energies

Article

# Distributed Optimal Coordination of a Virtual Power Plant with Residential Regenerative Electric Heating Systems

**Guixing Yang [1,2], Haoran Liu [3], Weiqing Wang [1,*], Junru Chen [1] and Shunbo Lei [3]**

1. Engineering Research Center of Ministry of Education for Renewable Energy Generation and Grid Connection Technology, Xinjiang University, Urumqi 830046, China; xjwkgl@126.com (G.Y.); junru.chen@xju.edu.cn (J.C.)
2. State Grid Xinjiang Electric Power Co., Ltd., Urumqi 830002, China
3. School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen 518172, China; leishunbo@cuhk.edu.cn (S.L.)

* Correspondence: wq59@xju.edu.cn

**Abstract:** Renewable energy sources play a key role in the transition towards clean and affordable energy. However, grid integration of renewable energy sources faces many challenges due to its intermittent nature. The controllability of aggregated regenerative electric heating load provides a method for the consumption of renewable energy sources. Based on the concept of a virtual power plant (VPP), this paper considers the cooperative energy management of aggregated residential regenerative electric heating systems. First, considering physical constraints, network constraints, and user comfort, comprehensive modeling of a VPP is given to maximize its social benefits. In addition, this VPP is investigated as a participant in day-ahead energy and reserve markets. Then, to solve this problem, a distributed coordination approach based on an alternating direction method of multipliers (ADMM) is proposed, which can respect the independence of users and preserve their privacy. Finally, the simulation results illustrate the effectiveness of our algorithm.

**Keywords:** virtual power plant; regenerative electric heating systems; ADMM



**Citation:** Yang, G.; Liu, H.; Wang, W.; Chen, J.; Lei, S. Distributed Optimal Coordination of a Virtual Power Plant with Residential Regenerative Electric Heating Systems. *Energies* **2023**, *16*, 4314. https://doi.org/10.3390/en16114314

Academic Editor: Konstantin Suslov

Received: 16 February 2023
Revised: 30 April 2023
Accepted: 22 May 2023
Published: 25 May 2023



## 1. Introduction

Modern power systems face an energy trilemma amid growing concerns over environmental issues and the depletion of fossil fuel resources. Transitioning to zero-carbon electricity generation while ensuring universal and affordable energy access poses significant challenges. However, there are two emerging technology trends that offer promising solutions: consumer-level control and communications and distributed energy resources (DERs). These trends provide new opportunities to address this crucial challenge effectively [1]. Consumer-level communication and control include energy management systems and the adoption of smart meters [2]. DERs encompass various components, such as distributed generation (DG), energy storage systems (ESSs), and flexible loads (FLs). A virtual power plant (VPP) is the primary vehicle for integrating DERs into the electrical distribution system [3]. VPPs have system controllability, visibility, and impact similar to a traditional generator.

With increasing pressure to increase DER penetration, how to manage numerous and heterogeneous DERs becomes an important and fundamental question. One approach is through a centralized coordination paradigm, wherein a central controller manages all DERs [4–7]. A coevolutionary version of the particle swarm optimization (PSO) algorithm is proposed to coordinate the scheduling of DERs [4]. A real-time active power control strategy is proposed to optimize the average generation cost of a VPP by dispatching DG to handle the load variation and intermittent active power output of non-controllable DG, such as wind turbines [5]. Aiming at the realization of VPP, an optimization algorithm based on the big bang big crunch method is proposed in [6]. The objective of these algorithms is to optimize the energy management of unbalanced distribution networks, with the primary aim of minimizing the reliance on purchasing electricity from the grid. This goal is achieved through the optimal scheduling of FLs and ESSs, and the optimal configuration of distributed renewable energy. For the optimal energy management problem of VPPs with different DGs and energy storage systems, an efficient optimization method based on the imperialist competition algorithm is proposed in [7]. Under the framework of the electricity market, DERs such as DGs and controllable loads have the opportunity to form VPP through aggregation and participate in the real-time operation of the transmission and distribution network. Ref. [8] proposed a direct load control-based algorithm for effectively managing a VPP comprising a large number of customers equipped with thermostatically controlled loads [9] consider a VPP with a dispatchable power plant, intermittent wind energy resources, and energy storage (ES). To maximize the profit of a VPP by selling and buying electricity in a day-ahead and balanced market. To solve this uncertainty problem, they proposed a two-stage stochastic mixed-integer linear programming model. Ref. [10] considers the optimal bidding strategy problem of a commercial VPP, which consists of DERs and electricity users. In order to simultaneously maximize day-ahead profit and minimize expected real-time production and unbalanced expenses, they developed a three-stage stochastic bi-level optimization model, the upper level aims to solve the profit maximization problem, and the lower level is responsible for the market-clearing problem. This bi-level is solved by using the mixed-integer linear programming model. In order to maximize the operating income of VPP with renewable energy sources, Ref. [11] also proposed a bi-level stochastic scheduling strategy based on a robust optimization method. However, with the increasing scale of the power system, the centralized framework faces significant challenges. These challenges include high communication and computational burden, lack of privacy, etc. These issues hinder the effectiveness of the centralized approach [12].

On the other hand, the distributed coordination method can overcome these limitations. Therefore, it is an effective method to solve the optimal coordination problem of DERs. Ref. [13] developed a cooperate control strategy aimed at effectively managing the output power of DGs in a distribution network system. To achieve this, the DGs are integrated into the system by aggregating them into a VPP. The proposed method is center-free, each DG only needs to communicate with its neighbors by using a local communication network. The VPP converges to the optimal output in a distributed manner according to the cost of the DGs and the necessary grid demand in the distribution network. Traditionally, the ESSs are aggregated into a fixed VPP. To reduce the power losses of the power network and improve voltage regulation. Ref. [14] developed a dynamic formation of VPPs. The clustering algorithm aggregates the ESSs into multiple VPPs according to the power demand and capacity of the ESSs. This distributed clustering algorithm improves the flexibility of the ESSs and then realizes energy trading, power dispatch, and optimal power flow based on VPP. In order to maximize the economic profit of a VPP, Ref. [15] proposed a distributed optimization algorithm based on the distributed primal-dual sub-gradient method, which can realize the collaborative decision-making of DERs in a VPP with limited communication, and its performance is similar to the centralized approach. Based on the alternating direction method of multipliers (ADMM) and consensus optimization, Ref. [16] developed a distributed optimization algorithm for the optimal dispatch problem of a VPP. This algorithm can maximize the economic benefits of two specific VPPs while satisfying power balance constraints, line transmission constraints, and local capacity constraints of DERs. To minimize the operation cost of a VPP, Ref. [17] developed a distributed approach for optimizing coordinated control of DERs in distribution networks with renewable energy. These devices provide active power at the aggregated level as required by the system operator. In order to solve this optimization problem, an improved algorithm based on ADMM is adopted. In this improved algorithm, by using the proximal operator, the initial update has a closed-form analytical solution, which greatly reduces the cost of a single agent. We reduce the amount of calculation and realize fast energy storage dispatch. A decentralized economic dispatch method and a lightweight architecture are proposed for VPPs with a large number of DERs [18]. This method has guaranteed convergence and can solve optimal dispatch problems quickly. This algorithm has strong robustness to large-scale DERs and randomness of parameters. Moreover, it has strong adaptability and can realize plug-and-play. To address the issue of delay in the existing consensus-based optimal dispatch algorithm, Ref. [19] proposed improvements by introducing new update rules and a reduction method. These enhancements aim to enhance the algorithm's performance and ensure convergence to the optimal solution in a distributed manner under specific conditions.

Noting that DERs are not only generators but also controllable loads connected to the network, most existing consensus-based algorithms aim to solve the optimal coordination problems of DERs in separate and independent time periods. However, these algorithms may inherently fail to fully capture the time-coupled operating characteristics of ES and flexible demand (FD). To solve this basic problem, Ref. [20] proposed a new consensus-based algorithm that incorporates additional consensus variables. These variables represent the relative maximum power constraints imposed on ES and FD resources, which drive consensual outcomes toward a feasible optimal solution and effectively mitigate the concentration of ES and FD responses over the same time period. To promote energy trading among residential buildings with ES, FLs, and renewable energy resources, Ref. [21] proposed a VPP energy management platform that leverages blockchain technology, which can protect the privacy of users and respect their independence. Typically, the practical communication network of power systems is affected by noise, communication delays, and dynamic switching of communication topology. In order to solve these problems, VPP is considered to participate in the power system, and a fully distributed robust optimal scheduling algorithm based on the dual decomposition and double consensus technologies is proposed [22]. The profit maximization problem of a VPP is solved by decomposing it into sub-problems. Moreover, in order to suppress the adverse effects of communication delay and noise, the algorithm proposes a monotonically decreasing uniform gain function. Ref. [23] introduced a decentralized coordination operation model for VPPs and Power-to-Hydrogen (P2H) energy systems, leveraging the Nash-Harsanyi bargaining game framework. The proposed model can capture the bargaining power and contributions of participants in the cooperative setting, ensuring a fair and reasonable distribution of profits. Furthermore, to enhance convergence efficiency and protect the privacy of each participant, an improved ADMM method is employed to solve the optimization problems in a distributed manner. In order to minimize the social utility loss of a VPP with photovoltaics and electric vehicles, Ref. [24] formulated the stochastic optimization problem. The charging dynamics of electric vehicles are temporally coupled constraints. In general, long-term optimization problems with temporally coupled constraints are difficult to solve well in real-time operations since the current control behavior affects the future feasible domain. To address this problem, a temporally coupled distributed online algorithm is proposed based on the dual ascent technique and Lyapunov optimization algorithm. However, existing studies [20–23] ignored power flows of the power grid, assuming all DERs are connected to an electrical node. In short, these optimization methods are network-agnostic [25]. While this simplifies the design of control and optimization algorithms, the results obtained with this approach may not be practical under time-varying operating conditions.

Greenhouse gas emissions were reduced by 30–40% due to the introduction of heating electrification [26]. Wherein regenerative electric heating (REH), as a promising residential heating form, is widely constructed in North China. REH is often considered as an alternative to traditional electric heating methods because of its energy efficiency and environmental benefits. REH works by storing heat energy in a thermal mass, such as ceramic bricks, during off-peak hours when electricity is cheaper. The stored heat energy can then be released during peak hours when electricity is more expensive, providing a cost-effective and energy-efficient heating solution. In addition to its cost and energy-saving benefits, REH also has environmental advantages. It can reduce carbon emissions by using off-peak electricity generated from renewable sources. Furthermore, REH systems are durable, require little maintenance, and have a long lifespan, making them a sustainable heating solution. In recent times, research on REH has been gaining increased attention from the scientific community. To achieve the optimal match between the supply and demand of REH, Ref. [27] proposed a novel multi-agent cooperative framework by utilizing deep reinforcement learning. In addition, Ref. [28] proposed an affine arithmetic-based model predictive control approach for optimizing the scheduling of REH, taking into account emergency residential building heating. Ref. [29] provided valuable insights into the demand response process of REH users in rural areas, taking into account the participation of load aggregators. However, the above studies [27–29] did not consider the reserve capacity of REH.

This paper focuses on the optimal coordination problem of REHs load within a VPP. To solve this problem, this paper proposes a distributed coordination algorithm to optimize the coordination of a VPP with residential REH systems. The primary aim of this study is to enhance the overall profitability of VPPs by managing the energy production and consumption of the VPP. The main contributions of this paper are as follows:

1. Optimal Coordination: This paper considers the optimal coordination of a VPP with residential REH systems. It involves coordinating the energy production and consumption of the VPP with the REH systems to minimize its own costs.
2. Reserve Capacity: This paper investigates the participation of the VPP in day-ahead energy and spinning reserve markets. By participating in these markets, the VPP can minimize its own costs and provide spinning reserve capacity to the grid.
3. Distributed Coordination Algorithm: To solve the optimal coordination problem, this paper proposes a distributed coordination algorithm based on ADMM. Compared to previous studies on the distributed coordination of VPPs, the proposed algorithm offers a more practical approach by considering the network constraints that exist in a VPP.

The remainder of this paper is organized as follows. Section 2 gives the mathematical model of the optimal coordination problem of a VPP with residential REH systems. Section 3 introduces the ADMM concept and its application in a VPP. Section 4 presents case studies to validate the effectiveness of the proposed algorithm. Lastly, Section 5 draws some conclusions.

## 2. Problem Formulation

In this section, we first introduce the architecture of the residential building. Then, the system model of distributed optimal coordination for a VPP is given. The schematic diagram of the VPP is shown in Figure 1. The VPP can provide energy and reserve services.

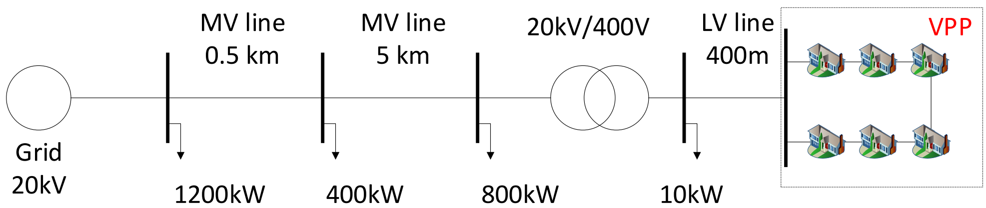



**Figure 1.** The schematic diagram of the VPP.

*Residential Building*

The architecture of a residential building is shown in Figure 2. The residential building is equipped with DGs and various loads, such as REH, FL, and inflexible loads. These components can be managed and dispatched via smart meters, which also connect the residential building and the power grid. Next, we will present the models of these two methods.

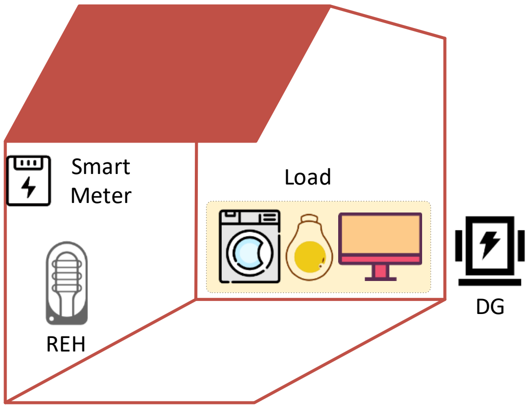



**Figure 2.** Architecture of a residential building.

In this paper, residential buildings in a VPP can obtain electric energy in two different ways. First, the user can use their DGs to generate energy. Second, the user can purchase electric energy from the local grid through traditional means.

Let $p_{i,t}^{\mathrm{DG}}$ denote the power generated by DG $i$ at time $t$. The generation cost of DG $i$ is formulated as a quadratic function:

$$C_{i,t}^{\mathrm{DG}} = \alpha_i (p_{i,t}^{\mathrm{DG}})^2 + \beta_i p_{i,t}^{\mathrm{DG}}.$$

where $\alpha_i$ and $\beta_i$ are the coefficient of the cost function. Moreover, DG involves three kinds of individual constraints: capacity constraints, ramp rate constraints, and spinning reserve constraints [30]. They can be expressed as follows:

$$p_i^{\mathrm{DG,min}} \leq p_{i,t}^{\mathrm{DG}} \leq p_i^{\mathrm{DG,max}}, \tag{1}$$

$$-DR_i \leq p_{i,t+1}^{\mathrm{DG}} - p_{i,t}^{\mathrm{DG}} \leq UR_i, \tag{2}$$

$$R_i^{\mathrm{DG}} + p_{i,t}^{\mathrm{DG}} \leq p_i^{\mathrm{DG,max}}, \tag{3}$$

where $p_i^{\mathrm{min}}$ and $p_i^{\mathrm{max}}$ are the generation limits of DG $i$; $UR_i$ and $DR_i$ represent the ramp-up rate limit and the ramp-down rate limit, respectively, of DG $i$; $R_i^{\mathrm{DG}}$ is the spinning reserve capacity of DG $i$.

The buying and selling of electricity between residential buildings and the grid involve transaction costs that can be formulated as follows:

$$C_{i,t}^{E} = \pi^E p_{i,t}^{\mathrm{E}},$$

where the $\pi^E$ is energy transaction price; $p_{i,t}^{\mathrm{E}}$ means the amount of power transactions between users $i$ and VPP at time $t$.

As shown in Figure 2, the residential buildings have various electric loads. They can be divided into three categories: adjustable load, flexible load, and inflexible load. The power consumption of the REH is contingent upon the preferences and requirements of the user. As such, the REH is considered an adjustable load, meaning that its power consumption can be dynamically adjusted and controlled based on the user's needs and preferences. Its structure is shown in Figure 3 [31].

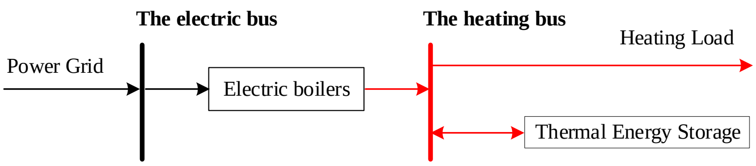



**Figure 3.** Structure of a regenerative electric heating system.

The relationship between the indoor temperature of a residential building and the power consumption of REH can be modeled as:

$$T_{i,t+1}^{in} = T_{i,t}^{in} + \frac{T_{i,t}^{out} - T_{i,t}^{in}}{\delta} + \frac{1}{\gamma}\left(p_{i,t}^{REH} + p_{i,t}^{r} - p_{i,t}^{a}\right)\Delta(t), \tag{4}$$

where $T_{i,t}^{in}$ is the indoor temperature of user $i$ at time $t$; $T_{i,t}^{out}$ is the outdoor temperature of user $i$ at time $t$; $\delta$ is the heat dissipation coefficient of residential building; $\gamma$ is the temperature coefficient of residential building; $p_{i,t}^{REH}$ is the power injection from the grid of REH $i$ at time $t$; $p_{i,t}^{a}$ and $p_{i,t}^{r}$ are charge and discharge power of REH $i$ at time $t$, respectively; $\Delta(t)$ is the range of time.

The dynamic model of the ESSs of REH can be modeled as:

$$E_{i,t+1}^{TS} = E_{i,t}^{TS}(1-u) + \left(\eta^{a} p_{i,t}^{a} - \frac{1}{\eta^{r}} p_{i,t}^{r}\right)\Delta(t), \tag{5}$$

where $E_{i,t}^{TS}$ is the energy of ESS $i$ at time $t$; $u$ is the coefficient of energy loss; $\eta^{a}$ and $\eta^{r}$ are the efficiency of charge and discharge power.

The user's cost can be expressed by the discomfort of indoor temperature, which is defined as the deviation between the actual indoor temperature and the user's preferred temperature. It can be modeled as:

$$C_{i,t}^{REH} = m(T_{i,t}^{in} - T_{i,t}^{ref})^2,$$

where $m$ indicates the user's sensitivity to the discomfort temperature; $T_{i,t}^{ref}$ is the user's preferred temperature.

In practice, it is essential to maintain the indoor temperature within a reasonable range to ensure occupant comfort. This range can be mathematically expressed as follows:

$$T_{i}^{\min} \le T_{i,t}^{in} \le T_{i}^{\max}. \tag{6}$$

where $T_{i}^{\min}$ and $T_{i}^{\max}$ represent the lower and upper bounds, respectively, on the indoor temperature that can be attained by the REH.

Moreover, the operational constraints of REH are:

$$0 \le p_{i,t}^{a} \le p^{a,\max}, \tag{7}$$

$$0 \le p_{i,t}^{r} \le p^{r,\max}, \tag{8}$$

$$0 \le E_{i,t}^{TS} \le E^{TS,\max}, \tag{9}$$

$$p_{t}^{a} + R^{ed} \le p^{R}, \tag{10}$$

$$p_{t}^{r} + R^{eu} \le p^{R}, \tag{11}$$

where $p^{a,\max}$ is the upper bound of the charging power of REH $i$; $p^{r,\max}$ is the upper bound of the discharging power of REH $i$; $E^{TS,\max}$ is the upper bound of energy that can store in ESS $i$; $R^{ed}$ and $R^{eu}$ are reserve energy capacity that can be used for charging or discharging; $p^{R}$ is the rated power of ESS of REH. Equations (7) and (8) mean that both charge and discharge process limits exist; the capacity to store the energy of ESSs also should be in a range, which is expressed by (9); Equations (10) and (11) mean that the charge and discharge power has to remain within the rated power.

Similar to REH, the discomfort cost of the FL can be expressed as follows:

$$C_{i,t}^{FL} = n(p_{i,t}^{FL} - p_{i,t}^{ref})^2,$$

which is defined as the deviation between the actual and preferred schedule of the FL, and $n$ indicates the user's sensitivity to the discomfort schedule; $p_{i,t}^{ref}$ is the user's preferred schedule of the FL.

The FL can be scheduled but the sum of power should meet the user's needs within the operation time $T$. Moreover, the power of the FL should be bounded. They are expressed as:

$$\sum_{t=1}^{T} p_{i,t}^{FL} = \sum_{t=1}^{T} p_{i,t}^{ref}, \tag{12}$$

$$p_{i,t}^{FL,min} \leq p_{i,t}^{FL} \leq p_{i,t}^{FL,max}, \tag{13}$$

where $p_{i,t}^{FL,min}$ and $p_{i,t}^{FL,max}$ are the upper and lower bounds of the power of the FL, respectively.

Moreover, denote $R_{i,t}^{FL}$ as the reserve capacity of flexible loads, which should be in a reasonable range. Hence, the constraints on the reserve capacity of flexible loads are:

$$p_{i,t}^{FL} - R_{i,t}^{FL} \geq p_{i,t}^{FL,min}, \tag{14}$$

$$R_{i,t}^{FL} \geq 0. \tag{15}$$

As shown in the [32,33], power flow constraints are crucial for the optimal coordination of VPPs. However, existing related studies [20–23] ignore this constraint. Generally, distribution networks have a radial structure. Therefore, the network constraints of a VPP are established according to LinDistFlow [34]:

$$P_{ij} - \sum P_{jk} = -p_j, \tag{16}$$

$$Q_{ij} - \sum Q_{jk} = -q_j, \tag{17}$$

$$v_i - v_j = r_{ij}P_{ij} + x_{ij}Q_{ij}, \tag{18}$$

where $P_{ij}$ and $Q_{ij}$ denote the active power and reactive power on line $(i,j)$, respectively; $p_j$ and $q_j$ are the net active power injection and the net reactive power injection of node $i$; and $v_i$ is the voltage of node $i$; $r_{ij}$ is the resistance of the distribution lines; $x_{ij}$ is the reactance of the distribution lines.

Moreover, the reserve requirements of a VPP are expressed as

$$\sum_{i=1}^{n} R_{i,t}^{DG} + R^{ed}\eta^{a} + R^{eu}\eta^{r} + R_{i,t}^{FL} + R_{i,t}^{E} \geq \sum_{i=1}^{n} R_{i,t}. \tag{19}$$

Therefore, the optimal coordination problem of a VPP can be formulated as follows:

$$\begin{aligned} &\min \sum_{t=1}^{T}\sum_{i=1}^{N} C_{i,t}^{E} + C_{i,t}^{DG} + C_{i,t}^{REH} + C_{i,t}^{FL}, \\ &S.t.(1)\text{–}(19). \end{aligned} \tag{20}$$

where $C_{i,t}^{E}$ is the transaction cost between the main grid and node $i$; $C_{i,t}^{DG}$ is the generation cost of distributed generation (DG); and $C_{i,t}^{REH}$ and $C_{i,t}^{FL}$ are the user's discomfort cost. Specifically, in the optimal coordination problem, the decision variable are $p_{i,t}^{\mathrm{DG}}, p_{i,t}^{a}, p_{i,t}^{r}, E_{i,t}^{TS}, p_{i,t}^{FL}, p_{i,t}^{\mathrm{E}}, R_{i}^{\mathrm{DG}}, R^{ed}, R^{eu}, R_{i,t}^{FL}, R_{i}^{\mathrm{E}}, T_{i,t}^{in}$. All of them are real variables. Moreover, there are 13 inequality constraints and 6 equality constraints.

**Remark 1.** *Note that the REH cannot be charged and discharged simultaneously. Therefore, we have to ensure that either $p_{i,t}^{a}$ or $p_{i,t}^{r}$ needs to be zero, i.e.,*

$$p_{i,t}^{a} p_{i,t}^{r} = 0. \tag{21}$$

*However, when $\eta^{a}\eta^{r} < 1$, there is no need to consider this nonconvex constraint [35].*

## 3. Distributed Coordinated Operation

In this section, a distributed method is developed to solve the optimal coordination problem with residential REH systems in a VPP.

### *3.1. ADMM Algorithm*

In this section, we will introduce the concept of ADMM. You can see more details in [36]. ADMM is a powerful approach for effectively addressing distributed convex optimization problems. This algorithm is based on a decomposition-coordination architecture where the solution of large global problems is achieved by effectively coordinating the solutions of small local subproblems. ADMM can be perceived as an attempt to merge the advantageous attributes of augmented Lagrangian methods and dual decomposition.

ADMM is a distributed optimization algorithm that adeptly combines the remarkable convergence properties offered by the method of multipliers with the inherent decomposability of dual ascent. Consider the following generic constrained optimization problem [36]

$$\begin{aligned} &\min f(x) + g(y), \\ &s.t. Ax + By = c. \end{aligned} \tag{22}$$

where $x \in \mathcal{R}^n$ and $y \in \mathcal{R}^m$, where $A \in \mathcal{R}^{p \times n}$, $B \in R^{p \times m}$ and $c \in \mathcal{R}^p$.

**Assumption 1.** *Suppose the functions $f$ and $g$ are convex.*

The optimal solution of the problem (22) is denoted by

$$p^\star = \inf\{f(x) + g(y) | Ax + By = c\}. \tag{23}$$

Denote the augmented Lagrangian function as

$$L_\rho(x, y, u) = f(x) + g(y) + u^T(Ax + By - c) + \frac{\rho}{2}||Ax + Bz - c||_2^2. \tag{24}$$

The ADMM consists of the iterations

$$x^{k+1} := \arg\min_x L_\rho(x, y^k, u^k) \tag{25}$$
$$y^{k+1} := \arg\min_y L_\rho(x^{k+1}, y, u^k) \tag{26}$$
$$u^{k+1} := u^k + \rho(Ax^{k+1} + By^{k+1} - c) \tag{27}$$

where $\rho > 0$. This algorithm comprises three essential steps: the step (25) aims at minimizing the variable $x$, the step (26) aims at minimizing the variable $y$, and the step (27) aims to update for the dual variables $u$.

For convenience, ADMM can be reformulated in an alternative form that involves the combination of quadratic and linear terms in the augmented Lagrangian, accompanied by the scaling of the dual variable. The residual is defined as $r = Ax + By - c$, we have

$$u^T r + \frac{\rho}{2}||r||_2^2 = \frac{\rho}{2}\left|\left|r + \left(\frac{1}{\rho}\right)u\right|\right|_2^2 - \left(\frac{1}{2\rho}\right)||u||_2^2, \tag{28}$$
$$= \frac{\rho}{2}||r + \nu||_2^2 - \left(\frac{\rho}{2}\right)||\nu||_2^2, \tag{29}$$

where $\nu = (1/\rho)u$ is the scale dual variable.

According to the scaled dual variable, the ADMM can be expressed as

$$
\begin{aligned}
x^{k+1} &:= \arg\min\Big(f(x) + \frac{\rho}{2}||Ax + By^k - c + \nu^k||_2^2\Big), \\
y^{k+1} &:= \arg\min\Big(g(y) + \frac{\rho}{2}||Ax^{k+1} + By - c + \nu^k||_2^2\Big), \\
u^{k+1} &:= \nu^k + Ax^{k+1} + By^{k+1} - c.
\end{aligned} \tag{30}
$$

The residual at iteration $k$ is denoted as $r^k = Ax^k + By^k - c$. Thus, the sum of the residuals is

$$
\nu^k = \nu^0 + \sum_{j=1}^{k} r^j. \tag{31}
$$

*3.2. Distributed Optimal Operation Decision of VPP*

Let $x_i$ be the decision variable. Then, the coordination optimization problem (20) can be expressed as the following general formulation:

$$
\sum f_i(x_i), \tag{32}
$$

$$
s.t. \sum Ax_i = b, \tag{33}
$$

$$
x \in \Xi \tag{34}
$$

Up to now, there are many advanced distributed algorithms that have been proposed [36–40]. Among them, ADMM is widely used in electric power systems, especially for optimal power flow problems [41]. Therefore, inspired by [42], this paper utilizes ADMM (30) as the primary method to solve the optimal coordination problem of the VPP in a distributed manner.

## 4. System Simulation

To evaluate the performance of the proposed distributed coordination method, a test system with 15 nodes is used. The single line diagram of a VPP is shown in Figure 4, where nodes {2,4,5,6,9,10,12,13} are connected with DGs. Moreover, the cost coefficients and the generation capacity limitations of DGs are presented in Table 1, which are referenced from [43]. Moreover, the parameters of the loads are shown in Figure 5. The result is based on Matlab 2018b and CPLEX 12.9.

**Table 1.** Parameters of the DGs.

| DG | $\alpha_i$ (MW$^2$h) | $\beta_i$ ($/MWh) | $p_i^{\mathrm{DG,min}}$ (MW) | $\overline{p}_i^{\mathrm{DG,max}}$ (MW) |
|---|---|---|---|---|
| DG1 | 0.04 | 10.5 | 20 | 85 |
| DG2 | 0.01 | 6.5 | 35 | 115 |
| DG3 | 0.01 | 9.2 | 50 | 110 |
| DG4 | 0.04 | 12.6 | 20 | 75 |
| DG5 | 0.01 | 7.2 | 25 | 80 |
| DG6 | 0.01 | 7 | 30 | 90 |
| DG7 | 0.01 | 10.1 | 30 | 105 |
| DG8 | 0.04 | 12.7 | 20 | 90 |

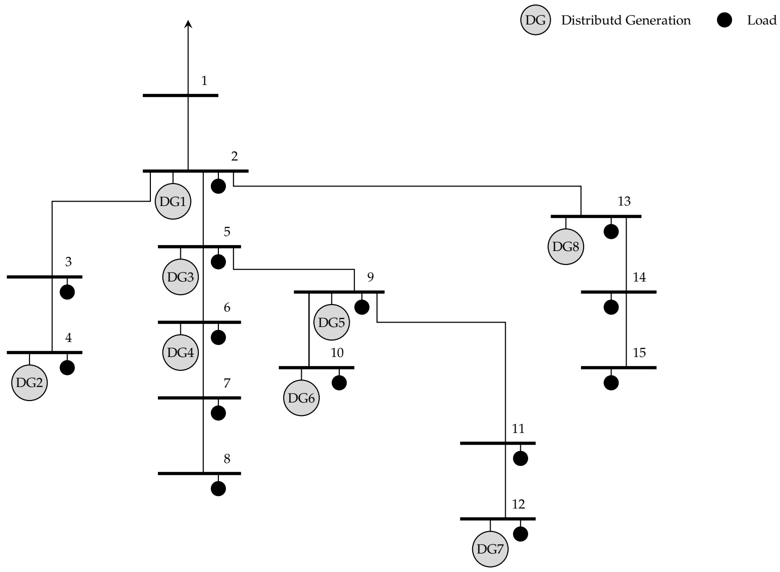



**Figure 4.** The single line diagram of a VPP.

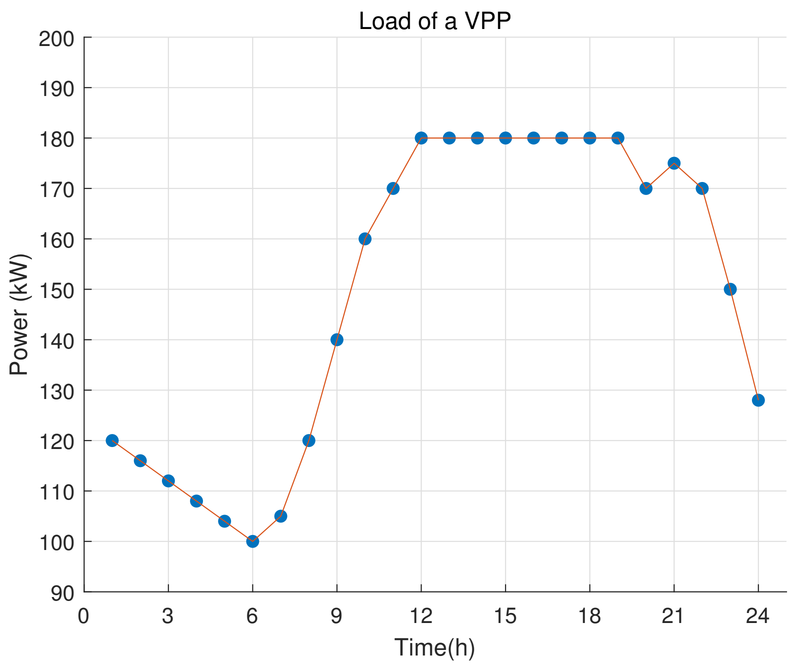



**Figure 5.** The parameters of loads.

In order to incentivize residential prosumers to actively manage their energy consumption and effectively reduce electricity costs, the VPP operates on the principles of Time of Use (TOU) tariffs. By implementing TOU tariffs, the VPP provides a structured pricing mechanism that incentivizes prosumers to adjust their energy usage patterns according to the varying electricity rates during different time periods. Specifically, the static TOU pricing scheme is employed as the basis for analysis and investigation [44]. The electricity price of the main grid is presented in Figure 6. The main grid implements a peak pricing scheme from 7:00 to 22:00, during which electricity is priced at its highest rate. Conversely, outside this time range, the grid employs an off-peak pricing structure, offering electricity at a lower cost. This time-dependent pricing strategy encourages consumers to shift their energy consumption activities to off-peak hours, thereby helping to alleviate the strain on the grid during peak periods and optimize the overall utilization of energy resources.

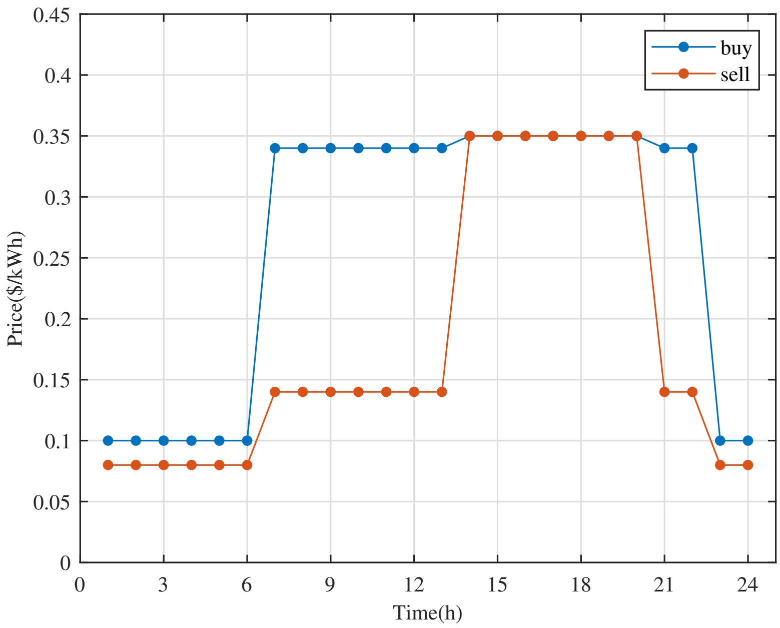



**Figure 6.** Electricity price.

The outdoor temperatures of residential buildings are shown in Figure 7a. The data comes from the outdoor temperature in winter in Beijing. People in different states have different requirements for indoor comfort temperatures. The optimum temperature for productivity is 22 °C. The optimum temperature for sleep is 15 °C. The comfort temperature range for working is from 20 to 24 °C. The comfort temperature range for sleeping is from 11 to 19 °C. The time for working is from 7:00 to 22:00, and the rest time is for sleeping and the indoor temperatures of residential buildings are shown in Figure 7b.

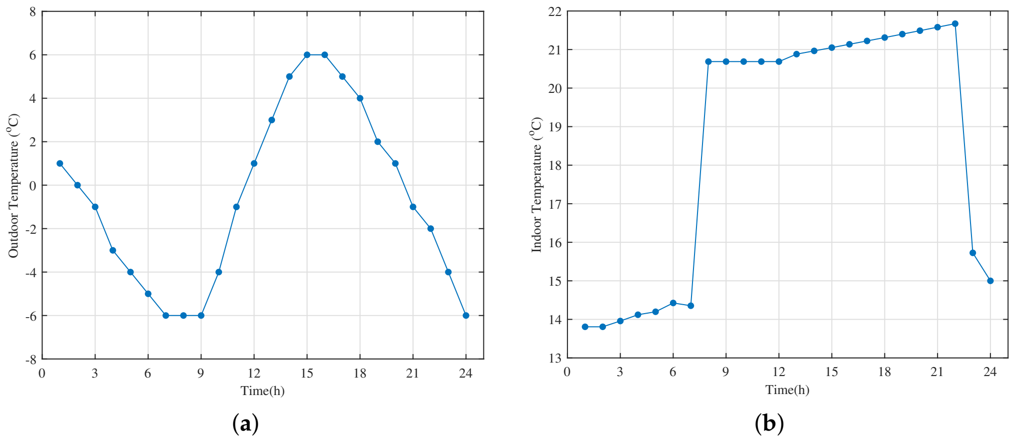
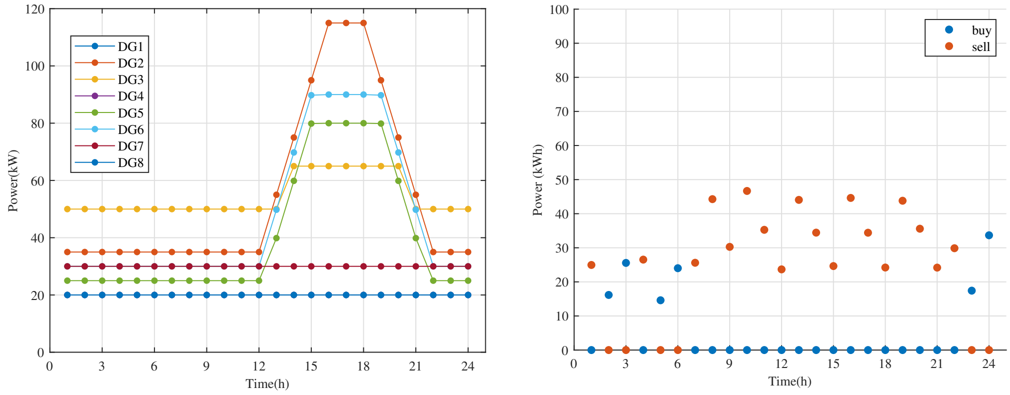



**Figure 7.** Outdoor and indoor temperature of the building: (**a**) Outdoor temperature; (**b**) Indoor temperature.

The power of DGs is shown in Figure 8a. DG produces less power during the off-peak demand for electricity but fully generates power during peak demand for electricity. The power transaction between the VPP and the main grid is shown in Figure 8b. The VPP strategically engages in the practice of procuring electric power from the main grid during periods of low demand, commonly known as off-peak hours, while subsequently selling electric power to the main grid during peak-demand periods. This dynamic approach empowers the VPP to optimize its operational profitability by capitalizing on the price differentials between off-peak and peak electricity demand. As shown in Figure 9, the storage system is charged and discharged several times. On the whole, the storage system is mainly discharged during the peak demand for electricity, and mainly charged during the off-peak demand for electricity. Therefore, thermal storage in residential REH systems has the potential for peak shaving.




(**a**) (**b**)

**Figure 8.** The electric power in a VPP: (**a**) The power of DG; (**b**) The transaction power with the electrical market.

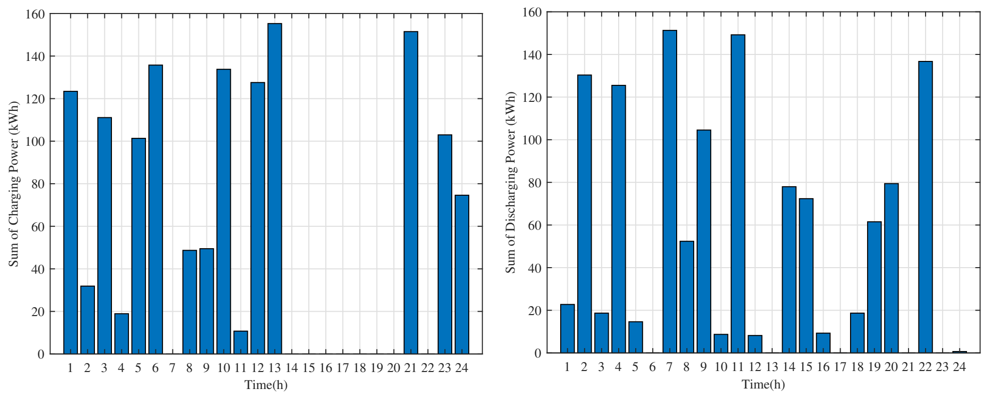



**Figure 9.** The power of thermal storage.

## 5. Conclusions

This paper tackled the intricate challenge of optimizing the coordination of residential regenerative electric heating (REH) systems using a virtual power plant (VPP) concept. This paper developed a comprehensive model considering the physical and network constraints of the VPP, while also ensuring user comfort. Moreover, the paper delved into investigating the VPP's potential as a participant in day-ahead energy and reserve markets. This would enable the VPP to optimize its operations and earn additional revenue by capitalizing on price signals. The simulation results reveal that thermal storage within REH systems holds significant potential for peak shaving. Moreover, to solve this complex coordination problem, the paper proposed a distributed method based on the alternating direction method of the multiplier. The simulation results serve as evidence of the efficacy of the algorithm in optimizing the VPP's operations in compliance with various constraints.

Building-integrated photovoltaics is an effective technology to achieve sustainable development energy. However, how to effectively integrate intermittent renewable energy into buildings is a great challenge. We will consider the impact of renewable sources in future research. In this way, the operating costs of the VPP are reduced while providing an environmentally friendly and sustainable operating mode.

**Author Contributions:** Conceptualization, G.Y. and H.L.; methodology, H.L.; software, G.Y.; validation, G.Y.; formal analysis, H.L.; investigation, G.Y.; resources, S.L.; data curation, G.Y.; writing—original draft preparation, H.L.; writing—review and editing, G.Y., S.L. and J.C.; visualization, G.Y.; supervision, W.W.; project administration, S.L.; funding acquisition, W.W. All authors have read and agreed to the published version of the manuscript.

**Funding:** This work was supported in part by the National Key R&D Program of China (2020YFC0827000), in part by the Guangdong Basic and Applied Basic Research Foundation (2022A1515110833).

**Data Availability Statement:** The data presented in this study are available in the article.

**Conflicts of Interest:** The authors declare no conflicts of interest.

## Abbreviations

The following abbreviations are used in this manuscript:

| | |
|---|---|
| ADMM | Alternating Direction Method of Multipliers |
| DER | Distributed Energy Resource |
| DG | Distributed Generation |
| ES | Energy Storage |
| FD | Flexible Demand |
| FL | Flexible Load |
| PSO | Particle Swarm Optimization |
| P2H | Power to Hydrogen |
| REH | Regenerative Electric Heating |
| TOU | Time of Use |
| VPP | Virtual Power Plant |

## References

1. Morstyn, T.; Farrell, N.; Darby, S.J.; McCulloch, M.D. Using peer-to-peer energy-trading platforms to incentivize prosumers to form federated power plants. *Nat. Energy* **2018**, *3*, 94–101. [CrossRef]
2. Han, D.M.; Lim, J.H. Smart home energy management system using IEEE 802.15.4 and zigbee. *IEEE Trans. Consum. Electron.* **2010**, *56*, 1403–1410. [CrossRef]
3. Pudjianto, D.; Ramsay, C.; Strbac, G. Virtual power plant and system integration of distributed energy resources. *IET Renew. Power Gener.* **2007**, *1*, 10–16. [CrossRef]
4. Pedrasa, M.A.A.; Spooner, T.D.; MacGill, I.F. Coordinated scheduling of residential distributed energy resources to optimize smart home energy services. *IEEE Trans. Smart Grid* **2010**, *1*, 134–143. [CrossRef]
5. Yuan, Y.; Wei, Z.; Sun, G.; Sun, Y.; Wang, D. A real-time optimal generation cost control method for virtual power plant. *Neurocomputing* **2014**, *143*, 322–330. [CrossRef]
6. Othman, M.M.; Hegazy, Y.; Abdelaziz, A.Y. Electrical energy management in unbalanced distribution networks using virtual power plant concept. *Electr. Power Syst. Res.* **2017**, *145*, 157–165. [CrossRef]
7. Kasaei, M.J. Energy and operational management of virtual power plant using imperialist competitive algorithm. *Int. Trans. Electr. Energy Syst.* **2018**, *28*, e2617. [CrossRef]
8. Ruiz, N.; Cobelo, I.; Oyarzabal, J. A direct load control model for virtual power plant management. *IEEE Trans. Power Syst.* **2009**, *24*, 959–966. [CrossRef]
9. Pandžić, H.; Morales, J.M.; Conejo, A.J.; Kuzle, I. Offering model for a virtual power plant based on stochastic programming. *Appl. Energy* **2013**, *105*, 282–292. [CrossRef]
10. Kardakos, E.G.; Simoglou, C.K.; Bakirtzis, A.G. Optimal offering strategy of a virtual power plant: A stochastic bi-level approach. *IEEE Trans. Smart Grid* **2016**, *7*, 794–806. [CrossRef]
11. Ju, L.; Tan, Z.; Yuan, J.; Tan, Q.; Li, H.; Dong, F. A bi-level stochastic scheduling optimization model for a virtual power plant connected to a wind-photovoltaic-energy storage system considering the uncertainty and demand response. *Appl. Energy* **2016**, *171*, 184–199. [CrossRef]
12. Liu, H.; Lei, S.; Hou, Y. An event-driven finite-time distributed optimization algorithm for economic dispatch in islanded microgrids. In Proceedings of the 2022 IEEE Power & Energy Society General Meeting (PESGM), Denver, CO, USA, 17–21 July 2022; pp. 1–5.
13. Xin, H.; Gan, D.; Li, N.; Li, H.; Dai, C. Virtual power plant-based distributed control strategy for multiple distributed generators. *IET Control Theory Appl.* **2013**, *7*, 90–98. [CrossRef]
14. Zhang, R.; Hredzak, B.; Fletcher, J. Dynamic aggregation of energy storage systems into virtual power plants using distributed real-time clustering algorithm. *IEEE Trans. Ind. Electron.* **2021**, *68*, 11002–11013. [CrossRef]

15. Yang, H.; Yi, D.; Zhao, J.; Dong, Z. Distributed optimal dispatch of virtual power plant via limited communication. *IEEE Trans. Power Syst.* **2013**, *28*, 3511–3512. [CrossRef]
16. Chen, G.; Li, J. A fully distributed ADMM-based dispatch approach for virtual power plant problems. *Appl. Math. Model.* **2018**, *58*, 300–312. [CrossRef]
17. Cai, D.; Wang, Z.; Wang, X.; Yang, W.; Wang, W.; Zhou, K.; Yu, D. Optimal dispatch of a virtual storage plant using inexact ADMM. *Front. Energy Res.* **2022**, *10*, 886845. [CrossRef]
18. Dong, L.; Fan, S.; Wang, Z.; Xiao, J.; Zhou, H.; Li, Z.; He, G. An adaptive decentralized economic dispatch method for virtual power plant. *Appl. Energy* **2021**, *300*, 117347. [CrossRef]
19. Lin, C.; Hu, B.; Shao, C.; Niu, T.; Cheng, Q.; Li, C.; Xie, K. An analysis of delay-constrained consensus-based optimal algorithms in virtual power plants. *ISA Trans.* **2022**, *125*, 189–197. [CrossRef]
20. Li, J.; Ye, Y.; Papadaskalopoulos, D.; Strbac, G. Distributed consensus-based coordination of flexible demand and energy storage resources. *IEEE Trans. Power Syst.* **2021**, *36*, 3053–3069. [CrossRef]
21. Yang, Q.; Wang, H.; Wang, T.; Zhang, S.; Wu, X.; Wang, H. Blockchain-based decentralized energy management platform for residential distributed energy resources in a virtual power plant. *Appl. Energy* **2021**, *294*, 117026. [CrossRef]
22. Naina, P.M.; Swarup, K.S. Double-consensus-based distributed energy management in a virtual power plant. *IEEE Trans. Ind. Appl.* **2022**, *58*, 7047–7056. [CrossRef]
23. Wang, X.; Zhao, H.; Lu, H.; Zhang, Y.; Wang, Y.; Wang, J. Decentralized coordinated operation model of VPP and P2H systems based on stochastic-bargaining game considering multiple uncertainties and carbon cost. *Appl. Energy* **2022**, *312*, 118750. [CrossRef]
24. Fan, S.; Liu, J.; Wu, Q.; Cui, M.; Zhou, H.; He, G. Optimal coordination of virtual power plant with photovoltaics and electric vehicles: A temporally coupled distributed online algorithm. *Appl. Energy* **2020**, *277*, 115583. [CrossRef]
25. Dall'Anese, E.; Guggilam, S.S.; Simonetto, A.; Chen, Y.C.; Dhople, S.V. Optimal regulation of virtual power plants. *IEEE Trans. Power Syst.* **2018**, *33*, 1868–1881. [CrossRef]
26. Tarroja, B.; Chiang, F.; AghaKouchak, A.; Samuelsen, S.; Raghavan, S.V.; Wei, M.; Sun, K.; Hong, T. Translating climate change and heating system electrification impacts on building energy use to future greenhouse gas emissions and electric grid capacity requirements in California. *Appl. Energy* **2018**, *225*, 522–534. [CrossRef]
27. Shen, R.; Zhong, S.; Zheng, R.; Yang, D.; Xu, B.; Li, Y.; Zhao, J. Advanced control framework of regenerative electric heating with renewable energy based on multi-agent cooperation. *Energy Build.* **2023**, *281*, 112779. [CrossRef]
28. Zhang, J.; Mu, Y.; Wu, Z.; Liu, Z.; Gao, Y.; Jia, H.; Li, H. Optimal scheduling method of regenerative electric heating for emergency residential building heating: An affine arithmetic-based model predictive control approach. *IET Energy Syst. Integr.* **2023**, *5*, 40–53. [CrossRef]
29. Zhong, S.; Wang, X.; Zhao, J.; Li, W.; Li, H.; Wang, Y.; Deng, S.; Zhu, J. Deep reinforcement learning framework for dynamic pricing demand response of regenerative electric heating. *Appl. Energy* **2021**, *288*, 116623. [CrossRef]
30. Wang, C.; Shahidehpour, S. Effects of ramp-rate limits on unit commitment and economic dispatch. *IEEE Trans. Power Syst.* **1993**, *8*, 1341–1350. [CrossRef]
31. Chen, W.; Shi, P.; Li, Y.; Cai, Q.; Chen, K.; Xiang, K. Optimal planning method for regenerative electric heating system considering the supply capability of distribution network. In Proceedings of the 2021 IEEE 5th Conference on Energy Internet and Energy System Integration (EI2), Taiyuan, China, 22–24 October 2021; pp. 1789–1793.
32. Yao, M.; Molzahn, D.K.; Mathieu, J.L. An optimal power-flow approach to improve power system voltage stability using demand response. *IEEE Trans. Control Netw. Syst.* **2019**, *6*, 1015–1025. [CrossRef]
33. Scarabaggio, P.; Carli, R.; Dotoli, M. Noncooperative equilibrium-seeking in distributed energy systems under ac power flow nonlinear constraints. *IEEE Trans. Control Netw. Syst.* **2022**, *9*, 1731–1742. [CrossRef]
34. Baran, M.; Wu, F. Optimal sizing of capacitors placed on a radial distribution system. *IEEE Trans. Power Deliv.* **1989**, *4*, 735–743.
35. Wu, D.; Yang, T.; Stoorvogel, A.A.; Stoustrup, J. Distributed optimal coordination for distributed energy resources in power systems. *IEEE Trans. Autom. Sci. Eng.* **2017**, *14*, 414–424. [CrossRef]
36. Boyd, S.; Parikh, N.; Chu, E.; Peleato, B.; Eckstein, J. Distributed optimization and statistical learning via the alternating direction method of multipliers. *Found. Trends® Mach. Learn.* **2011**, *3*, 1–122.
37. Carli, R.; Dotoli, M. Distributed alternating direction method of multipliers for linearly constrained optimization over a network. *IEEE Control Syst. Lett.* **2020**, *4*, 247–252. [CrossRef]
38. Yu, Y.; Açıkmeşe, B.; Mesbahi, M. Bregman parallel direction method of multipliers for distributed optimization via mirror averaging. *IEEE Control Syst. Lett.* **2018**, *2*, 302–306. [CrossRef]
39. Liu, H.; Fan, H.; Wang, B. Distributed event-triggered strategy for fixed-time economic dispatch in islanded microgrids. In Proceedings of the 2021 33rd Chinese Control and Decision Conference (CCDC), Kunming, China, 22–24 May 2021; pp. 1401–1406.
40. Liu, H.; Fan, H.; Wang, B.; Liu, L.; Lei, S. Event-triggered scheme for finite-time distributed economic dispatch in smart grids. *J. Frankl. Inst.* **2022**, *359*, 10602–10627. [CrossRef]
41. Molzahn, D.K.; Dörfler, F.; Sandberg, H.; Low, S.H.; Chakrabarti, S.; Baldick, R.; Lavaei, J. A survey of distributed optimization and control algorithms for electric power systems. *IEEE Trans. Smart Grid* **2017**, *8*, 2941–2962. [CrossRef]
42. Erseghe, T. Distributed optimal power flow using ADMM. *IEEE Trans. Power Syst.* **2014**, *29*, 2370–2380. [CrossRef]

43. Mashhour, E.; Moghaddas-Tafreshi, S.M. Bidding strategy of virtual power plant for participating in energy and spinning reserve markets—Part II: Numerical analysis. *IEEE Trans. Power Syst.* **2011**, *26*, 957–964. [CrossRef]
44. Nicolson, M.L.; Fell, M.J.; Huebner, G.M. Consumer demand for time of use electricity tariffs: A systematized review of the empirical evidence. *Renew. Sustain. Energy Rev.* **2018**, *97*, 276–289. [CrossRef]