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Determination of Critical Point of Particle Migration Direction in a Confined Shear Flow of Giesekus Fluids

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Abstract: Migration of a particle in a confined shear flow of Giesekus fluids is investigated numerically with the method of direct forcing/fictitious domain. We focus on the migration direction for the particle with initial lateral position y_0 and determination of critical point y_c of a particle moving towards the center line or wall. The effect of viscosity ratio μ_r , shear-thinning parameter α , Weissenberg number Wi , and blocking rate β on the value y_c is analyzed. The results showed that when $\mu_r \leq 0.5$, the particle will migrate towards the wall regardless of the value of y_0 . When $\mu_r > 0.5$, y_c increases with increasing μ_r , and some particles will migrate towards the center line with the increase in μ_r . The particle is more likely to migrate towards the center line at small values of Wi and α but at large values of μ_r . The impact of Wi and β on the particle migration direction is more obvious. The particle will migrate towards the wall for $\beta = 0.3$ and is more likely to migrate towards the wall with increasing β . α and Wi have little influence on the pressure distribution in the case of the same β and μ_r . The particle near the wall will migrate faster because large positive pressure and negative pressure appear around the particle.

Keywords: spherical particle; Giesekus fluid; shear flow; migration direction; critical point; numerical simulation



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1. Introduction

Particle migration in non-Newtonian fluids occurs in nature and many practical applications, such as chemical, mechanical, medical, and biological processes, so the dynamics of particle migrating in non-Newtonian fluids has received great attention. For example, blood flowing in the micro-circulation system includes the highly deformable red blood cells, which could transport oxygen and cause blood to have non-Newtonian characteristics [1]. Particle focusing in the microfluid device is an essential step before detecting, counting, analyzing, and sorting particles in the applications [2–4]. Hinder particle adhesion to the wall is important for the precious samples [5]. It also has applications in medical diagnostics, therapeutics [6], cell biology, and solar energy—for instance, the platelets are transfused to the patients during surgery; the cells are isolated to reduce the heterogeneity of the sample in order to study the characteristics of cells [7]; the particles are transferred from suspending medium to the others for washing or coating; and the metallic or non-metallic particles are mixed into the fluid of a solar thermal collector to improve the thermophysical property, enhance the efficiency of the renewable energy, and provide net environmental benefits [8].

Given the various applications described above, many scientists have investigated the particle migration in non-Newtonian fluids by using its rheological property and manipulated particles to migrate in an expected way, which did not work in Newtonian fluids. A particle in a pipe flow of Newtonian fluids would migrate to the lateral position between the wall and center of the pipe, and the particle of a different size would migrate to a different lateral position. Such migration phenomenon is not obvious for the submicron or nanoscale particles. Therefore, some scientists focused on the particle lateral migration

in non-Newtonian fluids for improving separation efficiency of particles. Yang et al. [9] revealed experimentally that there were many equilibrium positions, such as centerline and corners along the lateral direction under only viscoelastic effect in a rectangular microchannel. Wang et al. [10] revealed numerically that the particle would migrate to the center or the closest corner of the channel depending on the initial position, and these conclusions were the same as the experimental results [9] and the numerical results. [11]. What is more, the particle would more likely migrate to the corner as the Weissenberg number increases. D'Avino et al. [12] performed a two-dimensional simulation by neglecting the inertia effect and considering the viscoelastic effect alone; the results showed that the particle would move towards the closest wall in all situations, and there existed three regimes related to the particle-wall distance. D'Avino et al. [13] performed a three-dimensional simulation and found that wall confinement and viscoelasticity would induce particle migration. Caserta et al. [14] conducted an experiment and confirmed the above two numerical results. Liu et al. [15] showed that the particle always migrated towards the wall regardless of changes in fluid viscosity. The shear-thinning property, wall confinement, and Weissenberg number all facilitated the particle move to the wall.

When considering both effects of fluid inertia and elasticity, Wang et al. [10] indicated that the particle would move to the center of the shorter walls in Giesekus fluids. Raffiee et al. [16] performed the numerical simulation and found that under the condition of large elasticity and high inertial, the particle would aggregate to the off-center point along the axis, which was the same as the experimental results [17]. Jiang et al. [18] also found the particle focusing at the channel centerline, and they indicated that the large flow velocity and particle size could accelerate the particle focusing. Huang et al. [19] simulated numerically the particle migration in a two-dimensional Couette flow to study the inertia and elastic effect and found that the inertia caused the particle to move away from the wall, and the particle migration was controlled by the elastic normal stress. Liu et al. [20] studied numerically the migration of particles in a 3-D shear flow of Giesekus fluids and found that the particle migration could be classified into two patterns, i.e., “returning” and “passing”. The separatrix between these two patterns was dependent on the Weissenberg number and the shear-thinning effect. For three particles, the trajectory of two particles on both sides was different from that of the intermediate particle. The phenomenon of abnormal migration tended to be obvious with increasing the shear-thinning effect, solvent viscosity, and the block ratio but with decreasing the elastic effect [21]. Liu et al. [22] found that the elastic effect accelerated a variation in particle trajectory from the “returning” to the “passing” pattern when the elasticity number was large, while the inertial effect was just the opposite. A large extensional force caused two particles to pass over each other. Wall confinement accelerated a variation in particle trajectory from the “passing” to the “returning” pattern. Liu et al. [23] studied the migration of ellipsoidal particles in a confined shear flow of Giesekus fluids and found that the particle trajectory depended on initial orientation and position. The separatrix between the “returning” and “passing” pattern decreased with the increase in the Weissenberg number. The evolution of particle orientation depended on the initial orientation. The evolution of the particle orientation became slower with the increase in Weissenberg number.

For a confined shear flow, the migration direction and lateral equilibrium position of particles are related to the initial lateral position y_0 of particles, while the value of y_0 is related to the fluid property and blocking ratio. So far, the authors have not seen any report on the influence of fluid property and blocking ratio on y_0 when a particle moves in a confined shear flow of Giesekus fluid. Therefore, the purpose of this study is to investigate the migration direction (towards the center line or wall) for the particle with initial lateral y_0 , determine the critical point y_c of particle moving towards the center line or wall, and analyze the effect of viscosity ratio, shear-thinning parameter, Weissenberg number, and blocking rate on the value of y_c .

2. Equations and Numerical Method

The system is shown in Figure 1, where a particle with diameter of d migrates in a confined shear flow of viscoelastic Giesekus fluid. W , L , and H are used to represent the width, length, and height of the simulation domain, respectively. The origin of coordinates O is at the center of the simulation domain. Two planes move in parallel with the opposite direction at velocity of $U_0/2$, so the shear rate of the flow is $\dot{\gamma} = U_0/H$. The blocking rate is $\beta = d/H$.

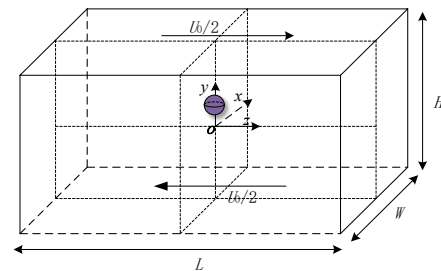


Figure 1. Diagram of 3D-confined flow and the coordinate system.

The numerical simulation is performed using the direct forcing/fictitious domain (DF-FD) method. The principle of DF-FD is that the space occupied by the particle is assumed to be filled with fluid (fictitious fluid) and a pseudo force acts on the particle to force the fictitious fluid to satisfy the constraint condition of rigid-body motion.

The fluid density and particle density are ρ_f and ρ_p , respectively; the viscosity from the contribution of non-Newtonian fluid and solvent viscosity are μ_p and μ_s , respectively; the particle volume and moment of inertia are V_p and J_p , respectively; and the particle velocity and angular velocity are $\mathbf{U}_p$ and $\boldsymbol{\omega}_p$, respectively. P and Ω are used to represent the particle domain and whole simulation domain, respectively, consisting of particle and fluid as shown in Figure 2, where ∂P is the boundary of particle.

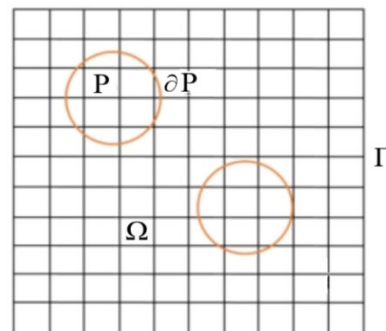


Figure 2. Diagram of different domains.

2.1. Fluid Equations

In Figure 2, the area Ω_f occupied by fluid is $\Omega_f = \Omega - P$. For incompressible flow, the fluid satisfies the continuity and momentum equations:

$$\nabla \cdot \mathbf{u} = 0 \text{ in } \Omega_f, \quad (1)$$

$$\rho_f \frac{d\mathbf{u}}{dt} = \nabla \cdot \boldsymbol{\sigma} \text{ in } \Omega_f, \quad (2)$$

in which ρ_f , $\mathbf{u}$, and $\boldsymbol{\sigma}$ are the fluid density, velocity, and stress tensor, respectively. According to the principle of DF-FD, it is assumed that the area occupied by the particle is filled with fluid, and a virtual volume force $\boldsymbol{\lambda}$ is used to ensure the virtual fluid satisfies the condition of

rigid-body motion. Therefore, by extending the fluid area from Ω_f to the entire calculation area Ω , the control equations satisfied by the flow in the area Ω can be written as:

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega, \quad (3)$$

$$\rho_f \frac{d\mathbf{u}}{dt} = \nabla \cdot \boldsymbol{\sigma} + \boldsymbol{\lambda} \quad \text{in } \Omega, \quad (4)$$

$$\mathbf{u} = \mathbf{U}_p + \boldsymbol{\omega}_p \times \mathbf{r} \quad \text{in } P, \quad (5)$$

where $\mathbf{r}$ is the position vector; the virtual volume force $\boldsymbol{\lambda}$ is exerted on the particle, i.e., $\boldsymbol{\lambda} \neq 0$ in particle area P , and $\boldsymbol{\lambda} = 0$ in the fluid area Ω_f . For incompressible flow, $\nabla \cdot \boldsymbol{\sigma} = -\nabla p + \mu \nabla^2 \mathbf{u}$ (p and μ are the fluid pressure and viscosity, respectively).

2.2. Particle Equations

The motion of particle can be divided into translation and rotation, which can be described by the Newton's second law:

$$m \frac{d\mathbf{U}}{dt} = (1 - \frac{1}{\rho_r}) m \mathbf{g} - \int_{\partial P} \mathbf{n} \cdot \boldsymbol{\sigma} d\mathbf{s}, \quad (6)$$

$$\frac{d(\mathbf{J} \cdot \boldsymbol{\omega}_p)}{dt} = - \int_{\partial P} \mathbf{r} \times (\mathbf{n} \cdot \boldsymbol{\sigma}) d\mathbf{s}, \quad (7)$$

where m is the particle mass; $\mathbf{J}$ is the rotational inertia of particles; $\mathbf{g}$ is acceleration of gravity; $\rho_r = \rho_p/\rho_f$ is the particle/fluid density ratio; and $\mathbf{n}$ is the outer normal direction of particle surface.

In particle area P , integrating Equation (4) and $\mathbf{r} \times$ Equation (4) and substituting it into Equations (6) and (7), respectively, then combining with Equation (5), we have:

$$(1 - \frac{1}{\rho_r}) m (\frac{d\mathbf{U}}{dt} - \mathbf{g}) = - \int_P \boldsymbol{\lambda} d\mathbf{x}, \quad (8)$$

$$(1 - \frac{1}{\rho_r}) \frac{d(\mathbf{J} \cdot \boldsymbol{\omega}_p)}{dt} = - \int_P \mathbf{r} \times \boldsymbol{\lambda} d\mathbf{x}. \quad (9)$$

2.3. Combined Equations

Using length H , velocity U_0 , time H/U_0 , pressure $\rho_f U_0^2$, and pseudo body force $\rho_f U_0^2/H$ as characteristic quantities to dimensionalize the equation, we have the following combined dimensionless continuity and momentum equations:

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega, \quad (10)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \frac{\mu_r \nabla^2 \mathbf{u}}{\text{Re}} - \nabla p + \frac{(1 - \mu_r) \nabla \cdot \mathbf{B}}{\text{ReWi}} + \boldsymbol{\lambda} \quad \text{in } \Omega, \quad (11)$$

$$\mathbf{u} = \mathbf{U} + \boldsymbol{\omega}_p \times \mathbf{r}, \quad \text{in } P, \quad (12)$$

$$(\rho_r - 1) V_p^* \frac{d\mathbf{U}}{dt} = - \int_P \boldsymbol{\lambda} d\mathbf{x}, \quad (13)$$

$$(\rho_r - 1) \mathbf{J}^* \frac{d\boldsymbol{\omega}_p}{dt} = - \int_P \mathbf{r} \times \boldsymbol{\lambda} d\mathbf{x}, \quad (14)$$

and constitutive equation of Giesekus fluids:

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{B} - \mathbf{B} \cdot \nabla \mathbf{u} - (\nabla \mathbf{u})^T \cdot \mathbf{B} + \frac{\alpha}{\text{Wi}} (\mathbf{B} - \mathbf{I})^2 + \frac{\mathbf{B} - \mathbf{I}}{\text{Wi}} = 0 \text{ in } \Omega, \quad (15)$$

in which $\mu_r = \mu_s / \mu_0$ is the viscosity ratio, with μ_0 being the total zero shear rate viscosity ($\mu_0 = \mu_s + \mu_p$); the Reynolds number is $\text{Re} = \rho_f U_0 H / \mu_0$; the Weissenberg number is $\text{Wi} = \lambda_t U_0 / H$, with λ_t being the fluid relaxation time; $\mathbf{B}$ is the configuration tensor and included in the fluid stress $\boldsymbol{\tau} = \mu_p (\mathbf{B} - \mathbf{I}) / \lambda_t$; $V_p^* = V_p / H^3$ is the particle volume; $J^* = J / \rho_p H^5$ is the dimensionless moment of inertia; and the mobility parameter α quantifies the degree of shear-thinning.

2.4. Sub-Problems

A fractional-step time approach is applied to decompose Equations (10)–(15) into the following three sub-problems.

(1) for fluid $\mathbf{u}^*$ and p^{n+1} :

$$\nabla \cdot \mathbf{u}^* = 0, \quad (16)$$

$$\frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} - \frac{\mu_r \nabla^2 \mathbf{u}^*}{2\text{Re}} = -\nabla p + \frac{1}{2} [3\mathbf{G}^n - \mathbf{G}^{n-1}] + \frac{\mu_r \nabla^2 \mathbf{u}^n}{2\text{Re}} + \boldsymbol{\lambda}^n, \quad (17)$$

$$\mathbf{G} = -\mathbf{u} \cdot \nabla \mathbf{u} + \frac{(1 - \mu_r) \nabla \cdot \mathbf{B}}{\text{ReWi}}. \quad (18)$$

Spatial discretization is performed using the projection approach based on finite difference in a uniform half-staggered grid, and all spatial derivatives are discretized using the second-order central difference.

(2) for particle $\mathbf{U}_p^{n+1}$, $\boldsymbol{\omega}_p^{n+1}$, $\boldsymbol{\lambda}^{n+1}$, and $\mathbf{u}^{n+1}$:

$$\rho_r V_p^* \frac{\mathbf{U}_p^{n+1}}{\Delta t} = (\rho_r - 1) V_p^* \frac{\mathbf{U}_p^n}{\Delta t} + \int_P \left(\frac{\mathbf{u}^*}{\Delta t} - \boldsymbol{\lambda}^n \right) d\mathbf{x}, \quad (19)$$

$$\rho_r \frac{J^* \boldsymbol{\omega}_p^{n+1}}{\Delta t} = (\rho_r - 1) \frac{J^* \boldsymbol{\omega}_p^n}{\Delta t} + \int_P \mathbf{r} \times \left(\frac{\mathbf{u}^*}{\Delta t} - \boldsymbol{\lambda}^n \right) d\mathbf{x}, \quad (20)$$

in which $\mathbf{U}_p^{n+1}$ and $\boldsymbol{\omega}_p^{n+1}$ can be calculated directly; $\boldsymbol{\lambda}^{n+1}$ and $\mathbf{u}^{n+1}$ are defined at the Lagrangian and Eulerian nodes, respectively:

$$\boldsymbol{\lambda}^{n+1} = \frac{\mathbf{U}_p^{n+1} + \boldsymbol{\omega}_p^{n+1} \times \mathbf{n} - \mathbf{u}^*}{\Delta t} + \boldsymbol{\lambda}^n, \quad (21)$$

$$\mathbf{u}^{n+1} = \Delta t (\boldsymbol{\lambda}^{n+1} - \boldsymbol{\lambda}^n) + \mathbf{u}^*. \quad (22)$$

$\mathbf{u}$ is transformed from Eulerian to Lagrangian nodes using the tri-linear interpolation function, and $\boldsymbol{\lambda}$ is transformed from Lagrangian to Eulerian nodes.

(3) constitutive equation for $\mathbf{B}$:

$$\frac{\mathbf{B}^{n+1} - \mathbf{B}^n}{\Delta t} + \mathbf{u}^{n+1} \cdot \nabla \mathbf{B}^n - \mathbf{B}^n \cdot \nabla \mathbf{u}^{n+1} - \left(\nabla \mathbf{u}^{n+1} \right)^T \cdot \mathbf{B}^n + \frac{\alpha}{\text{Wi}} (\mathbf{B}^n - \mathbf{I})^2 + \frac{\mathbf{B}^{n+1} - \mathbf{I}}{\text{Wi}} = 0. \quad (23)$$

When solving Equation (23), the scheme of first-order, central difference, and the third-order up-winding MUSCL are used for time, velocity gradient, and convective term, respectively.

2.5. Boundary Condition

The no-slip boundary condition is used at the wall and the particle surface, and the periodic condition is used at the streamwise (z) and crosswise (x) directions.

For a particle moving towards the wall, the simulation would stop when the particle's center is $d/2$ away from the wall. To balance calculation accuracy and efficiency, we set $L = 2H$, the grid size is $1/128$, and the time step is 5×10^{-4} . The independence test of grid and time step size are conducted by changing grid size from $1/136$ to $1/120$ and time step from 2×10^{-4} to 8×10^{-4} , respectively. When all normalized residuals for each quantity are less than 10^{-5} , the calculation is considered convergent. The numerical simulation is completed using a self-compiled program.

2.6. Validation

For verifying the feasibility of the method used here, we compared the present numerical results of particle angular velocity in a three-dimensional shear flow of an Oldroyd-B fluid, with the numerical results using the finite-element method and experimental data for the Boger fluid [24] as shown in Figure 3. It can be seen that the three match well.

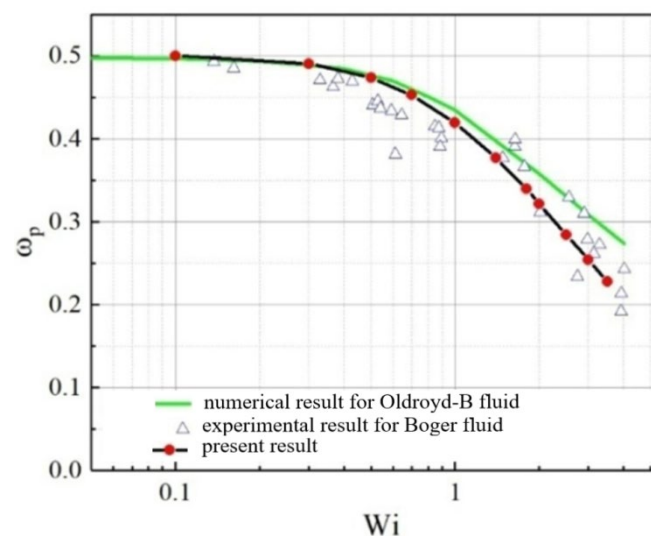


Figure 3. Comparison of particle angular velocity for different values of Wi .

3. Results and Discussion

The initial position of the particle is expressed as (x_0, y_0, z_0) , and the particle is at rest at the initial time. The particle migration direction (towards the center line or wall) is studied for different y_0 , and the critical value y_c (dividing point of particle moving towards the center line and wall) of y_0 is determined. In the simulation, we find that the critical value y_c is 0.146 under the condition of $\mu_r = 0.7$, $\alpha = 0.1$, $Wi = 0.3$, and $\beta = 0.2$. When $y_0 < 0.146$, the particle will migrate towards the central line, otherwise towards the wall. However, when one of μ_r , α , Wi , or β changes, y_c will change; in other words, μ_r , α , Wi , and β have an effect on the value of y_c . The effect is discussed below.

3.1. Effect of the Viscosity Ratio μ_r on the Critical Value y_c

Figure 4 displays the relationship between y_c and μ_r . We can see that when $\mu_r \leq 0.5$, the values of y_c are almost 0 in all cases, i.e., regardless of the value of initial lateral position y_0 , the particle will migrate towards the wall. When $\mu_r > 0.5$, the values of y_c increase with increasing μ_r , and some particles will migrate towards the center line with the increase in μ_r . For example, the particle with $y_c < 0.186$ (Point P₂) at $\mu_r = 0.9$, $\alpha = 0.1$, and $Wi = 0.7$ will migrate towards the center line. Therefore, the particle is more likely to migrate towards the center line in the high viscosity ratio μ_r . When $\mu_r > 0.5$, the values of y_c are dependent on α and Wi . For fixed α , the value of y_c rises with the decrease in Wi ; for fixed Wi , the value of y_c rises with decreasing α , that is, the particle is more likely to migrate towards the center line at small Wi and α .

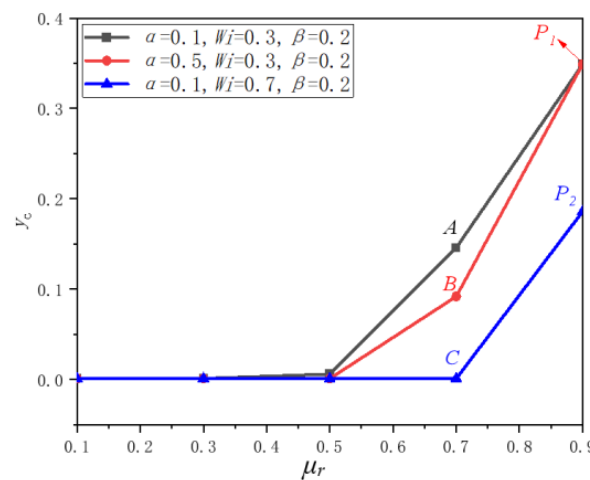


Figure 4. y_c as a function of μ_r for different values of α and Wi ($\beta = 0.2$).

The pressure contours for the three cases in Figure 4 are shown in Figure 5. The pressure is normalized with the average pressure, and positive and negative values indicate the pressures that are greater than or less than the average pressure. The figure shows the pressure distribution of a certain instantaneous flow, and the subsequent movement of particle is determined by the force acted upon the particle by the fluid that is linked to the instantaneous pressure distribution. The particle will migrate towards the region with lower pressure at the next moment. Comparing Figure 5a–c, we can see that the pressure distributions of the three cases are similar, indicating that α and Wi have little influence on the pressure distribution at the same β and μ_r .

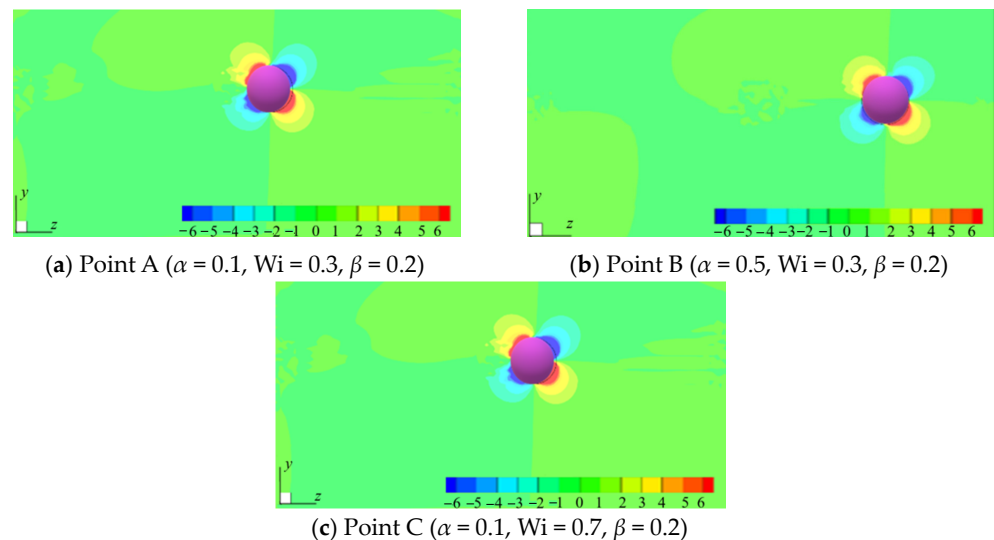


Figure 5. Pressure contours for three cases in Figure 3 ($\mu_r = 0.7$).

3.2. Effect of the Shear-Thinning Parameter α on the Critical Value y_c

α represents the degree of shear-thinning, where large α corresponds to a high degree of shear-thinning. Figure 6 shows y_c as a function of α for different values of μ_r and Wi . We can see that y_c is larger than 0 regardless of what value the parameter takes, indicating that particles may migrate to the wall or to the centerline. For the fluid without shear-thinning effect ($\alpha = 0$), the values of y_c are 0.147, 0.35, and 0.357, respectively. For the case of $\mu_r = 0.9$ and $Wi = 0.3$, as shown in red line, the values of y_c remain 0.35, i.e., the particle with $y_0 < 0.35$ and $y_0 \geq 0.35$ will migrate towards the centerline and wall, respectively. Moreover, the value of y_c is independent of α . For the case of $\mu_r = 0.7$ and $Wi = 0.3$ (black line), the

values of y_c are reduced with the increase in α , i.e., the particle is more likely to move towards the wall at large α . For example, the particle with $\alpha = 0$ will migrate towards the wall when $y_0 > 0.147$, while the particle with $\alpha = 0.7$ will migrate towards the wall when $y_0 > 0.061$. For fixed μ_r , the value of y_c increases with decreasing Wi (comparing black line and blue line); for fixed Wi , the value of y_c increases with increasing μ_r , that is, particle is more likely to migrate towards the center line at small Wi and large μ_r .

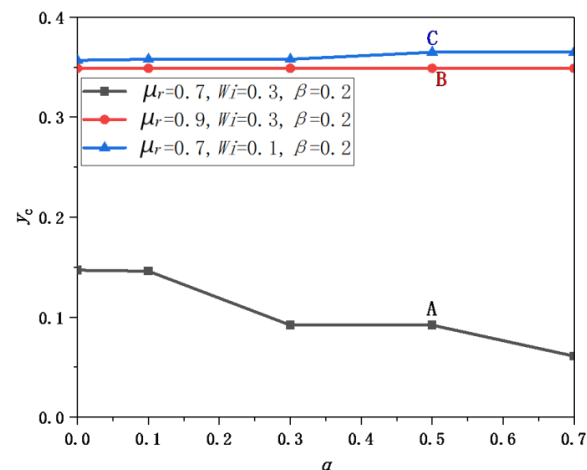


Figure 6. y_c as a function of α for different values of μ_r and Wi ($\beta = 0.2$).

Figure 7 displays the pressure contours at three cases in Figure 6. The explanation of the figure is the same as that of Figure 5. We can see that large red areas (large positive pressure) and large blue areas (large negative pressure) appear around the particle near the wall, showing that the particle near the wall will migrate faster because the particle will migrate towards the area with lower pressure at the next moment.

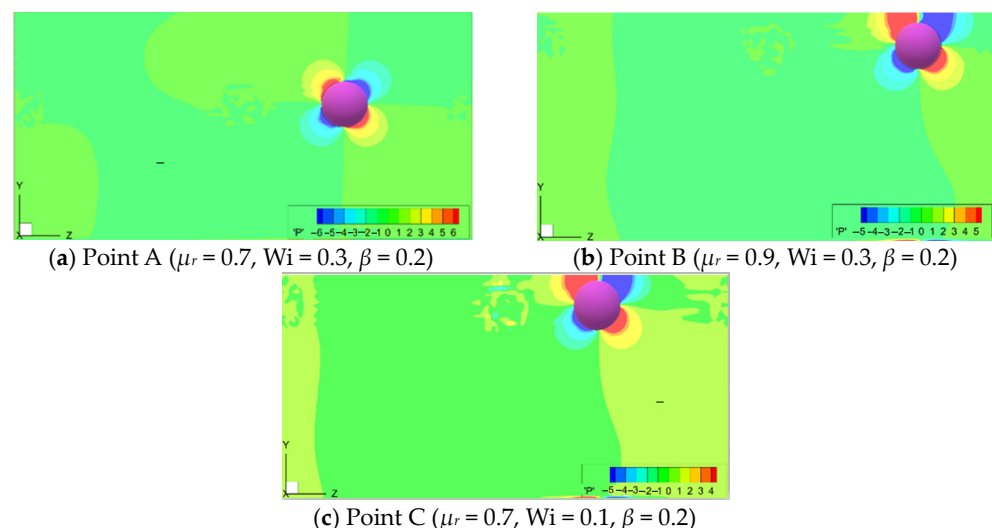


Figure 7. Pressure contours for three cases in Figure 6 ($\alpha = 0.5$).

3.3. Effect of the Weissenberg Number Wi on the Critical Value y_c

Wi is used for representing the relative importance of elastic and viscous force [25]. Figure 8 displays y_c as a function of Wi for different μ_r and α . It can be seen that the values of y_c are large (from 0.358 to 0.365) at $Wi = 0.1$, while they are small (≤ 0.006) for $0.5 < Wi < 1.0$; the values of y_c decrease with the increase in Wi in the range of $0.1 < Wi < 0.5$, indicating that the particle is more inclined to migrate towards the wall with the increases in Wi , and the effect of Wi on the particle migration direction is obvious. For fixed α , the

value of y_c rises with the increase in μ_r ; for fixed μ_r , the value of y_c rises with the decrease in α . Therefore, the particle is more likely to move towards the wall at small μ_r and large α .

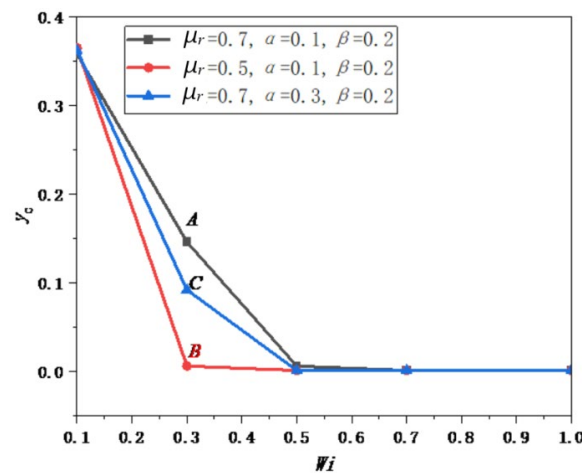


Figure 8. y_c as a function of Wi for different values of μ_r and α ($\beta = 0.2$).

The pressure contours for three cases in Figure 8 are shown in Figure 9. The description of the figure is the same as that of Figure 5. We can see that the particle in the three figures is located between the wall and the centerline, the pressure distributions are similar, i.e., the upper left corner and lower right corner have larger pressure, while the upper right corner and upper left corner have less pressure.

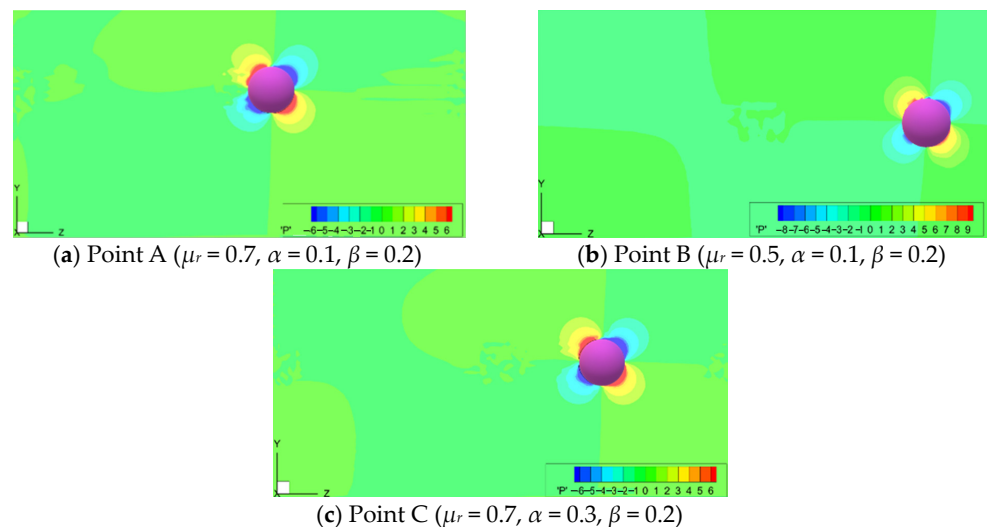


Figure 9. Pressure contours for three cases in Figure 7 ($Wi = 0.3$).

3.4. Effect of the Blocking Rate β on the Critical Value y_c

y_c as a function of β for different values of μ_r and Wi is given in Figure 10a, where we can see that β has a large effect on the particle migration direction. For $\beta = 0.2$, the values of y_c are 0.146 (Point A), 0.35 (Point B), and 0.358 (Point C), while the values of y_c are 0.001 for the three cases at $\beta = 0.3$, i.e., particle will migrate towards the wall for $\beta = 0.3$. The values of y_c decrease with the increase in β , although the change trend of y_c with β is different under different parameters. The particle is more inclined to migrate towards the wall with the increase in β . For fixed μ_r , the value of y_c increases with decreasing Wi ; for fixed Wi , the value of y_c rises with increasing μ_r . Therefore, the particle is more inclined to migrate towards the wall at large Wi and small μ_r .

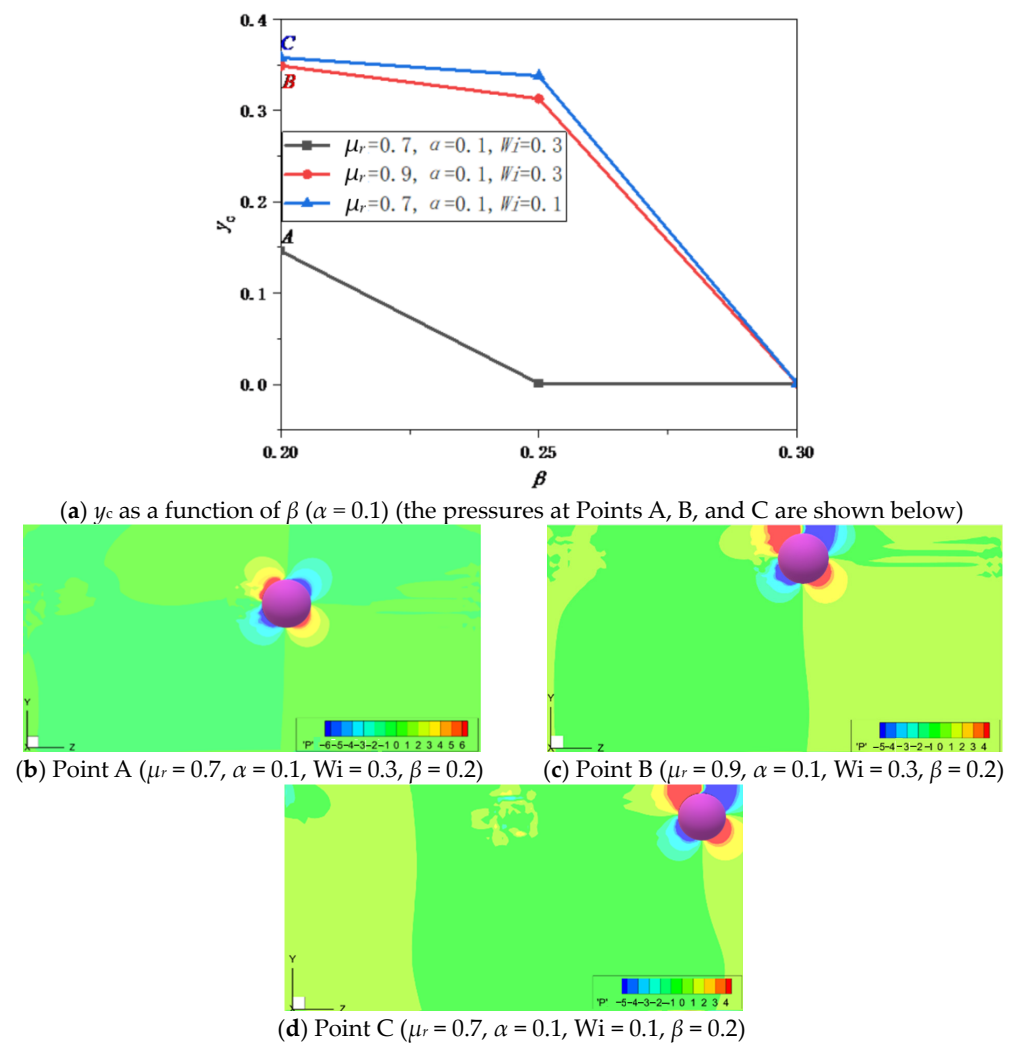


Figure 10. Variation of y_c and pressure contours.

Figure 10b–d shows the pressure contours for three cases in Figure 10a. The pressure distribution in Figure 10b–d is similar to that in Figure 7, that is, large positive pressure and large negative pressure appear around the particle near the wall surface so that the particle near the wall will migrate faster.

4. Conclusions

When a particle migrates in a confined shear flow, the determination of migration direction and final lateral equilibrium position of the particle is important. In this paper, the migration of a particle in a confined shear flow of Giesekus fluids is investigated numerically with the method of direct forcing/fictitious domain. The independence test of grid size and time step size are performed to ensure the reliability of the numerical results. We focused on the particle migration direction (towards the center line or wall) for the particle at initial position y_0 , and determination of dividing point y_c of particle moving towards the center line or wall. The effect of viscosity ratio μ_r , shear-thinning parameter α , Weissenberg number Wi , and blocking rate β on the value y_c is analyzed. Some main conclusions are summarized as follow.

- (1) When $\mu_r \leq 0.5$, regardless of the value of initial position y_0 , the particle will migrate towards the wall. When $\mu_r > 0.5$, the values of y_c increase with increasing μ_r , and some particles will migrate towards the center line with the increase in μ_r . In the case of large μ_r ($\mu_r = 0.9$), the value of y_c is independent of α . In the case of large Wi ($Wi > 0.5$), the values of y_c remain unchanged under different values of μ_r and α . In

other cases, the particle is more likely to migrate towards the center line at small Wi and α but at large μ_r . The reverse is also true, i.e., the particle is more inclined to migrate towards the wall at large Wi and α , but at small μ_r . The influence of Wi and β on the movement direction of the particle is more obvious. The particle will migrate towards the wall for $\beta = 0.3$ regardless of the value of μ_r and Wi , and it is more inclined to migrate towards the wall with increasing β .

- (2) The subsequent movement of particle is determined by the force exerted on the particle by the fluid, which is related to the instantaneous pressure distribution, and the particle will move towards the area with relative low pressure at the next moment. α and Wi have little influence on the pressure distribution in the case of the same β and μ_r . The particle near the wall will migrate faster because large positive pressure and negative pressure appear around the particle, and the particle will migrate towards the area with lower pressure at the next moment.

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