.*

(5) *Update pbest.* Compare each particle's objective function value with its *pbest*. If the current value is better than the *pbest* value, set the *pbest* value to the current value.
(6) *Update gbest*. Determine best *gbest* of the swarm as the minimum *pbest* of all particles.
(7) *Calculate the Mbest*, *constriction-expansion coefficient β* according to Equation (11) and Equation (12), respectively.
(8) *Calculate the local attractor according to the Selective probability operator proposed in this paper.*
(9) *Update the particle's position using Equation* (*9*)
(10) *Check if the stop criterion satisfied?*
(11) *If not*, *then go to step 2.*
(12) *Else*, *the searching process is stopped.*

4. Experimental Results

4.1. Benchmark Functions

To verify the performance of the proposed SQPSO, five benchmark functions (Sphere, Jason, Griewank, Rosenbrock and Rastrigrin) listed in Table 1 are conducted. These functions are all minimization problems with the minimum value to be zero. The results produced by the proposed SQPSO are compared with that of the EGA, DPSO, HPSO, IPSO and IQPSO in [17]. EGA is a modified genetic algorithm with elitism and adaptive mutation probability control, and DPSO, HPSO, IPSO are three types of revised version of PSO. IQPSO is an improved quantum-inspired particle swarm optimization, which is based on the principle of quantum rotation gates. Additionally, three algorithms are also used in this paper for comparison, which are PSO, DE and QPSO. For PSO, the acceleration coefficients c1 and c2 are set to 2, and the inertia weight decreased from 0.9 to 0.4 linearly [19]. The parameter of DE is set to F = 0.4, CR = 0.8 [20].

Table 1. Benchmark functions.

Name	Function	Dim	Range	Opt
Sphere	$f_1(x)=\sum_{i=1}^{n} x_i^2$	40	[−100,100]	0
Jason	$f_2(x)=\sum_{i=1}^{n}(x_i-i)^2$	40	[−100,100]	0
Griewank	$f_3(x)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2-\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})+1$	40	[−600,600]	0
Rosenbrock	$f_4(x)=\sum_{i=1}^{n}[100(x_{i+1}-x_i^2)^2+(x_i-1)^2]$	40	[−2.048,2.048]	0
Rastrigrin	$f_5(x)=\sum_{i=1}^{n}[x_i^2-10\cos(2\pi x_i)+10]$	40	[−5.12,5.12]	0

For QPSO and SQPSO, the coefficient β decreases from 1.0 to 0.5 linearly and the selective probability (SP) for SQPSO increases from 0.5 to 0.8 linearly. To compare the solution quality and convergence characteristics, 50 independent trial runs are performed for each benchmark function and mean function value and best function value are recorded. In order to make a fair comparison, the population size is set to 80 and population dimension is 40 for all the five benchmark functions. The maximum iteration number is set to 5000. All the algorithms are implemented in MATLAB 2008a and executed on an Intel Core2 Duo 1.66 GHz personal computer.

The numerical results in Table 2 show that the proposed SQPSO can achieve satisfactory performance. Specifically, both the sphere and Jason function have only one single optimal solution, so it is usually introduced to test the local search ability of the algorithm. From the results, it can be seen that the SQPSO outperforms all the other algorithms in terms of mean function value and best function value, which indicates SQPSO has strong local search ability. Rosenbrock is a mono-modal function and its optimal solution lies in a narrow area. The experimental results on Rosenbrock show that the mean function value of SQPSO is better than DPSO, HPSO, IPSO, PSO and QPSO. However, the best function value is inferior to other algorithms reported in [21]. Griewank and Rastrigrin are both multi-modal and they are usually used to compare the global search ability of the algorithm. As to Griewank, SQPSO can hit the minimum value zero and the mean function value is superior to other algorithms too. For Rastrigrin, both EGA and IQPSO give a better performance than SQPSO and the results of SQPSO are better than other methods.

Table 2. Mean value and best value for five benchmark functions with different approaches.

Function / Algorithm	f_1 (Sphere) Mean (Best)	f_2 (Jason) Mean (Best)	f_3 (Griewank) Mean (Best)	f_4 (Rosenbrock) Mean (Best)	f_5 (Rastrigrin) Mean (Best)
EGA [11]	2.743×10^{-10} (0)	8.865×10^{-8} (3.748×10^{-22})	1.042×10^{-4} (7.952×10^{-13})	0.84 (6.537×10^{-4})	2.257 (6.537×10^{-4})
DPSO [11]	5.403×10^{-7} (4.532×10^{-14})	2.595×10^{-6} (1.173×10^{-12})	1.322×10^{-3} (2.167×10^{-10})	28.094 (1.150×10^{-2})	28.826 (19.899)
HPSO [11]	1.319×10^{-6} (2.824×10^{-10})	6.735×10^{-3} (1.503×10^{-10})	2.546×10^{-3} (5.136×10^{-9})	28.995 (2.346×10^{-2})	29.956 (15.393)
IPSO [11]	1.524×10^{-7} (3.406×10^{-11})	1.350×10^{-5} (2.107×10^{-10})	2.224×10^{-3} (1.454×10^{-10})	27.13 (2.339×10^{-2})	31.906 (15.064)
IQPSO [11]	1.085×10^{-23} (0)	2.078×10^{-23} (0)	3.221×10^{-7} (0)	2.19×10^{-2} (2.717×10^{-9})	0.521 (1.075×10^{-4})
PSO	2.885×10^{-21} (1.774×10^{-23})	1.4526×10^{-21} (4.413×10^{-24})	8.0215×10^{-3} (0)	56.1057 (12.4904)	34.6046 (20.8941)
DE	1.3727×10^{-47} (3.9244×10^{-49})	1.0097×10^{-30} (0)	3.4506×10^{-4} (0)	12.4830 (6.6779)	56.7802 (14.9244)
QPSO	5.054×10^{-26} (7.333×10^{-31})	5.3011×10^{-30} (0)	8.1 (0)	48.4957 (25.2717)	25.7895 (13.9294)
SQPSO	$\mathbf{6.5759 \times 10^{-74}}$ ($\mathbf{1.8122 \times 10^{-89}}$)	**(0)** **(0)**	$\mathbf{2.217 \times 10^{-7}}$ **(0)**	**32.68016** **(14.7115)**	**13.7105** **(3.9798)**

In addition, compared with original QPSO without selective probability operator, the proposed SQPSO demonstrates good performance for all the five benchmark functions in terms of both the mean function value and best function value, which indicates that the SQPSO is an effective modification of QPSO.

4.2. ED Problem with Valve-Point Effects

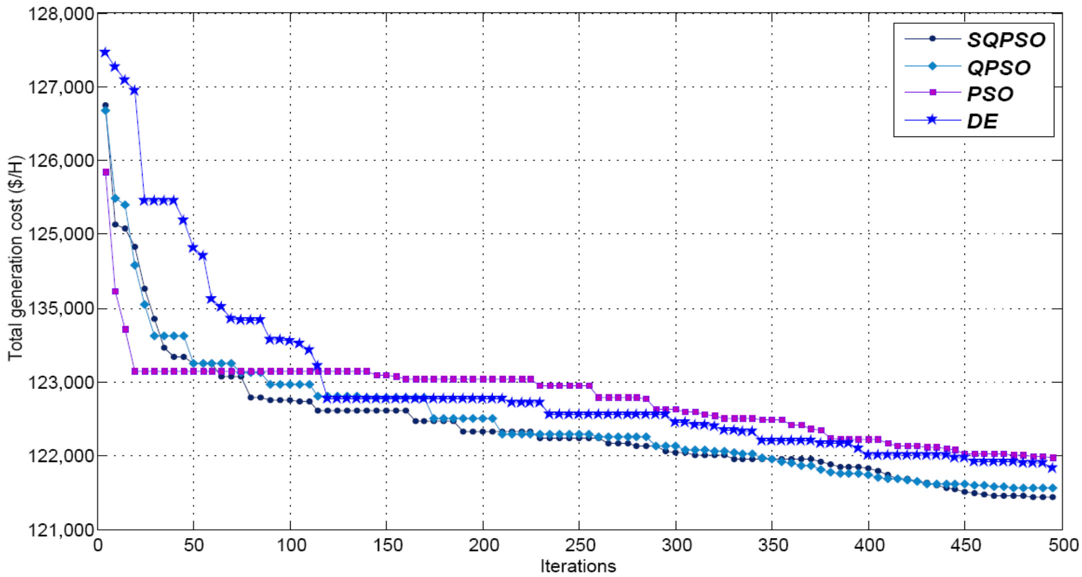
A large-scale power system of 40-generating units with quadratic cost function and valve-point effects is being considered here. Transmission losses are ignored and the total load demand of this text system is 10,500 MW. The system data can be found from [1]. One hundred independent runs are made for each method and population size is set to 80. The stopping criterion is set to 500. The result obtained from SQPSO is compared with some methods in the literature including IFEP [1], GA_PS_SQP [22], PC-PSO [23], SOH_PSO [23], NPSO [24] ,NPSO_LRS [24], PSO-GM [25], CBPSO_RVM [25], ICA-PSO [26], ACO [5], APSO(2) [27], HDE [28], ST-HDE [28] and IQPSO [29]. In addition, in order to compare the performance of the crossover operation in [18] with the proposed selective probability operator. The crossover operation [18] is introduced into QPSO, namely CQPSO, and the performance of CQPSO can be seen in the following results. The comparison results of SQPSO with other methods reported in literature are given in Table 3. The best solution of the SQPSO is 121,434.41 $/H, which is comparatively superior to most of the methods and the mean cost is better than other methods as well.

Table 3. Comparison results for ED problem with valve-point effects (40-unit system).

Methods	Generation cost($/H)			Standard Deviation
	Minimum	Mean	Maximum	
IFEP [1]	122,624.35	123,382	125,740.63	NR
GA-PS-SQP [22]	121,458.14	122,039	NR	NR
PC-PSO [23]	121,767.90	122,461.30	122,867.55	NR
SOH-PSO [23]	121,501.14	121,853.57	122,446.3	NR
NPSO [24]	121,704.74	122,221.37	122,995.10	NR
NPSO-LRS [24]	121,664.43	122,209.32	122,981.59	NR
PSO-GM [25]	121,845.98	122,398.38	123,219.22	258.44
CBPSO-RVM [25]	121,555.32	122,281.14	123,094.98	259.99
ICA-PSO [26]	121,422.17	121,428.14	121,453.56	NR
ACO [5]	121,532.41	121,606.45	121,679.64	45.58
APSO(2) [27]	121,663.52	122,153.67	122,912.40	NR
HDE [28]	121,813.26	122,705.66	NR	NR
ST-HDE [28]	121,698.51	122,304.30	NR	NR
IQPSO [21]	121,448.21	122,225.07	NR	NR
FCASO [30]	121,516.47	122,082.59	NR	NR
CASO [30]	121,865.63	122,100.74	NR	NR
CPSO-SQP [31]	121,458.54	122,028.16	NR	NR
CPSO [31]	121,865.23	122,100.87	NR	NR
DE	121,805.56	122,142.97	122,466.75	151.88
PSO	121,956.18	122,459.36	122,785.73	209.12
QPSO	121,487.27	121,750.48	121,991.99	111.68
CQPSO	121,463.39	121,732.98	121,778.74	79.38
SQPSO	**121,434.41**	**121,723.22**	**121,881.51**	**104.29**

The convergence characteristics of the SQPSO in comparison with PSO, DE, QPSO are shown in Figure 6. It is shown that PSO converges fastest among these methods while it suffers the premature convergence. Besides, DE is the slowest among the four methods, as DE involves a series of mutation, crossover and greedy selection operators, which leads to low convergence speed and increases the computational time as well. QPSO and SQPSO converge at nearly the same speed, however the SQPSO can produce a better solution as iteration increases, which indicates stronger searching ability. In addition, compared with CQPSO, SQPSO can outperform it almost in all aspects, which indicates that the proposed elective probability operator is improved compared with the crossover operation in [18].

Figure 6. Convergence characteristics for total generation costs (40-uint system).



The distribution of generation costs of the four algorithms for 100 runs is shown in Figure 7 which reflects the robustness of each algorithm. The curve of the SQPSO is at the bottom of the figure and stabilizes at a relatively intensive region, which means the distribution of the solution of SQPSO is much better than other methods. The detailed results of the best solution of DE, PSO, QPSO and SQPSO, for ED problem with valve-point effects are given in Table 4.

Figure 7. Distribution of generation costs of the four algorithms for 100 runs (40-unit system).

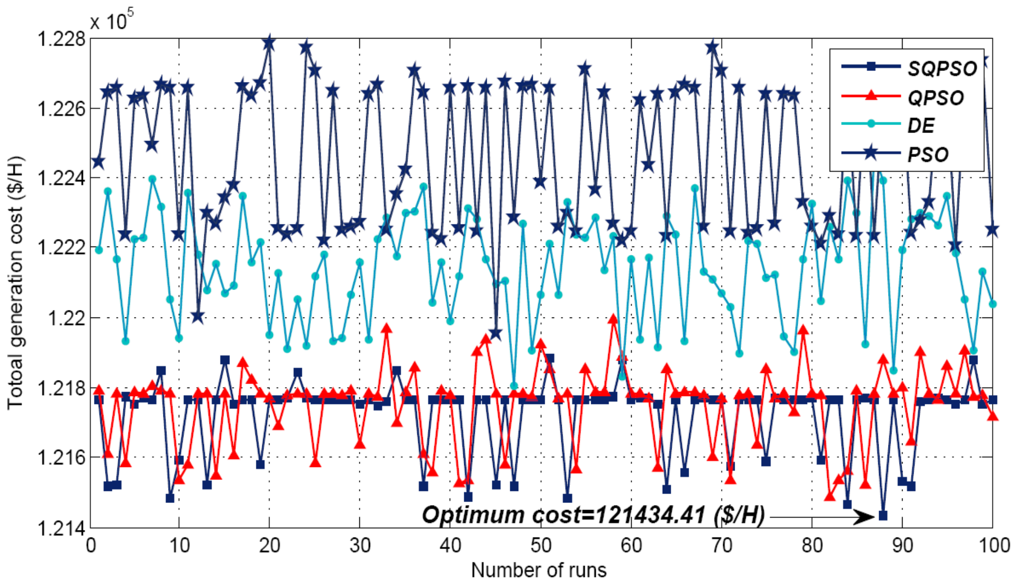


Table 4. Detailed results of the best solution of DE, PSO, QPSO and SQPSO, for ED problem with valve-point effects (40-unit system).

Unit	Methods				
	DE	**PSO**	**QPSO**	**CQPSO**	**SQPSO**
P_1	111.8012	113.9945	113.6426	113.9999	110.9173
P_2	111.5734	110.9343	111.9581	113.9999	111.7807
P_3	95.79661	100.748	97.56082	120.0000	97.56128
P_4	182.4958	179.1588	179.7457	179.7333	179.7005
P_5	87.27856	97.0000	88.53738	96.9999	93.37496
P_6	140.0000	140.0000	139.9981	140.0000	139.9862
P_7	300.0000	300.0000	299.989	300.0000	259.8548
P_8	285.2077	300.0000	284.9879	299.9999	284.9466
P_9	286.9856	299.9040	284.7968	293.3932	284.5976
P_{10}	130.0000	130.0000	130.0093	130.0000	130.0493
P_{11}	94.25143	94.0000	94.02522	94.0000	168.807
P_{12}	94.61699	94.0000	94.0286	94.0000	94.00315
P_{13}	125.7718	125.0000	125.0323	125.0000	214.7713
P_{14}	393.1819	393.9392	394.2728	394.2794	394.2986
P_{15}	395.1001	394.1116	394.2987	394.2794	304.61
P_{16}	393.7253	304.3765	394.3071	304.5196	394.2632
P_{17}	487.6391	500.0000	489.3179	489.2794	489.363
P_{18}	491.819	490.6004	489.2953	489.2795	489.5688
P_{19}	512.8806	513.8928	511.3082	511.2794	511.2797
P_{20}	511.7995	514.1406	511.3473	511.2794	511.3193
P_{21}	524.2502	524.3505	523.3044	523.2796	523.2616
P_{22}	523.9075	523.4735	523.3182	523.2796	523.3642
P_{23}	519.8336	529.2841	523.3638	523.2796	523.2587
P_{24}	527.6248	547.3133	523.3677	550.0000	523.3996
P_{25}	523.9776	522.9096	523.2928	523.2795	523.2836
P_{26}	523.2693	524.9206	523.3083	523.2798	523.2817
P_{27}	10.3912	10.0000	10.01133	10.0000	10.00975
P_{28}	10.0000	10.0000	10.08587	10.0000	10.0344
P_{29}	10.0335	10.0000	10.00228	10.0000	10.00645
P_{30}	92.73803	91.53567	90.21066	96.9999	88.52085
P_{31}	187.1519	190.0000	189.9984	190.0000	189.9972
P_{32}	189.9415	190.0000	189.9968	190.0000	189.9834
P_{33}	189.4094	190.0000	189.9988	190.0000	189.9822
P_{34}	197.3705	199.9374	199.9794	199.9999	165.321
P_{35}	199.2062	198.4492	199.9942	200.0000	199.9666
P_{36}	198.9157	200.0000	199.9942	200.0000	200.0000
P_{37}	109.5043	110.0000	110.0000	110.0000	110.0000
P_{38}	110.0000	110.0000	109.9926	110.0000	109.9984
P_{39}	108.1849	110.0000	109.9915	110.0000	109.992
P_{40}	512.3655	512.0254	511.3299	511.2794	511.2849
Total Demand	10,500	10,500	10,500	10,500	10,500
Total Cost	121,805.5647	121,956.1827	121,487.2762	121,463.3942	121,434.4071

4.3. The ED Problem with Multi-Fuel Option and Valve-Point Effects

In this section, the proposed SQPSO is applied to multi-fuel economic dispatch problem with valve-point effects. Transmission losses are ignored and system date can be found in [29]. The experimental results are also compared with other algorithms reported in literature, including CGA_MU [2], IGA_MU [2], ACO [5], ED-DE [32], ARCGA [33], PSO-GM [25], NPSO [24], NPSO-LRS [24], PSO-GM [25], CBPSO-RVM [25], APSO [27], GA [34], DSPSO–TSA [34], which are given in Table 5.

Table 5. Comparison of calculation results for multiple fuel ED problems with total demand of 2700 (MW).

Methods	**Generation cost ($/H)**			**Standard**	**Average**
	Minimum	**Mean**	**Maximum**	**Deviation**	**CPU times**
CGA_MU [2]	624.7193	627.6087	633.8652	NR	26.64
IGA_MU [2]	624.5178	625.8692	630.8705	NR	7.32
ACO [5]	623.9000	624.3500	624.7800	NR	8.35
ED-DE [32]	623.8290	623.8807	623.8894	NR	NR
ARCGA [33]	623.8281	623.8495	623.8814	NR	NR
NPSO [24]	624.1624	625.2180	627.4237	NR	NR
NPSO-LRS [24]	624.1273	624.9985	626.9981	NR	NR
PSO-GM [25]	624.3050	624.6749	625.0854	0.1580	NR
CBPSO-RVM [25]	623.9588	624.0816	624.2930	0.0576	NR
APSO [27]	624.0145	624.8185	627.3049	NR	0.52
GA [34]	624.5050	624.7419	624.8169	0.1005	18.3
TSA [34]	624.3078	635.0623	624.8285	1.1593	9.71
DSPSO–TSA [34]	623.8375	623.8625	623.9001	0.0106	3.44
DE	623.9280	624.0068	624.0653	0.0271	0.625
PSO	624.0120	624.2055	624.4376	0.0889	0.308
QPSO	623.8766	623.9639	624.4163	0.0688	0.315
CQPSO	623.8476	623.8652	623.8885	0.0151	0.318
SQPSO	**623.8319**	**623.8440**	**623.8605**	0.0107	0.324

It can be seen that SQPSO can get a minimum generation cost of 623.8319($/H), which is the best solution among all the methods. For the mean cost, SQPSO outperforms most of the methods expect for the ARCGA, which is slightly better than SQPSO, however the CUP times of ARCGA is almost three times that of SQPSO. When considering the average CPU time, the computational time for PSO, QPSO and SQPSO are at the same level, while the results of SQPSO is better than the other two methods. The detailed results of the best solution of DE, PSO, QPSO, CQPSO and SQPSO, for the multiple fuel ED problem with total demand of 2700 MW is given in Table 6.

Table 6. Detailed results of the best solution of DE, PSO, QPSO and SQPSO, for multiple fuel ED problem with total demand of 2700 MW.

Unit	CQPSO		DE		PSO		QPSO		SQPSO	
	Output (MW)	Fuel type	Output (MW)	Fuel type	Output (MW)	Fuel type	Output (MW)	Fuel type	Output (MW)	Fuel type
P_1	217.567	2	220.8058	2	220.8058	2	218.587	2	218.5939	2
P_2	211.7117	1	211.7154	1	211.7154	1	210.4723	1	211.2166	1
P_3	279.6489	1	280.7032	1	280.7032	1	280.7087	1	281.6653	1
P_4	240.5800	3	239.7713	3	239.7713	3	239.3708	3	238.9676	3
P_5	276.3749	1	277.2203	1	277.2203	1	279.6347	1	279.9345	1
P_6	239.6394	3	238.9671	3	238.9671	3	240.7144	3	239.2363	3
P_7	290.0985	1	289.0121	1	289.0121	1	290.1244	1	287.7275	1
P_8	240.8488	3	240.175	3	240.175	3	239.6396	3	239.6394	3
P_9	427.6622	3	425.4145	3	425.4145	3	423.8487	3	427.1502	3
P_{10}	275.8686	1	276.2151	1	276.2151	1	276.8994	1	275.8686	1
Pd	2,700		2,700		2,700		2,700		2,700	
Total Cost	623.8476		623.928		624.012		623.8766		623.8319	

The convergence characteristics and the distribution of generation costs of the SQPSO in comparison with PSO, DE, QPSO are shown in Figures 8 and 9. Clearly, SQPSO converges to the optimal solution faster than other three methods. It can reach the optimal region only in a few iterations, which shows powerful global search ability.

Figure 8. Convergence characteristics for total generation costs (multiple fuel options system).

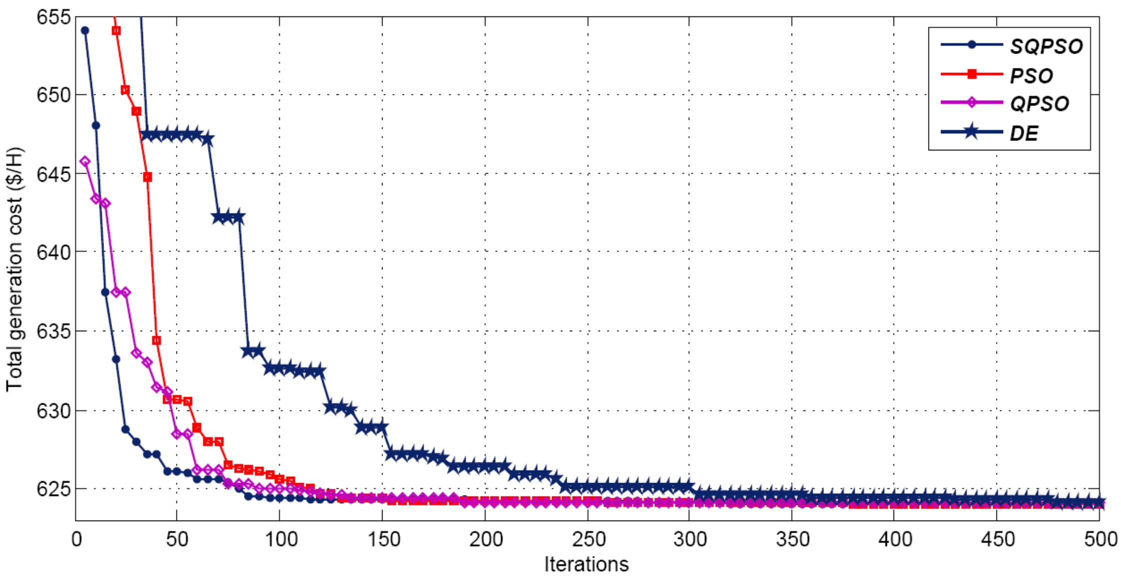
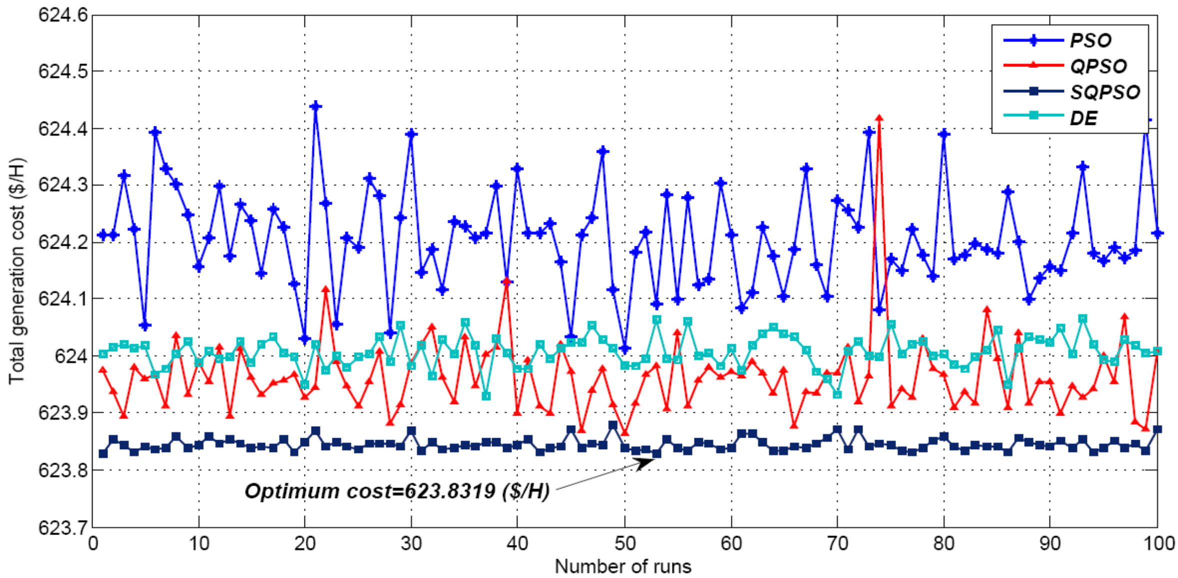


Figure 9. Distribution of generation costs for 100 runs (multiple fuel options system).



The results of different methods for the multiple fuel ED problems with total demand range from 2400 to 2600 MW are summarized in Table 7. It again shows that the SQPSO outperforms all the other methods.

Table 7. Comparison of calculation results for multiple fuel ED problem with total demand range from 2400–2600 MW.

Demand	Method	Generation cost ($/H)			Standard	Average
		Minimum	Mean	Maximum	Deviation	CPU times
2400	DE	481.9030	481.9527	482.0231	0.0285	0.4781
	PSO	482.0807	484.1717	491.5540	2.5598	0.3625
	QPSO	481.9235	483.4540	492.6059	2.2612	0.3562
	CQPSO	481.7469	481.7711	481.7974	0.0180	0.3683
	SQPSO	**481.7320**	**481.7440**	**481.7591**	**0.0068**	**0.3390**
2500	DE	526.4154	526.4771	526.5379	0.0244	0.5156
	PSO	526.4849	527.5594	535.1762	1.3761	0.3578
	QPSO	526.3758	527.5720	534.9611	1.4797	0.3328
	CQPSO	526.2537	526.2839	526.3229	0.0187	0.3453
	SQPSO	**526.2447**	**526.2556**	**526.2897**	**0.0079**	**0.3500**
2600	DE	574.5489	574.6371	574.9653	0.0916	0.5984
	PSO	574.6194	576.0185	589.1900	2.5451	0.3515
	QPSO	574.5857	575.7198	589.1281	2.0467	0.3315
	CQPSO	574.4492	574.6538	574.7928	0.1439	0.3576
	SQPSO	**574.3866**	**574.5076**	**574.7659**	**0.1640**	**0.3484**

5. Conclusions

An improved quantum-behaved particle swarm optimization called SQPSO is proposed in this paper, which introduces selective probability operator into the basic QPSO. The proposed SQPSO has been tested on five classic benchmark functions, as well as two ED problems with valve-point effects and multiple fuel options. It shows superior optimization performance in terms of the convergence rate and the robustness, compared with DE, PSO, CQPSO and QPSO. Additionally, SQPSO also shows competitive ability over other algorithms from the literature.

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