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Equilibrium between Environmental and Economic Objectives: An Activity-Based Costing Approach Application for Carbon Emissions Management in the Aluminum Alloy Wheel Industry

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Abstract: In the face of the increasingly dire threat of global climate change, reducing carbon emissions has become an urgent priority for governments and corporations worldwide. The aluminum alloy wheel manufacturing industry bears an even heavier burden for emission mitigation due to its high production volume, complex processes, and proportionally higher carbon footprint. With impending carbon taxes and trading policies looming, the industry urgently needs to strike a balance between maximizing profits and minimizing carbon emissions. Leveraging real-world industry data, this research develops four green Activity-Based Costing (ABC) models and utilizes optimization software to compare the following scenarios: non-continuous carbon tax, carbon tax with trading, tiered tax with exemptions, and exemptions combined with trading. Results demonstrate that integrating carbon trading and targeted tax reductions can improve corporate financial positions without severely compromising environmental goals. Although identifying optimal balance points remains a highly complex process, this study equips enterprises and policymakers with quantitative tools to navigate fluctuating carbon regulatory environments. As national policies progress, more multifaceted dynamic carbon tax models will likely provide more profound insights for sustainable development.

Keywords: activity-based costing (ABC); carbon tax; carbon right; carbon pricing; mathematical programming

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1. Introduction

Metz et al. [1] highlighted the severe implications of unchecked greenhouse gas emissions on climate change, stressing the critical need for immediate action towards reducing carbon emissions. Chen and He [2] advocated for the "cap and trade" system as a leading mechanism for mitigating carbon emissions. Following the European Union's initiation of a carbon trading scheme in 2005, This approach has become increasingly popular across different regions worldwide, such as Asia, South America, and North America. [2]. The EU's carbon emissions trading system requires companies to adapt their operational and strategic approaches to comply with carbon limits and the opportunity for trading [3], urging them to incorporate the costs and opportunities of carbon emissions trading into their strategic planning to enhance production efficiency and profit maximization [4,5]. With the presence of more than 20 carbon trading markets worldwide, predictions suggest the carbon market could reach a valuation of $2 trillion by 2025 [6], underlining a global dedication to diminishing greenhouse gas emissions, promoting environmental equity, and fulfilling a communal obligation towards protecting the environment and encouraging sustainable growth. In accordance with the goals outlined in the United Nations Framework Convention on Climate Change and its subsequent agreements, Taiwan implemented the "Greenhouse Gas Emission Reduction and Management Law" on 1 July 2015.

Additionally, in May 2021, DBS Bank, Standard Chartered Bank, Singapore Exchange, and Singapore's sovereign wealth fund Temasek unveiled the launch of Climate Impact X (CIX). This international carbon marketplace, based in Singapore, is focused on enabling the exchange of premium carbon credits and is specifically designed for this purpose [7]. CIX aims to utilize satellite surveillance, artificial intelligence, and blockchain technologies to increase the transparency, reliability, and effectiveness of carbon credits, aiming to make a substantial environmental contribution. The proponents of CIX argue that premium carbon credits are essential in closing the gap within climate mitigation efforts [7,8]. Traditionally, the relatively inexpensive cost of carbon emissions led numerous companies to prefer paying these expenses rather than investing in improvements to their production processes. However, recent significant changes in global climate conditions, along with rising carbon taxes, escalating carbon trading prices, and stricter emissions allowances, are transforming the economic landscapes for businesses [9]. This situation compels corporations to consider the advantages of retaining their existing production techniques versus the upfront costs associated with upgrading their production technologies to reduce carbon emissions, which could fundamentally change their profitability models, marking a crucial decision-making point for managers. The focus of this article is the escalating environmental concerns, such as air, water, and heavy metal pollution, exacerbated by human advancement and technology. Global warming, driven by CO_2 emissions, is causing alarming effects like the melting of Arctic glaciers, the depletion of the ozone layer, and significant climate shifts, posing direct threats to human existence. In response, governments and businesses worldwide are exploring viable solutions for environmental preservation. Highlighting the importance of industry's role, as per the 2018 Greenhouse Gas Emissions report published by the Executive Yuan's Environmental Protection Agency [10], the industrial processes sector is recognized as the second-largest contributor to emissions, following the sector of overall energy usage. In this context, this document examines a cost planning model for producing aluminum rims within the industry as a representative example. The article highlights the significant influence of the 2015 Paris Agreement and the United Nations Framework Convention on Climate Change (UNFCCC) as its legal underpinnings [11], which succeeded the Kyoto Protocol. This pact is designed to keep the increase in global average temperature under 2 degrees Celsius compared to pre-industrial times, with a more ambitious target of not exceeding 1.5 degrees. Discussions regarding how to put this into action are currently taking place in Poland. Economists suggest market mechanisms like carbon taxes and trading, like the European Union Allowance (EUA), to manage carbon emissions. This approach necessitates a synergistic effort between governments and industries to mitigate environmental harm.

Incorporating the goal of achieving net-zero carbon emissions by 2050 into the existing framework of production planning and control for the aluminum wheel sector necessitates a nuanced approach that seamlessly integrates with the objectives of Industry 4.0. This endeavor requires not only optimizing production to maximize profits and minimize costs but also embedding sustainable practices and carbon reduction strategies into the core of production cost models. Expanding upon Tsai et al.'s [12] foundational work, this involves refining the five models under Industry 4.0 to specifically address the implications of carbon taxes and trading mechanisms while also prioritizing the reduction of carbon emissions. Achieving net-zero carbon emissions by 2050 will demand a concerted effort to transition towards renewable energy sources, thereby reducing reliance on fossil fuels and diminishing the carbon footprint of manufacturing operations. Simultaneously, leveraging Industry 4.0 technologies such as the Internet of Things (IoT) and Artificial Intelligence (AI) can enhance energy management, making energy use more efficient and less environmentally detrimental. Implementing circular economy principles further supports this goal by minimizing waste and promoting the reuse and recycling of materials, reducing the demand for raw materials and the carbon emissions associated with their extraction and processing. This integrated approach ensures that the aluminum wheel sector not only advances in terms of technological innovation and economic efficiency but also moves towards sustainable manufacturing practices that are crucial for meeting the global challenge of net-zero carbon emissions by 2050.

Figure 1 illustrates the relationship between mathematical modeling and Industry 4.0 in the context of this study. The article is structured as follows: Section 2 explores global climate policies, the evolution of the green aluminum industry, and the application of green activity-based costing. Section 3 introduces the green production planning model under four ABC methodologies. Section 4 analyzes internal data from a sample company, forming the basis for the Section 3 models. Section 5 conducts single-period and multi-period sensitivity analyses, and the article concludes with a summary of the findings.

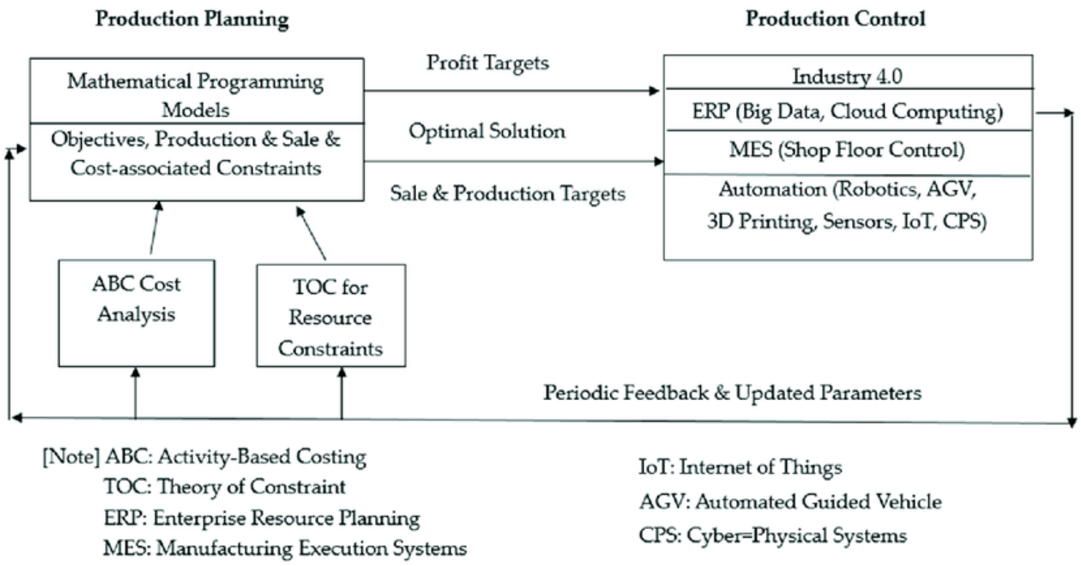


Figure 1. The relationship between mathematical modeling and Industry 4.0.

2. Background and Literature

2.1. Research Background

In light of the increasing urgency of global climate change issues, both national governments and international organizations are placing greater emphasis on carbon pricing mechanisms, including carbon taxes and emission trading schemes (ETS), as crucial strategies for reducing greenhouse gas emissions and fostering the transition to a sustainable economy. The rise in the European Union Emission Trading System (EU ETS) prices and its significant contribution to carbon tax revenue exemplifies this trend. According to the World Bank's "State and Trends of Carbon Pricing" report in 2023, despite challenges such as high inflation, fiscal stress, and energy crises, revenue from carbon taxes and ETS reached a historic high of approximately $95 billion. This achievement demonstrates that governments are prioritizing direct carbon pricing policies to reduce emissions even during economic downturns [13].

Furthermore, Sweden's carbon tax practice shows that while the tax has encouraged some businesses to reduce emissions, various exemptions and carve-outs mean that major polluters and exempt industries may not have significantly reduced their emissions. Consequently, this has prevented Sweden's carbon tax from realizing its full potential in reducing emissions. However, it is noteworthy that between 1990 and 2018, Sweden managed to reduce its greenhouse gas emissions by 27% [14].

Moreover, the International Carbon Price Floor (ICPF) proposed by the IMF highlights the potential of carbon pricing to significantly reduce global emissions, emphasizing that a unified approach could decrease emissions by up to 12.3% by 2030. This underscores the effectiveness of carbon pricing as a critical tool for global emissions reduction and supports the argument for broader implementation and coordination of carbon pricing mechanisms worldwide [15].

Additionally, the International Monetary Fund (IMF) further explores the technical choices and designs of carbon pricing mechanisms in its discussion "Carbon Taxes or Emission Trading Systems: Instrument Choice and Design". This includes administration, pricing levels, interaction with other mitigation tools, utilization of revenue to achieve efficiency and distribution goals, supportive measures for competitiveness concerns, expansion to broader emission sources, and global coordination. The report points out the significant practical advantages of carbon taxes, such as administrative ease, the certainty of price for investments, the potential to generate substantial revenue, and broader emission coverage, making them especially suitable for developing countries. However, ETS may be attractive due to its advantages in the political economy [16].

In conclusion, in response to escalating climate change concerns, the global emphasis on carbon pricing mechanisms like carbon taxes and ETS is growing, as seen in the EU ETS's revenue surge to $95 billion in 2023. Despite its potential, Sweden's carbon tax reveals limitations due to exemptions, though it achieved a 27% emissions reduction from 1990 to 2018. The IMF's International Carbon Price Floor suggests that a unified carbon pricing approach could cut global emissions by 12.3% by 2030, reinforcing carbon pricing's role in mitigating climate change. The IMF discussion also underscores carbon taxes' administrative simplicity and broad emission coverage, particularly benefiting developing countries while acknowledging ETS's political economy advantages.

2.2. Literature Review

The emergence of "green" as a predominant trend across industries signifies a shift towards products that are safer for both people and the planet. Emphasizing reusability, these green products are not only environmentally sustainable but also cost-effective. The evolution of the green enterprise concept redefines a company's raison d'être, encouraging a harmonious coexistence with the environment, as proposed by Hick [17]. Unlike the traditional corporate goals centered around maximizing economic profits or enhancing customer service highlighted by Shapiro [18], contemporary business strategies increasingly integrate environmental considerations as a core aspect.

Focusing on metal alloys, particularly in the context of nickel–aluminum bronze and manganese bronze, green alloys are identified for their environmentally friendly composition, characterized by extremely low or negligible lead content. Typically devoid of elements harmful to the environment or to human and animal health, these alloys are often crafted from recycled materials. The study by Das et al. [19] underscores the escalating use of aluminum alloys in transportation, highlighting the economic benefits derived from recycling aluminum-rich vehicles. They explore various aluminum recycling practices and systems, particularly for passenger cars, identifying potential components for recycling, evaluating the feasibility of direct recycling, and suggesting specific compositions for new recyclable alloys.

According to Continental Steel & Tube Company, high-quality green alloys possess attributes comparable to traditional alloys, including wear resistance, strength, machinability, and hardness, while being more eco-friendly. Complying with various industry standards, these alloys are particularly suited for applications where safety is paramount, as noted by Ascione [20]. The aluminum alloy industry is anticipated to increasingly utilize composite materials. In this realm, carbonized egg shells, as investigated in the study by Dwivedi et al. [21], are used as composite materials, offering improved mechanical properties, such as increased hardness, low density, high melting point, and superior abrasive performance compared to other materials like egg shells and calcium carbonate ($CaCO_3$) powder.

This study focuses on the sustainability of the aluminum life cycle, highlighting the challenges in tracking the metal's prolonged use, ownership transfers, and varying recycling processes, as noted by The Aluminum Association [22]. To improve sustainability, advancements in methods for quantifying material loss at the end of life are crucial.

This research employs a mathematical programming approach that combines Activity-Based Costing (ABC) with the Theory of Constraints (TOC) for a precise allocation of costs to distinct activities. This method provides a more detailed and accurate cost analysis than conventional techniques. ABC, which is supported by Kaplan's work, is used to enhance the allocation accuracy [23], and Malmi [24] helps estimate production and environmental costs by tracking resources used in manufacturing and assigning these costs to specific activities, as further explained by Cao et al. [25] and Lin et al. [26]. TOC, advocated by Kee and Schmidt [27] and Holmen [28], assists in optimal resource allocation and continuous system improvement.

The concept of green activity-based costing is applied across various industries, as seen in studies by Tsai and colleagues (2012–2019) in fields ranging from aviation to the pharmaceutical and textile industries. These studies employ diverse methodologies, including fuzzy methods and multi-criteria decision-making, to analyze data and present realistic industry profiles.

Traditional cost allocation methods in accounting often base the distribution of overhead costs on metrics tied to the volume of production, such as the number of hours spent on direct labor or machine usage. Yet, with the evolution of manufacturing technologies leading to more intricate processes, the share of overhead costs has increased significantly. This growth, along with the rising demand for tailored and varied products, has resulted in a surge of activities that do not directly correlate with production volume, including alterations in design and the replacement of molds, causing potential inaccuracies in the costing of products [29]. In reaction to these complexities, new costing approaches were formulated in the late 1960s and 1970s by researchers such as Cooper and Kaplan, tackling the challenge of indirect cost allocation, decision-making on product assortments, and the examination of cost drivers. By the 1980s, the introduction of mathematical models incorporating optimization principles marked a shift towards methodologies like Activity-Based Costing (ABC), Time-Driven Activity-Based Costing (TDABC), and the Theory of Constraints (TOC), among others. In 1988, Cooper and Kaplan unveiled the ABC approach, which implements a dual-step process for a more accurate assignment of costs to activities.

ABC methodology has been applied in various sectors, including aviation, construction, hospitality, logistics, and manufacturing [30–32]. It is also used in areas such as environmental management, quality improvement, outsourcing, software development, and project management [33,34]. This approach helps estimate costs associated with environmental initiatives, like carbon emissions management and pollution prevention, as defined by the United Nations Division for Sustainable Development [35]. ABC's detailed cost information supports managers in making informed decisions regarding sustainable management practices, such as pricing strategies, portfolio analysis, procurement, and outsourcing choices [36].

Goldatt and Cox [37] introduced the Theory of Constraints (TOC) in 1984, a concept explained by Radovilsky [38]. TOC is a management philosophy that addresses the efficient handling of limited resources through a series of steps aimed at identifying, leveraging, coordinating, enhancing, and re-evaluating constraints to gradually improve the output of a system. This method is especially beneficial for tackling bottlenecks in manufacturing, which can significantly influence the system's overall throughput.

Carbon emissions are segmented into three categories, encompassing direct emissions from production processes, indirect emissions from electricity consumption, and all other emissions from activities like waste management and material procurement [39]. This study highlights the importance of carbon emissions as a key limitation within production systems.

While ABC generally treats most production resources as variable expenses and serves as a tool for long-term analysis, it may not fully account for system constraints. On the contrary, TOC provides a short-term perspective by concentrating on the immediate distribution of limited resources, such as restrictions on raw material supplies and carbon emission quotas, which can affect ABC's cost estimations [40]. The Green Activity-Based Management (ABM) strategy expands on the principles of ABC, TOC, and the Critical Path Method (CPM) to include monitoring, measuring, modeling, and reporting environmental factors like carbon emissions. This comprehensive approach allows for the assessment of costs related to exceeding carbon emission limits and their effect on direct labor expenses and optimal product mix decisions [41].

In scenarios involving multiple products, linear programming (LP) methods have been utilized within cost-volume-profit (CVP) models to help identify the best product mix under certain production or sales limitations [42,43]. Furthermore, Fahimnia and Eshragh [44] developed a tactical supply chain planning model that merges economic goals with carbon emission objectives within a carbon taxation policy. Over the last two decades, numerous scholars have integrated environmental considerations into mathematical programming techniques to evaluate product portfolio choices [45–50]. (This study showcases the successful integration of ABC and TOC methodologies, which differ in their orientations—the former being more comprehensive and suited for long-term planning and the latter more focused and immediate. The ABC approach is flexible for assessing various decision-making scenarios at the executive level, including pricing policies and selections of product assortments [44,51,52], addressing environmental issues [52], and choosing strategies for green building projects [53]. The economic planning model incorporates TOC by using an integer linear programming method to create a diverse portfolio under capital constraints and applies TOC principles to determine the most advantageous product mix [40]. Employing the TOC approach to prioritize production processes proves to be versatile for a broad array of research topics [24], facilitating the identification of the most efficient product combinations. By taking environmental factors into account, it introduces a mathematical programming model that integrates various carbon emission costing models into decision-making for environmentally friendly product portfolios, thereby allowing managers to optimally use limited resources for maximum profitability.

3. Materials and Methods

Aluminum rim manufacturing typically involves four key stages, illustrated in Figure 2. This diagram uses squares to depict the stages of production, encompassing casting (o = 1), heat treatment (o = 2), computer numerical control (CNC) machining (o = 3, 4), and painting (o = 5), while ellipses indicate both the raw materials and the final products. Notably, the second phase of CNC machining is not always necessary; it is selectively employed for certain products that need additional machining for functional enhancement.

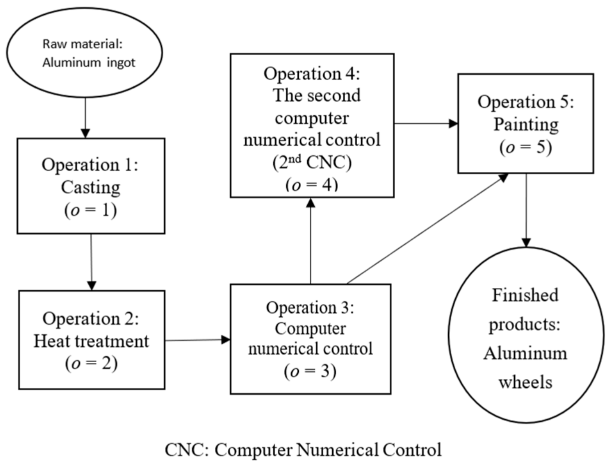


Figure 2. The process of typical aluminum wheel industry production.

3.1. The Assumptions of This Paper

In this research, we use an aluminum wheel manufacturing company as a case study. This company specializes in producing three types of wheels: automobile wheels (referred to as product 1, labeled P = 1), wheels for trucks (product 2, P = 2), and bespoke wheels tailored to specific customer requirements (product 3, P = 3). The production strategy for products 1 and 2 is based on a make-to-stock (MTS) approach, meaning these products are produced in larger quantities to maintain a certain level of inventory. Typically, the production volume for car wheels (product 1) is higher than that for truck wheels (product 2) due to greater demand. On the other hand, product 3, the custom wheel, is produced under a make-to-order (MTO) system, indicating that its production is triggered by specific customer orders, resulting in a more limited production quantity compared to the other two products. In order to ensure consistency, this study makes the following assumptions:

1. The ABC production process segregates activities into individual units and batch operations;
2. The period under consideration maintains a constant unit selling price;
3. Utilization of all machinery and labor is at a full 100%, with no expected failures or mishaps during this timeframe;
4. Utilization of all materials is efficient, ensuring no damage or waste occurs;
5. Material costs remain stable throughout this period, yet the company benefits from negotiated discounts with suppliers, directly influencing its profit margins. This discount agreement is in effect for the entire duration of this period;
6. Municipal laws permit the extension of work hours, encompassing additional shifts that incur overtime, compensated at rates of 133% and 166% of the standard pay for the second and third tiers of overtime, respectively;
7. The company incurs a carbon tax for each product, with the tax amount being proportional to the production volume;
8. The extent of carbon rights is subject to various government policies, but the company is free to engage in unrestricted carbon rights trading within the carbon market.
9. In this ABC model, outsourcing is not considered; all products, barring raw materials, are required to be manufactured in-house;
10. Trading of carbon rights is initiated only when the company's total emissions surpass the emission thresholds established by the government.

3.2. Basic Production Planning Model

In this part, we present a fundamental model for production planning based on the Activity-Based Costing (ABC) approach. This model encompasses an objective function, represented by Formula (1), along with a suite of essential cost functions and constraints that a company must consider to manufacture its products efficiently. These elements include functions for direct material costs, labeled (2) to (6); functions for unit-level operations, such as direct labor costs, numbered (7) to (12); functions related to batch-level operations, which cover material handling and setting costs, detailed in Equations (13)–(16); and finally, Equations (17) and (18) that govern machine-hour limitations. The objective function is as follows:

The company's maximum profit (π) = sales revenue of each product − the sum of the direct material costs of each product − direct labor costs − unit-level operating costs − batch-level operating costs − other fixed costs:

$$\begin{gathered}\pi=\sum_{i=1}^{n}S_iP_i-(DMC_1DQ_1+DMC_2DQ_2+DMC_3DQ_3-\\\sum_{i=1}^{n}\sum_{j=1}^{m}MC_jP_iq_{ij})-[HR_1+\omega_1(HR_2-HR_1)+\omega_2(HR_3-HR_1)]-\\\textstyle\sum_{i=1}^{n}d_oS_{io}B_{io}-d_oH_oB_o-F\end{gathered}\tag{1}$$

Symbol description:

π	The maximum profit achievable by the company.
S_{i}	The per-unit selling price of the ith product.
P_{i}	The production output of the ith product.
DMC_1, DMC_2, DMC_3	The per-unit expense of materials ($DMC1$), the first-stage discounted cost ($DMC2$), the second-stage discounted cost ($DMC3$)
DQ_1, DQ_2, DQ_3	The amount used under standard operating conditions ($DQ1$), the amount of the initial segment in the discounted area ($DQ2$), and the amount for the second portion of the discount. ($DQ3$)
MC_j	The cost per unit of the material. j.
q_{ij}	The amount of material j consumed when producing one unit of product i.
HR_1, HR_2, HR_3	Direct labor costs under normal circumstances (HR1), overtime costs in the first period (HR2), and overtime costs in the second period (HR3).
ω_0, ω_1, ω_2	SOS2 variables refer to a group of variables that must have positive values, with the condition that no more than two variables in this group can have values greater than zero.
d_o	The cost incurred for performing one unit of operation 'o'.
H_{o}	Batch operation ($o \in$ B) demand under material handling operation.
B_o	The count of batch processes within the scope of material handling operations. ($o \in$ B).
S_{io}	This refers to the market demand for product i when it is in the setting operation phase. (o ∈ B).
B_{io}	The quantity of batch production for product i during the setup operation. (o ∈ B).
F	The additional expenses that the company in the example incurs and remains constant.

In the modern business context, discounts on raw materials are prevalent. These discounts often arise when either the buyer possesses significant bargaining leverage over the seller or when the seller, aiming to establish a long-term partnership, is open to negotiating terms with the buyer. The negotiation process can lead to the establishment of a contract with three different pricing tiers: the standard price, the first level of discounted price, and an even lower second discounted price, as depicted in Figure 3. The second group $DMC_1DQ_1 + DMC_2DQ_2 + DMC_3DQ_3$ of Formula represents the material discount cost formula. Equations (2)–(6) represent the relevant restrictive expressions of the material discount function. α0, α1, α2 are SOS1 variables: when one of the variables is set to 1, the other variables must be zero. If α0 = 1, then α1 = 0, α2 = 0, see Equation (6); DQ1 = 0, DQ3 = 0, see Equations (4) and (5). DQ2 > α1MQ1, DQ2 ≤ α1MQ2, see Equation (3), which means that the cost of materials and the purchased quantity represent the normal purchasing situation without any discount, and the cost and quantity are DMC1DQ1 and DQ1, respectively. On the other hand, if α1 = 1, then α0 = 0, α2 = 0, see Equation (6); DQ1 = 0, DQ3 = 0, see Equations (3)–(5). DQ2 > α1MQ1, DQ2 ≤ α1MQ2, see Formula (4). This means that the cost of the material and the quantity purchased are in the first part of the scenario involving discounts on purchases: the price and amount are DMC2MQ2 and DQ2, respectively. When the purchase quantity is greater than MQ2, the second stage of purchase discount will be enabled. And there is no upper limit in this case, which means that the cost will always be DMC3DQ3.

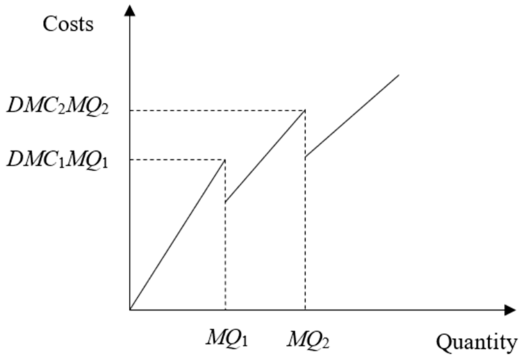


Figure 3. Direct material discount function.

Constraints:

$$\sum_{i=1}^{n} q_{i1}P_i = DQ_1 + DQ_2 + DQ_3 \quad (2)$$

$$0 \leq DQ_1 \leq \alpha_0 MQ_1 \quad (3)$$

$$\alpha_1 MQ_1 < DQ_2 \leq \alpha_1 MQ_2 \quad (4)$$

$$\alpha_2 MQ_2 < DQ_3 \quad (5)$$

$$\alpha_0 + \alpha_1 + \alpha_2 = 1 \quad (6)$$

Symbol description:

MQ_1, MQ_2, MQ_3	The maximum purchase quantity under normal circumstances (*MQ*1), the first-tier discount (*MQ*2), and the second-tier discount situation (*MQ*3).
$\alpha_0, \alpha_1, \alpha_2$	SOS1 variable: if one variable is assigned a value of 1, then all other variables must be set to zero precisely.

Direct labor typically signifies the workforce employed in manufacturing processes. The third set of equations in (1) can be rephrased as. $HR_1 + \omega_1(HR_2 - HR_1) + \omega_2(HR_3 - HR_1)$ Denotes the cost function for direct labor at the individual unit level. The correlation diagram and the associated constraint formulas are presented in Figure 4 and Equations (7)–(12). In these equations, both β1 and β2 are SOS1 variables. When one of these variables is set to 1, the remaining variables must be precisely set to zero. ω0, ω1, and ω2 are all SOS2 variables, which must be a set of positive variables, and at most, two variables can be non-zero. If β1 = 1, then β2 = 0, see Equation (12). If ω0 and ω1 $\leq$ 1, see Equations (8) and (9), then ω0 + ω1 = 1, which means that the use of man-hours and related costs between (CH1, HR1) and (CH2, HR2), the exact working hours are ω0CH1 + ω1CH2, and the cost is ω0HR1 + ω1HR2; this also means that the first period of overtime has been adopted. On the other hand, if β2 = 1, then β1 = 0, see Equation (12), ω1 and ω2 $\leq$ 1 (such as Equations (9) and (10)), ω1 + ω2 = 1, which means labor hours and related costs between (CH2, HR2) and (CH3, HR3), the exact working hours are ω1CH2 + ω2CH3, and the cost is ω1HR2 + ω2HR3; This implies that the company has opted for a second overtime period. In the function calculating direct labor costs, the cost of regular labor is treated as a constant expense, indicating that regardless of the number of hours worked, the labor cost represented by HR1 will see an increase.

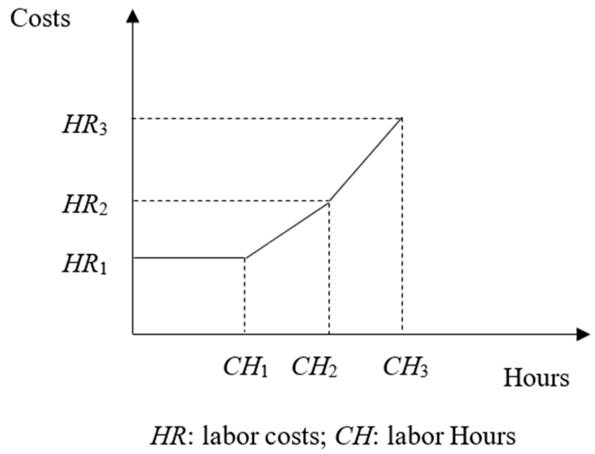


Figure 4. Direct labor cost function.

Constraints:

$$\sum_{i=1}^{n}\sum_{o=1}^{m} u_{io}P_i \leq CH_1 + \omega_1(CH_2 - CH_1) + \omega_2(CH_3 - CH_1) \tag{7}$$

$$\omega_0 - \beta_1 \leq 0 \tag{8}$$

$$\omega_1 - \beta_1 - \beta_2 \leq 0 \tag{9}$$

$$\omega_2 - \beta_2 \leq 0 \tag{10}$$

$$\omega_0 + \omega_1 + \omega_2 = 1 \tag{11}$$

$$\beta_1 + \beta_2 = 1 \tag{12}$$

Symbol description:

u_{io}	This is the amount of labor time needed to produce one unit of product i during operation o.
CH_1, CH_2, CH_3	This refers to the highest number of direct labor hours available under typical working conditions (CH_1), the first period of overtime (CH_2), and the second period of overtime (CH_3).
β_1, β_2	SOS1 variable: when one of the variables is set to 1, the other variables must be exactly zero.

In the context of our research, it is posited that the scope of material handling is strictly limited to transferring materials from the inventory to the production line, thereby rendering these operations independent of the actual production tasks. Nonetheless, each job encompasses specific preparatory actions. For instance, in CNC machining, the necessary parameters can be pre-programmed into the computer ahead of the manufacturing stage. Similarly, in painting tasks, the paint is premixed in readiness for subsequent application. The items in the fourth and fifth groups of Equation (1) $d_o H_o B_o$ and $\sum_{i=1}^{n} d_o S_{io} B_{io}$ represent the material handling and setting cost function, and the related constraints are Equations (13)–(16).

Constraints of material handling operations:

$$\sum_{i=1}^{n} q_{i1}P_i \leq \rho_o B_o (o = 6) \tag{13}$$

$$H_o B_o \leq A_o \ (o = 6) \tag{14}$$

Constraints of setting work:

$$P_i \leq \tau_{io} B_{io} (i = 1 \ldots 3, o = 7) \tag{15}$$

$$\sum_{i=1}^{n} S_{io} B_{io} \leq A_o (o = 7) \tag{16}$$

Symbol description:

ρ_o	In simpler terms, this refers to the specific amount of materials needed for each set of operations in the material handling process ($o \in$ B).
A_o	Capacity of batch-level operations ($o \in$ B).
τ_{io}	The required quantity of each batch of batch-level operations for the production of i products under the setting work ($o \in$ B).

Equations (17) and (18) pertain to the constraints regarding the machine's capacity limitations. In Formula (17), CPo signifies the operational capacity of the machine during a specific operation labeled as "o", while rhio represents the number of machine hours required to manufacture one unit of product "i" during operation "o". In this ABC model, CNC operations are categorized into two distinct phases: the initial CNC processing (o = 3) and the subsequent CNC processing (o = 4). Notably, both of these operations utilize the same CNC machine tool, and the second CNC processing is regarded as an optional step. Equation (18) is designed to ensure compliance with this condition, where CPCNC denotes the capability of the CNC machine tool.

Constraints:

$$\sum_{i=1}^{n} rh_{io} P_i \leq CP_o (o = 1, 2, 5) \tag{17}$$

$$\sum_{i=1}^{n} (rh_{i3} + rh_{i4}) P_i \leq CP_{CNC} \tag{18}$$

Symbol description:

rh_{io}	The machine-hour demand for producing a unit of product i under o operation.
CP_o	Machine capacity for operation o (o = 1, 2, 5).
CP_{CNC}	CNC machine tool capacity.

3.3. Objective Function for Producing Maximum Profit

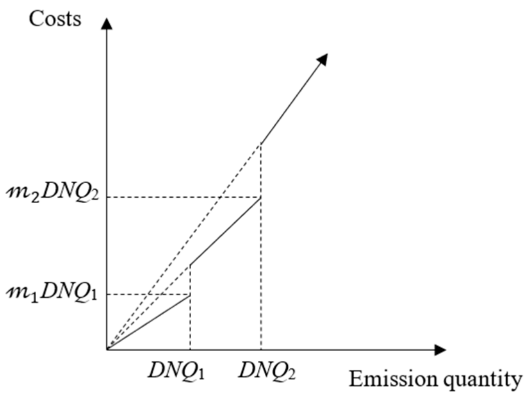
In this section, we will introduce two variations of the carbon tax model alongside the fundamental production planning model outlined in Section 3.2. These two variations encompass the following aspects: the carbon tax model with a complete progressive tax rate and no allowances and the carbon tax model with a complete progressive tax rate but with the inclusion of allowances. Each of these models consists of two sub-models. The first sub-model defines the cost function related to carbon tax, while the second sub-model extends the first one by incorporating the cost function associated with carbon rights. The ultimate goal of this study is to maximize the overall profit (π). Now, let us delve into Model 1-1: Carbon Tax Function with Full Progressive Tax Rates Without Allowances.

$$\begin{gathered}
\pi = \sum_{i=1}^{n} S_i P_i - (DMC_1 DQ_1 + DMC_2 DQ_2 + DMC_3 DQ_3 - \\
\sum_{i=1}^{n} \sum_{j=1}^{n} MC_j P_i q_{ij}) - [HR_1 + \omega_1 (HR_2 - HR_1) + \omega_2 (HR_3 - HR_1)] - \\
\sum_{i=1}^{n} d_o S_{io} B_{io} - d_o H_o B_o - \\
(m_1 NQ_1 + m_2 NQ_2 + m_3 NQ_3) - F
\end{gathered} \tag{19}$$

Symbol description:

m_1,m_2,m_3	Carbon tax rate for the first stage of carbon tax cost (m_1), carbon tax rate for the second stage of carbon tax cost (m_2), carbon tax rate for the third stage of carbon tax cost (m_3).
NQ_1,NQ_2,NQ_3	The carbon emission quantity of the first segment (NQ_1), the carbon emission quantity of the second segment (NQ_2) and the carbon emission quantity of the third segment (NQ_3).

In the specific section under discussion, model 1-1 is analyzed in terms of a carbon tax, characterized as a non-continuous function without any exemptions. This model also takes into account a carbon emission ceiling mandated by the government, which, for the purpose of this model, is set at 28,000 tons of CO_2. The model proposes that the tax payable by a company is directly proportional to its production volume. To illustrate these relationships and their associated restrictions, refer to the graphical representations and their corresponding constraints presented in Figure 5 and (20) through (25). In these equations, ν_1, ν_2 and ν_3 these are all SOS1 (Special Ordered Set Type 1) variables, meaning that if one variable is selected or activated (set to 1), all the other variables in the set must remain inactive (set to zero). If ν_1 = 1, then ν_2, ν_3= 0, see Equation (24); $0 \leq NQ_1 \leq DNQ_1$, see Equation (21); $NQ_2, NQ_3 = 0$, see (22) to (23), it means that the carbon tax cost is $m_1 NQ_1$, and the carbon emission is NQ_1, which also means that the first stage of the carbon tax function is used. On the other hand, if ν_2 = 1, then ν_1, ν_3 = 0, see Equation (24); $DNQ_1 < NQ_2 \leq DNQ_2$, see Equation (22); $NQ_1,\ NQ_3 = 0$, see Equations (22) and (23). This means that the carbon tax cost is $m_2 NQ_2$, and the carbon emission is NQ_2, which also means that the second stage of the carbon tax function is used. Furthermore, if ν_3 = 1, then ν_1, ν_2 = 0, see Equation (24), $NQ_3 > DNQ_2$, see Equation (23), $NQ_1,\ NQ_2 = 0$, see (22) to (23), it means that the tax cost is $m_3 NQ_3$, and the carbon emission is NQ_3, which also means that the third-stage carbon tax function is used. In addition, the carbon tax can also describe the following function f7 (CCE).



m: the tax rate; DNQ: carbon emission quantity

Figure 5. Carbon tax function with full progressive tax rates without allowances.

Constraints:

$$\sum_{i=1}^{n} e_i P_i = NQ_1 + NQ_2 + NQ_3 \leq GCE \tag{20}$$

$$0 \leq NQ_1 \leq \nu_1 DNQ_1 \tag{21}$$

$$\nu_2 DNQ_1 < NQ_2 \leq \nu_2 DNQ_2 \tag{22}$$

$$\nu_3 DNQ_2 < NQ_3 \tag{23}$$

$$\nu_1 + \nu_2 + \nu_3 = 1 \tag{24}$$

$$\nu_1, \nu_2, \nu_3 = 0, 1 \tag{25}$$

Functions:

$$f_7(ETC) = \begin{cases} m_1 CCE,\ 0 \leq CCE \leq DNQ_1 \\ m_2 CCE,\ DNQ_1 < CCE \leq DNQ_2 \\ m_3 CCE,\ CCE > DNQ_2 \end{cases}$$

Symbol description:

DNQ_1, DNQ_2, DNQ_3	Maximum ceiling for carbon emissions in the first segment (DNQ_1), carbon emissions in the second segment (DNQ_2), and in the third segment (DNQ_3)
v_0, v_1, v_2	SOS1 variables: when one of the variables is set to 1, the others must be exactly zero

3.3.1. Carbon Tax Function with Full Progressive Tax Rates without Allowances (with Carbon Trading)

$$\begin{gathered} \pi = \sum_{i=1}^{n} S_i P_i - (DMC_1 DQ_1 + DMC_2 DQ_2 + DMC_3 DQ_3 - \\ \sum_{i=1}^{n}\sum_{j=1}^{n} MC_j P_i q_{ij}) - [HR_1 + \omega_1(HR_2 - HR_1) + \omega_2(HR_3 - HR_1)] - \\ \sum_{i=1}^{n} d_o S_{io} B_{io} - d_o H_o B_o - \\ \left\{ \begin{array}{c} [(m_1 NQ_1 + m_2 NQ_2 + m_3 NQ_3) - r(GCE - CCE)]_1 + \\ [(m_1 NQ_1 + m_2 NQ_2 + m_3 NQ_3) + r(CCE - GCE)]_2 \end{array} \right\} - F \end{gathered} \tag{26}$$

Symbol description:

m_1, m_2, m_3	The tax rate applied to carbon emissions for the initial phase of the carbon tax expense ($m1$), the tax rate assigned to carbon emissions in the second phase of the carbon tax expenditure ($m2$), the tax rate applied to carbon emissions during the third phase of calculating carbon tax costs ($m3$).
NQ_1, NQ_2, NQ_3	The carbon emission quantity of the first segment (NQ_1), the carbon emission quantity of the second segment (NQ_2) and the carbon emission quantity of the third segment (NQ_3).
Ø_1, Ø_2	SOS1 variables: when one of the variables is set to 1, the others must be exactly zero.
r	unit carbon cost.
CCE	The aggregate amount of carbon dioxide released by the company.
GCE	Maximum carbon emissions cap set by the government.

The formula $[(m_1 NQ_1 + m_2 NQ_2 + m_3 NQ_3) - r(GCE - CCE)]\text{Ø}_1 + (m_1 NQ_1 + m_2 NQ_2 + m_3 NQ_3) + r(CCE - GCE)\text{Ø}_2$ in model 1-2, the objective function (26) presented in this subsection extends the carbon tax model discussed in Section 3.3.1, which previously had a complete progressive tax structure without any mention of allowances. It now incorporates the model for carbon rights trading as well. Alongside utilizing the fundamental cost functions (26) through (20), this objective function also includes segments that make use of the carbon tax cost functions (21) to (26) in combination with the carbon rights trading mechanism (27) to (30). MBR stands for the maximum purchase of carbon rights. If Ø1 = 1, then Ø2 = 0, see Equation (30); CQ1 $\geq$ 0 and CQ1 $\leq$ GCE, see Equation (28), which means that carbon emissions are lower than the limit set by the government, so the company will not need to buy additional carbon rights. On the other hand, if Ø2 = 1, then Ø1 = 0, see Equation (30); CQ2 > GCE and CQ2 $\leq$ (GCE + MBR), see Equation (29); this implies that when a company exceeds the government-imposed upper limit on carbon emissions, they must acquire extra carbon rights to support their ongoing production processes. Additionally, the carbon entitlement function can also encompass the following functionality.

$$f_8(CCE - GCE).$$

Section 3.3.1 primarily focuses on the government's carbon emission cap without discussing the carbon trading mechanism. This subsection, on the other hand, delves into the impact of the carbon trading feature on profits. The carbon entitlement function represents an additional carbon cost, and each industry is allocated a specific government-mandated carbon emission allowance. As long as a company's total carbon emissions remain within the government-imposed limits, it will not incur any carbon entitlement fees, and any surplus carbon allowances can be traded on the market. However, if companies wish to increase their production output, they must acquire additional carbon allowances through the carbon market. Within this subsection, we assume that the cost of purchasing carbon rights is directly proportional to the quantity needed. You can refer to Figure 6 for relevant diagrams and equations that illustrate these concepts (27) to (30).

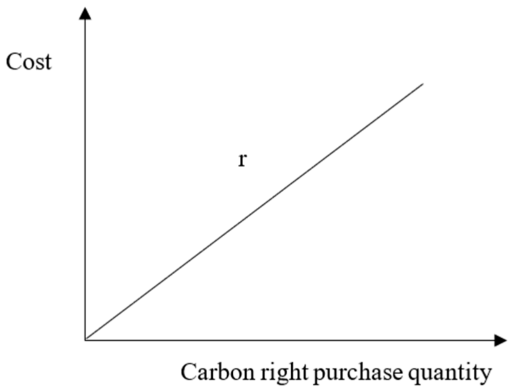


Figure 6. Linear carbon entitlement cost function.

Constraints:

$$\sum_{i=1}^{n} e_i P_i = CQ_1 + CQ_2 = CCE \tag{27}$$

$$0 \leq CQ_1 \leq GCE\varnothing_1 \tag{28}$$

$$GCE\varnothing_2 < CQ_2 \leq (GCE + MBR)\varnothing_2 \tag{29}$$

$$\varnothing_1 + \varnothing_2 = 1 \tag{30}$$

Functions:

$$f_8(CCE - GCE) = r(CCE - GCE)$$

Symbol description:

CQ_1, CQ_2	Determine if the business should acquire carbon credits, if $CQ_1 > 0$, the company does not need to purchase carbon rights; conversely, if $CQ_2 > 0$, the company must purchase carbon rights.
MBR	Maximum number of carbon rights purchased.

The combination of carbon tax and carbon entitlement function is illustrated below:

$$f_9(CCE) = \begin{cases} f_7(CCE),\ 0 \leq CCE \leq GCE \\ f_7(CCE) + f_8(CCE - GCE),\ CCE > GCE \end{cases}$$

3.3.2. Model C: Carbon Tax Function with Full Progressive Tax Rate with Allowances

$$\begin{gathered}\pi=\sum_{i=1}^{n} S_i P_i-(DMC_1 DQ_1+DMC_2 DQ_2+DMC_3 DQ_3- \\ \sum_{i=1}^{n}\sum_{j=1}^{n} MC_j P_i q_{ij})-[HR_1+\omega_1(HR_2-HR_1)+\omega_2(HR_3-HR_1)]- \\ \sum_{i=1}^{n} d_o S_{io} B_{io}-d_o H_o B_o- \\ [\delta_1 sr_1(w_1-DYQ_0)+\delta_2 sr_2(w_2-DYQ_0)+\delta_3 sr_3(w_3-DYQ_0)]-F\end{gathered} \tag{31}$$

Symbol description:

sr_1, sr_2, sr_3	The initial price set for the carbon tax (sr_1), the second carbon tax price (sr_2), and the third carbon tax price (sr_3).
$\delta_0, \delta_1, \delta_2, \delta_3$	SOS2 variable refers to a group of variables that are positive, and within this set, no more than two variables can have values greater than zero.
DYQ_0	The amount of carbon emissions from the tax-free allowance (DYQ_0).
w_1, w_2, w_3	The amount of carbon emissions occurring in the initial segment (w_1), carbon emissions in the second segment (w_2), and the carbon emissions in the third segment (w_3).

The 2-1 model in this Section treats the carbon tax as an interrupted function without any exemptions and takes into account a carbon emission ceiling, which is fixed at 28,000 tons (CO_2) for this particular model. The quantity of products manufactured by a firm dictates the magnitude of carbon tax it is required to pay. Illustrations of these relationships and their respective limitations are presented in Figure 7 and through Equations (32)–(38). δ_0, δ_1, δ_2, and δ_3 are all SOS1 variables, and when one of these variables is set to 1, the other variables must be exactly zero. If δ_0 = 1, then δ_1, δ_2, δ_3 = 0, see Equation (37); $w_0 \geq 0$ and $w_0 \leq$ DYQ0, see Equation (33); $w_1, w_2, w_3 = 0$, see Equations (34)–(36), which means that, in this case, the carbon tax will remain at 0. On the other hand, if $\delta_1 = 1$, then δ_0, δ_2, δ_3 = 0, see Equation (37); $DYQ_0 < w_1 \leq DYQ_1$, see Equation (34); $w_0, w_2, w_3 = 0$, see Equations (34)–(36), which means that the cost and emission quantity will be between (DYQ1, 0) and (DYQ 1, $sr_1(DYQ_1 - DYQ_0)$), the complete carbon tax is $\delta_1 sr_1(w_1 - DYQ_0)$, and the carbon emission is w_1, which also means that the first stage of the carbon tax function is used. Furthermore, if $\delta_2 = 1$, then $\delta_0, \delta_1, \delta_3 = 0$, see Equation (37); $DYQ_1 < w_2 \leq DYQ_2$, see Equation (35); $w_0, w_1, w_3 = 0$, see Equation (34) to (36). This means that the carbon tax cost is $\delta_2 sr_2(w_2 - DYQ_0)$, and the carbon emission is w_2, which also means that the second stage carbon tax function is adopted. In addition, if $\delta_3 = 1$, then $\delta_0, \delta_1, \delta_2 = 0$, $w_3 > DYQ_2$, see Equation (36); $w_0, w_1, w_2 = 0$, see Equations (34)–(36), which means the carbon tax cost tax is $\delta_3 sr_3(w_3 - DYQ_0)$, and the carbon emission is w_3, which also means that the first stage carbon tax function is used. The carbon tax can also account for the following function f10 (CCE).

Constraints:

$$\sum_{i=1}^{n} e_i P_i = w_0 + w_1 + w_2 + w_3 \leq GCE \tag{32}$$

$$0 \leq w_0 \leq \delta_0 DYQ_0 \tag{33}$$

$$\delta_1 DYQ_0 < w_1 \leq \delta_1 DYQ_1 \tag{34}$$

$$\delta_2 DYQ_1 < w_2 \leq \delta_2 DYQ_2 \tag{35}$$

$$\delta_3 DYQ_2 < w_3 \tag{36}$$

$$\delta_0 + \delta_1 + \delta_2 + \delta_3 = 1 \tag{37}$$

$$\delta_0, \delta_1, \delta_2, \delta_3 = 0, 1 \tag{38}$$

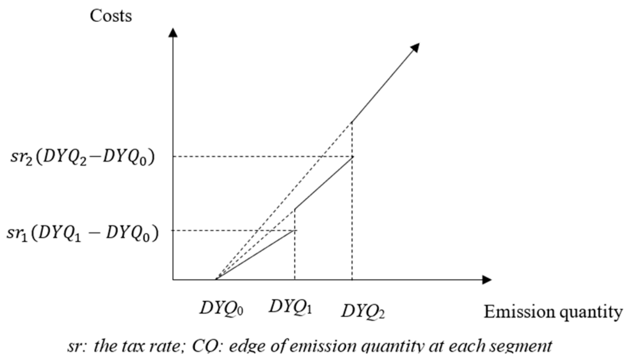


Figure 7. Carbon tax function with full progressive tax rates with allowances.

Functions:

$$f_{10}(CCE) = \begin{cases} sr_1(CCE - DYQ_0),\ 0 \le CCE \le DYQ_1 \\ sr_2(CCE - DYQ_0),\ DYQ_1 < CCE \le DYQ_2 \\ sr_3(CCE - DYQ_0),\ CCE > DYQ_2 \end{cases}$$

Symbol description:

$DYQ_0, DYQ_1, DYQ_2, DYQ_3$	Carbon emissions of exemptions (DYQ_0), carbon emissions of the first stage (DYQ_1), the second stage (DYQ_2), and carbon emissions of the third stage (DYQ_3).
$\delta_0, \delta_1, \delta_2, \delta_3$	SOS1 variables: when one of the variables is set to 1, the others must be exactly zero.

3.3.3. Carbon Tax Function with Full Progressive Tax Rates with Allowances (with Carbon Trading)

$$\begin{gathered} \pi = \sum_{i=1}^{n} S_i P_i - (DMC_1 DQ_1 + DMC_2 DQ_2 + DMC_3 DQ_3 - \\ \sum_{i=1}^{n}\sum_{j=1}^{n} MC_j P_i q_{ij}) - [HR_1 + \omega_1(HR_2 - HR_1) + \omega_2(HR_3 - HR_1)] - \\ \sum_{i=1}^{n} d_o S_{io} B_{io} - d_o H_o B_o - \\ \left\{ \begin{array}{c} \begin{bmatrix} (\delta_1 sr_1(w_1 - DYQ_0) + \delta_2 sr_2(w_2 - DYQ_0) + \delta_3 sr_3(w_3 - DYQ_0)) \\ -r(GCE - CCE) \end{bmatrix}_1 + \\ \begin{bmatrix} (\delta_1 sr_1(w_1 - DYQ_0) + \delta_2 sr_2(w_2 - DYQ_0) + \delta_3 sr_3(w_3 - DYQ_0)) \\ +r(CCE - GCE) \end{bmatrix}_2 \end{array} \right\} - F \end{gathered} \tag{39}$$

Symbol description:

sr_1, sr_2, sr_3	The first carbon tax price (sr_1), the second carbon tax price (sr_2), and the third carbon tax price (sr_3).
$\delta_0, \delta_1, \delta_2, \delta_3$	The SOS2 variable refers to a group of variables that are all positive, and within this set, no more than two variables can have values other than zero.
DYQ_0	The quantity of carbon emissions that falls within the limit allowed without incurring tax (DYQ_0).
w_1, w_2, w_3	Carbon emissions in the first segment (w_1), carbon emissions in the second segment (w_2), and the carbon emissions in the third segment (w_3).
$\varnothing_1, \varnothing_2$	SOS1 variables: when one of the variables is set to 1, the others must be exactly zero.
r	unit carbon cost.
CCE	The company's total carbon emissions.
GCE	Maximum carbon emissions cap set by the government.

In this subsection, the part which is used to illustrate $(\delta_1 sr_1(w_1 - DYQ_0) + \delta_2 sr_2(w_2 - DYQ_0) + \delta_3 sr_3(w_3 - DYQ_0)) - r(GCE - CCE)\text{Ø}_1 + (\delta_1 sr_1(w_1 - DYQ_0) + \delta_2 sr_2(w_2 - DYQ_0) + \delta_3 sr_3(w_3 - DYQ_0)) + r(GCE - CCE)\text{Ø}_2$ in the part of the objective function (39) in model 2-2, in addition to using the basic cost function (39) to (21), the carbon tax cost function (32) to (37) is also used. Plus, the carbon rights trading function, which is the parts of Equations (27)–(30) introduced in the previous Section 3.3.2. MBR stands for the maximum purchase of carbon rights. If Ø1 = 1, then Ø2 = 0, see Equation (30); CQ1 $\geq$ 0 and CQ1 $\leq$ GCE, see Equation (31), which means that carbon emissions are lower than the limit set by the government, so the company will not need to buy additional carbon rights. On the other hand, if Ø2 = 1, then Ø1 = 0, see Equation (30); CQ2 > GCE and CQ2 $\leq$ (GCE + MBR), see Equation (29); in other words, this situation implies that the company's carbon emissions exceed the government's set upper limit, necessitating the purchase of additional carbon credits to continue their production activities. Moreover, the carbon entitlement function is designed to encompass additional related factors. $f_{11}(CCE - GCE)$.

Symbol description:

CQ_1, CQ_2	Decide whether the company wants to purchase carbon rights; if CQ_1 > 0, the company does not need to purchase carbon rights; conversely, if CQ_2 > 0, the company must purchase carbon rights.
MBR	Maximum number of carbon rights purchased.

The combination of carbon tax and carbon entitlement function is illustrated below:

$$f_{12}(CCE) = \begin{cases} f_{10}(CCE),\ 0 \leq CCE \leq GCE \\ f_{10}(CCE) + f_{11}(CCE - GCE),\ CCE > GCE \end{cases}$$

3.4. Methods and Material: Multi-Period Production Decision-Making Model

This part of the research shifts focus from analyzing a single time frame to exploring multiple time periods. In this multi-period methodology, the study treats variables as changeable over several periods, indicated by adding a 't' subscript to the variables in the mathematical equations. For parameters that are known, the 't' subscript is applied only to those that change from one period to the next within the model's framework.

The current chapter builds upon the model discussed in the previous chapter by incorporating a multi-phase approach. In Section 3.3, the study outlines the basic production model. Then, Sections 3.5 and 3.6 concentrate on the multi-phase models, particularly regarding the expenses related to carbon taxes. These sections also explore two distinct scenarios within a multi-period framework:

Analyzing how the option to save or borrow carbon emission permits or rights affects production choices and the profitability of each model.

Investigating the impact of imposing a collective period limit on material needs on the production strategies and financial success of each model.

By adopting this multi-period viewpoint, the research aims to offer a deeper and more detailed understanding of the effects of carbon taxation and material limitations on production processes and economic results over an extended period.

3.4.1. Objective Function

$$\begin{aligned} \pi = \sum_{t=1}^{T}\sum_{i=1}^{n} S_i P_{it} - \sum_{t=1}^{T}(DMC_1 DQ_{1t} + DMC_2 DQ_{2t} + DMC_3 DQ_{3t} - \sum_{i=1}^{T}\sum_{j=1}^{m} MC_j P_{it} q_{ij}) \\ - \sum_{t=1}^{T}[HR_1 + \omega_{1t}(HR_2 - HR_1) + \omega_{2t}(HR_3 - HR_1)] - \\ \sum_{t=1}^{T}\sum_{i=1}^{n}(d_o S_{io} B_{iot} + d_o H_o B_{ot}) - \sum_{i=1}^{T} F_t \end{aligned} \quad (40)$$

3.4.2. Direct Material Discount Function

This has been explained in Section 3.1, and this section lists the multi-phase mode.

Constraints:

$$\sum_{i=1}^{n} q_{i1} P_{it} = DQ_{1t} + DQ_{2t} + DQ_{3t},\ t = 1,2\ldots T \quad (41)$$

$$0 \le DQ_{1t} \le \alpha_{0t} MQ_1,\ t = 1,\ 2\ldots T \quad (42)$$

$$\alpha_{1t} MQ_1 < DQ_{2t} \le \alpha_{1t} MQ_2,\ t = 1,\ 2\ldots T \quad (43)$$

$$\alpha_{2t} MQ_2 < DQ_{3t},\ t = 1,\ 2\ldots T \quad (44)$$

$$\alpha_{0t} + \alpha_{1t} + \alpha_{2t} = 1,\ t = 1,\ 2\ldots T \quad (45)$$

In the case of multiple periods, the material requirement quantity specifies the upper limit quantity for the total period (UDQ):

$$\sum_{t=1}^{T} DQ_{1t} + DQ_{2t} + DQ_{3t} \le UDQ,\ t = 1,2\ldots T \quad (46)$$

3.4.3. Unit-Level Operations: Direct Labor Cost Function

This has been explained in 3.2, and this section lists the multi-phase mode.
Constraints:

$$\sum_{i=1}^{n}\sum_{o=1}^{m} u_{io} P_{it} \le CH_1 + \omega_{1t}(CH_2 - CH_1) + \omega_{2t}(CH_3 - CH_1),\ t = 1,2\ldots T \quad (47)$$

$$\omega_{0t} - \beta_{1t} \le 0,\ t = 1,\ 2\ldots T \quad (48)$$

$$\omega_{1t} - \beta_{1t} - \beta_{2t} \le 0,\ t = 1,\ 2\ldots T \quad (49)$$

$$\omega_{2t} - \beta_{2t} \le 0,\ t = 1,\ 2\ldots T \quad (50)$$

$$\omega_{0t} + \omega_{1t} + \omega_{2t} = 1,\ t = 1,\ 2\ldots T \quad (51)$$

$$\beta_{1t} + \beta_{2t} = 1,\ t = 1,\ 2\ldots T \quad (52)$$

3.4.4. Batch-Level Operations: Material Handling and Setting Cost Functions

This has been explained in Section 3.2, and this section lists the multi-phase mode.
Constraints of material handling operations:

$$\sum_{i=1}^{n} q_{i1} P_{it} \le \rho_o B_{ot}(o = 6),\ t = 1,2\ldots T \quad (53)$$

$$H_o B_{ot} \le A_o(o = 6),\ \ ,\ t = 1,\ 2\ldots T \quad (54)$$

Constraints of setting work:

$$P_{it} \le \tau_{io} B_{iot}(i = 1\ldots 3, o = 7),\ t = 1,\ 2\ldots T \quad (55)$$

$$\sum_{i=1}^{n} S_{io} B_{iot} \le A_o(o = 7),\ t = 1,2\ldots T \quad (56)$$

3.4.5. Constraints of Machine Hours

This has been explained in Section 3.2, and this section lists the multi-phase mode.
Constraints:

$$\sum_{i=1}^{n} rh_{io} P_i \le CP_o(o = 1,2,5),\ t = 1,2\ldots T \quad (57)$$

$$\sum_{i=1}^{n} (rh_{i3} + rh_{i4}) P_i \le CP_{CNC},\ t = 1,2\ldots T \quad (58)$$

3.5. Multi-Phase Model of Model 1

3.5.1. Carbon Tax Function with Full Progressive Tax Rates without Allowances

$$\begin{aligned}\pi = \sum_{t=1}^{T}\sum_{i=1}^{n} S_iP_{it} - \sum_{t=1}^{T}(DMC_1DQ_{1t} + DMC_2DQ_{2t} + DMC_3DQ_{3t} - \sum_{i=1}^{T}\sum_{j=1}^{m} MC_jP_{it}q_{ij}) \\ - \sum_{t=1}^{T}[HR_1 + \omega_{1t}(HR_2 - HR_1) + \omega_{2t}(HR_3 - HR_1)] - \\ \sum_{t=1}^{T}\sum_{i=1}^{n}(d_oS_{io}B_{iot} + d_oH_oB_{ot}) - \\ \sum_{t=1}^{T}(m_{1t}NQ_1 + m_{2t}NQ_2 + m_{3t}NQ_3) - \sum_{i=1}^{T} F_t\end{aligned} \tag{59}$$

This has been explained in 3.3.1, and this section lists the multi-phase mode.
Constraints:

$$\sum_{i=1}^{n} e_iP_{it} = NQ_{1t} + NQ_{2t} + NQ_{3t},\ \mathrm{t} = 1,2\ldots\mathrm{T} \tag{60}$$

$$0 \le NQ_{1t} \le \nu_{1t}DNQ_1,\ \mathrm{t} = 1,2\ldots\mathrm{T} \tag{61}$$

$$\nu_{2t}DNQ_1 < NQ_{2t} \le \nu_{2t}DNQ_2,\ \mathrm{t} = 1,2\ldots\mathrm{T} \tag{62}$$

$$\nu_{3t}DNQ_2 < NQ_{3t},\ \mathrm{t} = 1,2\ldots\mathrm{T} \tag{63}$$

$$\nu_{1t} + \nu_{2t} + \nu_{3t} = 1,\ \mathrm{t} = 1,2\ldots\mathrm{T} \tag{64}$$

In the scenario where production decisions span over multiple periods, the formula is specifically necessary only in the 1-1 model. This requirement emerges from the lack of a carbon rights trading system, leading to a constraint where the aggregate carbon emissions across all periods must not exceed the cumulative emission limit set by the government. In situations where it is possible to either store or borrow carbon emission quotas or rights, the formula for limiting the carbon emissions of an enterprise through multiple phases is defined such that it should not surpass the overall carbon emission ceiling (GCE) as determined by the government:

$$\sum_{i=1}^{n} e_iP_{it} \le \sum_{i=1}^{T} GCE_t,\ \mathrm{t} = 1,2\ldots\mathrm{T} \tag{65}$$

Functions:

$$f_7(ETC) = \begin{cases} m_1CCE_t,\ 0 \le CCE_t \le DNQ_1 \\ m_2CCE_t,\ DNQ_1 < CCE_t \le DNQ_2 \\ m_3CCE_t,\ CCE_t > DNQ_2 \end{cases}$$

3.5.2. Carbon Tax Function with Full Progressive Tax Rates without Allowances (with Carbon Trading)

$$\begin{aligned}\pi = \sum_{t=1}^{T}\sum_{i=1}^{n} S_iP_{it} - \sum_{t=1}^{T}(DMC_1DQ_{1t} + DMC_2DQ_{2t} + DMC_3DQ_{3t} - \sum_{i=1}^{T}\sum_{j=1}^{m} MC_jP_{it}q_{ij}) \\ - \sum_{t=1}^{T}[HR_1 + \omega_{1t}(HR_2 - HR_1) + \omega_{2t}(HR_3 - HR_1)] - \\ \sum_{t=1}^{T}\sum_{i=1}^{n}(d_oS_{io}B_{iot} + d_oH_oB_{ot}) - \\ \left\{\begin{array}{l} \left[\sum_{t=1}^{T}(m_{1t}NQ_1 + m_{2t}NQ_2 + m_{3t}NQ_3) - r\left(\sum_{i=1}^{T} GCE_t - \sum_{t=1}^{T} CCE_t\right)\right]\lambda_1 + \\ \left[\sum_{t=1}^{T}(m_{1t}NQ_1 + m_{2t}NQ_2 + m_{3t}NQ_3) + r\left(\sum_{t=1}^{T} CCE_t - \sum_{i=1}^{T} GCE_t\right)\right]\lambda_2 \end{array}\right\} \\ - \sum_{i=1}^{T} F_t\end{aligned} \tag{66}$$

In the multi-period production decision model, the 1-2 model incorporates a cost function for carbon rights, distinguishing it from the single-phase model outlined in Section 3.3.2. This approach necessitates evaluating whether the aggregate carbon emissions over multiple periods fall below or exceed the limits imposed by government regulations. The cap on total carbon emissions across multiple periods serves as the criteria for businesses to determine the necessity of purchasing carbon rights. Within this framework, λ_1 and λ_2 function as SOS1 variables, meaning if one is assigned the value of 1, the other must be set to exactly zero. For instance, if λ_1 equals 1, then λ_2 must be 0, as shown in Equation (70), with ACQ1 being greater than or equal to 0 and less than or equal to MQ as per Equation (68), indicating that total carbon emissions are below the government's threshold, thus negating the need for additional carbon rights purchases. Conversely, if λ_2 is assigned a value of 1, rendering λ_1 as 0 (refer to Equation (70)), and ACQ2 exceeds MQ but remains within MQ plus TMBR as per Equation (69); this implies that carbon emissions surpass the government's upper limit, necessitating companies to buy extra carbon rights for ongoing production. Additionally, the variance between GCEt and CCEt carbon emissions is analyzed each period to assess whether enterprises need to store or borrow carbon emissions in that particular period.

Constraints:

$$\sum_{t=1}^{T}\sum_{i=1}^{n} e_i P_{it} = ACQ_1 + ACQ_2 = \sum_{i=1}^{T} CCE_t, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{67}$$

$$0 \leq ACQ_1 \leq MQ\lambda_1,\ \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{68}$$

$$MQ\lambda_2 < ACQ_2 \leq (MQ + TMBR)\lambda_2, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{69}$$

$$\lambda_1 + \lambda_2 = 1, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{70}$$

Function:

$$f_8(CCE_t - GCE_t) = r(CCE_t - GCE_t)$$

Symbol description:

ACQ_1, ACQ_2	In the multi-period model, the enterprise evaluates the option of purchasing carbon credits. IF ACQ1 > 0, the company does not need to purchase carbon rights; on the contrary, if ACQ2 > 0, the company must purchase carbon rights.
MQ	Equivalent to the sum of $GCE_1 + GCE_2 + \ldots GCE_{\mathrm{t}}$.
$TMBR$	The highest quantity of carbon credits a firm can buy over multiple planning periods.
$\lambda_1, \lambda_2,$	In an SOS1 variable set, if any single variable is assigned the value 1, all remaining variables in the set are constrained to be precisely zero.

3.6. Multi-Phase Model of Model 2

3.6.1. Carbon Tax Function with Full Progressive Tax Rates with Allowances

$$\begin{aligned}\pi = &\sum_{t=1}^{T}\sum_{i=1}^{n} S_i P_{it} - \sum_{t=1}^{T}(DMC_1 DQ_{1\mathrm{t}} + DMC_2 DQ_{2\mathrm{t}} + DMC_3 DQ_{3\mathrm{t}} - \sum_{i=1}^{T}\sum_{j=1}^{m} MC_j P_{it} q_{ij}) \\ &- \sum_{t=1}^{T}[HR_1 + \omega_{1\mathrm{t}}(HR_2 - HR_1) + \omega_{2\mathrm{t}}(HR_3 - HR_1)] - \\ &\sum_{t=1}^{T}\sum_{i=1}^{n}(d_{\,o} S_{io} B_{iot} + d_o H_o B_{ot}) - \\ &\sum_{t=1}^{T}[\delta_{1\mathrm{t}} sr_{1\mathrm{t}}(w_{1t} - DYQ_{\,0}) + \delta_{2t} sr_{2\mathrm{t}}(w_{2t} - DYQ_{\,0}) + \delta_{3\mathrm{t}} sr_{3\mathrm{t}}(w_{3t} - DYQ_0)] - \sum_{i=1}^{T} F_t\end{aligned} \tag{71}$$

This has been explained in Section 3.3.3, and this Section lists the multi-phase mode.

Constraints:

$$\sum_{i=1}^{n} e_i P_{it} = w_{0t} + w_{1t} + w_{2t} + w_{3t}, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{72}$$

$$0 \leq w_{0t} \leq \delta_{0t} DYQ_0, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{73}$$

$$\delta_{1t} DYQ_0 < w_{1t} \leq \delta_{1t} DYQ_1, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{74}$$

$$\delta_{2t} DYQ_1 < w_{2t} \leq \delta_{2t} DYQ_2, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{75}$$

$$\delta_{3t} DYQ_2 < w_{3t}, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{76}$$

$$\delta_{0t} + \delta_{1t} + \delta_{2t} + \delta_{3t} = 1, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{77}$$

In the context of multi-period production decision-making, the requirement to implement this specific formula is unique to the 2-1 model. This necessity arises due to the absence of carbon rights trading, which imposes a limitation that the cumulative carbon emissions across multiple periods must not surpass the government's established maximum limit for total carbon emissions. Therefore, under circumstances where enterprises are unable to store or borrow carbon quotas or rights, the formula for restricting multi-phase enterprise carbon emissions is defined as being less than or equal to the government-mandated total carbon emission cap (GCE).

$$\sum_{i=1}^{n} e_i P_{it} \leq \sum_{i=1}^{T} GCE_t, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{78}$$

Functions:

$$f_{10}(CCE) = \begin{cases} sr_1(CCE_t - DYQ_0),\ 0 \leq CCE_t \leq DYQ_1 \\ sr_2(CCE_t - DYQ_0),\ DYQ_1 < CCE_t \leq DYQ_2 \\ sr_3(CCE_t - DYQ_0),\ CCE_t > DYQ_2 \end{cases}$$

3.6.2. Carbon Tax Function with Full Progressive Tax Rates with Allowances (with Carbon Trading)

$$\begin{gathered}
\pi = \sum_{t=1}^{T} \sum_{i=1}^{n} S_i P_{it} - \sum_{t=1}^{T} (DMC_1 DQ_{1t} + DMC_2 DQ_{2t} + DMC_3 DQ_{3t} - \sum_{i=1}^{T} \sum_{j=1}^{m} MC_j P_{it} q_{ij}) \\
- \sum_{t=1}^{T} [HR_1 + \omega_{1t}(HR_2 - HR_1) + \omega_{2t}(HR_3 - HR_1)] - \\
\sum_{t=1}^{T} \sum_{i=1}^{n} (d_o S_{io} B_{iot} + d_o H_o B_{ot}) - \\
\left\{ \begin{array}{l}
\left[\begin{array}{c} \sum_{t=1}^{T} \left(\begin{array}{c} \delta_{1t} sr_{1t}(w_{1t} - DYQ_0) + \delta_{2t} sr_{2t}(w_{2t} - DYQ_0) \\ + \delta_{3t} sr_{3t}(w_{3t} - DYQ_0) \end{array} \right) \\ -r \left(\sum_{t=1}^{T} GCE_t - \sum_{i=1}^{T} CCE_t \right) \end{array} \right] \lambda_1 + \\
\left[\begin{array}{c} \sum_{t=1}^{T} \left(\begin{array}{c} \delta_{1t} sr_{1t}(w_{1t} - DYQ_0) + \delta_{2t} sr_{2t}(w_{2t} - DYQ_0) \\ + \delta_{3t} sr_{3t}(w_{3t} - DYQ_0) \end{array} \right) + \\ r \left(\sum_{t=1}^{T} CCE_t - \sum_{i=1}^{T} GCE_t \right) \end{array} \right] \lambda_2
\end{array} \right\} \\
- \sum_{i=1}^{T} F_t
\end{gathered} \tag{79}$$

In the 2-2 model of the multi-period production decision-making approach, a carbon rights cost function is introduced. This model requires an assessment of whether the total carbon emissions over multiple periods are either below or above the limits set by governmental regulations. The total carbon emission ceiling for multiple periods is utilized as a criterion for businesses to determine their need for purchasing carbon rights. In this model, λ_1 and λ_2 are designated as SOS1 variables, where if one is activated (set to 1), the other must be deactivated (set to 0). Specifically, if λ_1 equals 1, then λ_2 must be 0, as illustrated in Equation (73); $ACQ_1 \geq 0$ and $ACQ_1 \leq$ MQ, as per Equation (71), implies that the total carbon emission is lower than the government's limit; hence, companies will not need to buy additional carbon rights. Conversely, if λ_2 is set to 1, making λ_1 equal to 0, as shown in Equation (73); then ACQ_2 > MQ and $ACQ_2 \leq$ (MQ + TMBR), as per Equation (72), indicates that carbon emissions exceed the government's upper limit, obliging companies to purchase extra carbon rights for further production. Additionally, the disparity between GCEt and CCE_t carbon emissions can be compared in each period to determine if enterprises need to store or borrow carbon emissions in that period.

Constraints:

$$\sum_{t=1}^{T}\sum_{i=1}^{n} e_i P_{it} = ACQ_1 + ACQ_2 = \sum_{i=1}^{T} CCE_t, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{80}$$

$$0 \leq ACQ_1 \leq MQ\lambda_1, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{81}$$

$$MQ\lambda_2 < ACQ_2 \leq (MQ + TMBR)\lambda_2, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{82}$$

$$\lambda_1 + \lambda_2 = 1, \mathrm{t} = 1, 2 \ldots \mathrm{T} \tag{83}$$

Functions:

$$f_{11}(CCE_t - GCE_t) = r(CCE_t - GCE_t)$$

Symbol description:

ACQ_1, ACQ_2	In the multi-period framework, the decision to buy carbon credits hinges on specific conditions. Should ACQ_1 be greater than zero, the company is exempt from the need to acquire carbon credits; however, if ACQ_2 exceeds zero, the firm is obligated to purchase carbon rights.
MQ	Equivalent to the sum of GCE_1 + GCE_2 + . . .GCE_t.
TMBR	Maximum number of carbon rights purchased in multi-period model
λ_1, λ_2,	SOS1 variables: when one of the variables is set to 1, the others must be exactly zero.

4. Results and Discussion

4.1. Results

In this analysis, this study evaluates the most effective product mix for each model using real-world data, focusing on a globally recognized company known for its commitment to environmental sustainability.

The introduction of carbon taxes and carbon rights by local governments is anticipated to significantly influence the financial performance of this company, potentially altering its existing product lineup. In light of these upcoming policies, this study demonstrates how businesses, using the example of this particular company, can identify viable product combinations under new carbon tax and carbon rights frameworks. The investigation includes an examination of various combinations of carbon taxes and carbon rights across different models. This comparative approach aims not only to aid companies in understanding the differential impacts of these models but also to guide governments in selecting the most appropriate policies. For tackling the complexities of these scenarios, LINGO18 software is identified as the most suitable tool.

Through this methodology, the paper provides an insightful exploration of how carbon-related policies can reshape business strategies, offering a valuable resource for both corporate decision-making and governmental policy planning.

4.1.1. Sample Data

The company's primary products are car rims, truck rims, and custom rims. Both car and truck rims are produced as make-to-stock items. Each product requires two materials:

aluminum ingot (m = 1) and paint (m = 2), with discounts applicable to aluminum ingots. The production process involves seven stages, five of which are unit-level operations, including casting, heat treatment, CNC machining, and painting. The remaining two are batch-level operations: material handling and setup.

The company's operations are also guided by government labor policies. In case of workforce shortages, an overtime system is implemented in two stages with increased salary rates of 1.33 and 1.66 times the regular pay. Each production operation has a specific capacity limit, and carbon emissions are calculated in tons, as detailed in the company's operational tables.

4.1.2. Data Analysis

The evaluation and comparison of models are conducted using the data from Tables 1–3, focusing on the optimal outcomes presented in Tables 4–7 and considering the values in the objective function and associated constraints. Tables 4–7 presents the data for our analysis, where the asterisk (*) is used to denote multiplication between the respective values.

4.1.3. Carbon Tax Function with Full Progressive Tax Rate without Allowance (Model 1)

Table 4 displays the fundamental production planning model's best solution, objective function, and constraints under the ABC method for model 1-1. This model yields a maximum profit of \$26,588,110, with production outputs for three products being 2006, 3624, and 5914 units, respectively. For aluminum ingots, a phase 2 material discount is applied ($\alpha 2 = 1$), leading to a purchase of 151,680 units at \$79 each. The labor is at a phase 2 utilization rate ($\beta 1 = 1$), indicating the activation of the initial overtime stage. There are 2167 batches for material handling and 1003, 1812, and 5914 batches for setting up the three products. The total labor time is 61,628 h, costing \$9,361,820; carbon emissions hit the government's maximum limit of 27,999, resulting in a carbon tax of \$9,799,650.

In model 1-2, as depicted in Table 5, a carbon entitlement function is added. This model achieves a highest profit of \$28,418,590, which is an increase of \$1,830,480 from model 1. The output of the three products adjusts to 2000, 6910, and 5257 units. Notably, the production of truck rims (product 2) increases substantially due to the carbon rights trading system. The purchase of aluminum ingots remains in phase 2 ($\alpha 2 = 1$), with 210,770 units bought at \$79 each. Labor utilization remains in the second stage ($\beta 1 = 1$), implying continued overtime work. Batch-level tasks include 3011 material handling batches and 1000, 3455, and 5257 batches for setting up each product. Labor usage totals 74,092 h, costing \$12,664,780; carbon emissions reach 32,591, hitting the government's ceiling and necessitating the purchase of an additional 4591 carbon rights units. The carbon tax costs \$11,406,850, and the carbon rights purchase amounts to \$1,147,750. This model demonstrates that the adoption of a carbon rights trading system by the government can increase corporate profits and provide greater flexibility compared to the standard model.

4.1.4. Carbon Tax Function with Full Progressive Tax Rate with Allowances (Model 2)

Table 6 presents the optimal solution, the objective function, and the limitations of the fundamental production planning model employing the ABC method in model 2-1. This model's peak profit is \$28,338,110, with production outputs for three products at 2006, 3624, and 5914 units, respectively. For aluminum ingots, a discount is applied at phase 2 ($\alpha 2 = 1$), leading to the procurement of 151,680 units at a unit price of \$79. Labor utilization is at stage two ($\beta 1 = 1$), indicating the commencement of the first stage of overtime. The model includes 2167 batches of material handling and 1003, 1812, and 5914 batches for setting up each product. Total labor hours amount to 61,628, incurring a cost of \$9,361,820; carbon emissions hit the government's upper limit of 27,999, resulting in a carbon tax expense of \$8,049,650.

Table 1. Example data.

					Merchandise SKU		Accessible
		o		**Car rim**	**Truck Rim**	**Custom Rims**	**Capacity**
Max/min production volume			P_i	>2000	>1000	>2000, 6000	
Selling price			S_i	4000	6000	<8000	
			Direct materials at the unit level				
Aluminum ingot (m = 1)	Refer to the "Discounted material costs" section for the unit price.		q_{i1}	10	20	10	
Coating (m = 2)	$MC_2 = \$50/\text{unit}$		q_{i2}	2	3	4	
			Discounted material cost				
Quantity	$MQ_1 = 80{,}000$	$MQ_2 = 250{,}000$	>250,000				
Cost	$DMC_1 = \$70$	$DMC_2 = \$69$	$DMC_3 = \$67$				
			Unit level work				
machine hours	Casting	1	rh_{i1}	2	3	2	$CP_1 = 46{,}200$
	Heat treatment	2	rh_{i2}	3	4	3	$CP_2 = 50{,}400$
	CNC	3	rh_{i3}	1	1	1	$CP_{CNC} = 18{,}900$
	CNC 2nd	4	rh_{i4}	0	0	0.9	
	Painting	5	rh_{i5}	0.1	0.1	0.2	$CP_5 = 2070$
Labor hours	Casting	1	u_{i1}	1.2	1.7	1.2	
	Heat treatment	2	u_{i2}	1.5	2	1.5	
	CNC	3	u_{i3}	1	1	1.6	
	CNC 2nd	4	u_{i4}	0	0	1	
	Painting	5	u_{i5}	0.3	0.3	0.7	

Table 2. Example data (continued from the table above).

					Product Item		Available Capacity
		o		**Car Rim**	**Truck Rim**	**Custom Rims**	
			Batch level operation				
Material handling	$d_6 = \$2500/\text{batch}$	6	H_6		1		$A_6 = 8800$
			ρ_6		70		
Setting	$d_7 = \$200/\text{batch}$	7	S_{i7}	1	1	2.5	$A_7 = 17{,}600$
			τ_{i7}	2	2	1	

Table 2. *Cont.*

			o	**Product Item**			**Available Capacity**
				Car Rim	**Truck Rim**	**Custom Rims**	
			Direct labor cost				
Cost	HR_1 = \$7,022,400	HR_2 = \$14,018,400	HR_3 = \$23,337,600				
Labor hours	CH_1 = 52,800	CH_2 = 79,200	CH_3 = 105,600				
Salary rate	\$133/h	\$177/h	\$ 221/h				
Carbon tax with full progressive tax rate without allowance (Model 1)				e_i	1.5	2	3
Cost	m_1 DNQ_1	m_2 DNQ_2	m_3DNQ_3				
Quantity	DNQ_1 = 10,000	DNQ_2 = 20,000	DNQ_3 > 20,000				
Tax rate	m_1 = \$250/unit	m_2 = \$300/unit	m_3 = \$350/unit				
Carbon tax with full progressive tax rate with allowance (Model 2)				e_i	1.5	2	3
Cost	$sr_1(DYQ_1–DYQ_0)$	$sr_2(DYQ_1–DYQ_0)$	$sr_3(DYQ_3–DYQ_0)$				
Quantity	DYQ_0 = 5000	DYQ_1 = 15,000	DYQ_2 = 25,000		DYQ_3 > 25,000		
Tax rate	sr_1 = \$250/unit	sr_2 = \$300/unit	sr_3 = \$350/unit				

Table 3. Example data (continued from the table above).

			o	**Car Rim**	**Product Item Truck Rim**	**Custom Rims**	**Available Truck Rim**
			Linear carbon right cost				
Carbon right cost	r = \$250/unit					Maximum carbon emissions cap set by the government	GCE = 28,000
Quantity	MBR = 100,000						
Other fixed costs	F = 10,000,000						
			The approach or method used for decision-making in multi-stage production processes.				
The upper limit of three-phase carbon emission	GCE_1 = 28,000	GCE_2 = 25,200	GCE_3 = 22,400				
The upper limit of three-phase materials	UDQ = 39,000						

Table 4. The Ideal result, Object function, and associated limitations or conditions of the framework 1-1.

[Ideal result]	
$\pi = 26{,}588{,}110$; $P_1 = 2006$; $P_2 = 3624$; $P_3 = 5914$; $DQ_1 = 0$; $DQ_2 = 151{,}680$; $DQ_3 = 0$; $\omega_0 = 0.6656061$; $\omega_1 = 0.3343939$; $\omega_2 = 0$; $B_6 = 2167$; $B_{17} = 1003$; $B_{27} = 1812$; $B_{37} = 5914$; $\nu_1 = 0$; $\nu_2 = 0$; $\nu_3 = 1$; $\alpha_1 = 0$; $\alpha_2 = 1$; $\alpha_3 = 0$; $\beta_1 = 1$; $\beta_2 = 0$; $CCE = 27{,}999$	
[Object function]	
Maximum $\pi = 4000{*}P_1 + 6000{*}P_2 + 8000{*}P_3 - (70{*}DQ_1 + 69{*}DQ_2 + 67{*}DQ_3 - 100{*}P_1 + 150{*}P_2 + 200{*}P_3) - [7{,}022{,}400 + \omega_1{*}6{,}996{,}000 + \omega_2{*}16{,}315{,}200] - 2500{*}B_6 - 200{*}B_{17} - 200{*}B_{27} - 500{*}B_{37} - (250{*}m_1 + 300{*}m_2 + 350{*}m_3) - 10{,}000{,}000$	
[Constraints]	
Direct material discount:	Direct labor:
$10{*}P_1 + 20{*}P_2 + 10{*}P_3 = DQ_1 + DQ_2 + DQ_3$	$4{*}P_1 + 5{*}P_2 + 6{*}P_3 = 52{,}800 - \omega_1{*}26{,}400 - \omega_2{*}52{,}800$
$0 \le DQ_1 \le \alpha_0{*}200{,}000$	$\omega_0 - \beta_1 \le 0$
$\alpha_1{*}200{,}000 < DQ_2 \le \alpha_1{*}500{,}000$	$\omega_1 - \beta_1 - \beta_2 \le 0$
$\alpha_2{*}500{,}000 < DQ_3$	$\omega_2 - \beta_2 \le 0$
$\alpha_0 + \alpha_1 + \alpha_2 = 1$	$\omega_0 + \omega_1 + \omega_2 = 1$
	$\beta_1 + \beta_2 = 1$
Batch Level: Material Handling	
$10{*}P_1 + 20{*}P_2 + 10{*}P_3 \le 70{*}B_6$	Batch Level: Settings
$1{*}B_6 \le 8800$	$P_1 \le 2{*}B_{17}$
	$P_2 \le 2{*}B_{27}$
Machine hours:	$P_3 \le 1{*}B_{37}$
o = 1: $2{*}P_1 + 3{*}P_2 + 2{*}P_3 \le 46{,}200$	$1{*}B_{17} + 1{*}B_{27} + 2.5{*}B_{37} \le 17{,}600$
o = 2: $3{*}P_1 + 4{*}P_2 + 3{*}P_3 \le 50{,}400$	
o = 3, 4: $1{*}P_1 + 1{*}P_2 + 1.9{*}P_3 \le 18{,}900$	Min/Max demand:
o = 5: $0.1{*}P_1 + 0.1{*}P_2 + 0.2{*}P_3 \le 2070$	$P_1 > 2000$; $P_2 > 1000$; $6000 > P_3 > 2000$
	A carbon levy employing a fully graduated rate system inclusive of exemptions:
	$1.5{*}P_1 + 2{*}P_2 + 3{*}P_3 = NQ_1 + NQ_2 + NQ_3$
	$1.5{*}P_1 + 2{*}P_2 + 3{*}P_3 \le 28{,}000$
	$0 \le NQ_1 \le 10{,}000{*}\nu_1$
	$10{,}000{*}\nu_2 \le NQ_2 \le 20{,}000{*}\nu_2$
	$20{,}000{*}\nu_3 \le NQ_3$
	$\nu_1 + \nu_2 + \nu_3 = 1$

Table 5. The Ideal result, Object function, and associated limitations or conditions of the framework 1-2.

[Ideal result]	
$\pi = 28{,}418{,}590$; $P_1 = 2000$; $P_2 = 6910$; $P_3 = 5257$; $DQ_1 = 0$; $DQ_2 = 210{,}770$; $DQ_3 = 0$; $\omega_0 = 0.1934848$; $\omega_1 = 0.8065152$; $\omega_2 = 0$; $B_6 = 3011$; $B_{17} = 1000$; $B_{27} = 3455$; $B_{37} = 5257$; $\nu_1 = 0$; $\nu_2 = 0$; $\nu_3 = 1$; $\alpha_1 = 0$; $\alpha_2 = 1$; $\alpha_3 = 0$; $\beta_1 = 1$; $\beta_2 = 0$; $GCE = 28{,}000$; $\varnothing_1 = 0$; $\varnothing_2 = 1$; $CQ_1 = 0$; $CQ_2 = 32{,}591$	
[Object function]	
Maximum $\pi = 4000{*}P_1 + 6000{*}P_2 + 8000{*}P_3 - (70{*}DQ_1 + 69{*}DQ_2 + 67{*}DQ_3 - 100{*}P_1 + 150{*}P_2 + 200{*}P_3) - [7{,}022{,}400 + \omega_1{*}6{,}996{,}000 + \omega_2{*}16{,}315{,}200] - 2500{*}B_6 - 200{*}B_{17} - 200{*}B_{27} - 500{*}B_{37} - (250{*}m_1 + 300{*}m_2 + 350{*}m_3) - 250{*}(GCE - CCE))^{*} \varnothing_1 - (250{*}m_1 + 300{*}m_2 + 350{*}m_3 + 250{*}(CCE - GCE))^{*} \varnothing_2 - 10{,}000{,}000$	
[Constraints]	
Direct material discount:	Direct labor:
$10{*}P_1 + 20{*}P_2 + 10{*}P_3 = DQ_1 + DQ_2 + DQ_3$	$4{*}P_1 + 5{*}P_2 + 6{*}P_3 = 52{,}800 - \omega_1{*}26{,}400 - \omega_2{*}52{,}800$
$0 \le DQ_1 \le \alpha_0{*}200{,}000$	$\omega_0 - \beta_1 \le 0$
$\alpha_1{*}200{,}000 < DQ_2 \le \alpha_1{*}500{,}000$	$\omega_1 - \beta_1 - \beta_2 \le 0$
$\alpha_2{*}500{,}000 < DQ_3$	$\omega_2 - \beta_2 \le 0$
$\alpha_0 + \alpha_1 + \alpha_2 = 1$	$\omega_0 + \omega_1 + \omega_2 = 1$
	$\beta_1 + \beta_2 = 1$

Table 5. *Cont.*

Batch Level: Material Handling

$10{*}P_1 + 20{*}P_2 + 10{*}P_3 \leq 70{*}B_6$
$1{*}B_6 \leq 8800$

Machine hours:
o = 1: $2{*}P_1 + 3{*}P_2 + 2{*}P_3 \leq 46{,}200$
o = 2: $3{*}P_1 + 4{*}P_2 + 3{*}P_3 \leq 50{,}400$
o = 3, 4: $1{*}P_1 + 1{*}P_2 + 1.9{*}P_3 \leq 18{,}900$
o = 5: $0.1{*}P_1 + 0.1{*}P_2 + 0.2{*}P_3 \leq 2070$

Linear carbon rights:
$1.5{*}P_1 + 2{*}P_2 + 3{*}P_3 = CQ_1 + CQ_2 = CCE$
$0 \leq CQ_1 \leq 28{,}000{*}\varnothing_1$
$28{,}000{*}\varnothing_1 < CQ_2 \leq 128{,}000{*}\varnothing_2$
$\varnothing_1 + \varnothing_2 = 1$

Batch Level: Settings
$P_1 \leq 2{*}B_{17}$
$P_2 \leq 2{*}B_{27}$
$P_3 \leq 1{*}B_{37}$
$1{*}B_{17} + 1{*}B_{27} + 2.5{*}B_{37} \leq 17{,}600$

Min/Max demand:
$P_1 > 2000$; $P_2 > 1000$; $6000 > P_3 > 2000$

A carbon levy employing a fully graduated rate system inclusive of exemptions:
$1.5{*}P_1 + 2{*}P_2 + 3{*}P_3 = NQ_1 + NQ_2 + NQ_3$
$1.5{*}P_1 + 2{*}P_2 + 3{*}P_3 \leq 28{,}000$
$0 \leq NQ_1 \leq 10{,}000{*}\nu_1$
$10{,}000{*}\nu_2 \leq NQ_2 \leq 20{,}000{*}\nu_2$
$20{,}000{*}\nu_3 \leq NQ_3$
$\nu_1 + \nu_2 + \nu_3 = 1$

Table 6. The Ideal result, Object function, and associated limitations or conditions of the framework 2-1.

[Ideal result]

$\pi = 28{,}338{,}110$; $P_1 = 2006$; $P_2 = 3624$; $P_3 = 5914$; $DQ_1 = 0$; $DQ_2 = 151{,}680$; $DQ_3 = 0$; $\omega_0 = 0.6656061$; $\omega_1 = 0.3343939$; $\omega_2 = 0$; $B_6 = 2167$; $B_{17} = 1003$; $B_{27} = 1812$; $B_{37} = 5914$; $\delta_0 = 0$; $\delta_1 = 0$; $\delta_2 = 0$; $\delta_3 = 1$; $\alpha_1 = 0$; $\alpha_2 = 1$; $\alpha_3 = 0$; $\beta_1 = 1$; $\beta_2 = 0$; $CCE = 27{,}999$

[Object function]

Maximum $\pi = 4000{*}P_1 + 6000{*}P_2 + 8000{*}P_3 - (70{*}DQ_1 + 69{*}DQ_2 + 67{*}DQ_3 - 100{*}P_1 + 150{*}P_2 + 200{*}P_3) - [7{,}022{,}400 + \omega_1{*}6{,}996{,}000 + \omega_2{*}16{,}315{,}200] - 2500{*}B_6 - 200{*}B_{17} - 200{*}B_{27} - 500{*}B_{37} - (\delta_1{*}250{*}(w_1 - 5000) + \delta_2{*}300{*}(w_2 - 5000) + \delta_3{*}350{*}(w_3 - 5000)) - 10{,}000{,}000$

[Constraints]

Direct material discount:
$10{*}P_1 + 20{*}P_2 + 10{*}P_3 = DQ_1 + DQ_2 + DQ_3$
$0 \leq DQ_1 \leq \alpha_0{*}200{,}000$
$\alpha_1{*}200{,}000 < DQ_2 \leq \alpha_1{*}500{,}000$
$\alpha_2{*}500{,}000 < DQ_3$
$\alpha_0 + \alpha_1 + \alpha_2 = 1$

Direct labor:
$4{*}P_1 + 5{*}P_2 + 6{*}P_3 = 52{,}800 - \omega_1{*}26{,}400 - \omega_2{*}52{,}800$
$\omega_0 - \beta_1 \leq 0$
$\omega_1 - \beta_1 - \beta_2 \leq 0$
$\omega_2 - \beta_2 \leq 0$
$\omega_0 + \omega_1 + \omega_2 = 1$
$\beta_1 + \beta_2 = 1$

Batch Level: Material Handling

$10{*}P_1 + 20{*}P_2 + 10{*}P_3 \leq 70{*}B_6$
$1{*}B_6 \leq 8800$

Machine hours:
o = 1: $2{*}P_1 + 3{*}P_2 + 2{*}P_3 \leq 46{,}200$
o = 2: $3{*}P_1 + 4{*}P_2 + 3{*}P_3 \leq 50{,}400$
o = 3, 4: $1{*}P_1 + 1{*}P_2 + 1.9{*}P_3 \leq 18{,}900$
o = 5: $0.1{*}P_1 + 0.1{*}P_2 + 0.2{*}P_3 \leq 2070$

Batch Level: Settings
$P_1 \leq 2{*}B_{17}$
$P_2 \leq 2{*}B_{27}$
$P_3 \leq 1{*}B_{37}$
$1{*}B_{17} + 1{*}B_{27} + 2.5{*}B_{37} \leq 17{,}600$

Min/Max demand:
$P_1 > 2000$; $P_2 > 1000$; $6000 > P_3 > 2000$

A carbon levy employing a fully graduated rate system inclusive of exemptions:
$1.5{*}P_1 + 2{*}P_2 + 3{*}P_3 = w_0 + w_1 + w_2 + w_3$
$1.5{*}P_1 + 2{*}P_2 + 3{*}P_3 \leq 28{,}000$
$0 \leq w_0 \leq 5000{*}\delta_1$
$5000{*}\delta_1 \leq w_1 \leq 15{,}000{*}\delta_1$
$15{,}000{*}\delta_1 \leq w_2 \leq 25{,}000{*}\delta_1$
$25{,}000{*}\delta_1 \leq w_3$
$\delta_0 + \delta_1 + \delta_2 + \delta_3 = 1$

Table 7. The Ideal result, Object function, and associated limitations or conditions of the framework 2-2.

[Ideal result]

π = 30,168,590; P_1 = 2000; P_2 = 6910; P_3 = 5257; DQ_1 = 0; DQ_2 = 210,770; DQ_3 = 0; ω_0 = 0.1934848; ω_1 = 0.8065152; ω_2 = 0; B_6 = 3011; B_{17} = 1000; B_{27} = 3455; B_{37} = 5257; δ_0 = 0; δ_1 = 0; δ_2 = 0; δ_3 = 1; α_1 = 0; α_2 = 1; α_3 = 0; β_1 = 1; β_2 = 0; GCE = 28,000; $\varnothing_1$ = 0; $\varnothing_2$ = 1; CQ_1 = 0; CQ_2 = 32,591

[Goal function]

Maximum $\pi = 4000{*}P_1 + 6000{*}P_2 + 8000{*}P_3 - (70{*}DQ_1 + 69{*}DQ_2 + 67{*}DQ_3 - 100{*}P_1 + 150{*}P_2 + 200{*}P_3) - [7{,}022{,}400 + \omega_1{*}6{,}996{,}000 + \omega_2{*}16{,}315{,}200] - 2500{*}B_6 - 200{*}B_{17} - 200{*}B_{27} - 500{*}B_{37} - (\delta_1{*}250{*}(w_1 - 5000) + \delta_2{*}300{*}(w_2 - 5000) + \delta_3{*}350{*}(w_3 - 5000) - 250{*}(GCE - CCE)){*}\varnothing_1 - (\delta_1{*}250{*}(w_1 - 5000) + \delta_2{*}300{*}(w_2 - 5000) + \delta_3{*}350{*}(w_3 - 5000) + 250{*}(CCE - GCE)){*}\varnothing_2 - 10{,}000{,}000$

[Constraints]

Direct material discount:

$10{*}P_1 + 20{*}P_2 + 10{*}P_3 = DQ_1 + DQ_2 + DQ_3$
$0 \le DQ_1 \le \alpha_0{*}200{,}000$
$\alpha_1{*}200{,}000 < DQ_2 \le \alpha_1{*}500{,}000$
$\alpha_2{*}500{,}000 < DQ_3$
$\alpha_0 + \alpha_1 + \alpha_2 = 1$

Direct labor:

$4{*}P_1 + 5{*}P_2 + 6{*}P_3 = 52{,}800 - \omega_1{*}26{,}400 - \omega_2{*}52{,}800$
$\omega_0 - \beta_1 \le 0$
$\omega_1 - \beta_1 - \beta_2 \le 0$
$\omega_2 - \beta_2 \le 0$
$\omega_0 + \omega_1 + \omega_2 = 1$
$\beta_1 + \beta_2 = 1$

Batch Level: Material Handling

$10{*}P_1 + 20{*}P_2 + 10{*}P_3 \le 70{*}B_6$
$1{*}B_6 \le 8800$

Batch Level: Settings

$P_1 \le 2{*}B_{17}$
$P_2 \le 2{*}B_{27}$
$P_3 \le 1{*}B_{37}$
$1{*}B_{17} + 1{*}B_{27} + 2.5{*}B_{37} \le 17{,}600$

Machine hours:

o = 1: $2{*}P_1 + 3{*}P_2 + 2{*}P_3 \le 46{,}200$
o = 2: $3{*}P_1 + 4{*}P_2 + 3{*}P_3 \le 50{,}400$
o = 3, 4: $1{*}P_1 + 1{*}P_2 + 1.9{*}P_3 \le 18{,}900$
o = 5: $0.1{*}P_1 + 0.1{*}P_2 + 0.2{*}P_3 \le 2070$

Min/Max demand:

$P_1 > 2000$; $P_2 > 1000$; $6000 > P_3 > 2000$

Linear carbon rights:

$1.5{*}P_1 + 2{*}P_2 + 3{*}P_3 = CQ_1 + CQ_2 = CCE$
$0 \le CQ_1 \le 28{,}000{*}\varnothing_1$
$28{,}000{*}\varnothing_1 < CQ_2 \le 128{,}000{*}\varnothing_2$
$\varnothing_1 + \varnothing_2 = 1$

A carbon levy employing a fully graduated rate system inclusive of exemptions:

$1.5{*}P_1 + 2{*}P_2 + 3{*}P_3 = w_0 + w_1 + w_2 + w_3$
$1.5{*}P_1 + 2{*}P_2 + 3{*}P_3 \le 28{,}000$
$0 \le w_0 \le 5000{*}\delta_1$
$5000{*}\delta_1 \le w_1 \le 15{,}000{*}\delta_1$
$15{,}000{*}\delta_1 \le w_2 \le 25{,}000{*}\delta_1$
$25{,}000{*}\delta_1 \le w;$
$\delta_0 + \delta_1 + \delta_2 + \delta_3 = 1$

Model 2-2, presented in Table 7, introduces an enhanced carbon entitlement feature. This model achieves a maximum profit of $30,168,590. In other words, this indicates that the company has achieved an improvement of $1,830,480 in efficiency or profitability over the first model. Particularly noteworthy is the increase in the production of truck rims, with volumes at 2000, 6910, and 5257 units, respectively. (product 2) due to the carbon rights trading scheme. The aluminum ingots are still purchased under a phase 2 discount ($\alpha 2 = 1$), with 210,770 units bought at $79 each. Labor utilization continues at the second stage ($\beta 1 = 1$), signifying ongoing overtime work. The model involves 3011 material handling batches and 1000, 3455, and 5257 batches for the setup of each product. Labor hours reach 74,092, costing $12,664,780; carbon emissions are at 32,591, maxing out the government's limit and necessitating the purchase of an additional 4591 units of carbon rights. The carbon tax amounts to $9,656,850, and the expense for carbon rights is $1,147,750. This model demonstrates that with a carbon rights trading mechanism implemented by the government, businesses can achieve higher profits and greater flexibility compared to traditional models.

4.1.5. Model Comparison

In this section, we conduct a comparative assessment between Model 1 and Model 2, as outlined in Table 8. The comparison encompasses various aspects, including profitability, production volume, carbon emissions, and the associated expenses related to carbon taxes and carbon rights.

Table 8. Model 1 and model 2 comparison of single-phase model.

Model	Gain	Volume of Merchandise	CO_2 Discharge	Emissions Tax	Emission Credits	Price of Emission Permits	Amount of Supplies Bought	Workforce Expenditure	Working Hours
1-1	26,588,110	P_1 = 2006; P_2 = 3624; P_3 = 5914	27,999	9,799,650	0	0	151,680	9,361,820	61,628
1-2	28,418,590	P_1 = 2000; P_2 = 6910; P_3 = 5257	32,591	11,406,850	4591	1,147,750	210,770	12,664,780	74,092
2-1	28,338,110	P_1 = 2006; P_2 = 3624; P_3 = 5914	27,999	8,049,650	0	0	151,680	9,361,820	61,628
2-2	30,168,590	P_1 = 2000; P_2 = 6910; P_3 = 5257	32,591	9,656,850	4591	1,147,750	210,770	12,664,780	74,092

Profitability: Among the models, Model 2-2 emerges as the most profitable option. This observation mirrors real-world scenarios, where the application of a non-continuous carbon tax structure with exemptions and the incorporation of carbon rights trading leads to enhanced profitability. The profitability ranking, in descending order, is as follows: Model 2-2, Model 2-1, Model 1-2, and Model 1-1.

Production Volume: Models 1-1 and 2-1 yield identical production volumes, possibly due to the consistent upper limit on carbon emissions imposed in both models. This suggests that maintaining a uniform upper limit results in a similar product mix. Similarly, Models 1-2 and 2-2 also generate an equivalent quantity of goods. Carbon Emissions: Models 1-1 and 2-1 adhere to government-mandated carbon emission limits, while Models 1-2 and 2-2 exceed these limits. The latter models compensate by purchasing carbon rights, allowing the company to maximize profits by increasing production.

The trading system for carbon credits, as outlined in Section 4.2, provides corporations with the ability to alter their production rates. In our research, the carbon trading tax has been established at $250. Simulations using the LINGO program suggest a preference for increased production when acquiring carbon credits. Nevertheless, with a rise in the carbon trading tax, selling carbon credits might become a more lucrative option for companies than increasing their output. Carbon Tax Cost: When comparing models with carbon rights trading, model 1-2 incurs the highest carbon tax cost. This model, featuring a discontinuous carbon tax function without tax allowance and carbon rights trading, applies different tax rates based on emission levels. Consequently, this model experiences relatively higher costs.

Cost of Carbon Rights: Both models 1-2 and 2-2 incur a carbon rights cost of 4951. For aspects like material purchase quantity, labor cost, and labor hours, Models 1-1 and 2-1 yield identical results.

Overall, this comparative analysis highlights how different configurations of carbon tax and trading mechanisms can significantly impact a company's financial performance, production decisions, and environmental footprint. This understanding is crucial for companies navigating carbon regulations and for policymakers aiming to balance environmental objectives with economic realities.

4.1.6. Review

This study investigates the sensitivity of margins and product mix to changes in the production environment, as depicted in Table 8. Fluctuations in resources, such as employee turnover or machine malfunctions, greatly influence the outcome. Companies often opt for the most profitable model, with model 2-2 being preferred due to favorable tax exemptions. However, from a governmental standpoint, the main aim is carbon emission reduction, rendering models 1-1 and 2-1 equally effective. Yet, for maximizing carbon tax revenue, model 1-1 stands out as more impactful on government finances.

The carbon tax rates used in this research are derived from World Bank Group data and pricing in China. It is essential for governments to tailor these rates to their country's fair market values, as carbon taxation is becoming increasingly prevalent. This necessitates managerial vigilance regarding the effects of carbon taxes and rights on business operations.

The implementation of carbon taxes and the utilization of emissions trading rights have a significant influence on a firm's profitability and its range of products. With global trends moving toward carbon emission pricing, companies are advised to reduce their carbon emissions during production to minimize the financial impact of these measures. Governments, on the other hand, grapple with the challenge of determining whether carbon tax rates and emission limits should be industry-specific, taking into account that some industries may naturally produce higher carbon emissions due to their scale or inherent characteristics.

Deciding whether to implement a carbon tax, establish carbon trading, or a combination of both is a critical decision for governments. This choice will significantly influence businesses and, potentially, consumers. A high tax rate could prompt companies to shift production to countries with lower or no carbon taxes, which might generate significant revenue for environmental protection but could also impact economic activities and global competitiveness. On the other hand, a low tax rate may not effectively achieve environmental objectives. Additionally, if the tax is reflected in product pricing, it could lead to more complex economic repercussions.

Therefore, it is crucial for policymakers to find an equilibrium in carbon tax policies that aligns environmental objectives with economic realities, ensuring that environmental goals are met without negatively impacting economic activities and global competitiveness.

4.2. Discussion

Integrating the goal of achieving net-zero carbon emissions by 2050 into our discussion, this study highlights the pivotal role of Activity-Based Costing (ABC) and Industry 4.0 technologies in facilitating strategic planning for businesses transitioning towards this environmental milestone. By evaluating and adjusting production processes to reduce carbon emissions and improve resource efficiency, alongside leveraging technological innovations to optimize production, minimize energy consumption, and enhance material utilization, the research outlines a practical roadmap for organizations. This approach not only emphasizes the balance between environmental management and economic benefits but also offers a strategic perspective on achieving sustainable development.

This research's primary contribution is offering governments and enterprises a thorough insight into various scenarios involving carbon taxes and rights. It presents four models: incremental tax rates without allowances, both with and without trading, and incremental tax rates with allowances, again with and without trading. It compares each model, helping companies prepare for post-policy effects and governments to understand the impact on businesses.

While the production costs of green products may sometimes exceed those of conventional offerings, there is potential for this cost differential to diminish through economies of scale and the adoption of new technologies. For instance, continual advancements in battery technologies for electric vehicles can provide more affordable options to consumers over time. Additionally, governmental subsidies and tax policies may further promote the cost-effectiveness of green products. When evaluated across the entire lifecycle, the long-term environmental and social benefits of green products could offset their initially higher input costs, conferring overall cost-effectiveness. However, it should be acknowledged that some industries and regional contexts may continue to face economic barriers in transitioning to fully sustainable solutions without sufficient structural support systems in areas like research funding, policy incentives, and public awareness campaigns. Assuming that digital assembly sites are widely used, we have reason to expect that production efficiency may be improved because automated processes reduce bottlenecks and errors, and assembly tasks can be completed quickly; at the same time, product quality may also be improved because digital processes reduce manual work. The errors caused are expected to reduce the finished product defect rate; in addition, reducing direct human-computer interaction is expected to reduce operational safety hazards. Of course, the above-predicted benefits need to be verified by subsequent research. This article provides a preliminary exploration of the potential effectiveness of imagining digital applications.

Based on our research into managing carbon emissions in the aluminum alloy wheel industry, incorporating Activity-Based Costing (ABC) enhances cost precision, which supports better production and environmental management decisions. Utilizing carbon tax and emissions trading schemes helps balance environmental and economic objectives. Adopting Industry 4.0 technologies can optimize production processes, reduce emissions, and maximize profitability. Continuous improvement, through the Theory of Constraints (TOC), identifies and resolves system bottlenecks, improving operational efficiency and environmental performance. Integrating carbon management into strategic planning allows for taking advantage of trading opportunities and mitigating costs related to carbon taxes and emissions limits, enhancing sustainability practices while maintaining or improving profitability.

This study is mainly based on the internal data provided by actual enterprises. By using LINGO optimization software to solve the quantitative models, the optimal solutions are obtained. In order to simulate various constraint conditions in real situations, we set up multiple data restrictions related to hub manufacturing, such as the upper limit of hub production volume, carbon emission limits, raw material procurement quota limits, and related economic constraints, including labor costs and processing time requirements. Then, we use LINGO software to solve the maximum profit value against these complex constraint relationships and preset data restriction conditions. The final results can help us evaluate the impacts of different carbon tax and carbon trading mechanisms on corporate economic and environmental outcomes.

5. Conclusions

The aluminum alloy wheel industry serves as an exemplary case study for examining the impacts of carbon emissions and carbon taxation due to multiple compelling reasons. Firstly, as a critical downstream application of aluminum alloy materials, the industry experiences substantial market demand, making the study of its carbon footprint and the effects of carbon taxation highly relevant and practical. The production process of aluminum alloy wheels is inherently complex, involving multiple stages that lend themselves well to the application of activity-based costing methods for detailed cost analysis. This complexity also indicates a significant potential for energy efficiency improvements through the adoption of new energy technologies, aligning with the principles of sustainable development.

Manufacturers in this sector are increasingly subject to stringent environmental regulations and policies aimed at reducing carbon emissions. Investigating the economic impacts of carbon taxation and carbon trading within this context can provide actionable insights and recommendations for navigating these challenges. The high level of data transparency within the aluminum alloy wheel industry facilitates modeling and comparative analysis, enhancing the reliability and relevance of research findings.

The industry's substantial production volume means that policy changes have a pronounced impact, making it an ideal candidate for studying the broader effects of environmental policies on industrial sectors. The aluminum alloy wheel industry's characteristics and the pressures it faces can reflect the broader situation for industrial enterprises, making it a potent example of how industries can adapt to and mitigate the impacts of carbon emissions regulations.

Choosing the aluminum alloy wheel industry for this case study is justified by its significant demand, complex production processes, the potential for energy efficiency improvements, the pressure from environmental policies, high data transparency, the noticeable impact of policies, and its representativeness of industrial enterprises' challenges and opportunities in the face of carbon emissions regulations. This research aims to provide a comprehensive analysis that can offer valuable insights not only for the aluminum alloy wheel industry but also for other sectors facing similar challenges.

This paper explores the challenges and complexities surrounding global efforts to combat climate change, particularly in light of the goals set forth in the 2015 Paris Agreement within the UNFCCC framework. It is noteworthy that nations actively implementing climate-related policies are responsible for less than 20% of the total global greenhouse gas emissions. This underscores the formidable task of effectively implementing strategies to combat climate change.

Building upon the influential 2019 research conducted by Tsai et al. [11], this research investigates the application of eco-friendly Activity-Based Costing (ABC) in the aluminum alloy rim industry within the context of Industry 4.0. The study introduces four distinct eco-friendly ABC production planning models: one involving a carbon tax with a comprehensive progressive rate, excluding exemptions; a similar model that incorporates carbon trading mechanisms; a carbon tax model with a complete progressive rate, including allowances; and a variant that combines carbon trading. These models take into account essential cost elements such as direct labor, material costs, batch-level activities such as material handling and setup, and constraints related to machine labor. Models 1-2 and 2-2 specifically focus on the integration of carbon rights trading mechanisms. To handle complex calculations efficiently, the LINGO software tool is employed. Findings from the study reveal the potential advantages for companies in integrating a carbon rights trading mechanism, possibly leading to increased profits. This is demonstrated in a scenario applying a 5000-unit tax exemption. The paper posits that the implementation of carbon rights trading by governments could lead to setting tax-free quotas based on average enterprise carbon emissions, offering a more equitable and manageable approach.

Nevertheless, the study recognizes certain limitations within these models, such as the exclusion of factors like recycling systems, which could reintegrate waste into production, thereby cutting material costs and carbon footprints. Firstly, the model does not consider factors such as waste recycling systems that could impact costs and carbon emissions. Additionally, the linearized model for carbon trading rights is overly simplistic and unable to fully reflect complexities. These limitations may affect the applicability of the research findings and conclusions. The simplistic linear approach of the carbon rights model in this analysis also restricts its intricacy; thus, future research might delve into more advanced carbon rights models to better gauge their impact on corporate profits.

The paper suggests future research avenues that concentrate on more elaborate models, accurately reflecting the complexities of environmental policies, industry adherence, and economic incentives. Such future studies could yield a more profound understanding of how environmental regulations can be fine-tuned for both ecological and economic sustainability, building upon the foundational work of Tsai et al. [11]. This research lays a positive foundation. Future studies will construct more sophisticated models that accurately reflect the intricate relationships between environmental policies, industry compliance, and economic incentives. According to the research results, companies can evaluate combinations of different operating strategies and identify optimal solutions when facing carbon tax adjustments. Moreover, when setting industry-specific carbon tax levels, policymakers should consider factors such as industry differences, technical status, and cost-bearing capabilities to strike a balance between emission reduction targets and economic vitality. This will help provide theoretical support for the dual goal of environmental and economic sustainability. For example, dynamic carbon tax models can be established to track the impact of multi-year carbon tax reforms on corporate performance. Alternatively, differentiated carbon tax models can be developed to guide policymaking.

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References

1. Metz, B.; Davidson, O.; Bosch, P.; Dave, R.; Meyer, L. *Climate Change 2007: Mitigation of Climate Change*; Cambridge University Press: Cambridge, MA, USA, 2007.
2. Chen, L.; He, F. Efficiency Analysis of Carbon Emission Quotas. *J. Sci. Ind. Res.* **2017**, *76*, 461–464.
3. Ellerman, A.D.; Convery, F.J.; De Perthuis, C. *Pricing Carbon: The European Union Emissions Trading Scheme*; Cambridge University Press: Cambridge, MA, USA, 2010.
4. Kunsch, P.; Springael, J. Simulation with system dynamics and fuzzy reasoning of a tax policy to reduce CO_2 emissions in the residential sector. *Eur. J. Oper. Res.* **2008**, *185*, 1285–1299. [CrossRef]
5. Zhang, L.; Yang, W.; Yuan, Y.; Zhou, R. An integrated carbon policy-based interactive strategy for carbon reduction and economic development in a construction material supply chain. *Sustainability* **2017**, *9*, 2107. [CrossRef]
6. Liu, W.; Xie, W.; Qiu, P. The impact of carbon emission cap and carbon trade mechanism on the order decision with stochastic demand. *Int. J. Shipp. Transp. Logist.* **2015**, *7*, 347–376. [CrossRef]
7. Daga, A. New Global Carbon Exchange to Launch in Singapore by Year-End. Reuters. Available online: https://www.reuters.com/business/sustainable-business/singapores-dbs-stanchart-team-up-global-carbon-credit-exchange-2021-05-20/ (accessed on 20 May 2021).
8. Du, S.; Qian, J.; Liu, T.; Hu, L. Emission allowance allocation mechanism design: A low-carbon operations perspective. *Ann. Oper. Res.* **2020**, *291*, 247–280. [CrossRef]
9. Fang, C.; Ma, T. Technology adoption with carbon emission trading mechanism: Modeling with heterogeneous agents and uncertain carbon price. *Ann. Oper. Res.* **2021**, *300*, 577–600. [CrossRef]
10. Environmental Protection Administration, Executive Yuan. *2018 Republic of China National Greenhouse Gas Inventory Report*; Environmental Protection Administration, Executive Yuan: Taipei, Taiwan, 2018.
11. United Nations Framework Convention on Climate Change (UNFCCC). *Adoption of the Paris Agreement. I: Proposal by the President (Draft Decision)*; United Nations Office: Geneva, Switzerland, 2015.
12. Tsai, W.H.; Chu, P.Y.; Lee, H.L. Green activity-based costing production planning and scenario analysis for the aluminum-alloy wheel industry under industry 4.0. *Sustainability* **2019**, *11*, 756. [CrossRef]
13. World Bank. State and Trends of Carbon Pricing. 2023. Available online: https://www.worldbank.org/en/research (accessed on 1 January 2024).
14. Parry, I.W.H.; Black, S.; Zhunussova, K. *Carbon Taxes or Emissions Trading Systems? Instrument Choice and Design*; International Monetary Fund: Washington, DC, USA, 2022; 25p.
15. Jonsson, S.; Ydstedt, A.; Asen, E. *Looking Back on 30 Years of Carbon Taxes in Sweden*; Fiscal Fact No. 727; The Tax Foundation: Washington, DC, USA, 2020.
16. World Economic Forum. Global Carbon Pricing Can Reduce Emissions and Pay for Itself. Available online: https://www.weforum.org/press/2021/11/global-carbon-pricing-can-reduce-emissions-and-pay-for-itself/ (accessed on 1 January 2024).
17. Hick, S. Morals Maketh the money. *Aust. CPA* **2000**, *70*, 72–73.
18. Shapiro, J. *Modeling the Supply Chain*; Nelson Education: Toronto, ON, Canada, 2006.
19. Das, S.K.; Green, J.A.S.; Kaufman, J.G. The development of recycle-friendly automotive aluminum alloys. *JOM* **2007**, *59*, 47–51. [CrossRef]
20. Ascione, A. *What is the Difference between Hastelloy® and Incoloy®?* Continental Steel & Tube Company: Fort Lauderdale, FL, USA, 2015.
21. Dwivedi, S.P.; Sharma, S.; Mishra, R.K. Characterization of waste eggshells and $CaCO_3$ reinforced AA2014 green metal matrix composites: A green approach in the synthesis of composites. *Int. J. Precis. Eng. Manuf.* **2016**, *17*, 1383–1393. [CrossRef]
22. The Aluminum Association. Aluminum: The element of sustainability. In *A North American Aluminum Industry Sustainability Report*; The Aluminum Association: Arlington, VA, USA, 2011; p. 33.
23. Kaplan, R.S. Management accounting for advanced technological environments. *Science* **1989**, *245*, 819–823. [CrossRef]
24. Malmi, T. Activity-based costing diffusion across organizations: An exploratory empirical analysis of Finnish firms. *Account. Organ. Soc.* **1999**, *24*, 649–672. [CrossRef]

25. Cao, P.; Toyabe, S.; Kurashima, S.; Okada, M.; Akazawa, K. A modified method of activity-based costing for objectively reducing cost drivers in hospitals. *Methods Inf. Med.* **2006**, *45*, 462–469. [PubMed]
26. Lin, B.Y.J.; Chao, T.H.; Yao, Y.; Tu, S.M.; Wu, C.C.; Chern, J.Y.; Chao, S.H.; Shaw, K.Y. How can activity-based costing methodology be performed as a powerful tool to calculate costs and secure appropriate patient care? *J. Med. Syst.* **2007**, *31*, 85–90. [PubMed]
27. Kee, R.; Schmidt, C. A comparative analysis of utilizing activity-based costing and the theory of constraints for making product-mix decisions. *Int. J. Prod. Econ.* **2000**, *63*, 1–17. [CrossRef]
28. Holmen, J.S. ABC vs. TOC: It's a matter of time. *Strateg. Financ.* **1995**, *76*, 37.
29. Kim, Y.-W.; Ballard, G. Activity-Based Costing and Its Application to Lean Construction. In Proceedings of the 9th Annual Conference of the International Group for Lean Construction, Singapore, 6–8 August 2001.
30. Tsai, W.H.; Hsu, J.L.; Chen, C.H.; Chou, Y.W.; Lin, S.J.; Lin, W.R. Application of ABC in hot spring country inn. *Int. J. Manag. Enterp. Dev.* **2010**, *8*, 152–174. [CrossRef]
31. Tsai, W.-H. Activity-based costing model for joint products. *Comput. Ind. Eng.* **1996**, *31*, 725–729. [CrossRef]
32. Tsai, W.H.; Yang, C.H.; Chang, J.C.; Lee, H.L. An activity-based costing decision model for life cycle assessment in green building projects. *Eur. J. Oper. Res.* **2014**, *238*, 607–619. [CrossRef]
33. Fichman, R.G.; Kemerer, C.F. Activity-Based Costing for Component-Based Software Development. *Inf. Technol. Manag.* **2002**, *3*, 137–160. [CrossRef]
34. Roztocki, N. Using the Integrated Activity-Based Costing and Economic Value Added Information System for Project Management. In Proceedings of the 7th Americas Conference on Information Systems, Boston, MA, USA, 3–5 August 2001.
35. AlMaryani, M.A.H.; Sadik, H.H. Strategic management accounting techniques in Romanian companies: Some survey evidence. *Procedia Econ. Financ.* **2012**, *3*, 387–396. [CrossRef]
36. Almeida, A.; Cunha, J. The implementation of an activity-based costing (ABC) system in a manufacturing company. *Procedia Manuf.* **2017**, *13*, 932–939. [CrossRef]
37. Goldratt, E.M.; Cox, J. *The Goal: Excellence in Manufacturing*; North River Press: Great Barrington, MA, USA, 1984.
38. Radovilsky, Z.D. A quantitative approach to estimate the size of the time buffer in the theory of constraints. *Int. J. Prod. Econ.* **1998**, *55*, 113–119. [CrossRef]
39. Pember, A.; Lemon, M. *Measuring and Managing Environmental Sustainability: Using Activity-Based Costing/Management (ABC/M)*; The Consortium for Advanced Management—International (CAM—I): Austin, TX, USA, 2012.
40. Cooper, R.; Kaplan, R.S. Measure costs right: Make the right decision. *CPA J.* **1988**, *60*, 38.
41. Wesumperuma, A.; Ginige, A.; Ginige, A.; Hol, A. Green Activity Based Management (ABM) for Organisations. In Proceedings of the Australasian Conference on Information Systems, ACIS 2013, Melbourne, Australia, 4–6 December 2013.
42. Jaedicke, R.K. Improving breakeven analysis by linear programming techniques. *NAA Bull.* **1961**, *42*, 5–12.
43. Balakrishnan, J.; Cheng, C.H. Discussion: Theory of constraints and linear programming: A re-examination. *Int. J. Prod. Res.* **2010**, *38*, 1459–1463. [CrossRef]
44. Fahimnia, B.; Sarkis, J.; Choudhary, A.; Eshragh, A. Tactical supply chain planning under a carbon tax policy scheme: A case study. *Int. J. Prod. Econ.* **2015**, *164*, 206–215. [CrossRef]
45. Demeere, N.; Stouthuysen, K.; Roodhooft, F. Time-driven activity-based costing in an outpatient clinic environment: Development, relevance and managerial impact. *Health Policy* **2009**, *92*, 296–304. [CrossRef]
46. Hsieh, C.L.; Tsai, W.H.; Chang, Y.C. Green activity-based costing production decision model for recycled paper. *Energies* **2020**, *13*, 2413. [CrossRef]
47. Schulze, M.; Seuring, S.; Ewering, C. Applying activity-based costing in a supply chain environment. *Int. J. Prod. Econ.* **2012**, *135*, 716–725. [CrossRef]
48. Tsai, W.H.; Lin, W.R.; Fan, Y.W.; Lee, P.L.; Lin, S.J.; Hsu, J.L. Applying a mathematical programming approach for a green product mix decision. *Int. J. Prod. Res.* **2012**, *50*, 1171–1184. [CrossRef]
49. Zheng, C.W.; Abu, M.Y. Application of activity based costing for palm oil plantation. *J. Mod. Manuf. Syst. Technol.* **2019**, *2*, 1–14. [CrossRef]
50. Kee, R. The sufficiency of product and variable costs for production-related decisions when economies of scope are present. *Int. J. Prod. Econ* **2008**, *114*, 682–696. [CrossRef]
51. Dekker, R.; Bloemhof, J.; Mallidis, I. Operations research for green logistics—An overview of aspects, issues, contributions, and challenges. *Eur. J. Oper. Res.* **2012**, *219*, 671–679. [CrossRef]
52. Qian, L.; David, B.A. Parametric cost estimation based on activity-based costing: A case study for design and development of rotational parts. *Int. J. Prod. Econ.* **2008**, *113*, 805–818. [CrossRef]
53. Plenert, G. Optimizing theory of constraints when multiple constrained resources exist. *Eur. J. Oper. Res.* **1993**, *70*, 126–133. [CrossRef]