



Article A Fully Coupled Hydro-Mechanical Approach for Multi-Fracture Propagation Simulations

Yinghao Deng ^{1,2,3}, Di Wang ^{1,2}, Yan Jin ^{3,*} and Yang Xia ^{3,*}

- State Key Laboratory of Shale Oil and Gas Enrichment Mechanisms and Effective Development, Beijing 102206, China
- ² State Energy Center for Shale Oil Research and Development, Beijing 100728, China
- ³ State Key Laboratory of Petroleum Resources and Prospecting, Beijing 102249, China
- * Correspondence: jiny@cup.edu.cn (Y.J.); xiayang@vip.126.com (Y.X.)

Abstract: Hydraulic fracturing is a complex nonlinear hydro-mechanical coupled process. Accurate numerical simulation is of great significance for reducing fracturing costs and improving reservoir development benefits. The aim of this paper is to propose an efficient numerical simulation method for the fracturing-to-production problem under a unified framework that has good convergence and accuracy. A hydro-mechanical coupled fracturing model (HMFM) is established for poroelastic media saturated with a compressible fluid, and the local characteristics of the physical field are fully considered. Each fracture is explicitly characterized using the discrete fracture model (DFM), which can better reflect the physical characteristics near fractures. Based on the extended finite element method (XFEM) and the Newton-Raphson method, a fully coupled approach named Unified Extended Finite Element (UXFEM) is developed, which can solve the nonlinear system of equations that describe the solution under a unified framework. UXFEM can accurately capture the local physical characteristics of different physical fields on the orthogonal structured grids. It realizes the gridfracture decoupling, and fractures can propagate in any direction, which shows greater flexibility in simulating fracture propagation. The fully coupled approach can better reflect the essential relationship between pressure, stress, and fracture, which is beneficial to studying hydro-mechanical coupled problems. To validate the UXFEM, UXFEM is compared with the classical KGD model, analytic solution, and COMSOL solution. Finally, based on UXFEM, the interference phenomenon and fracturing-to-production study are carried out to prove the broad practical application prospect of this new fully coupled approach.

Keywords: hydro-mechanical; fracture propagation; production; fully coupled; stress interference

1. Introduction

Fracturing, as an effective technical means, can significantly improve the development efficiency of low-permeability and ultra-low-permeability reservoirs, such as shale and tight sandstone. The reservoir often develops various discontinuous structures such as natural fractures, faults, and caves, resulting in significant multi-scale characteristics of the reservoir [1–3]. Moreover, all physical processes in the subsurface are highly nonlinear and complex multiphysics problems [4,5], which poses considerable challenges to studying fracture propagation and other related research.

As the most traditional research method, experiments have played a considerable role in promoting the study of fracture propagation [2,3,6–10]. However, the limitations of physical simulation are also prominent, especially for complex multiphysics coupling problems. The experimental cost is relatively high, and the results are often obtained at the laboratory scale, which cannot accurately reflect the underground situation. Furthermore, some complex problems are often unable to be experimentally studied. In this context, numerical simulation technology is applied to the design before fracturing, monitoring during fracturing, and post-fracturing dynamic production studies. Many scholars



Citation: Deng, Y.; Wang, D.; Jin, Y.; Xia, Y. A Fully Coupled Hydro-Mechanical Approach for Multi-Fracture Propagation Simulations. *Energies* **2023**, *16*, 1601. https://doi.org/10.3390/en16041601

Academic Editor: Reza Rezaee

Received: 4 January 2023 Revised: 27 January 2023 Accepted: 1 February 2023 Published: 5 February 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). have conducted extensive and meaningful research on the fracture propagation problem in petroleum engineering and proposed some numerical simulation methods. Those methods mainly include the finite element method (FEM) and its derivative methods, the boundary element method (BEM), and the discrete element method (DEM).

FEM is a flexible, effective, and widely used numerical method [11–17]. Fractures are highly coupled with grids, and fractures must be set along the boundary of mesh grids. Therefore, fractures must propagate along the grid boundary or continuously reconstruct mesh grids, which requires tremendous calculation. At the same time, the shape function of FEM is continuous. It is often difficult to accurately describe various discontinuities when describing the fluid pressure and the solid displacement fields. Thus, it is often necessary to use local mesh refinement to achieve high-accuracy calculations, which also increases the calculation burden. The characteristics of fracture geometry and physical field are challenging to characterize efficiently and accurately in traditional FEM, and the limitations of FEM are significant.

Some FEM-based extension methods have been proposed successively, such as nodesplitting FEM [18,19], generalized FEM [20,21], and XFEM [22–27]. Node-splitting FEM allows FEM to describe the fracture width simply. Still, the fracture propagation path must follow the grid boundary, which cannot accurately describe the fracture propagation and interaction. XFEM is a method based on the partition of unity method (PUM), which uses enrichment functions to capture the physical characteristics of fracture walls and fracture tips. XFEM is an efficient method for solving discontinuous problems, which can well solve strong and weak discontinuity problems. The core idea is to capture various discontinuities with the help of enrichment functions constructed based on analytical solutions or asymptotic analytical solutions, and fractures and grids are independent of each other. XFEM, based on the discrete fracture model, can explicitly characterize each fracture. Many scholars have constructed enrichment functions to describe fracture surfaces, fracture tips, and intersecting fractures based on physical field characteristics and analytical solution characteristics, which can efficiently deal with complex fracture mechanical behavior. However, most of the current XFEM models cannot accurately consider the fluid flow and exchange between fractures and matrix, usually ignoring the fluid leak-off term during fracturing stimulation or replacing it with an empirical leak-off equation, which cannot meet the simulation requirements. Moreover, XFEM also has considerable troubles in the numerical simulation of complex fracture flow, and it is difficult to achieve a fully coupled approach under a unified numerical framework.

BEM [28–34] is a dimensionality reduction research method based on Green's formula. It can calculate fracture aperture and stress more accurately and conveniently than FEM and is suitable for simulating fracture propagation with complex topologies. However, this method faces two main problems. One is that it cannot accurately describe the influence of fractures on fluid/solid physical fields. The other one is that it is difficult to consider the leak-off during fracture propagation [35,36].

DEM [37] is a numerical simulation method specially used to solve the problem of discontinuous media. It can deal with fracture intersecting, branching, merging, and kinking problems. However, calibrating/updating particle properties remains a complex technical challenge. The particle number needs to be large enough to achieve sufficient accuracy, which prevents this technique from being widely used in large-scale models [36].

Different methods have different advantages and different defects. In order to synthesize their advantages and compensate for each other's weaknesses, some hybrid numerical simulation methods are also widely used. Recently, Li et al. [36,38–40] developed a meaningful thermal/hydro-mechanical (THM) model. They solved it using the hybrid numerical simulation method, realizing the two-dimensional and three-dimensional simulation of the construction of a complex fracture network. Settgast et al. [41] developed a fully coupled finite element/finite volume approach for simulating field-scale hydraulically driven fractures in three dimensions. Guo et al. [42] adopted the mixed finite element/displacement discontinuity method to solve for the spatial–temporal evolutions of pore pressure and in situ stress because of parent-well production and injection and models the fracture propagation during infill-well completion based on updated heterogeneous in situ stresses. Recently, a coupled simulation strategy combining the embedded discrete fracture method (EDFM) and the XFEM has been developed to simulate the fluid-driven fracture propagation process in porous media. EDFM and XFEM are used to simulate fracture-related fluid mechanics and solid mechanics, respectively, with information exchanged under the iterative numerical coupling scheme, and it realizes two-dimensional and three-dimensional hydraulic fracturing fracture propagation simulation [43–45]. Liu et al. [46] developed a hydro-mechanical model for non-planar hydraulic fracture propagation in ductile formations, which is solved by the hybrid extended finite element/finite volume method. Zhang et al. [47] combined the extended finite element/phase-field method to solve the discontinuous and continuous hydraulic fracturing formulations.

However, there are still many insurmountable problems in developing realistic simulation tools for the hydraulic fracturing process. The problems of numerical simulation of fracture propagation are mainly reflected in the contradiction between calculation efficiency and calculation accuracy, reflected explicitly in several aspects: (1) There are strict requirements for grid division; (2) The fracture propagation path is not arbitrary, and the fracture morphology of simulation results is distorted; (3) Matrix flow, fracture flow, and solid deformation coupling are challenging, and the leak-off term during the fracturing process is often ignored or simplified; (4) The nonlinearity is strong, and the fully coupled approach is scarce; (5) There are still risks and challenges in the convergence and stability of numerical calculation. The reasons for this contradiction are complex and diverse, such as rock heterogeneity, unclear mechanism of fracture propagation, and aggravation of heterogeneity caused by the intersection of fractures and artificial stimulation measures. In recent years, some new methods and models have been proposed and achieved good results, such as Peridynamics [48–51].

This paper uses the discrete fracture model (DFM) to characterize each fracture [52–56]. A fully coupled numerical approach for solving nonlinear hydro-mechanical coupled models is established based on XFEM. In this method, the solid rock deformation obeys quasistatic linear elasticity, characterized by the stress balance equation. The matrix flow follows Darcy's law, and the fracture fluid flow obeys the cubic law. In the derivation, this paper uses the normal flow velocity discontinuity term in the weak form of matrix and fracture flow equations to automatically characterize the fluid leak-off term. This paper uses XFEM to compile a fully coupled UXFEM solver for the abovementioned HMFM. This approach solves the fluid pressure field and the solid displacement field under the same framework, realizing the unification and full coupling of the model and solving.

According to the different physical characteristics of solid deformation and fluid flow, UXFEM adopts different enrichment functions and successfully realizes the numerical simulation of hydro-mechanical fracture propagation. The advantages of XFEM are entirely inherited in UXFEM. Fractures are decoupled from mesh grids, and fractures are allowed to propagate in any direction and can be deflected at any angle without mesh reconstruction or other special processing. In this paper, the maximum circumferential tensile stress criterion is used as the fracture propagation criterion, the displacement-based method is used to calculate the stress intensity factors (SIFs), and fractures always propagate along the direction of the maximum circumferential stress. When UXFEM solves nonlinear mathematical models, it adopts Newton–Raphson iterative linearization to deal with nonlinear systems. With the advantages of XFEM, UXFEM can achieve the balance between calculation efficiency and accuracy to a certain extent, which has practical significance for the actual engineering-scale fracturing simulation research.

2. Unified Extended Finite Element Method

2.1. Fracture Description

This paper uses pair of orthogonal level set functions f(x), g(x) to describe each fracture explicitly, and some enrichment functions are constructed based on level set func-

tions. The fracture surface is defined as f(x) = 0, and the fracture tip is defined as f(x) = 0, g(x) = 0.

The core of XFEM is to capture the discontinuity of the physical field by using enrichment functions with discontinuous properties and enriched degree of freedoms (DOF) on the basis of FEM, which makes the description of the physical field characteristics independent of grids, bringing a lot of conveniences. The fracture is a strong discontinuity for the displacement field, while the fracture is a weak discontinuity for the fluid pressure field. That means the pressure on the two sides of the fracture is continuous, but the pressure derivative is discontinuous. In this paper, the set of all nodes is denoted as N_{all} , and the enriched nodes, including surface nodes and tip nodes, are denoted as N_s and N_t , respectively (Figure 1).



Figure 1. Enriched node-set.

2.2. Solid Displacement Field Approximation and Enrichment Functions

In the XFEM framework, the displacement approximation is written as:

$$u(\mathbf{x}) = \sum_{i \in N_{all}} N_i(\mathbf{x}) u_i + \sum_{j \in N_s} N_j(\mathbf{x}) (H(f(\mathbf{x})) - H((f(\mathbf{x}_j))) a_j + \sum_{q \in N_t} N_q(\mathbf{x}) \sum_{l=1}^4 (F_l(\mathbf{x}) - F_l(\mathbf{x}_q)) b_q^l$$
(1)

where $N_i(\mathbf{x})$, $N_j(\mathbf{x})$, $N_q(\mathbf{x})$ are standard FEM shape function; u_i is standard displacement degrees of freedom (DDOF) for nodes N_{all} ; a_j^n , b_{ql}^m are added enriched DDOFs for nodes N_s and N_t respectively; $H(\cdot)$ is the Heaviside step function; $F_l(\cdot)$ is fracture tip displacement enrichment function.

For expression brevity, we combine the last two terms in Equation (1) and rewrite Equation (1) as:

$$u(\mathbf{x}) = \sum_{i \in N_{all}} N_i(\mathbf{x}) u_i + \sum_{j \in N_{enr}} \Psi_j(\mathbf{x}) \widetilde{u}_j = \mathbf{N} \cdot \mathbf{U}$$
(2)

where $N_{enr} = \{N_s, N_t \times 4\}$; Ψ_j and \tilde{u}_j denote enrichment functions and its added enriched DDOFs; **N** is the combination of standard FEM shape function N_i and enrichment function Ψ_j ; and **U** is the combination of standard DDOFs and added enriched DDOFs.

For the element completely penetrated by the fracture, the displacement field is discontinuous on both sides, which is a strong discontinuity. The Heaviside step function is used to enrich the surface nodes.

$$H(f(\mathbf{x})) = \begin{cases} 1, f(\mathbf{x}) \ge 0\\ 0, f(\mathbf{x}) < 0 \end{cases}$$
(3)

For fracture tips, the stress has the singularity of $O(r^{-\frac{1}{2}})$. For isotropic linear elastic materials, tip nodes can be enriched by the fracture tip displacement enrichment function [22,57,58]:

$$[F_l(r,\theta), l = 1, 2, 3, 4] = \left[\sqrt{r}\sin\frac{\theta}{2}\sqrt{r}\cos\frac{\theta}{2}\sqrt{r}\sin\frac{\theta}{2}\sin\theta\sqrt{r}\cos\frac{\theta}{2}\sin\theta\right]$$
(4)

where r, θ are polar coordinates with origin at the fracture tip and x_1 axis oriented into the body and parallel to the fracture surfaces (Figure 2).



Figure 2. Polar coordinate of a fracture tip.

2.3. Fluid Pressure Field Approximation and Enrichment Functions

In the XFEM framework, the pressure approximation is written as:

$$p(\mathbf{x}) = \sum_{i \in N_{all}} N_i(\mathbf{x}) p_i + \sum_{a \in N_s} N_a(\mathbf{x}) (\phi_s(\mathbf{x}) - \phi_s(\mathbf{x}_a)) \widetilde{p}_a + \sum_{b \in N_t} N_b(\mathbf{x}) (\phi_t(\mathbf{x}) - \phi_t(\mathbf{x}_b)) \widetilde{p}_b^m$$
(5)

where $N_i(\mathbf{x})$, $N_a(\mathbf{x})$, $N_b(\mathbf{x})$ are standard FEM shape function; p_i is standard pressure degrees of freedom (PDOFs) for node N_{all} ; a_j^n , b_{ql}^m are added enriched PDOFs for nodes N_s and N_t respectively; $\phi_s(\cdot)$ is the modified level set absolute value function; $\phi_t(\cdot)$ is fracture tip pressure enrichment function.

For expression brevity, we combine the last two terms in Equation (5) and rewrite Equation (5) as:

$$p(\boldsymbol{x}) = \sum_{i \in N_{all}} N_i(\boldsymbol{x}) p_i + \sum_{j \in N_{enr}} \Phi_j(\boldsymbol{x}) \widetilde{p}_j = \boldsymbol{H} \cdot \boldsymbol{P}$$
(6)

where $N_{enr} = \{N_s, N_t\}$; Ψ_j and \tilde{u}_j denote enrichment functions and their added enriched PDOFs; *H* is the combination of standard FEM shape function N_i and enrichment function Φ_j ; and *P* is the combination of standard PDOFs and added enriched PDOFs.

For the fracture surface, the pressure is continuous, but the pressure gradient is discontinuous. Moës [59] uses the modified level set absolute value function to capture the fluid pressure characteristics, avoiding the appearance of blending elements.

$$\phi_{s}(\boldsymbol{x}) = \sum_{j} \left| f(\boldsymbol{x}_{j}) \right| N_{j}(\boldsymbol{x}) - \left| \sum_{j} f(\boldsymbol{x}_{j}) N_{j}(\boldsymbol{x}) \right|$$
(7)

According to the study by Chen et al. [60], the pressure gradient has the singularity of $O(r^{-\frac{1}{2}})$. A new enrichment function was constructed based on the asymptotic analytic solution [61–63].

$$\phi_t(\mathbf{x}) = \sqrt{r} \cos \frac{\theta}{2} \tag{8}$$

2.4. XFEM Discretization

This paper only considers the quasi-static process with infinitesimal strain in porous media, which is homogeneous, isotropic, and linearly elastic. The porous medium domain Ω , with boundary Γ , Γ_t , Γ_u (Figure 3). The fracture Γ_f consisting of two surfaces is embedded in the domain Ω . In this paper, fluid flow and matrix deformation are coupled based on poroelasticity [64], matrix and fracture flow are coupled based on discontinuous flow on the fracture surface, and fracture deformation and fracture flow are coupled using the cubic law. The coupled governing equations are Equation (A4) and (A21) in Appendix A. For the detailed formula derivation, please refer to the Appendix A.



Figure 3. Illustration of a domain.

According to the basic theory of XFEM, the fracture opening can be expressed as:

$$w_f = \llbracket \boldsymbol{u} \rrbracket \cdot \boldsymbol{n}_f = \left(\sum_{i \in N_s} N_i(x) a_i + 2 \sum_{q \in N_t} \sqrt{r} N_q(x) b_q \right) \cdot \boldsymbol{n}_f = \llbracket N \rrbracket \cdot \boldsymbol{U} \cdot \boldsymbol{n}_f$$
(9)

where $[\![u]\!]$ denotes the displacement jump across the fracture; a_i and b_q are discontinuous enriched DDOFs; $[\![N]\!]$ denotes the discontinuous shape function matrix.

By substituting the displacement and pressure approximation expressions into the weak form of the governing equations, the discrete calculation format can be obtained as follows:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ M_{uu} & M_{u\widetilde{u}} & M_{pp} & M_{p\widetilde{p}} \\ M_{\widetilde{u}u} & M_{\widetilde{u}\widetilde{u}} & M_{\widetilde{p}p} & M_{\widetilde{p}\widetilde{p}} \end{bmatrix} \begin{bmatrix} \dot{U} \\ \dot{\tilde{U}} \\ \dot{\tilde{P}} \\ \dot{\tilde{P}} \\ \dot{\tilde{P}} \end{bmatrix} + \begin{bmatrix} K_{uu} & K_{u\widetilde{u}} & K_{pp} & K_{p\widetilde{p}} \\ K_{\widetilde{u}u} & K_{\widetilde{u}\widetilde{u}} & K_{\widetilde{p}p} & K_{\widetilde{p}p} \\ 0 & 0 & KK_{pp} & KK_{pp} \end{bmatrix} \begin{bmatrix} U \\ \tilde{U} \\ P \\ \tilde{P} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_1 \\ F_2 \\ \tilde{F}_2 \end{bmatrix}$$
(10)

Equation (10) can be simplified to the following form:

$$\begin{bmatrix} 0 & 0 \\ M_u & M_p \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{U}} \\ \dot{\boldsymbol{P}} \end{bmatrix} + \begin{bmatrix} K_u & K_p \\ 0 & KK_p \end{bmatrix} \begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{P} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$
(11)

where

$$K_{u} = \int_{\Omega} \boldsymbol{B}^{T} : \boldsymbol{C} : \boldsymbol{B} d\Omega$$
$$K_{p} = -\int_{\Omega} \boldsymbol{B}^{T} \alpha_{\varsigma}^{T} \boldsymbol{H}^{T} d\Omega - \int_{\Gamma_{f}} [\![N]\!] \boldsymbol{n}_{f} \boldsymbol{H}^{T} d\Gamma$$
$$F_{1} = \int_{\Gamma_{t}} \boldsymbol{N} \cdot \overline{\boldsymbol{\sigma}} d\Gamma - \int_{\Omega} \boldsymbol{N} \cdot (\boldsymbol{\sigma}_{0} + \alpha p_{0} \boldsymbol{I}) d\Omega$$

$$M_{u} = \int_{\Omega} \alpha \rho H_{\varsigma} B d\Omega + \int_{\Gamma_{f}} \rho H n_{f} [\![N]\!]^{T} d\Gamma$$

$$M_{p} = \int_{\Omega} \rho Q^{-1} H H^{T} d\Omega + \int_{\Gamma_{f}} \frac{\rho w}{K_{l}} H H^{T} d\Gamma$$

$$KK_{p} = \int_{\Omega} \frac{\rho k_{m}}{\mu} D^{T} D d\Omega + \int_{\Gamma_{f}} \frac{\rho w^{3}}{12\mu} D_{s} D_{s}^{T} d\Gamma$$

$$F_{2} = \int_{\Gamma_{f}} H Q_{m} d\Gamma - \int_{\Gamma_{q}} H \overline{q} d\Gamma$$

$$\varsigma = [1, 1, 0]$$

$$N = [N^{u} N^{\widetilde{u}}]^{T}, B = [\nabla N^{u} \nabla N^{\widetilde{u}}]$$

$$H = [N^{p} N^{\widetilde{p}}]^{T}, D = [\nabla N^{p} \nabla N^{\widetilde{p}}]$$

where H_s is the total shape function at the point source, D_s is the directional derivative of H_s along the tangential direction of the fracture, i.e., $D_s = \partial H_s / \partial s$. The superscripts "u" and " \tilde{u} " correspond to standard DDOFs and enriched DDOFs, respectively, and the superscripts "p" and " \tilde{p} " correspond to standard PDOFs and enriched PDOFs, respectively. Equation (11) is a set of coupled nonlinear equations, which are solved by the Newton– Rapson method.

3. Fracture Propagation and Solution Strategy

3.1. Fracture Propagation Criterion

In practice, the hydraulic fracture usually propagates in a mixed mode. The maximum circumferential stress criterion is adopted as the fracture propagation criterion. It is assumed that the fracture propagates when the effective stress intensity factor K_e along that direction reaches the fracture toughness K_{IC} , and it will deflect by an angle of [65]:

$$\theta = 2\arctan\left(\frac{-2K_{II}/K_I}{1+\sqrt{1+8(K_{II}/K_I)^2}}\right)$$
(12)

The effective stress intensity factor K_e is expressed as:

$$K_e = \cos\frac{\theta}{2} \left(K_I \cos^2\frac{\theta}{2} - 1.5K_{II} \sin\theta \right) \ge K_{IC}$$
(13)

The displacement-based approach is used to calculate the SIFs. K_I and K_{II} are determined by the discontinuous displacement of the fracture tip element. The relational equation is [66].

$$\begin{cases} K_I = \frac{0.806\sqrt{\pi}ED_n}{4(1-\nu^2)\sqrt{r}} \\ K_{II} = \frac{0.806\sqrt{\pi}ED_s}{4(1-\nu^2)\sqrt{r}} \end{cases}$$
(14)

where *E* is Young's modulus, ν is Poisson's ratio, D_n and D_s are normal and tangential discontinuous displacements of fractures. It should be noted that the calculation models in this paper are all under the plane strain condition.

3.2. HMFM Fracturing Simulation Process

Fracture propagation is a highly nonlinear hydro-mechanical coupled process. In this paper, the fully coupled approach is implemented. The fully coupled approach forms a single large system of equations that solve for all of the displacement/pressure unknowns at once. Figure 4 summarizes the fully coupled implementation of the solution of the HMFM.



Figure 4. UXFEM flow chart.

4. Results and Discussions

4.1. Validation 1: Comparison between UXFEM and KGD Model

The KGD model is one of the most commonly used fracturing models. The viscositydominated hydraulic pressure is calculated using the approximate solution of the KGD model derived by Detournay [67]. Table 1 lists the parameters used by the model. The fracture length and width at the wellbore position are compared with those calculated by UXFEM and plane strain KGD model, as shown in Figure 5. It can be seen that the evolution of fracture length and width calculated by UXFEM in this paper can well fit the KGD model. The calculation error increases with the fracture length because the infinite reservoir assumption cannot be satisfied with the fracture propagation.

Table 1. Parameters for validation 1.

Parameters	Value	Unit
Young modulus	60	GPa
Poisson ratio	0.25	/
Fracture toughness	1.2	MPa·m ^{0.5}
Fluid viscosity	50	mPa·s
Injection rate	0.01	m²/s
Convergence tolerance	0.01	/



Figure 5. Comparison of KGD and UXFEM, (**a**) Comparison of fracture length, and (**b**) Comparison of fracture width at the wellbore.

4.2. Validation 2: Comparison of KI and KII in Numerical Simulation and Analytical Solution

Rice [68] deduces the analytical expressions of the SIFs at the fracture tip under hydrostatic pressure and horizontal stress. For the single fracture propagation in an infinite domain, the SIFs are expressed as:

$$\begin{cases} K_I = \sqrt{\pi L_f} \left[p - \left(\sigma_H \sin^2 \beta + \sigma_h \cos^2 \beta \right) \right] \\ K_{II} = \frac{\sqrt{\pi L_f}}{2} \left[\sigma_H - \sigma_h \right] \sin 2\beta \end{cases}$$
(15)

where L_f is the half length of the fracture; p is hydrostatic pressure in the fracture; S is the inclination angle of fracture and maximum horizontal principal stress. σ_H and σ_h are the maximum horizontal stress and the minimum horizontal stress, respectively.

In order to approximate the infinite formation and eliminate the influence of the boundary, the model size is set to 200×200 m, the initial half-length of the fracture is 5 m, and the domain is discretized into structured grids (159 × 159). Table 2 lists the parameters used by the model. Calculate the SIFs (K_I and K_{II}) of different inclination angles, and the results are compared, as shown in Figure 6. By comparison, the SIFs calculated by UXFEM in this paper can well fit the analytical solution. The maximum error of KI is not more than 2%, which proves the method's accuracy.

Table 2. Parameters for validation 2.

Parameters	Value	Unit
Young's modulus	60	GPa
Poisson's ratio	0.25	/
Maximum horizontal in-situ stress	60	MPa
Minimum horizontal in-situ stress	55	MPa
Hydrostatic pressure applied to the fracture surface	65	MPa
Half-length of the fracture	5	m



Figure 6. Calculation results comparison of stress intensity factor between UXFEM and analytical solution.

4.3. Validation 3: Comparison between UXFEM and COMSOL Multiphysics[®] 5.6

The purpose of this section is to show the accuracy and feasibility of our fully coupled approach. We consider a quasi-static mud loss case of a shale oil reservoir. The simulation parameters are shown in Table 3. The geometry (Figure 7) is built to carry out the simulation process. The domain is discretized into 25,860 unstructured grids in COMSOL and 3481 structured grids in UXFEM. Figure 8 shows the displacement field, effective stress component $\sigma'_{y'}$ and pressure field calculated by COMSOL and UXFEM. In Figure 8, the results calculated by UXFEM and COMSOL are in good agreement. From Figure 8a,d, the displacement at the injection point is the largest, implying that the fracture width is the largest at the injection point. Figure 8b,e shows that there is an obvious stress concentration at the fracture tip. Figure 8c,f shows that the pressure is higher along the fracture, and a small low-pressure area appears around the tips. Figure 9 shows that fracture widths calculated by UXFEM and COMSOL can be matched. Calculating the L₂ error norm:

$$\left\|F - F^{h}\right\|_{L_{2}} = \left[\int_{\Omega} (F - F^{h}) d\Omega\right]^{0.5}$$
(16)

where F^h is the COMSOL solution and F is the UXFEM solution.

The displacement L_2 error, effective stress component $\sigma'_y L_2$ error, and pressure L_2 error are 0.0083%, 0.041%, and 0.078%, respectively. A very good agreement between the results proves the effectiveness and accuracy of this fully coupled approach.

Table 3. Parameters for Validation 3.

Parameters	Value	Unit
Initial reservoir pressure	10	MPa
Pressure at injection point	25	MPa
Maximum horizontal in-situ stress	20	MPa
Minimum horizontal in-situ stress	15	MPa
Young's modulus	40	GPa
Poisson's ratio	0.2	/
Matrix permeability	$1 imes 10^{-18}$	m ²
Initial matrix porosity	0.15	/
Bulk modulus of the solid	50	GPa
Bulk modulus of the fluid	2.5	GPa
Fluid density	1000	kg/m ³
Fluid viscosity	10	mPa⋅s
Biot coefficient	0.85	/



Figure 7. Geometric model for the comparison between UXFEM and COMSOL.



Figure 8. Distributions of the displacement, Effective stress component σ'_y and pressure at 1000 s, (a) Displacement distribution calculated by COMSOL, (b) Effective stress component σ'_y distribution calculated by COMSOL, (c) Pressure distribution calculated by COMSOL, (d) Displacement distribution calculated by UXFEM, (e) Effective stress component σ'_y distribution calculated by UXFEM, (f) Pressure distribution calculated by UXFEM.



Figure 9. Comparison of the fracture width between UXFEM and COMSOL.

4.4. Case Study1: Propagation of Three Parallel Fractures

A 200 m \times 200 m two-dimensional model is established to study the fracture deflection propagation caused by the stress shadow effect. The initial half-length of the fracture was 5 m, the distance between the three fractures was 14 m or 21 m, and the midpoints of the three fractures are the injection points. The injection rate at injection points is 0.01 m²/s. Other parameters are listed in Table 4.

Parameters Value Unit Initial reservoir pressure 50 MPa Maximum horizontal in situ stress 60 MPa Minimum horizontal in situ stress 55 MPa -17 Matrix permeability 1×10 m² Initial matrix porosity 0.1 1 Initial fracture length 10 m m^2/s Injection rate at injection points 0.01 Bulk modulus of the fluid 2.5 GPa kg/m³ Fluid density 1000 Fluid viscosity 50 mPa∙s MPa·m^{0.5} 5 Fracture toughness K_{IC} Biot coefficient 0.85 /

Table 4. Parameters for case study.

The final morphology of the simulated fracture, displacement field, and effective stress component σ'_y are shown in Figure 10. For the model of three fractures, the middle fracture is squeezed and suppressed by the same on both sides. The middle fracture almost propagates along a straight line, and its length is significantly shorter than the other two fractures. Due to the stress interference between fractures, the side fractures deflect in the opposite direction, away from the middle fracture [69]. Fractures will actively choose to propagate along the path with the least resistance. As the fracture spacing is smaller, the fracture deflection angle is larger, indicating that the fracture spacing is an important factor affecting the stress interference intensity.



(a)





(d)

Figure 10. Simulation results with fracture spacing = 14 or 21 m after 280 s, (**a**) Fracture final morphology of spacing = 14 m, (**b**) Displacement of spacing = 14 m, and (**c**) Effective stress component σ'_y of spacing = 14 m, (**d**) Fracture final morphology of spacing = 21 m, (**e**) Displacement of spacing = 21 m, and (**f**) Effective stress component σ'_y of spacing = 21 m.

4.5. Case Study2: Propagation in Multiple En Échelon Fractures

En échelon fractures are common in geology and are caused by the mechanical interaction between their near-tip stress fields [36,70]. To study the evolution of En échelon fractures, a 200×200 m geometry model is built. The model contains three prefabricated fractures, with an initial fracture length of 10 m and a vertical spacing of 16 m. The initial parameters of the three fractures are completely the same. The injection point is located at the midpoint of the fracture (The red points in Figure 11), and the injection rate is set to $0.01 \text{ m}^2/\text{s}$. Other parameters of the model are the same as those in 5.4 (Table 4). See Figure 11 for the calculation results. It can be seen that after the fractures overlap, the propagation of the middle fracture is restrained, and the tendency of mutual attraction between adjacent fractures is becoming more and more obvious. Enhancement or impediment of the growth ahead of an individual fracture front leads to asymmetric fracture geometry [36].

The stress interference of parallel fractures has been studied previously, and the fractures tend to mutually repel each other. However, for such parallel fractures with incomplete left-right overlap, the stress shadow effect will cause the stress direction to change, and the overlapping tips will appear as a phenomenon of mutual attraction. In the process of hydraulic fracturing, we hope that hydraulic fractures can form complex shapes. However, this phenomenon may lead to fracture collision and reduce the fracturing effect, which needs to be avoided. Therefore, the optimization of the overlapping length and spacing of fractures is an important job.



Figure 11. Sequential growth of En échelon fractures: trajectory, pressure, and effective stress component σ'_{y} after 20 s, 70 s, and 170 s.

4.6. Case Study3: Multi-Well and Multi-Cluster Simultaneous Fracturing and Production

The problem considered here is the fracturing-production problem of 7 hydraulic fractures generated by two horizontal wells (#1 and #3) and one vertical well (#2), numbered 1 to 7, respectively. The schematic diagram of the initial fractures and the model is shown in Figure 12. The size of the model is $200 \text{ m} \times 100 \text{ m}$. Two horizontal wells are along the straight-line x = 60 m and x = 140 m, respectively. The vertical well is located in the middle of the model. Table 4 lists the detailed parameters.



Figure 12. Multi-well and multi-cluster simultaneous fracturing model.

This study aims to analyze the interaction between evolved multi-fractures and local stress-field-pressure fields and help explore the interference phenomenon's influence on fracture morphology and production. Taking well #1 as an example, it can be seen that the fractures are subject to obvious stress interference from Figures 13 and 14, and the stress interference is manifested in two points:

- (1) The fractures will undergo obvious diversion. For well #1, Frac. 1–3 have different degrees of deflection. The deflection of the two Frac. 1 and 3 are more prominent. In the early stage, the middle fracture Frac. 2 does not deflect obviously due to the interference of superimposed stress. In the later stage of fracturing, Frac. 2 and 4 attract each other, showing En échelon fracture morphology [69].
- (2) There are noticeable differences in fracture length and width. The length of Frac. 2 and 6 of the two horizontal wells is shorter, followed by Frac. 3 and 5, and the longest is Frac. 1 and 7. Frac. 4 cannot propagate far enough and even the front edge of the fracture tends to close. Although Frac. 1 and 7 have propagated long enough, one side of the fracture tends to close under the effect of stress shadow. These phenomena are very likely to lead to the reduction of fracturing efficiency and cost waste.

The complex morphology of fractures is the result of stress interference. Studies have found that 25–30% of the fractures do not bring production [71,72], indicating that 25–30% of the fractures are not opened or closed after being opened, which is also the effect of stress interference. Therefore, stress interference has both positive and negative effects. Numerical simulation can be used to find the best design for well placement, perforation, and fracturing so that the fractures can be fully opened and stabilized to maximize the benefits of fracturing.







Figure 14. Fracture trajectory, effective stress component $\sigma'_{y'}$ and fracture width distribution after fracturing.

Change the bottom hole condition to a constant pressure of 25 MP, and proppant instead of the hydraulic pressure supports the fracture. Assuming that the fracture has high conductivity and obeys Darcy's law, it is only necessary to change the fracture flow coefficient $\frac{w^2}{12\mu}$ in Equation (A4) into the hydraulic fracture permeability. The stress-sensitive parameters are brought into the governing equation for calculation during production. For the estimation method of stress-sensitive parameters, see the study of Lu et al. [73]. The pressure change in the production process is shown in Figure 15, which shows that there is almost no production interference between fractures and wells within 30 days, and obvious interference can be observed after 300 days. The calculated cumulative production. The early production increases rapidly because the reservoir near the well is fully stimulated. However, the reservoir's extremely low permeability makes it difficult to maintain rapid production.



Figure 15. Pressure field after 30, 300, 600, and 900 days.



Figure 16. Cumulative production curve during the hydro-mechanical coupled production process.

Four calculation examples with different well-spacing (40 m, 60 m, 80 m, and 100 m) are simulated for comparison to verify the effect of well-spacing on fracture morphology and production. After 60 s injection, the fracture morphology and displacement field are

as Figure 17. The final morphology of fractures under the stress interference is different with different well-spacing. Then, realize conversion from hydraulic fracturing to production, as shown in Figure 18, and the cumulative production curve after 2000 days is plotted in Figure 19. Figure 18 shows that small well-spacing will lead to slow pressure drop near boundaries, resulting in a low production rate. Thus, the cumulative production of small well-spacing is smaller than that of large well-spacing, which can be explained by the small well-spacing leading to a poor fracturing effect. Therefore, reasonable well-spacing is a crucial factor affecting the formation of a complex fracture network and efficient production.



Figure 17. Displacement field after 60 s (well-spacing = 100, 80, 60, and 40 m).



Figure 18. Pressure field after 2000 days (well-spacing = 100, 80, 60, and 40 m).



Figure 19. Comparison of cumulative production curves with different well-spacing.

5. Conclusions

Aiming at solving the fracturing-to-production problem under the unified framework, a fully coupled approach UXFEM for solving the HMFM under the same framework is established based on the extended finite element method and Newton-Rapson method. This technology inherits all the excellent characteristics of XFEM and can fully reflect the mechanical mechanism of fracture propagation. Meanwhile, it also avoids some potential problems caused by the iterative coupling method, such as convergence reduction. The effectiveness and accuracy of UXFEM in HMFM research are validated by comparing it with the results of classical KGD models, analytical solutions, and COMSOL Multiphysics[®] 5.6. To study the interference problem during the fracturing process, three examples are used to illustrate the effect of stress shadow on fracturing. At the same time, after simultaneous fracturing, this paper also successfully realized the fracturing-to-production analysis. The research shows that hydraulic fractures are interfered with by the stress between wells and fractures, which plays an essential role in forming the final fracture network morphology and cumulative production. Proper well location design, perforation plan, and production plan are the keys to determining whether the stress interference can be used reasonably to increase the fracturing effects and reservoir development efficiency. This research has certain practical significance for fracturing design and production prediction.

Author Contributions: Data curation, D.W.; Methodology, Y.D.; Supervision, Y.J. and Y.X.; Writing—Original draft, Y.D. All authors have read and agreed to the published version of the manuscript.

Funding: This work was financially supported by the National Natural Science Foundation of China (Grant NO. U19B6003-05 and 51874321).

Data Availability Statement: Not applicable.

Acknowledgments: The financial support provided by the National Natural Science Foundation of China is highly appreciated.

Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

For a quasi-static process, ignoring the effect of gravity, the stress balance equation of the rock matrix is [74]:

 ∇

$$\cdot \sigma = 0$$
 (A1)

where σ is the Cauchy stress tensor. Considering the effective stress, the Cauchy stress tensor σ can be written as:

$$\sigma = \sigma' + \sigma_0 - \alpha (p - p_0)I \tag{A2}$$

where σ_0 is the initial total stress; p_0 is the initial pressure; α is the Biot coefficient; I is the identity matrix; σ' is the effective stress; $\sigma' = C : \varepsilon$. C is the elasticity tensor; ε is the strain, which can be calculated from the geometric equation.

The boundary condition is:

$$\begin{cases} u = \overline{u} & \text{on } \Gamma_u \\ \sigma \cdot n = \overline{\sigma} & \text{on } \Gamma_f \\ \sigma \cdot n_f = -pn_f & \text{on } \Gamma_f \end{cases}$$
(A3)

where Γ_u , Γ_t and Γ_f are displacement boundary, traction boundary, and fracture surfaces, respectively; *n* and *n*_f are the normal direction to the related boundary.

Taking δu as the trial function, the weak form of the solid matrix deformation governing equation can be obtained from Equations (A1)–(A3),

$$\int_{\Omega} \nabla \delta \boldsymbol{u} \cdot (\boldsymbol{C} : \boldsymbol{\varepsilon} + \boldsymbol{\sigma}_0 - \boldsymbol{\alpha}(\boldsymbol{p} - \boldsymbol{p}_0)\boldsymbol{I}) d\Omega - \int_{\Gamma_f} [\![\delta \boldsymbol{u}]\!] \cdot \boldsymbol{p} \boldsymbol{n}_f d\Gamma - \int_{\Gamma_t} \delta \boldsymbol{u} \cdot \overline{\boldsymbol{\sigma}} d\Gamma = 0 \quad (A4)$$

The governing equation for mass conservation of a compressible-fluid flow is:

$$\frac{dm}{dt} + \nabla \cdot (\rho \boldsymbol{v}_m) = 0 \tag{A5}$$

where *m* denotes the fluid mass, v_m is the matrix flux. ρ is the fluid density.

The constitutive relations for poroelasticity material are written as [64]

$$\delta m = \rho(\alpha \delta e + \frac{\delta p}{Q}) \tag{A6}$$

where *e* is the volumetric strain, $e = \nabla \cdot u$; *Q* is the Biot modulus, $Q = \left[\frac{\phi}{K_l} + \frac{\alpha - \phi}{K_s}\right]^{-1}$. where K_l is the fluid modulus; ϕ is the porosity of the matrix.

Combine Equation (A5) and (A6), the matrix fluid flow governing equation can express as:

$$\alpha \rho \nabla \cdot \dot{u} + \frac{\rho}{Q} \frac{\partial p}{\partial t} + \nabla \cdot (\rho v_m) = 0 \tag{A7}$$

The boundary condition is:

$$\begin{cases} \rho \boldsymbol{v}_m \cdot \boldsymbol{n} = \overline{q} & \text{on } \Gamma_q \\ \llbracket \rho \boldsymbol{v}_m \cdot \boldsymbol{n}_f \rrbracket = \rho \boldsymbol{v}_d & \text{on } \Gamma_f \end{cases}$$
(A8)

where Γ_q is the flow boundary, \overline{q} is the mass flux; $[\rho v_m \cdot n_f]$ denotes the normal discontinuous flow on the fracture surface; v_d denotes the fluid transfer from the fracture into the matrix, which can be interpreted as the leak-off effect.

Taking δp as the trial function, the weak form of the matrix flow governing equation can be obtained from Equations (A7) and (A8),

$$\int_{\Omega} \alpha \rho \delta p \nabla \cdot \dot{u} d\Omega + \int_{\Omega} \rho Q^{-1} \delta p \dot{p} d\Omega + \int_{\Gamma_q} \delta p \bar{q} d\Gamma - \int_{\Gamma_f} \delta p \rho v_d d\Gamma - \int_{\Omega} \nabla (\delta p) \rho v_m d\Omega = 0$$
(A9)

Matrix flow obeys Darcy's law, $v_m = -\frac{k_m}{\mu}\nabla p$. Thus, Equation (A9) can be rewritten as:

$$\int_{\Gamma_f} \delta p \rho v_d d\Gamma = \int_{\Omega} \alpha \rho \delta p \nabla \cdot \dot{\boldsymbol{u}} d\Omega + \int_{\Omega} \rho Q^{-1} \delta p \dot{p} d\Omega + \int_{\Gamma_q} \delta p \bar{q} d\Gamma + \int_{\Omega} \frac{\rho k_m}{\mu} \nabla(\delta p) \nabla p d\Omega$$
(A10)

Compressible fluid flow in fracture satisfies mass conservation and cubic law [75]:

$$\frac{\partial(\rho w)}{\partial t} + \nabla \cdot (\rho q_f) = Q_m \tag{A11}$$

$$q_f = -\frac{w^3}{12\mu}\frac{\partial p}{\partial s} = v_f w \tag{A12}$$

where q_f is the fluid flux in the fracture; w is the fracture aperture; v_f is the average fluid velocity of fracture section; and Q_m is the mass source term in the fracture.

$$\frac{\partial(\rho w)}{\partial t} = \frac{\partial \rho}{\partial t} w + \rho \frac{\partial w}{\partial t} = \frac{\rho w}{K_l} \frac{\partial p}{\partial t} + \rho \frac{\partial w}{\partial t}$$
(A13)

where $\frac{1}{K_l} = \frac{1}{\rho} \frac{\partial \rho}{\partial p}$. Combine Equation (A11) and (A13):

$$\frac{\rho w}{K_l} \frac{\partial p}{\partial t} + \rho \frac{\partial w}{\partial t} + \nabla \cdot \left(\rho q_f\right) = Q_m \tag{A14}$$

The boundary condition is:

$$\boldsymbol{q}_f \cdot \boldsymbol{n}_f = w \boldsymbol{v}_d \tag{A15}$$

Taking δp as the trial function, the weak form of the fracture flow governing equation can be obtained from Equation (A14),

$$\int_{\Omega_f} \delta p \frac{\rho w}{K_l} \frac{\partial p}{\partial t} d\Omega + \int_{\Omega_f} \delta p \rho \frac{\partial w}{\partial t} d\Omega + \int_{\Omega_f} \delta p \nabla \cdot (\rho q_f) d\Omega - \int_{\Omega_f} \delta p Q_m d\Omega = 0$$
(A16)

The third term in Equation (A16) can be rewritten as:

$$\int_{\Omega_f} \delta p \nabla \cdot (\rho \boldsymbol{q}_f) d\Omega = \int_{\Omega_f} \nabla \cdot (\delta p \rho \boldsymbol{q}_f) d\Omega - \int_{\Omega_f} \nabla \delta p \cdot \rho \boldsymbol{q}_f d\Omega = \int_{\Gamma_f} \delta p \rho \boldsymbol{q}_f \cdot \boldsymbol{n}_f d\Gamma + \int_{\Omega_f} \frac{\partial (\delta p)}{\partial s} \cdot \frac{\rho w^3}{12\mu} \frac{\partial p}{\partial s} d\Omega$$
(A17)

For fluid flow in the fracture, the fluid exchange (leak-off) term can be written as:

$$\int_{\Gamma_f} \delta p \rho \boldsymbol{q}_f \cdot \boldsymbol{n}_f d\Gamma = \int_{\Gamma_f} \delta p \rho w \boldsymbol{v}_d d\Gamma$$
(A18)

Thus,

$$\int_{\Gamma_f} \delta p \rho w v_d d\Gamma = -\int_{\Omega_f} \frac{\partial (\delta p)}{\partial s} \cdot \frac{\rho w^3}{12\mu} \frac{\partial p}{\partial s} d\Omega - \int_{\Omega_f} \delta p \frac{\rho w}{K_l} \frac{\partial p}{\partial t} d\Omega - \int_{\Omega_f} \delta p \rho \frac{\partial w}{\partial t} d\Omega + \int_{\Omega_f} \delta p Q_m d\Omega$$
(A19)

Since the fracture aperture is much smaller than its length, the variation of fluid pressure across the fracture width can be ignored. The fracture flow governing equation can be obtained from Equation (A19).

$$\int_{\Gamma_f} \delta p \rho v_d d\Gamma = -\int_{\Gamma_f} \frac{\partial (\delta p)}{\partial s} \cdot \frac{\rho w^3}{12\mu} \frac{\partial p}{\partial s} d\Gamma - \int_{\Gamma_f} \delta p \frac{\rho w}{K_l} \frac{\partial p}{\partial t} d\Gamma - \int_{\Gamma_f} \delta p \rho \frac{\partial w}{\partial t} d\Gamma + \int_{\Gamma_f} \delta p Q_m d\Gamma$$
(A20)

Based on Equation (A10) and (A20), the weak form of the coupled matrix-fracture flow governing equation is:

$$\int_{\Omega} \alpha \rho \delta p \nabla \cdot \dot{u} d\Omega + \int_{\Omega} \rho Q^{-1} \delta p \dot{p} d\Omega + \int_{\Gamma_q} \delta p \bar{q} d\Gamma + \int_{\Omega} \frac{\rho \kappa_m}{\mu} \nabla (\delta p) \nabla p d\Omega + \int_{\Gamma_f} \frac{\partial (\delta p)}{\partial s} \cdot \frac{\rho w^3}{12\mu} \frac{\partial p}{\partial s} d\Gamma + \int_{\Gamma_f} \delta p \frac{\rho w}{\kappa_l} \dot{p} d\Gamma + \int_{\Gamma_f} \delta p \rho \frac{\partial w}{\partial t} d\Gamma - \int_{\Gamma_f} \delta p Q_m d\Gamma = 0$$
(A21)

References

- 1. Liu, W.; Yao, J.; Chen, Z.; Liu, Y. Effect of quadratic pressure gradient term on a one-dimensional moving boundary problem based on modified Darcy's law. *Acta Mech. Sin.* **2016**, *32*, 38–53. [CrossRef]
- Bunger, A.P.; Kear, J.; Jeffrey, R.G.; Prioul, R.; Chuprakov, D. Laboratory investigation of hydraulic fracture growth through weak discontinuities with active ultrasound monitoring. In Proceedings of the 13th ISRM International Congress of Rock Mechanics, Montreal, QC, Canada, 10–13 May 2015.
- Guo, J.; Luo, B.; Lu, C.; Lai, J.; Ren, J. Numerical investigation of hydraulic fracture propagation in a layered reservoir using the cohesive zone method. *Eng. Fract. Mech.* 2017, 186, 195–207. [CrossRef]
- 4. Yang, D.; Zhou, Y.; Xia, X.; Gu, S.; Xiong, Q.; Chen, W. Extended finite element modeling nonlinear hydro-mechanical process in saturated porous media containing crossing fractures. *Comput. Geotech.* **2019**, *111*, 209–221. [CrossRef]
- Olson, J.E.; Wu, K. Sequential versus simultaneous multi-zone fracturing in horizontal wells: Insights from a non-planar, multifrac numerical model. In Proceedings of the SPE Hydraulic Fracturing Technology Conference, The Woodlands, TX, USA, 6–8 February 2012; OnePetro: Richardson, TX, USA, 2012.
- 6. Zoback, M.D.; Rummel, F.; Jung, R.; Raleigh, C.B. Laboratory hydraulic fracturing experiments in intact and pre-fractured rock. *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.* **1977**, *14*, 49–58. [CrossRef]
- 7. Bunger, A.P.; Gordeliy, E.; Detournay, E. Comparison between laboratory experiments and coupled simulations of saucershaped hydraulic fractures in homogeneous brittle-elastic solids. *J. Mech. Phys. Solids* **2013**, *61*, 1636–1654. [CrossRef]
- Guo, T.; Zhang, S.; Qu, Z.; Zhou, T.; Xiao, Y.; Gao, J. Experimental study of hydraulic fracturing for shale by stimulated reservoir volume. *Fuel* 2014, 128, 373–380. [CrossRef]
- 9. Hou, B.; Chang, Z.; Fu, W.; Muhadasi, Y.; Chen, M. Fracture initiation and propagation in a deep shale gas reservoir subject to an alternating-fluid-injection hydraulic-fracturing treatment. *SPE J.* **2019**, *24*, 1839–1855. [CrossRef]
- 10. Yang, H.; Guo, Y.; Wang, L.; Bi, Z.; Guo, W.; Zhao, G.; Yang, C. Study on the Stimulation Effectiveness Evaluation of Large-Scale Hydraulic Fracturing Simulation Experiment based on Optical Scanning Technology. *SPE J.* **2022**, *27*, 2941–2959. [CrossRef]
- 11. Advani, S.H.; Lee, J.K. Finite element model simulations associated with hydraulic fracturing. SPE J. 1982, 22, 209–218.
- 12. Devloo, P.R.; Fernandes, P.D.; Gomes, S.M.; Bravo, C.M.A.A.; Damas, R.G. A finite element model for three dimensional hydraulic fracturing. *Math. Comput. Simul.* 2006, 73, 142–155. [CrossRef]
- 13. Wangen, M. Finite element modeling of hydraulic fracturing in 3D. Comput. Geosci. 2013, 17, 647–659. [CrossRef]
- 14. Bao, J.Q.; Fathi, E.; Ameri, S. A coupled finite element method for the numerical simulation of hydraulic fracturing with a condensation technique. *Eng. Fract. Mech.* **2014**, *131*, 269–281. [CrossRef]
- 15. Chen, Z.; Jeffrey, R.G.; Zhang, X.; Kear, J. Finite-element simulation of a hydraulic fracture interacting with a natural fracture. *SPE J.* **2017**, *22*, 219–234. [CrossRef]
- Pakzad, R.; Wang, S.; Sloan, S.W. 3D finite element modelling of fracturing in heterogeneous rock: From pure solid to coupled fluid/solid analysis. In Proceedings of the ISRM European Rock Mechanics Symposium-EUROCK, St. Petersburg, Russia, 22–26 May 2018; Society of Petroleum Engineers: Richardson, TX, USA, 2018.
- 17. Pezzulli, E.; Nejati, M.; Salimzadeh, S.; Matthai, S.K.; Driesner, T. Finite element simulations of hydraulic fracturing: A comparison of algorithms for extracting the propagation velocity of the fracture. *Eng. Fract. Mech.* **2022**, 274, 108783. [CrossRef]
- 18. Ji, L.; Settari, A.; Sullivan, R.B. A novel hydraulic fracturing model fully coupled with geomechanics and reservoir simulation. *SPE J.* **2009**, *14*, 423–430. [CrossRef]
- 19. Kim, J.; Moridis, G.J. Numerical analysis of fracture propagation during hydraulic fracturing operations in shale gas systems. *Int. J. Rock Mech. Min.* **2015**, *76*, 127–137. [CrossRef]
- Duarte, C.A.; Reno, L.G.; Simone, A. A high-order generalized FEM for through-the-thickness branched cracks. Int. J. Numer. Methods Eng. 2007, 72, 325–351. [CrossRef]
- Gupta, V.; Kim, D.J.; Duarte, C.A. Analysis and improvements of global–local enrichments for the generalized finite element method. *Comput. Methods Appl. Mech. Eng.* 2012, 245, 47–62. [CrossRef]
- 22. Belytschko, T.; Black, T. Elastic crack growth in finite elements with minimal remeshing. *Int. J. Numer. Meth. Eng.* **1999**, 45, 601–620. [CrossRef]
- Belytschko, T.; Moës, N.; Usui, S.; Parimi, C. Arbitrary discontinuities in finite elements. Int. J. Numer. Meth. Eng. 2001, 50, 993–1013. [CrossRef]
- Moës, N.; Dolbow, J.; Belytschko, T. A finite element method for crack growth without remeshing. *Int. J. Numer. Meth. Eng.* 1999, 46, 131–150. [CrossRef]

- Mohammadnejad, T.; Andrade, J.E. Numerical modeling of hydraulic fracture propagation, closure and reopening using XFEM with application to in-situ stress estimation. *Int. J. Numer. Anal. Met.* 2016, 40, 2033–2060. [CrossRef]
- Xia, Y.; Jin, Y.; Chen, M.; Chen, K.P. An enriched approach for modeling multi-scale discrete-fracture/matrix interaction for unconventional-reservoir simulations. SPE J. 2019, 24, 349–374. [CrossRef]
- Zheng, H.; Pu, C.; Xu, E.; Sun, C. Numerical investigation on the effect of well interference on hydraulic fracture propagation in shale formation. *Eng. Fract. Mech.* 2020, 228, 106932. [CrossRef]
- Olson, J.E. Predicting Fracture Swarms The Influence of Subcritical Crack Growth and the Crack-Tip Process Zone on Joint Spacing in Rock; Geological Society, Special Publications: London, UK, 2004; Volume 231, pp. 73–88.
- 29. Weng, X.; Kresse, O.; Cohen, C.; Kresse, O.; Cohen, C.-E.; Wu, R.; Gu, H. Modeling of hydraulic-fracture-network propagation in a naturally fractured formation. *SPE Prod. Oper.* **2011**, *26*, 368–380.
- Castonguay, S.T.; Mear, M.E.; Dean, R.H.; Schmidt, J.H. Predictions of the growth of multiple interacting hydraulic fractures in three dimensions. In Proceedings of the SPE Annual Technical Conference and Exhibition, New Orleans, LA, USA, 30 September– 2 October 2013; Society of Petroleum Engineers: Richardson, TX, USA, 2013.
- Hou, B.; Chen, M.; Cheng, W.; Diao, C. Investigation of hydraulic fracture networks in shale gas reservoirs with random fractures. *Arab. J. Sci. Eng.* 2016, 41, 2681–2691. [CrossRef]
- 32. Tang, H.; Winterfeld, P.H.; Wu, Y.S.; Huang, Z.Q.; Di, Y.; Pan, Z.; Zhang, J. Integrated simulation of multi-stage hydraulic fracturing in unconventional reservoirs. *J. Nat. Gas. Sci. Eng.* **2016**, *36*, 875–892. [CrossRef]
- Kumar, D.; Ghassemi, A. Three-dimensional poroelastic modeling of multiple hydraulic fracture propagation from horizontal wells. Int. J. Rock Mech. Min. 2018, 105, 192–209. [CrossRef]
- Shen, B.; Shi, J. A numerical scheme of coupling of fluid flow with three-dimensional fracture propagation. *Eng. Anal. Bound. Elem.* 2019, 106, 243–251. [CrossRef]
- Salimzadeh, S.; Paluszny, A.; Zimmerman, R.W. Three-dimensional poroelastic effects during hydraulic fracturing in permeable rocks. *Int. J. Solids Struct.* 2017, 108, 153–163. [CrossRef]
- Li, S.; Firoozabadi, A.; Zhang, D. Hydromechanical modeling of non-planar three-dimensional fracture propagation using an iteratively coupled approach. J. Geophys. Res. Solid Earth. 2020, 125, e2020JB020115. [CrossRef]
- 37. Cundall, P.A. A computer model for simulating progressive, large-scale movement in blocky rock system. In Proceedings of the International Symposium on Rock Mechanics, Nancy, France, 4–6 October 1971.
- Li, S.; Feng, X.T.; Zhang, D.; Tang, H. Coupled thermo-hydro-mechanical analysis of stimulation and production for fractured geothermal reservoirs. *Appl. Energy* 2019, 247, 40–59. [CrossRef]
- Li, S.; Zhang, D.; Li, X. A new approach to the modeling of hydraulic-fracturing treatments in naturally fractured reservoirs. SPE J. 2017, 22, 1064–1081. [CrossRef]
- 40. Li, S.; Li, X.; Zhang, D. A fully coupled thermo-hydro-mechanical, three-dimensional model for hydraulic stimulation treatments. J. Nat. Gas. Sci. Eng. 2016, 34, 64–84. [CrossRef]
- 41. Settgast, R.R.; Fu, P.; Walsh, S.D.; White, J.A.; Annavarapu, C.; Ryerson, F.J. A fully coupled method for massively parallel simulation of hydraulically driven fractures in 3-dimensions. *Int. J. Numer. Anal. Met.* **2017**, *41*, 627–653. [CrossRef]
- 42. Guo, X.; Wu, K.; An, C.; Tang, J.; Killough, J. Numerical Investigation of Effects of Subsequent Parent-Well Injection on Interwell Fracturing Interference Using Reservoir-Geomechanics-Fracturing Modeling. *SPE J.* **2019**, *24*, 1884–1902. [CrossRef]
- Zeng, Q.; Liu, W.; Yao, J. Hydro-mechanical modeling of hydraulic fracture propagation based on embedded discrete fracture model and extended finite element method. *J. Petrol. Sci. Eng.* 2018, 167, 64–77. [CrossRef]
- 44. Wang, C.; Huang, Z.; Wu, Y.S. Coupled numerical approach combining X-FEM and the embedded discrete fracture method for the fluid-driven fracture propagation process in porous media. *Int. J. Rock. Mech. Min.* **2020**, *130*, 104315. [CrossRef]
- Ren, G.; Younis, R.M. An integrated numerical model for coupled poro-hydro-mechanics and fracture propagation using embedded meshes. *Comput. Methods Appl. Mech. Eng.* 2021, 376, 113606. [CrossRef]
- Liu, W.; Yao, J.; Zeng, Q. A numerical hybrid model for non-planar hydraulic fracture propagation in ductile formations. J. Petrol. Sci. Eng. 2021, 196, 107796. [CrossRef]
- 47. Zhang, J.; Yu, H.; Xu, W.; Lv, C.; Micheal, M.; Shi, F.; Wu, H. A hybrid numerical approach for hydraulic fracturing in a naturally fractured formation combining the XFEM and phase-field model. *Eng. Fract. Mech.* **2022**, *271*, 108621. [CrossRef]
- Ouchi, H.; Katiyar, A.; Foster, J.T.; Sharma, M.M. A peridynamics model for the propagation of hydraulic fractures in heterogeneous, naturally fractured reservoirs. In Proceedings of the SPE Hydraulic Fracturing Technology Conference, The Woodlands, TX, USA, 3–5 February 2015; OnePetro: Richardson, TX, USA, 2015.
- Rabczuk, T.; Ren, H. A peridynamics formulation for quasi-static fracture and contact in rock. *Eng. Geol.* 2017, 225, 42–48. [CrossRef]
- Qin, M.; Yang, D.; Chen, W.; Xia, X. Hydraulic fracturing network modeling based on peridynamics. *Eng. Fract. Mech.* 2021, 247, 107676. [CrossRef]
- Diehl, P.; Lipton, R.; Wick, T.; Tyagi, M. A comparative review of peridynamics and phase-field models for engineering fracture mechanics. *Comput. Mech.* 2022, 69, 1259–1293. [CrossRef]
- Moinfar, A.; Sepehrnoori, K.; Johns, R.T.; Varavei, A. Coupled geomechanics and flow simulation for an embedded discrete fracture model. In Proceedings of the SPE Reservoir Simulation Symposium, The Woodlands, TX, USA, 18–20 February 2013; Society of Petroleum Engineers: Richardson, TX, USA, 2013.

- 53. Zidane, A.; Firoozabadi, A. An efficient numerical model for multicomponent compressible flow in fractured porous media. *Adv. Water Resour.* **2014**, *74*, 127–147. [CrossRef]
- Garipov, T.T.; Karimi-Fard, M.; Tchelepi, H.A. Discrete fracture model for coupled flow and geomechanics. *Comput. Geosci.* 2016, 20, 149–160. [CrossRef]
- McClure, M.W.; Babazadeh, M.; Shiozawa, S.; Huang, J. Fully Coupled Hydromechanical Simulation of Hydraulic Fracturing in 3D Discrete-Fracture Networks. SPE J. 2016, 21, 1302–1320. [CrossRef]
- 56. Wei, S.; Kao, J.; Jin, Y.; Shi, C.; Xia, Y.; Liu, S. A discontinuous discrete fracture model for coupled flow and geomechanics based on FEM. *J. Petrol. Sci. Eng.* **2021**, 204, 108677. [CrossRef]
- 57. Dolbow, J.E. An Extended Finite Element Method with Discontinuous Enrichment for Applied Mechanics. Ph.D. Dissertation, Northwestern University, Evanston, IL, USA, December 1999.
- Gordeliy, E.; Peirce, A. Enrichment strategies and convergence properties of the XFEM for hydraulic fracture problems. *Comput. Methods Appl. Mech. Eng.* 2015, 283, 474–502. [CrossRef]
- Moës, N.; Cloirec, M.; Cartraud, P.; Remacle, J.F. A computational approach to handle complex microstructure geometries. Comput. Methods Appl. Mech. Eng. 2003, 192, 3163–3177. [CrossRef]
- Chen, K.P.; Jin, Y.; Chen, M. Pressure-gradient singularity and production enhancement for hydraulically fractured wells. *Geophys. J. Int.* 2013, 195, 923–931. [CrossRef]
- Xia, Y.; Jin, Y.; Huang, Z.; Lu, Y.; Wang, H. An Extended Finite Element Method for Hydro-Mechanically Coupled Analysis of Mud Loss in Naturally Fractured Formations. In Proceedings of the 53rd US Rock Mechanics/Geomechanics Symposium, New York, NY, USA, 23–26 June 2019; Society of Petroleum Engineers: Richardson, TX, USA, 2019.
- 62. Xia, Y.; Jin, Y.; Oswald, J.; Chen, M.; Chen, K. Extended finite element modeling of production from a reservoir embedded with an arbitrary fracture network. *Int. J. Numer. Methods Fluids* **2018**, *86*, 329–345. [CrossRef]
- 63. Liu, P.; Liu, F.; She, C.; Zhao, L.; Luo, Z.; Guan, W.; Li, N. Multi-phase fracturing fluid leakoff model for fractured reservoir using extended finite element method. *J. Nat. Gas. Sci. Eng.* **2016**, *28*, 548–557. [CrossRef]
- 64. Coussy, O. Poromechanics; John Wiley & Sons: Hoboken, NJ, USA, 2004.
- Schöllmann, M.; Richard, H.A.; Kullmer, G.; Fulland, M. A new criterion for the prediction of crack development in multiaxially loaded structures. *Int. J. Fract.* 2002, 117, 129–141. [CrossRef]
- 66. Olson, J.E.; Taleghani, A.D. Modeling simultaneous growth of multiple hydraulic fractures and their interaction with natural fractures. In Proceedings of the SPE Hydraulic Fracturing Technology Conference, The Woodlands, TX, USA, 19–21 January 2009; Society of Petroleum Engineers: Richardson, TX, USA, 2009.
- 67. Detournay, E. Propagation regimes of fluid-driven fractures in impermeable rocks. Int. J. Geomech. 2004, 4, 35–45. [CrossRef]
- 68. Rice, J.R. Mathematical analysis in the mechanics of fracture. Fract. Adv. Treatise 1968, 2, 191–311.
- 69. Liu, X.; Rasouli, V.; Guo, T.; Qu, Z.; Sun, Y.; Damjanac, B. Numerical simulation of stress shadow in multiple cluster hydraulic fracturing in horizontal wells based on lattice modelling. *Eng. Fract. Mech.* **2020**, *238*, 107278. [CrossRef]
- 70. Schultz, R. Discontinuity Patterns and Their Interpretation. In *Geologic Fracture Mechanics*; Cambridge University Press: Cambridge, UK, 2019; pp. 170–208.
- Miller, C.; Waters, G.; Rylander, E. Evaluation of production log data from horizontal wells drilled in organic shales. In Proceedings of the North American Unconventional Gas Conference and Exhibition, The Woodlands, TX, USA, 14–16 June 2011; Society of Petroleum Engineers: Richardson, TX, USA, 2011.
- 72. Lecampion, B.; Desroches, J. Simultaneous initiation and growth of multiple radial hydraulic fractures from a horizontal wellbore. *J. Mech. Phys. Solids* **2015**, *82*, 235–258. [CrossRef]
- 73. Lu, Y.; Wei, S.; Xia, Y.; Jin, Y. Modeling of geomechanics and fluid flow in fractured shale reservoirs with deformable multicontinuum matrix. *J. Petrol. Sci. Eng.* **2021**, *196*, 107576. [CrossRef]
- 74. Hughes, T.J.R. The Finite Element Method: Linear Static and Dynamic Finite Element Analysis; Prentice-Hall: Englewood Cliffs, NJ, USA, 1987.
- Witherspoon, P.A.; Wang, J.S.; Iwai, K.; Gale, J.E. Validity of cubic law for fluid flow in a deformable rock fracture. *Water Resour. Res.* 1980, *16*, 1016–1024. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.