



# **Phase Change Materials Energy Storage Enhancement Schemes and Implementing the Lattice Boltzmann Method for Simulations: A Review**

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Abstract: Utilizing phase change materials (PCMs) is one of the most effective methods of storing thermal energy and is gaining popularity in renewable energy systems. In order to analyze PCM performance, various numerical methods have been deployed to study the transient behaviour during phase changes. PCMs' low thermal conductivity prevents their use as pure PCMs in industrial applications. There are various efficient methods of enhancing PCM thermal conductivity, which are addressed in this article. On the other hand, the lattice Boltzmann method (LBM) is very inclusive in the numerical simulation of complex fluid flows, thermal transport, and chemical interactions because of its ability to simply represent various complex physical phenomena, suitability for parallel programming, and easy coding and implementation. Many numerical studies have been conducted on PCMs using the LBM. This study aims to review these studies and categorize them in a way so that one may thoroughly understand the LBM's capabilities in the simulation of PCMrelated investigations. First, PCM characteristics and applications are presented, then the LBM implementation in PCM problems is addressed. Afterward, the fabrication and types of PCMs are mentioned. Next, the improvement of thermal energy storage methods of PCMs is stated. Furthermore, governing equations are reviewed. Lastly, the opportunities and challenges of the LBM in PCMs are discussed.

**Keywords:** thermal energy storage; performance enhancement; lattice Boltzmann method (LBM); porous media; nanoparticle; phase change material (PCM)

# 1. Introduction

Thermal energy could be stored in two different ways: sensible and latent. Energy storage using sensible heat uses a solid or liquid medium without a phase change as an energy carrier, while latent heat energy storage (LHTES) uses the phase change enthalpy during melting or solidification to store energy. LHTES provides higher heat storage capacity with a lower weight and volume of the material than the sensible type. Using phase change materials (PCMs) for storing latent heat is the top-rated on the list of developed, applied, and investigated energy storage methods [1,2]. In PCMs, supplying or extracting energy is performed via a slight temperature change during the phase change, and the phase change occurs in an intermediate temperature range [3]. Another application of PCMs is thermal management and temperature control [4], for example, the thermal protection of electronic devices against pulsed power loads [5] or battery temperature management [6]. The worst thing about PCMs blocking their way to becoming a reliable method of energy storage is their low thermal conductivity. Hence, abundant studies were conducted to investigate the improvement of the thermal conductivity of PCMs, which is thoroughly addressed in Section 3 of this review. The most applicable methods are implementing porous media as composite PCMs, nanoparticle-enhanced PCMs (NPEPCMs), and using



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extended surfaces, such as fins. A schematic of some studies in these fields is presented in Figure 1.

**Figure 1.** Schematic of the domains of enhanced PCM problems: (**a**) implementing porous media [7], (**b**) using nanoparticle-enhanced PCMs [8], and (**c**) using finned cavities [9].

When it comes to studying the phase change process of enhanced PCMs, the complex geometries while simulating the domain are challenging to overcome. Generally, to simulate problems of this type, the finite volume method (FVM) is deployed, such as the study conducted by Acharya et al. [10] on ceramic matrices. Also, Farahani et al. [11], Selimefendigil et al. [12], and Ouri et al. [13] worked on the magnetic field effect on nanoparticle-embedded PCMs in a tube/cavity. Liu et al. [14] investigated the effect of fin arrangements on the melting process of PCMs. Selimefendigil and Oztop [15] studied a PCM-filled cavity under the presence of a rotating cylinder. All these and many more studies corroborated that the FVM is still a suitable method of discretization when studying enhanced PCMs. As a result, existing phase transition mathematical models are essentially continuum-based. A growing consensus has emerged in recent decades that serious problems may arise in the classical areas of macroscopic physics, despite the existence of known fundamental equations. [16]. Therefore, in many cases, due to the complexity of the geometry, which leads to huge computational costs, challenging convergence, and limitations on parallel solving, the lattice Boltzmann method (LBM) could an appropriate choice to simulate the phase change process.

As opposed to conventional computational fluid dynamics (CFD) methods which deploy numerical discretization with partial differential equations, the LBM is based on the particle interactions of a physical system on a microscopic scale; then, macroscopic quantities are obtained from these particle distribution functions. This concept would lead to simplified coding, feasibility in parallelization, and easier handling of complex geometries [17]. To simulate complex macroscopic transport phenomena, the lattice Boltzmann method deploys straightforward kinetic models. In order to recover continuum fluid dynamics models at the macroscopic limit, lattice gas models all share the common goal of defining a fictitious simplified micro-world. The benefits of the LBM's mesoscopic spirit include its simple calculation procedures, parallel computation, and the easy yet reliable handling of complex geometries. McNamara and Zanetti [18] first proposed the LBM. Benzi et al. [16], in their book, mentioned two possibilities: the variables in lattice gas automata are Boolean populations expressing whether links between particles in a lattice exist or not. An arrangement of collision rules with appropriate conservation laws serves as the basis of evolution. After deriving the LB equation from the theory of lattice gas cellular automata [19], Chen and Doolen [20] implemented the LBM for fluid flows.

A thorough investigation of the various applications and most practical types of PCMs was performed by Sharma et al. [21]. In this review, PCM materials were categorized by their thermal, physical, kinetic, and chemical properties, and finally, their economic efficiency. As a conclusion of this study, the choice of the PCM took various factors into account, such as having large latent heat capacity and suitable operating temperatures, high

thermal conductivity, high density, insignificant changes in volume, no phase separation, no corrosion, chemical stability, compatibility with the container, nontoxicity, nonflammable and nonpolluting, low cost or affordability, and recyclability. Another matter that was addressed in many papers is PCM drawbacks and methods to enhance them, which are discussed from the viewpoint of the influence of operational parameters [22] and the effect of nano-PCMs [23–25].

The major disadvantages of PCMs are the ones sorted by Zhang et al. [4] as follows: supercooling, phase separation (for non-homogenous PCMs in case of the existence of non-pure components), and low thermal conductivity (for pure PCMs).

Zhang et al. [4] also managed to address some solutions for each of the abovementioned drawbacks, as follows:

- Decrease supercooling by incorporating a nucleation agent into the PCMs. This causes
  heterogeneous nucleation, which creates a crystal structure similar to solid PCMs.
  This would cause solidification on the nucleation agent's surface, reducing the need
  for additional cooling. In order to lessen the degree of supercooling, cold finger
  technology, supersonic treatment, and mechanical impulsion were also researched [4];
- Avoid phase separation: Gelling or thickening agents could be implemented to prevent phase separation. By increasing the viscosity of the liquid phase, phase separation may be slowed;
- Improving thermal conductivity: According to Jegadheeswaran and Pohekar [26], the
  most popular techniques include using extended surfaces, such as fins, using multiple
  PCM units, improving thermal conductivity, and creating micro-encapsulated PCMs
  or even combining two or more of the above-mentioned methods [27]. The most
  practical method, which also has drawbacks, such as increasing the weight of the
  system, is remembered as using extended surfaces (fins).

As complementary studies, Liu et al. [28] reviewed the thermal conductivity improvement of phase change materials. In Section 4 of the current review, the methods that enhance the performance of PCMs are discussed with a focus on LBM-related studies.

In this review study, first, PCM applications are mentioned, then LBM implementation in numerical studies is reviewed in Section 2. The fabrication of PCMs is an important topic, and the current study also gathers information about this in Section 3. Then the enhancement methods of PCMs are addressed by concentrating on the LBM-related studies in Section 4. The knowledge of the governing equations of the phase transition and heat transfer is inevitable to simulate the phase change phenomenon and is addressed in Section 5. Then, the trend of LBM phase change equations is overhauled to implement the LBM in the simulation of PCM-related issues in Section 6. The comparison of the capabilities and strengths between the LBM and other conventional numerical methods is mentioned in Section 7, and an overall conclusion is presented in Section 8.

## 2. PCM Applications and Implementing the LBM in PCM Studies

The concept of phase change heat transfer has a wide range of engineering applications, such as casting, and is not limited to solid and liquid heat transfer for energy storage. However, the current study focuses on PCMs for LHTES.

Phase change materials have drawn attention in various systems encountering energy storage. PCMs may be used as cooling and heating aids in buildings while demanding lower energy charges by using solar systems [29,30]. PCMs with melting temperatures below 15 °C may be used as cold energy storage to cool and ventilate room air. On the other hand, PCMs with melting temperatures around 35~45 °C are commonly used for battery thermal management. PCMs with a melting temperature higher than 90 °C are used for temperature reduction to prevent fires. Molten salt-based PCMs [31], whose melting temperatures are around the range of 200 to 1000 °C, may be implemented for applications demanding high temperatures, such as furnaces, advanced solar energy systems, and high-temperature thermal shielding [25].

Some investigations have focused on practical PCM applications [32,33], which described PCM usage in waste heat recovery [34], battery thermal management [35–38], solar power plants [39,40], medical treatments [41], pharmaceuticals [42], energy transformation [22], photovoltaic cells [25,43–45], thermal management in mobile phones [46] (in particular, microelectronics) [47], and military electronic systems [48]. Recently, some investigations were conducted to check the feasibility of using PCMs in astronautic applications considering gravity effects [43]. Some of these applications are illustrated in Figure 2.



Figure 2. A schematic showing various PCM applications.

Recently, phase transition problems were also investigated using the LBM, and a detailed survey of these works is addressed in Section 4, where LBM studies on the energy storage of PCMs are reviewed. The LBM is inherently transient, so it is a superior means for studying the phase change process and its transient nature [49]. In recent years, the LBM was implemented in simulation and numerical investigations of heat transfer in the liquid-solid phase change [50–54]. Further investigations were conducted to use the LBM to study solid and liquid phase change applications, such as melting ice slurries [55], complex heat storage [49], heat transfer in PCMs with specific conditions, such as the presence of porous media [7,53,54,56,57], nanoparticle-enhanced PCMs [8,58–60], encapsulated PCMs [61], and last but not least, studies on gravity's effect in PCMs and its usage feasibility in aeronautic industries [62,63].

## 3. Fabrication and Thermal Performance of PCMs

There are several studies on PCM applications. In the current study, the applications of PCMs are reviewed based on the type of PCM. In order to carry out this goal, Sharma et al. [21] categorized PCMs into three different types, which are addressed in this section.

## 3.1. Organic PCMs

Organic materials are of two compounds: paraffin and non-paraffin. Paraffin wax is formed from a mixture of mainly straight-chain alkanes that release latent heat when the  $(CH_3)$ - chain crystallizes. Paraffin compounds are safe, cheap, and non-corrosive materials. They have little volume change when changing phases. Lastly, systems using paraffin as a PCM have a very long freeze-melt cycle [21,64].

Non-paraffin organic PCMs have the most extensive list among all types of PCMs, with varying properties. Some studies have categorized these PCMs [65] and defined

esters, alcohols, etc., as the proper materials for this group. Their drawback is that they are flammable; therefore, their working temperatures are low. Also, they are mildly corrosive.

## 3.2. Inorganic PCMs

Salt hydrates and metallics are the two main groups of materials forming inorganic PCMs. The principal characteristic of inorganic PCMs is that they are not significantly supercooled, which avoids decreasing the heat of fusion during cycling. If PCMs are cooled below their melting point, they remain in a liquid state. This is a metastable state which is called supercooled. The salt hydrate's general chemical formula is  $AB.nH_2O$ . Here, the dehydration of these substances is the solid-liquid phase change. The most important properties of this type of PCM are the high latent heat of fusion per unit of volume, high thermal conductivity, low price, and non-corrosive nature [21,66]. Although salt hydrates are suitable for energy storage, their major disadvantage is that they melt incongruently. This means that melting happens when the process of dissolving the salt hydrate in water is not yet complete [67,68].

Metallics is the category that consists of not only low melting temperature metals but also metal eutectics. They have less volume and higher weights than other PCMs. They have high thermal conductivity, so enhancement methods are not required on these types of PCMs, which is addressed in Section 4.

#### 3.3. Eutectics

Eutectics are composed of two or more components that freeze and melt homogenously. These components could be both organic, both inorganic, or one is organic, while the other one is inorganic. While melting, both components liquefy simultaneously. Gallium is one of the most famous eutectics. Its main property is that it melts and freezes without segregation [69]. Some examples of various types of PCMs are illustrated in Figure 3.



**Figure 3.** Different types of PCMs, including organic PCMs, such as paraffin wax and ice, inorganic PCMs, such as salt hydrates and gallium, and eutectic PCMs.

#### 4. Improvement of Energy Storage in PCMs

As mentioned in Section 1, PCM performance improvement may be briefed as the thermal conductivity enhancement of the PCM material. Therefore, many studies have been established to find methods to reach more significant thermal conductivities of PCMs. These studies also have led to collateral effects, such as lower volume changes during the phase change, more uniform temperature distributions, shorter charging times, and more. Here, some effective methods for PCM improvements are summarized.

## 4.1. Porous Media as Composite PCMs

A reliable scheme to improve PCM thermal conductivity is using highly porous structures as catalysts with high thermal conductivities. In general, implementing porous media and composite PCMs could be two different methods of PCM enhancement. However, all known composite PCMs are usually porous media. Therefore, here, these two methods are combined and defined as one single coupled method.

Since this paper concentrates on lattice Boltzmann method applications in PCM studies, there are investigations in the simulation of porous media-filled cavities of PCMs. Wu et al. [7] studied the solid-liquid phase change problem considering convection heat transfer in porous media. Deploying a generalized model of the LB based on the Darcy-Brinkman-Forchheimer model considers the existence of the porous medium. The aim of this study and other correlated studies [53,57] was to illustrate the agreement between this model and previous analytical and numerical models. Another type of study in this scope has focused on the geometrical characteristics of the porous medium used to enhance the energy storage of PCMs. A recent study on this topic was from Mabrouk et al. [70], which investigated the porosity effects on this realm.

#### 4.1.1. Metal Foam Composite PCMs

A key process in the provision of composite PCMs is impregnating PCMs into porous metals [4]. High thermal conductivity and the specific surface ratios of metal foams are the main reasons why they have attracted attention [71]. Metal foams significantly affect the phase change heat transfer of melting and solidification [72]. They shorten the charging time and construct a more uniform temperature distribution [73–76]. Adding metal substances to PCMs would lead to greater thermal conductivity [77]. Most of the disquisitions in this area have focused on foam geometry. Some has followed the LBM method, e.g., Gaedtke et al. [78] and Mabrouk et al. [70]. Others such as Sai Dinesh et al. [79,80] and Parida et al. [81] who investigated the porosity and pore size of metal foams implemented other schemes that simulate the flow through metal foams. They concluded that even for the same porosity value, a smaller pore size results in faster melting and a higher heat transfer rate. Furthermore, for lower porosities, the melting time is reduced due to the higher thermal conductivity (see Figure 4).



**Figure 4.** The melting front for different porosities of metal foams after 163 s performed by Parida et al. [81]; (a) 50% porosity, (b) 60% porosity, and (c) 70% porosity.

In a work conducted by Hu et al. [82], the pore density was explained. Low porosity and high pore density composite PCMs melt faster than high porosity and low pore density ones. As this method has the most visible impact on the melting process of PCMs, there are various experimental investigations in this area. For instance, Mohamed Moussa and Karkri [83] conducted a study on the effects of different heat condition types in metal foam-enhanced PCMs. The thermal behavior of a PCM filled with a low porosity metal foam was the target of an experimental study by Hu et al. [84], and more recently, the geometrical characteristics of a metal foam were investigated in a set of experiments which were led by Iasiello et al. [85]. A major concern of the studies in this area is whether the solution domain is three or two-dimensional. In three-dimensional domains, one could thoroughly study the effect of metal foam geometry as it has various conduction paths from behind and in front of the 2D section. However, in 2D studies, these conduction paths would not be declared. This phenomenon that would distinguish the conduction paths is critical in some applications to be observed, such as photovoltaic panels' energy storage enhancement by adding metal foam-embedded PCMs, which was the main concern of a recent study by Li et al. [86]. Some problems addressed were energy storage tanks enhanced by PCMs impregnated with metal foams [87]. Despite the latter applications, which require 3D simulation studies, there are multiple applications that implement metal matrixes (not necessarily foams) that do not have complex geometries (such as open-cell irregular foams) and could be simplified in 2D studies without significantly affecting the results. Shirbani et al. [88] addressed these applications and implemented the LBM to study the effects of the geometrical characteristics of a solid matrix on the melting process of a PCM. Their main goal was to reach the best pore arrangement in cylindrical-shaped pores in a solid matrix along with the best pore diameter to reach the most enhanced performance of a PCM during melting. They concluded that a bigger mean pore diameter would help the natural convection, and then the arrangement would not affect the melting time. However, for a small pore mean diameter, the arrangement and conduction path thickness would be important.

#### 4.1.2. Composite PCMs with Carbon Materials

Carbon nanomaterials, expanded graphite (EG), and graphene-based materials are examples of carbon materials that have been extensively studied and used as thermal conductivity-enhancing materials [4]. Carbon substances are suitable to formulate a solution with barely corrosive PCMs, such as salt hydrates. The fabrication processes of carbon/graphite foam composite PCMs [89–92] and graphene aerogel [93] are the same as metal foams, called the vacuum impregnation process. More applicable studies have been conducted recently, such as Polytetrafluoroethylene (PTFE)-based PCMs [94] and gypsum-based PCMs [95], which both use carbon fibers as the enhancement agent for the PCM.

#### 4.2. Nanoparticle-Enhanced PCMs

A typical disadvantage of PCMs is their low thermal conductivity. Khodadadi et al. [96,97] were among the first researchers who came up with the idea of dispersing high thermal conductivity nanoparticles into PCMs. According to the results of their study, the application diversity of PCMs could be increased when the thermal conductivity of the PCM is improved.

Recent studies were conducted to report LBM implementation on the simulation of nanoparticle-enhanced phase change material (NEPCM) heat transfer. Jourabian et al. [60] studied the LBM application in simulating ice melting enhanced with Cu nanoparticles. They presented a good concurrence between their numerical results and those of previous numerical and experimental methods. They also declared that adding nanoparticles would hasten the melting process. Then, Feng et al. [59] conducted a study on the same material with a novel LB method and different boundary conditions. Their results approved the previously mentioned work, and they concluded that by increasing the nanoparticles' volume fraction, the stored energy and molten fraction would increase, in line with faster development in the temperature field and melting interface.

Ibrahem et al. [8], in their work, could add one more characteristic change in PCMs when improved by adding nanoparticles. They explained that the presence of nanoparticles enhanced the movement of the phase change front, summarized in Figure 5. Ren et al. [58] studied the melting process of nanoparticle and metal foam-enhanced PCMs in a heat pipe. They concluded that increasing the porosity of metal foams or increasing the nanoparticle volume fraction would speed up the melting process of nanoparticle PCMs. Moreover,

they mentioned that the immersed boundary enthalpy-based lattice Boltzmann method would be a suitable method for a numerical study on this matter. Recent studies addressed the complex geometries and combinations of various PCM enhancement methods. For instance, in the work published by Jourabian et al. [98], the geometry was an inclined elliptical annulus, and the NEPCM was combined with porous media and a metal matrix. They concluded that for inclined geometries, it is better to use the solid matrix in PCMs to have stable stratification. Recently Peng and Sadaghiani [99] studied the effect of alumina nanoparticle-enhanced PCMs around a cylindrical battery in a rectangular finned domain. Adding a slope to the geometry of the later problem, meaning the study of an inclined finned domain consisting of hybrid nanoparticle-enhanced PCMs, is what Laouer et al. [100] studied. They reported the effect of the number of fins and their optimum length ratios. Adding fins to a domain-containing PCM is another way of increasing the heat transfer coefficient and is reviewed in the next section.



**Figure 5.** Interface positions in the Ibrahem et al. [8] study. (**a**) Pure water, (**b**) nano-PCM No.1, and (**c**) nano-PCM No.2.

## 4.3. Effect of Adding Fins

A common, straightforward, and applicable way to improve the thermal conductivity of PCMs is through the use of extended surfaces called fins. However, this has some drawbacks, such as increased system weight [4]. Here, the main goal is to review LBMbased studies on the effects of fins on the phase change behavior of PCMs.

Talati and Taghilou [101,102] investigated the LBM application on solidification in a finned container. In their study, they defined the optimum aspect ratio of the container to maximize the solidification time, which was 0.5. Also, they concluded that changing the fin's material would not significantly impact the results. As a complementary study to their previous work, they conducted a study on the melting of a PCM in the same conditions [103]. As an additional result, they found that moving a solid body would accelerate the melting process.

As a novel study, Ren and Chan [9] implemented a multi-relaxation time (MRT)-LBM to simulate the heat transfer of finned PCMs. Their first major result was the development of the method that they implemented. The secondary results illustrated that by increasing the number of fins, the melting process was completed faster (see Figure 6). Furthermore, the melting rate grew when the length of the inner fins expanded, despite a limit. They found, however, that fewer long fins were more effective than more short fins.



**Figure 6.** Comparison of melting fronts for a cavity (**a**) without fins and (**b**) with fins from the study published by Ren and Chan [9].

A phase change material coupled with conduction fins was also simulated using the MRT-LBM for transient melting heat transferrin, a work published by Gao et al. [103]. Their novelty was the coupling of the MRT-LBM with the representative element volume scale (REV-scale). Their results indicated the same conclusion as that of Ren and Chan [9], as previously addressed. Recent studies corresponding to adding fins in geometries to enhance the energy storage of PCMs are also about a combination of the enhancement methods, such as the NEPCM. Regardless of the simulation method, Chu et al. [104] incorporated the NEPCMs by using a finned tank to enhance the energy storage of PCMs. The focus of their work was energy loss in various cases of the layouts of fins. As a recent study of the combined enhancement methods of PCMs, Yang et al. [105] decided to investigate the effect of adding a metal foam to a finned shell and tube consisting of a PCM and concluded that this action decreased the nonuniformity of solidification and temperature distribution; the authors also mentioned that this hybrid method would lead to a 90.5% reduction in the total energy released compared to a pure PCM unit.

#### 5. Phase Change Governing Equations

The general governing equations for thermo-fluidic transport encountering phase changes (melting and solidification), assuming the flow to be Newtonian, laminar, and incompressible, are [49,106]:

$$\nabla .(\boldsymbol{u}) = 0 \tag{1}$$

$$\rho \left[ \frac{\partial u}{\partial t} + \nabla . (uu) \right] = -\nabla p + \nabla . (\mu \nabla u) + \rho f$$
<sup>(2)</sup>

$$\left(\rho C_p\right) \left[\frac{\partial}{\partial t} + \nabla .(\boldsymbol{u}T)\right] = \nabla .(k\nabla T) + q \tag{3}$$

In these equations,  $\rho$ , T, p, u,  $C_p$ , k, and  $\mu$  are, respectively, the fluid density, temperature, pressure, fluid velocity vector, specific heat, thermal conduction coefficient, and dynamic viscosity of fluid. f is the external body force defined per unit of mass and considering the Boussinesq approximation, the thermal expansion factor  $\beta$ , the vector of gravitational acceleration g, and  $T_{ref}$  as a reference temperature this force may be defined as  $f = g\beta (T - T_{ref})$ .

The source term that appeared in Equation (3) (i.e., q) is the latent heat due to a phase transition and is specified by the following relation:

$$q = -\left[\frac{\partial(\rho\Delta H)}{\partial t} + \nabla .(\rho u\Delta H)\right]$$
(4)

where *H* is the latent enthalpy of the cell subjected to a phase transition that is being computed.  $f_l$  denotes the liquid fraction obtained from  $f_l = \Delta H/L$ , and *L* indicates the latent heat of fusion. The second term in Equation (4) may be omitted when the substance is pure [49,51]:

$$q = -\frac{\partial(\rho\Delta H)}{\partial t} = -\frac{\partial(\rho L f_l)}{\partial t}$$
(5)

To have a more accurate understanding of the fundamental equations under phase change conditions, Su and Davidson [51] separated the energy equations for each phase and described the source term of each differently. Accordingly, one could rewrite the continuity, momentum, and energy equations as follows:

$$\nabla \boldsymbol{.} \boldsymbol{u}_f = \boldsymbol{0} \tag{6}$$

$$\rho_f \left[ \frac{\partial u_f}{\partial t} + \nabla \left( u_f u_f \right) \right] = -\nabla p_f + \mu_f \nabla^2 u_f + \rho_f g \beta \left( T - T_{ref} \right)$$
(7)

$$\left(\rho C_p\right)_f \left[\frac{\partial T_f}{\partial t} + \nabla \cdot \left(\boldsymbol{u}_f T_f\right)\right] = \nabla \cdot \left(k_f \nabla T_f\right) + q_{L_f} + q_{sf} \tag{8a}$$

$$\left(\rho C_p\right)_s \frac{\partial T_s}{\partial t} = \nabla . \left(k_s \nabla T_s\right) + q_{L_f} - q_{sf} \tag{8b}$$

The fluid and solid phases are addressed with the *f* and *s* subscripts.  $q_{sf}$  shows the transferred heat from two different phases, meaning solid to liquid, while  $q_{L_f}$  is the solid-liquid phase transition latent heat. From a more detailed point of view, a PCM could be considered a porous medium. Accordingly, melting and solidification would be distinguished by their initial porosity. For solidification,  $\varepsilon_0 = 1$ , would represent the liquid phase, and for melting,  $\varepsilon_0 = 0$ , represents the solid phase. Considering a PCM as a porous medium, Equation (5) may be represented as:

$$q_{L_f} = -\frac{\partial \rho L_f \varepsilon}{\partial t} \tag{9}$$

where  $L_f$  is the latent heat per unit of mass and  $\varepsilon$  is the volume fraction of the fluid phase or the porosity:

$$\varepsilon = \frac{V_f}{V_s + V_f} \tag{10}$$

Based on the definition of porosity which was presented in Equation (10), the energy equations for the solid and liquid phases (Equations (8a,b)) could be merged:

$$\left[\varepsilon(\rho C_p)_f + (1-\varepsilon)(\rho C_p)_s\right] \left[\frac{\partial T}{\partial t} + \nabla \cdot \left(\varepsilon u_f T\right)\right] = \left[\varepsilon k_f + (1-\varepsilon)k_s\right] \nabla^2 T - \left[\varepsilon(\rho L_f)_f + (1-\varepsilon)\left(\rho L_f\right)_s\right] \frac{\partial \varepsilon}{\partial t}$$
(11)

The effective transport parameters of the solid-liquid mixture are expressed in terms of porosity:

$$\rho_m = \varepsilon \rho_f + (1 - \varepsilon) \rho_s \tag{12}$$

$$C_{pm} = \frac{(\rho C_p)_m}{\rho_m} = \frac{\varepsilon (\rho C_p)_f + (1 - \varepsilon) (\rho C_p)_s}{\varepsilon \rho_f + (1 - \varepsilon) \rho_s}$$
(13)

$$\alpha_m = \frac{\varepsilon k_f + (1 - \varepsilon)k_s}{\varepsilon (\rho C_p)_f + (1 - \varepsilon)(\rho C_p)_s}$$
(14)

These are representing the effective density, heat capacity, and thermal diffusivity, respectively. One should consider that to establish Equation (9), the temperature range for the melting of a PCM should be within  $\Delta T_m$ , which is described as  $\Delta T_m = T_{m_{high}} - T_{m_{low}}$ .

Otherwise, Equation (9) should be replaced with  $C_{pm} = \frac{L_f}{\Delta T_m}$ . Therefore, the momentum and energy equations would be reduced to:

$$\frac{\partial u}{\partial t} + \nabla .(uu) = -\frac{1}{\rho_f} \nabla p_f + \nu_f \nabla^2 u + \rho_f g \beta \left(T - T_{ref}\right)$$
(15)

$$\frac{\partial T}{\partial t} + \nabla .(\varepsilon \boldsymbol{u}T) = \alpha_m \nabla^2 T - \left(\frac{L_f}{C_{pm}}\right) \frac{\partial \varepsilon}{\partial t}$$
(16)

where the PCM is embedded in a porous material (such as metal foam) due to its porous medium being, the Carman-Kozeny equation with the percolation model of Darcy-Brinkmann-Forchheimer is implemented in the low liquid fraction areas as the governing equations for internal flow [50,84,107,108]. The model could be expressed in terms of the generalized Navier-Stokes (N-S) equation, and this equation could be predigested to the standard N-S equation under certain conditions.

For a flow in porous media that is laminar and incompressible in the absence of viscous dissipation, a chemical reaction, or radiative heat transfer and in the presence of natural convection, the Navier-Stokes equations are as follows [53,57,109]:

 $\nabla$ 

$$.u = 0 \tag{17}$$

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) \left(\frac{u}{\varepsilon}\right) = -\frac{1}{\rho} \nabla(\varepsilon p) + \nu_e \nabla^2 u + F$$
(18)

These equations are the continuity and Darcy-Brinkmann-Forchheimer equations, respectively.  $v_e \nabla^2 u$  is responsible for representing the presence of solid boundaries and is called the Brinkmann term.  $\varepsilon$ ,  $v_e$ , and F are the porosity of the porous medium, effective viscosity, and the body force induced by the presence of the porous medium and other external forces, defined by:

$$F = -\frac{\varepsilon \nu_f}{K} u - \frac{\varepsilon F_{\varepsilon}}{\sqrt{K}} |u| u + \varepsilon G, \quad K = \frac{\varepsilon^2 d_p^2}{150(1-\varepsilon)^2}$$
(19)

In this relation,  $-\frac{\varepsilon v_f}{K} u$  is the linear (Darcy), and  $-\frac{\varepsilon F_{\varepsilon}}{\sqrt{K}} |u| u$  is the nonlinear (Forchheimer) drag force due to the porous medium.  $F_{\varepsilon}$  is the geometric function, K is the porous medium permeability,  $v_f$  is the fluid kinematic viscosity, G is the body forced-induced by an external force, and  $d_p$ . is the solid particle diameter. If G is created only by gravity, the Boussinesq approximation of buoyancy would be:

$$G = -g\beta(T - T_m) \tag{20}$$

The geometric function,  $F_{\varepsilon}$ , is presented in Ergun's relation [110]:

$$F_{\varepsilon} = \frac{1.75}{\sqrt{150\varepsilon^3}} \tag{21}$$

The energy equation for melting in the presence of a porous medium with natural convection could be rewritten in its volume averaged form [57,111]:

$$\sigma \frac{\partial T}{\partial t} + u \cdot \nabla T = \nabla (\alpha_e \nabla T) - \varepsilon \frac{L_a}{C_{pf}} \frac{\partial \gamma_f}{\partial t}$$
(22)

where,  $L_a$  is the melting, latent heat,  $\alpha_e$  is the apparent thermal diffusivity,  $\gamma_f$  is the liquid fraction in a pore space,  $C_{pf}$  specifies the liquid-specific heat, and  $\sigma$  is the thermal capacity ratio and is defined by:

$$\sigma = \frac{\varepsilon \rho [\gamma_f C_{pf} + (1 - \gamma_f) C_{ps}] + (1 - \varepsilon) (\rho C_p)_m}{\rho C_{pf}}$$
(23)

The liquid and solid phases of a PCM and the solid matrix properties are referred to as the *f*, *s*, and *m* subscripts.  $\alpha_e = \frac{k_e}{\rho c_p}$  is the effective thermal diffusivity, and the effective thermal conductivity, *k*<sub>e</sub>, is approximated by:

$$k_e = \varepsilon \Big[ \gamma_f k_f + \Big( 1 - \gamma_f \Big) k_s \Big] + (1 - \varepsilon) k_m \tag{24}$$

At last, in Equation (22), by deploying the enthalpy-based method, the liquid fraction could be related to the temperature and then solved [112,113]. The enthalpy,  $H_p$ , which is thoroughly addressed in Section 6.2 is defined as:

$$H_p = c_p T + \gamma_f L_a \tag{25}$$

Regarding the phase change in the porous medium, Kazmierczak [114] first studied the phenomenon of the natural convection melting of a PCM in the porous medium based on Darcy's model. Then, Beckermann and Viskanta [111] established a mathematical model based on the Brinkmann-Forchheimer equation, which is more appropriate.

#### 6. Thermal Lattice Boltzmann Method in PCMs

The lattice Boltzmann method (LBM) is implemented to solve numerical problems due to the use of the bounce-back boundary condition as a no-slip condition and its convenience in parallel computing [115]. There are two types of LBMs used in solving phase change problems: the phase-field method and the enthalpy-based method, which are thoroughly discussed in the following sections. Fabritiis et al. [116] proposed a model to solve the problem of phase transitions consisting of a chemical term. Then, Miller [117] modified Fabritiis et al.'s model by tracing the interface of a solid/liquid phase using the phase-field method. Further investigations [118–122] were accomplished to solve the anisotropic liquid-solid phase change, flows with binary-alloy solidification, anisotropic crystal growth, and dendritic growth by convection.

Jiaung et al. [112] first proposed the enthalpy-based LBM. In their study, the latent heat of a PCM appeared as a source term in the LB equation, and they extended it. However, the main debility of this method was the number of iterations required to calculate the temperature and liquid fraction during each time step. Therefore, Chatterjee and Chakraborty [123–126], in their investigations, used the iterative scheme suggested by Brent et al. [127] to simplify the iterations. As a subset of the enthalpy-based LBM, Fuentes et al. [128] developed and validated the MRT-LBM, which has more accuracy than the other methods. However, the low convergence rate is the weakness of this method, which is discussed in Section 6.3.

## 6.1. Phase-Field Scheme

In the pursuant section, the phase-field method governing equations mentioned in Section 6 are presented based on the study conducted by Miller [121]. For every scheme that mounts on the LB equation, there is a general form of LBE and its corresponding source terms that represent the scheme. The LBM basis of the phase-field method is the single-relaxation time Bhatnagar, Gross, and Krook (BGK) model [129]. The general LB equation is presented as follows [130,131]:

$$f_i(\vec{x} + \vec{e_i}\Delta t, t + \Delta t) - f_i(\vec{x}, t) = -\Omega_{ij}\Delta t \left[ f_j - f_j^{eq} \right](\vec{x}, t) + S_i\Delta t$$
(26)

In the above relation,  $\Omega_{ij} \Delta t = 1/\tau_f$  where  $\tau_f$  is the relaxation time of the fluid [115]. In Equation (26), the left-hand side is the collision term, and the first term on the right-hand side is called the streaming step. The discrete distribution function  $f_i(\vec{x}, t)$  is responsible for representing the moving of particles alongside the directions of the lattice.  $\vec{c_i}$  indicates the discrete lattice speed along the *i* direction. To determine the direction, first, the LBM type should be determined.  $D_n Q_m$  represents the dimension and directions of the lattice by n, m, respectively. There are several types of n and m combinations, such as  $D_2Q_5$ ,  $D_2Q_7$ ,  $D_2Q_9$ ,  $D_3Q_{15}$ . For instance, Figure 7 shows the discretization of the D2Q9 model.



Figure 7. Discretization of space for the D2Q9 model (discrete velocity directions) [49].

Among all of the 2D methods the  $D_2Q_9$ , due to its proper convergence and appropriate accuracy, is the most common method [19]. Furthermore, Su et al. [52] mentioned for twodimensional flow structures, 2D simulations could capture the coefficient of heat transfer accurately. Therefore, the D2Q9 lattice dimension is suitable, in spite of the fact that the change in fluid patterns from spirals to recirculations may occur. *eq* is a superscript referring to the equilibrium distribution function:

$$f_e^i = \rho \left| A_i + B_i u_i + C_i u^2 + D_i u_i^2 + E_i u_i u^2 + F_i u_i^3 + G_i u_i^2 u^2 + H_i u^4 \right|$$
(27)

In this relation,  $\rho$  is the fluid density, and for the Navier-Stokes equations with heat transfer to be reproduced,  $u_i = uc_i$  and  $A_i, B_i, \ldots$  are chosen [116]. Here, the key macroscopic quantities that are reproduced are as follows:

$$\rho = m \sum_{i} f_{i} (mass \ density)$$

$$\rho u = m \sum_{i} f_{i}c_{i}(current \ density)$$

$$\rho T = m \sum_{i} f_{i}(c_{i} - u)^{2}(temperature)$$

$$q = m \sum_{i} f_{i}(c_{i} - u)^{3}(heat \ flux)$$
(28)

When the LB equation is applied to the phase change problem, the interaction of the fluid with the melting front is encoded by the source term  $S_i$ . Only a fraction  $1 - r(\phi)$  is transmitted to the next link for any lattice site. This step is called reflection and is provided by [121]:

$$\widetilde{f}_{i}(\vec{x}+\vec{e}_{i}\Delta t,t+\Delta t) = \left(1-r(\vec{x},t)\right)f_{i}(\vec{x},t) + r(\vec{x},t) + r(\vec{x},t)f_{\overline{i}}(\vec{x},t)$$
(29)

The overbar denotes the mirror conjugate to direction *i*, meaning  $\vec{e}_i + \vec{e}_{\bar{i}} = 0$ . Considering Equation (29) to update Equation (26) leads to the following:

$$f_i(\vec{x} + \vec{e}_i \Delta t, t + \Delta t) = \tilde{f}_i - (1 - r(\vec{x}, t))\Omega_{ij}\Delta t \left[ f_j - f_j^{eq}(\vec{x}, t) \right] + \tilde{S}_i \Delta t$$
(30)

The reflectivity  $r(\phi)$  in the case of full liquid or full solid cells should vary between 0 and 1, respectively. Consider  $r(\vec{x}, t)$  as a step function and assume it is on the grid, then Equation (30) would be reduced to the bounce-back boundary condition's standard version. Here, a continuous solution-like phase-field function represents a diffuse boundary, so a proper  $r(\phi)$  is [121]:

$$r(\phi) = \frac{1}{2} \left\{ 1 - \tanh\left(\frac{d}{l_m}\right) \right\} = \frac{1}{2} \left\{ 1 - \tanh\left(\frac{2\delta_w}{l_m}\operatorname{atanh}(\phi)\right) \right\}$$
(31)

where *d* is the distance from the interface ( $\phi = 0.5$ ). For liquid and solids, it is positive and negative, respectively.  $l_m = \frac{v}{c_s}$  is the molecular mean free path and  $\delta_w$  is the diffuse interface width, which is proportional to the curvature of the interface between phase  $\xi$ . The LBM is a mesoscopic approach, so the spatial ordering is  $\delta_w > \Delta x > l_m$ . At last, what remains to define is the source term of the momentum, which includes external and body forces, including buoyancy [117,121]:

$$\widetilde{S}_i = (1 - r(\phi))S_i^l + r(\phi)s_i^s$$
(32)

In the solid phase, the sources are denoted with *s*, and for the liquid phase, it is designated with *l*. For example, a buoyancy source term appears in the liquid phase and eliminates in the solid phase.

#### 6.2. Single relaxation Time (SRT) Enthalpy-Based Model

To determine the flow velocity field, the general equations that were mentioned in Section 6.1 are still valid for the enthalpy-based SRT LBM.

$$f_i(x+e_i\Delta t,t+\Delta t) = f_i(x,t) - \frac{1}{\tau_f} \Big[ f_i(x,t) - f_i^{eq}(x,t) \Big] + \Delta t F_i$$
(33)

In the above equation, the relaxation time  $\tau_f$  is obtained from  $\nu / (c_s^2 \Delta t) + 0.5$ . In which  $\nu$  is the kinetic viscosity, and for  $c_s^2 = 1/3$  and  $\Delta t = 1$ , it is equal to  $\tau_f = 3\nu + 0.5$ . The equilibrium particle distribution function for the enthalpy-based model is as follows [130]:

$$f_i^{eq} = \omega_i \rho \left[ 1 + \frac{e_i \cdot u}{c_s^2} + \frac{(e_i \cdot u)}{2c_s^2} - \frac{u^2}{2c_s^2} \right]$$
(34)

In the above relation, the weight factor  $\omega_i$  is determined in each lattice direction *i*. Additionally,  $c_s$  is the sound speed in lattice units. The weight factors and the lattice speed for the  $D_2Q_9$  model are as below [49]:

$$\omega_{i} = \begin{cases} \frac{4}{9} & i = 0\\ \frac{1}{9} & i = 1, 2, 3, 4\\ \frac{1}{36} & i = 5, 6, 7, 8 \end{cases}$$
(35)

$$e_i = \begin{bmatrix} 0 & 1 & 0 - 1 & 0 & 1 - 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \end{bmatrix}$$
(36)

The source term  $S_i$  of Equation (26) and  $F_i$  in Equation (33) are usually called the force terms and are obtained from [132]:

$$S_i = F_i = \omega_i \left( 1 - \frac{1}{2\tau_f} \right) \left( \frac{\boldsymbol{e}_i - \boldsymbol{u}}{c_s^2} + \frac{\boldsymbol{e}_i - \boldsymbol{u}}{c_s^4} \boldsymbol{e}_i \right) F$$
(37)

where *F* represents the external body forces, for instance, buoyancy  $F = \rho g \beta (T - T_{ref})$ . Afterward, the macroscopic quantities would be derived as:

$$\rho = \sum_{0}^{8} f_i \tag{38a}$$

$$\rho u = \sum_{0}^{8} e_i f_i + \frac{\Delta t}{2} F \tag{38b}$$

These are densities of the fluid and flow, respectively.

Obviously, for any problem dealing with energy, the temperature distribution is one of the most important parameters. Thus, when dealing with an LBM thermal lattice, the Boltzmann governing equations which were proposed by Huang et al. [133], should implement:

$$g_i(\vec{x} + \vec{e_i}\Delta t, t + \Delta t) - g_i(\vec{x}, t) = -\frac{1}{\tau_g} \Big[ g_j - g_j^{eq} \Big](\vec{x}, t)$$
(39)

This part of the LBM method is called the temperature field governing equation. Where  $\tau_g = \alpha / (c_s^2 \Delta t) + 0.5 = 3\alpha + 0.5$  is the temperature relaxation time and is different from the fluid relaxation time. In which  $\alpha$  is the thermal diffusivity of the flow. It should be noted that the Chapman-Enskog expansion was utilized on the equations of momentum and energy to derive Equations (33) and (39) [134]. When the equilibrium temperature distribution function follows the relation expressed in Equation (40) [135]:

$$g_{i}^{eq} = \begin{cases} H - C_{p}T + \omega_{i}C_{p}T\left(1 - \frac{u^{2}}{2c_{s}^{2}}\right) & i = 0\\ \omega_{i}\rho\left[1 + \frac{e_{i}\cdot u}{c_{s}^{2}} + \frac{(e_{i}\cdot u)}{2c_{s}^{2}} - \frac{u^{2}}{2c_{s}^{2}}\right] & i \neq 0 \end{cases}$$
(40)

Then, the total enthalpy is calculated from:

$$\sum_{0}^{8} g_i = H \tag{41}$$

On the other hand, if Equation (42) is used to determine the equilibrium temperature distribution function, then the temperature is derived from Equation (43) when D2Q5 is implemented [136]:

$$g_i^{eq} = T\omega_i^T \left[1 + \frac{e_i \cdot u}{c_s^2}\right] \tag{42}$$

$$\sum_{0}^{4} g_i = T \tag{43}$$

In which  $\omega_i^T$  is a weighting factor; these factors' values would vary from Equation (35), as the reference [136] selected D2Q5 model. The temperature and liquid fraction would then be calculated using the enthalpy [136,137]:

$$f_{l} = \begin{cases} 0 & H < H_{S} \\ \frac{H - H_{S}}{H_{l} - H_{S}} & H_{S} \le H \le H_{l} \\ 1 & H > H_{l} \end{cases}$$
(44)

$$T = \begin{cases} \frac{H}{C_p} & H < H_S \\ T_s \frac{H - H_S}{H_l - H_S} (T_l - T_s) & H_S \le H \le H_l \\ T_l + \frac{H - H_l}{C_p} & H > H_l \end{cases}$$
(45)

Finally, a source term is introduced to Equation (39) to apply the phase transition, and with  $\Delta t = 1$ , it could be rewritten as [136,138]:

$$g_i^{n,k+1}(\vec{x} + \vec{e_i}) - g_i(\vec{x}) = -\frac{1}{\tau_g} \Big[ g_i(x) - g_i^{eq}(x) \Big] - \omega_i \frac{L}{C_p} \Big[ f_l^{n,k}(x) - f_l^{n-1}(x) \Big]$$
(46)

In which *n*, the same as *t* and *k*, is the iteration counter.

# 6.3. Multiple-Relaxation Time (MRT) Enthalpy-Based Model

In the MRT-LBM enthalpy-based model, the velocity field LBM governing equations are the same as SRT, while only the temperature field is obtained from the MRT-LBM. The MRT enthalpy-based equation for temperature distribution is [139]:

$$g_i(\vec{x} + \vec{e}_i \Delta t, t + \Delta t) - g_i(\vec{x}, t) = -\Lambda_{ik} \left[ g_k - g_k^{eq} \right](\vec{x}, t)$$
(47)

where this time, there is a relaxation matrix  $\Lambda = \Lambda_{ik}$  for the velocity field [9]. For the momentum space, the collision step is completed as:

S

$$\boldsymbol{m}(\boldsymbol{x},t+\Delta t) = \boldsymbol{m}(\boldsymbol{x},t) - \boldsymbol{s}[\boldsymbol{m}(\boldsymbol{x},t) - \boldsymbol{m}^{eq}(\boldsymbol{x},t)]$$
(48)

$$m = Mg_i = (m_0, m_1, \dots, m_8)^T, m^{eq} = Mg_i^{eq} = (m_0^{eq}, m_1^{eq}, \dots, m_8^{eq})^T$$
(49)

$$= M\Lambda M^{-1} \tag{50}$$

*M* is the transformation matrix and is provided in:

Also,  $m^{eq}$  is the equilibrium function in a momentum space and is obtained from [9,140]:

$$\boldsymbol{m}^{eq} = (H, -4H + 2C_{p,ref}T + 3C_pT\frac{u^2}{c^2}, 4H - 3C_{p,ref}T -3C_pT\frac{u^2}{c^2}, C_pT\frac{u}{c}, -C_pT\frac{u}{c}, C_pT\frac{v}{c}, -C_pT\frac{v}{c}, C_pT\frac{u^2 - v^2}{c^2}, C_pT\frac{uv}{c^2})^T$$
(52)

The relaxation matrix *S* in the momentum space is provided by [128]:

$$\boldsymbol{S} = diag(s_0, s_e, s_\varepsilon, s_j, s_q, s_j, s_q, s_e, s_e)$$
(53)

The diagonal matrix presented above should be in a way that its relaxation parameters satisfy  $s_0 = 1$ ,  $s_j = 1/\tau_s$  and  $0 < s_{e,\varepsilon,q} < 2$ , where:

$$\tau_{s} = \begin{cases} 3\frac{\lambda_{pcm}}{\rho_{f}c_{p,ref}} + 0.5 & \text{in the PCM region} \\ \\ 3\frac{\lambda_{w}}{\rho_{f}c_{p,ref}} + 0.5 & \text{in the solid wall and fins} \end{cases}$$
(54)

For better numerical stability and to avoid numerical diffusion, the relaxation parameters should satisfy the following equation [9,139]:

$$\left(\frac{1}{s_e} - \frac{1}{2}\right)\left(\frac{1}{s_j} - \frac{1}{2}\right) = \frac{1}{4}$$
(55)

There is a function called the post-collision distribution, which is the result of a collision process in the velocity field. This function could be obtained from an inverse transformation as follows:

$$g_i(\mathbf{x}, t + \Delta t) = \mathbf{M}^{-1} \mathbf{m}(\mathbf{x}, t + \Delta t)$$
(56)

Then streaming process completes:

$$g_i(\mathbf{x} + \mathbf{e}_i \Delta \mathbf{t}, \mathbf{t} + \Delta \mathbf{t}) = g_i(\mathbf{x}, t + \Delta t)$$
(57)

At last, the total enthalpy based on the boundary conditions could be derived from [9,141]:

$$\sum_{0}^{8} g_i = H \tag{58}$$

# 7. Numerical Performance of the LBM

Among all the efforts that have been taken to study LBM applications in phase change material simulations, some explicitly mentioned the comparison between the LBM and conventional numerical methods, including the finite volume method (FVM). However, the verification section of each study somehow mentioned this matter. The mentioned comparisons and the advantages of the LBM could be summarized as follows:

- 1. Compared to the finite volume method, the LBM efficiently reduces the running time [101].
- 2. It may be an alternative robust numerical technique to simulate fluid flows and intricate physical processes [103];
- 3. Its numerical results agree reasonably with the experimental studies [103] and analytical solutions [102];
- 4. As opposed to conventional computational fluid dynamics (CFD) approaches which limit the mushy zone by a constant between 10<sup>4</sup> to 10<sup>7</sup>, by using the volumetric LB scheme in the LB model, the empirical mushy zone constant could be avoided in the absence of any free parameters, thereby avoiding the empirical mushy zone [61];
- 5. The ability of parallel coding allows a faster running time for the LBM. For instance, Ren and Chan [9] indicated that the parallel code responds 29 times faster than the non-parallel code. This is true for a  $600 \times 600$  grid with GPU GTX TITAN;
- 6. Li et al. [140] compared the numerical stability of the SRT and MRT against the FVM and revealed that the MRT is superior in numerical stability;
- 7. To briefly compare the SRT and MRT in all aspects, one could say that the MRT works better with low thermal conductivity and diffusivity, which would lead to stable and reliable results [140].

# 8. Conclusions

Phase change materials (PCMs) are being used in various applications. Although, one of the main drawbacks that prevent researchers from implementing PCMs in some industrial and commercial applications is the low thermal conductivity. Using composite PCMs, nanoparticle-enhanced PCMs or extended surfaces are the major options to overcome this disadvantage. The lattice Boltzmann method (LBM) is a robust, strong, straightforward, and stable alternative numerical method that could be implemented in such investigations. The LBM is suitable for complex geometries or flows and would produce stable and reliable results, according to the results of numerous studies that used this method. The results agree well with both experimental and traditional numerical methods. In some instances, the maximum error was only approximately 0.1% [140]. Various problems in the simulation of the phase transition of PCMs were solved successfully by different types of LBMs. However, like any numerical simulation method, the LBM has its own limitations. The LBM is strongly dependent on the relaxation time, and as all the quantities are dimensionless in this method, it is highly sensitive to the dimensionless numbers of the problem, including Reynolds (if the velocity needs to be studied), Rayleigh (in case of natural convection), Prandtl (which affects the viscosity and consequently the relaxation time directly). Therefore, LBM results are suitable only for a particular range of dimensionless numbers, and this is one of the biggest challenges while implementing the LBM. To overcome this limitation, one should use the MRT-LBM to overcome the limitations of the relaxation time by adjusting the MRT's parameters (i.e., the magic parameter) [142]. In this review paper, the recent advances in the applications of PCMs and the numerical simulation of phase changes with the LBM were presented and discussed.

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