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A Critical Review of State-of-the-Art Optimal PMU Placement Techniques

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Abstract: Phasor measurement unit (PMU) technology is a need of the power system due to its better resolution than conventional estimation devices used for wide-area monitoring. PMUs can provide synchronized phasor and magnitude of voltage and current measurements for state estimation of the power system to prevent blackouts. The drawbacks of a PMU are the high cost of the device and its installation. The main aspect of using PMUs in electrical networks is the property to observe the adjacent buses, thereby making it possible to observe the system with fewer PMUs than the number of buses through their optimal placement. In the last two decades, exhaustive research has been done on this issue. Considering the importance of this field, a comprehensive review of the progress achieved until now is carried out and the limitations of existing reviews in the literature are highlighted. This paper can be seen as a major attempt to provide an up-to-date review of the research work carried out in this all-important field of PMU placement and indicates that some perspectives of optimal PMU placement still need attention. Eventually, the work will open a new standpoint for the research community to fill the research gap.

Keywords: phasor measurement unit (PMU); synchrophasors; optimization; optimal PMU placement (OPP); complete network observability (CNO); incomplete network observability (INO); single PMU or line outage ($N-1$ contingency)

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1. Introduction

The power system is structured using electrical components that are responsible for the conversion of different forms of energy into electrical energy along with its transmission, distribution, and utilization. Voltage, current, phasors, and frequency are important electrical quantities of a power system. Except for frequency, all three electrical quantities are distinct in different parts of a power system. Proper working of all the parts in the power system is indispensable. For proper working of a power system, monitoring these electrical quantities is crucial [1] for optimal operation, reliability, security, contingency analysis, and restoration of a power system. A monitoring system performs three basic activities, which include determining the network topology, observability, and state estimation of a power system. Conventionally, power system quantities like voltage and current are monitored through supervisory control and data acquisition (SCADA) systems. The SCADA system is not sufficient for monitoring the modern power system [2–4] because of its low resolution, and it is not capable of giving phasor, frequency, and rate of change of frequency information. A report concerning the 14 August 2003 cascading blackout incident in the northeastern United States, published by the North American Electric Reliability Corporation (NERC), showcased that the phase angle between western Michigan and Cleveland did increase steadily for an hour, as shown in Figure 1. If the information regarding phase angle had been available at the proper time, the operators could have taken appropriate actions to prevent the cascading blackout [5,6].

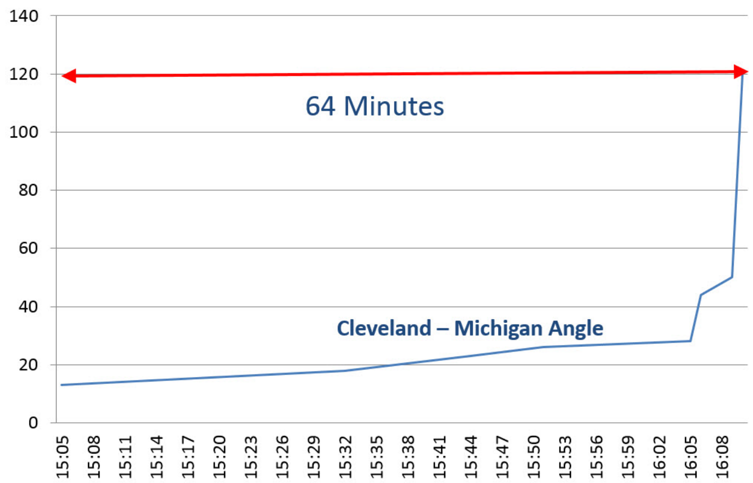

Figure 1. Phase angle divergence between Cleveland and Michigan.

Modern power systems consist of renewable energy sources, intelligent electronic devices, advanced protection, and control devices; therefore, fast and robust monitoring systems should be installed for the operation and safe control of the power system. Monitoring phasor information is also critical along with the magnitudes of voltage and current. In 1980, the concept of phasor monitoring in a power system was introduced by Prof. Phadke at Virginia Tech, USA. For the first time, the concept of the phasor measurement unit (PMU) was established in 1988 and its application was introduced by Macrodyne corporation [7]. Commercial production of PMUs started in 1991 by Macrodyne in collaboration with Virginia Tech [8].

The applications of the PMU in a power system are implementation of smart grid, integration of renewable sources, static to dynamic control, real-time control, frequency control, state estimation, adaptive protection, automated energy management system (EMS), thermal overload monitoring, stability monitoring, system restoration, and post-disturbance analysis [9–12] in a power system. Wide range monitoring of power systems for reliable operation and control is called wide-area measurement system (WAMS), which is not possible without PMU integration. A brief comparison of SCADA and PMU technology in WAMS is given in the section below.

1.1. SCADA versus PMU

The drawbacks of the SCADA system for modern power systems are slow sampling rate, no time synchronization, and no phase angle information. PMUs are the backbone of WAMS, as they are faster [13] and give phasor information. The communication protocol used for the SCADA system is International Electro-technical Commission (IEC) 60870-5 series, and the protocol used for PMU is National Standard of the People's Republic of China/Recommended (GB/T) 26865.2-2011 [14]. Table 1 summarizes the difference between SCADA and PMU technology used in WAMS [15,16].

Table 1. Different attributes of SCADA and PMU in WAMS.

Attribute	SCADA	PMU
Universal time synchronization	✗	✓
Local estimation of phase angles	✗	✓
Reporting rate	Once in (4 to 6) s	(10/12/15/20/30/60 frames)/s
Data flow latency	High	Negligible
State view of power system	Steady	Dynamic
Total input/output channels	100+ Analog and digital	10 Phasors, 32+ analog and digital
Communication method	Serial communication	Network communication

1.2. Existing Surveys in OPP

During the last two decades, the work done in optimal PMU placement (OPP) that is presented in different review articles in different ways is remarkable. In literature, one of the earliest reviews by Yuill et al. in 2011 [17] reviewed meta-heuristic and deterministic techniques only in OPP. Based on literature published through 2011, the integer linear programming technique was claimed to be the most adaptable technique in OPP in all scenarios. Manousakis et al. [18] reviewed different formulations and summarized published work in OPP along with future trends. In 2014, Aminifar et al. [19] provided a systematic review on the structural design of PMUs, placement of PMUs, application of PMUs, and function of WAMS.

In 2016, Negash et al. [20] claimed to review mathematical and artificial intelligence techniques for OPP. The claim is false, and correct classification is present in our paper. Mohanta et al. [8] systematically reviewed PMUs as sensors and briefly explained formulation and techniques used in OPP. Sefid et al. [2] reviewed OPP in the smart grid but discussed very few published articles related only to OPP. Misclassification of meta-heuristic techniques presented is correctly presented in our paper. Baba et al. [21] reviewed a few published articles available in the literature about OPP for network observability. Johnson et al. [22] reviewed different properties of OPP formulation and shortcomings of the techniques in OPP. This work is appreciable, but the challenges discussed for future research by Johnson et al. are not claimable, as the past research provides us with global optimum value, solutions with ZIBs, $N-1$ contingency, channel limitation, and numerical observability. These all so-called challenges discussed by the author are available with references, and true challenges are discussed in our paper.

Abdulkareem et al. [23] reviewed a few published articles with benefits and drawbacks of each approach in OPP. After reviewing the fast growth in OPP, this paper resolves limitations of existing reviews in the literature by presenting a flow chart to have a complete picture of all the optimization techniques used for OPP in the literature. We have classified optimization techniques into two major categories: conventional and non-conventional techniques. We have thoroughly discussed, with tables, the most common techniques and briefly the less common techniques used in OPP. We have called attention to all the objective functions, constraints, and the most important PMU installation schemes, so one can have a comprehensive understanding by just skimming through this article. Furthermore, different tools to solve the OPP are also discussed in this study. At the end of this paper, we point towards the research gap to provide direction for future research.

The main contributions of the paper are as follows:

1. A comprehensive review of state-of-the-art optimal PMU placement techniques is presented.
2. We have outlined the strengths and weaknesses of research efforts in the PMU placement problem. Critical discussion is also made on the previously published review articles in optimal PMU placement.
3. A taxonomy of optimization techniques, used in optimal PMU placement, is presented that gives a framework for grouping different optimization algorithms. The literature discussed in this paper follows the same taxonomy.

4. A summary of commonly used techniques like linear programming, genetic algorithm, and particle swarm optimization is presented. A comparative analysis of common algorithms used in the PMU placement problem is also provided in tables.
5. Comprehensive discussion on testbed systems, optimization solvers, and pros and cons of different techniques in the literature of optimal PMU placement is provided.
6. An extensive list of references covering the work of the last two decades is provided. Important missing gaps in this field of research are also discussed in detail.

The organization of this paper is as follows. Taxonomy of techniques used to solve OPP, fundamental statements of the objective function, and observability constraints are given in Section 2. OPP using conventional and non-conventional optimization techniques is discussed in Sections 3 and 4. Testbed systems, optimization solvers, pros and cons of different algorithms, and detailed future scope are discussed in Section 5. Finally, the conclusion of this work is given in Section 6.

2. Optimal PMU Placement

Optimization is a branch of mathematics that helps in finding the best solution to any physical phenomena translated into a mathematical equation. Optimization theory provides algorithms to solve properly stated mathematical equations using computer-aided programs [24,25]. It is bifurcated into conventional and non-conventional techniques. Conventional techniques are linear programming (LP), non-linear programming (NLP), dynamic programming (DP), and combinational optimization, etc. Non-conventional techniques are particle swarm optimization, evolutionary algorithm, genetic algorithm, simulated annealing algorithm, and tabu search methods [26], etc.

The optimization for PMU placement in the power system is called the optimal PMU placement (OPP) problem. In OPP, if PMUs are placed on every single bus in the power system, then they provide redundant information about power system parameters. However, due to the high cost of the PMU device, its installation, phasor data concentrator (PDC), communication infrastructure, upgrading of old substations (if not compatible), and additional cost to deal with big data makes PMU installation uneconomical. Moreover, as a consequence of Ohm's Law, when a PMU is placed at a bus, neighboring buses also become observable. This implies that it is feasible to have complete or maximum observability (numerical as well as topological) with minimum PMU devices by establishing PMUs at specified buses only [27] using optimization for large networks. Optimization also assists in resolving different contingencies and constraints in PMU placement planning. To solve the constraints of communication availability, conventional measurements (CMs), faults, and burdens on the line or transformer, modern techniques outperform the conventional techniques [18]. In OPP, the most commonly used techniques are linear programming, particle swarm optimization, genetic algorithm, and greedy search algorithm. Optimization techniques used for OPP are shown in Figure 2.

In OPP problem formulation, different objective functions, constraints, contingencies, and installation schemes are used. The most common problem formulated for OPP is a binary integer linear problem. Problem formulation is briefly presented on the basis of objective function and constraints.

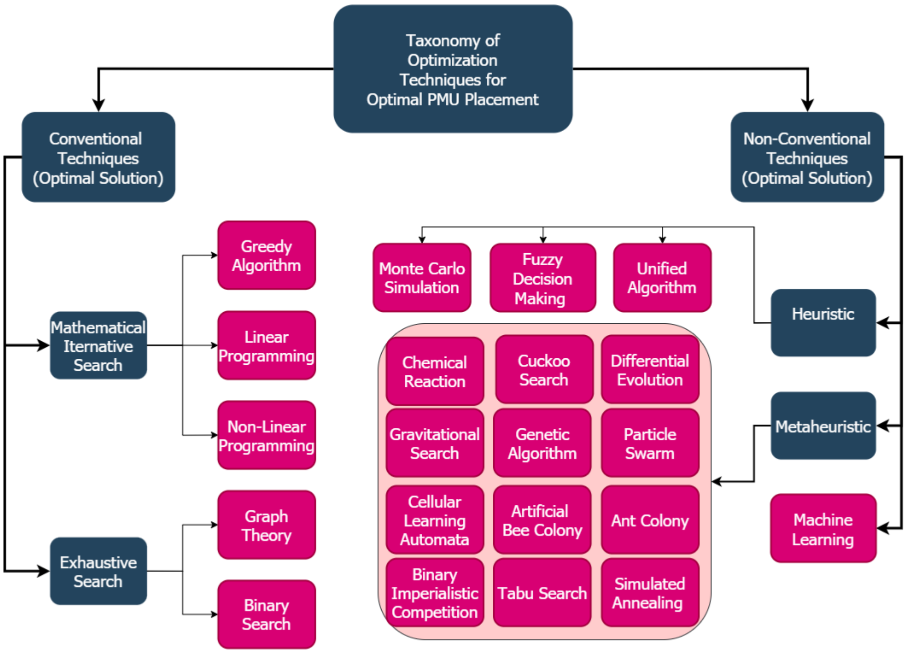

Figure 2. Taxonomy of optimization techniques used in optimal PMU placement.

2.1. Fundamental Statement of Objective Function

There are two most common objective functions used in literature that minimize the number of installed PMUs to minimize the cost and maximize the redundancy by incorporating the observability into the objective function. Minimizing the number of PMUs is given in Equation (1), as stated below:

$$F_1 = \min \sum_{i=1}^{m} P_i \times C_i \tag{1}$$

In Equation (1), the variable P is the PMU device to be installed on a bus i, m is the total number of buses, C is the cost of device that can be a constant if all the devices are of the same price, and P_i is a binary decision variable.

$$P_i = \begin{cases} 1 & \text{If device P is installed on bus i} \\ 0 & \text{If device P is not installed on bus i} \end{cases}$$

In Equation (2), the objective function is to maximize the redundancy by incorporating observability as a part of the objective function, where O_i is the number of times bus i is observed via installed PMUs.

$$F_2 = \max \sum_{i=1}^{m} O_i \tag{2}$$

As a multi-objective problem, both of the objective functions can be combined easily by changing the sign of function F_2 and treating it as a minimization function. The combined formulation is given in Equation (3), as stated below:

$$F_3 = \sum_{i=1}^{m} P_i - \frac{1}{N \times \max(O_i^{allPMU} + 1)} \sum_{i=1}^{m} O_i - 0.35 \tag{3}$$

where N is the total count of buses in the power system, O_i^{allPMU} is the number of times that bus i is observed when all the buses are having PMUs. The coefficient in the second term of Equation (3), makes sure that function F_1 has higher importance than function F_2 in the OPP. Further investigation on different objective functions and constraints (linear and non-linear) can be found in review article [22] on OPP.

2.2. Fundamental Statement of Observability Constraint

In power system monitoring, observability is defined such that all the measurements provide enough information for state estimation. In the state estimation, an optimal estimate of the power system's current state is important, which is provided using data from measurements and network topology. For state estimation, important state variables are bus voltage (magnitude and angle). If these two variables are known, all remaining variables such as line current (magnitude and phasor), active and reactive power in lines, and active and reactive power of load can be calculated easily [28–30]. Conventionally, state estimation data are voltage magnitude and angles, active and reactive power flows, and power injections that have a non-linear relationship with states of the power grid. These state estimators are non-linear, time-consuming, and solved using iterative methods. The most commonly used technique is weighted least square, as shown in Equation (4). The use of synchrophasors measurement makes the estimator linear and the solution non-iterative, which can be obtained from PMU devices in reality [31].

$$z = h(x) + e \tag{4}$$

In Equation (4), z is a measurement vector made up with traditional measurements; $h(x)$ is a measurement function made up of non-linear relationships between the measurement vector and the state vector x, and e is an error vector.

The observability of the power system is explored by two major techniques. One is numerical observability and the other is topological observability. Numerical observability has the drawback that it needs to solve complex Jacobean matrix calculations, which make the solution complicated, resulting in lesser use of this technique. Topological observability is a much more common way in which the full rank of spanning tree is obtained [27]. Approaches to observability can be classified as topological, numerical, or hybrid (combined topological and numerical) approaches. The most commonly used constraint is the topological observability in the literature. Complete topological observability of the network, under normal conditions, is given in Equations (5) and (6). Here, 'O' is the observability vector consisting of observability expressions o_is, each standing for the ith observability constraint.

$$O = AX \tag{5}$$

$$O \geq u \tag{6}$$

$$A = \begin{bmatrix} 1 & 0 & \dots & 1 \\ 1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & \dots & 1 \end{bmatrix}$$

whose elements are given below:

$$A_{ij} = \begin{cases} 1 & \text{if either } i = j \text{ or } i\&j \text{ buses are joined by a branch} \\ 0 & \text{Otherwise} \end{cases}$$

$$X = \begin{bmatrix} x_1 & x_2 & \dots x_N \end{bmatrix}^T \quad \text{and} \quad u = \begin{bmatrix} 1 & 1 & \dots 1 \end{bmatrix}^T$$

where A is the node incidence matrix or binary connectivity matrix, with size $N \times N$; X is a row vector having size $N \times 1$ with elements x_i, $i = 1, 2, ..., N$, and u is a row vector $N \times 1$ consisting of ones, representing a bus observable by one PMU.

3. OPP Using Conventional Optimization

Conventional techniques that are used in optimal PMU placement are classified as mathematical iterative search and exhaustive search techniques. Details of articles that used conventional techniques are discussed below.

3.1. Mathematical Iterative Algorithms

In mathematical iterative search, an initial value is used to make a sequence of the improved estimate, in which the nth estimate is taken from the previous value. To solve the mathematical problem of x variable, we put the value of x_0 to find the value of x_1. The solution to the problem is converged when, after successive iterations, the value of x_{n-1} becomes equal to the value of x_n up to three decimal places. Mathematical iterative technique-based PMU placement is discussed below:

3.1.1. Greedy Algorithm Based OPP

The greedy search algorithm comprises many algorithms that try to find the quickest solution instead of finding the optimal solution of any optimization problem [32]. These algorithms provide the local optimum value of the solution in most cases and give global optimum solutions in very few problems. Work done in OPP using greedy search algorithm is discussed below:

(a) Information Theoretic Approach (ITA)

In ITA, also known as the data analytic approach, a set of candidate models is examined to discover the model probability that is closer to the truth than all others models in the set. Li et al. [33] used greedy ITA for OPP and used mutual information (MI) among states of the system and measurements. MI was used as an objective function and incorporated observability and uncertainty reduction.

(b) Posterior Cramér-Rao Bound (PCRB)

The basic tool to find estimation performance along with the Markovian model is the Posterior Cramér-Rao Bound. It defines the bounds for the variance of a biased estimator.

Yang et al. [34] used greedy approach PCRB for OPP. To crack the optimization problem, the greedy search algorithm was used. For submodular and non-submodular objective functions, the greedy search algorithm guarantees a solution and provides better results, respectively.

3.1.2. Linear Programming (LP) Based OPP

Linear programming is a technique to maximize or minimize a linear mathematical problem [35]. The cost is used as an objective function considering the constraints on decision variables. In LP, the decision variable cannot be negative and used to solve NP-complete problems. PMU placement using LP techniques are discussed below:

(a) Binary Integer Linear Programming (BILP)

The BILP technique is used to solve a system of linear equalities or inequalities in binary unknowns (0 or 1).

Binary unknowns are used where the formulated problem has solutions in discrete binary form (Yes or No).

Abbasy et al. [36] proposed the OPP technique using BILP by considering both conventional existing injection flow (IF) measurements and loss of one or more PMUs. Enshaee et al. [37] used BILP for CNO and increased measurement redundancy by incorporating outage scenarios of a PMU or a line to solve OPP. PMU channel limitation and the effect of zero injection buses (ZIBs) is also incorporated.

Amare et al. in [38] used BILP to find OPP with CNO and considered $N-1$ contingency. unobservable buses depth, multi-stage sequential appointment, critical buses monitoring, existing location of PMU, unsuitable locations for PMU, and information about the critical buses are inputs to the BILP for cracking the problem.

Rezaeiankoochi et al. [39] used BILP to solve OPP and considered post-disturbance coherency at unrelated depths of observability. To accomplish these, dissimilar probabilistic parameters are used to create scenarios. Subsequently, the subtractive clustering algorithm was used to find central buses that are constructed on the likeness of the post-disturbance variation. Afterward, by selecting each bus as a central bus, different scenarios are solved for placing PMUs.

Ahmed et al. [13] used BILP for OPP and considered $N-1$ contingency for complete network observability of the system.

Mahaei et al. [40] used BILP for OPP and considered CNO along with the multistage installation of PMUs, two adjacent injection measurements, and $N-1$ contingency.

Bei et al. in [41] proposed generalized procedure to solve the OPP and different situations like: loss of PMU, zero and non-zero injection buses and power flow measurement device on branches. PMU failure was also investigated using linear estimator-based measurements from PMU, computational performance and bad data processing.

Sarailoo et al. [42] proposed the addition of new PMUs in the network with already installed PMUs for robustness, maximizing data availability profile and avoiding communication interruption and transmission faults. A three-stage scalable synchrophasor availability constrained placement algorithm was proposed to solve OPP using integer linear programming (ILP). Gou [43] proposed generalized OPP formulation considering CNO and INO along with the redundant placement of PMU using ILP. Kavasseri et al. [44] proposed an economical solution for the shared placement of PMU and CMs. The problem was primarily modeled as nonlinear integer programming and then adapted to equivalent integer linear programming via the Boolean method. Results require lesser PMUs, and the methodology was economical as compared to conventional measurement in fixed locations or absent. Sodhi et al. [45] proposed OPP in different stages to limit the burden of investment using ILP and multi-criteria decision-making (MCDM) techniques. ILP approach was utilized to perceive the optimum place of the PMUs for comprehensive observability of system even in case of a branch or PMU contingency, and MCDM helps to prioritize the locations considering utility concern to locate weights for three criteria for installation in different stages.

Dobakhshari et al. [46] proposed OPP with conventional equipment. This study also incorporates a solitary line or PMU contingency with bursting observability. In finding the least number of PMUs circuit equations of PMU, conventional measurement, and network topology to find a global optimal solution, which was different from the previous methods. Communication limitation was also incorporated. Gou et al. [47] proposed OPP using a unified algorithm along with bad data detection and observability analysis. Each iteration tries to obtain optimum location by making the power system observable or making critical buses non-critical.

The cost of synchrophasors is evolving day by day. In total expenses, the price of the advancement of a substation is much more than PMU. In such a scenario, the consideration of minimizing the PMU number is not enough, and minimizing the substation number is also essential. This issue was solved by Pal et al. [48] using ILP based on dual-use line relays (DULRs), which function in minimizing the overall cost of the system. Simultaneously, optimization of communication infrastructure, cybersecurity infrastructure, labor cost, and device cost was discussed.

For consideration of ZIBs, conventional measurements (CMs) are constructed by logically summing up bus observability functions in [49]. The limitations caused by ZIBs and CMs are identified that were previously not considered. Singh et al. [50] proposed OPP using ILP with two contingencies. One contingency was based on voltage stability and the other was based on intense islanding. To fulfill two objectives, two contingencies were considered via a single multi-objective function to minimize PMUs and maximize observability. Gou [51] proposed OPP using ILP with and without conventional power flow and injection measurement.

Optimization of substation number by placing simultaneously traditional PMUs and dual-use line relays was accomplished in [52]. Redundancy in the measurement of serious elements and estimating the transformer tap ratios were incorporated in the study via the general optimal substation coverage (GOSC) algorithm. Techno-economic benefits are achieved using the GOSC algorithm. Rashidi et al. in [53] used Lyapunov exponent for full system observability, improved actual stability monitoring, and assessment of the system. Firstly, the maximization of the redundancy of critical buses was achieved. Secondly, each bus's role in the system stability and critical buses was recognized using the Lyapunov exponent founded approach. Rakpenthai et al. [54] used ILP for optimization of redundant measurements and heuristic methods for arranging the measurement positions for PMUs placement.

Monitoring of current and voltage phasors by placing PMUs on the branch was accomplished in [55], with optimal PMU placement keeping the entire network observable using the ILP technique. Huang et al. [56] proposed OPP, which gives controlled islanding of the power system for network observability under the typical state and controlled islanding state. Maximization of redundancy and minimization of PMU number are the objective functions that are combined by the weighting variable. Consideration of zero injection bus, PMU failure, and line outage are also included for system observability.

PMUs used for monitoring need to be patched after some time to avoid vulnerabilities causing shut down of PMUs for some time. If PMUs are placed redundantly, then a set of PMUs can be placed offline while maintaining the complete system observability. The challenge in optimizing the patching plan to patch all the PMUs in the smallest number of rounds and maintaining observability all the time was discussed in [57]. The case problem was cracked by the ILP technique, and large networks are solved using a greedy heuristic algorithm. Mousavian et al. [58] proposed OPP in two stages using ILP. In the first stage CNO and in the second stage $N-1$ contingency along with the switching of transmission lines were taken into account. Huang et al. [59] proposed OPP considering steady-state readiness of synchrophasor information at each bus to meet the defined level and communication channels limitation. A Markov model was built for evaluating the synchrophasor availability. In [60], estimation theory criteria are used as a base for PMU placement. PMU reading of both current-voltage and conventional state estimation under the Bayesian framework was used to place PMUs. Convex optimization relaxation was used to achieve less computation. Numerical optimality was guaranteed by convex relaxation bypassing the combinatorial search.

Junhyung Bae [61] proposed optimal placement of PMUs and accounted for the network protection against cyberattacks. Redundant PMUs are allocated on the vulnerable buses for this purpose. The formulation of the problem is binary and is solved using the binary integer linear programming technique.

Chen et al. [62] proposed optimal PMU placement with improved redundancy using the BILP technique. Effect of ZIB, conventional measurements, contingency analysis, and channel limitation of PMUs is incorporated. The redundancy level of each bus is optimized by assigning weights to an auxiliary variable.

Koochi et al. [63] proposed optimal PMU placement with complete network observability and backup protection to detect faults on the transmission line. Channel capacity of different companies of PMUs is also considered in the formulation.

Hyacinth et al. [64] proposed optimal PMU placement with maximized measurement redundancy based on the three major attributes in deciding placement locations: the degree of the vertex (DOV), the average neighborhood degree of the vertex (ANDOV), and the bus observability index (BOI) in the first stage and minimized the number of PMUs in next stage.

Baba et al. [65] proposed optimal PMU placement using BILP to maximize measurement redundancy and minimize the number of PMUs considering contingency, channel limitation, and ZIB (named as pure transit node).

Elimam et al. [66] proposed optimal PMU placement using ILP and considered network impedance parameters (series and shunt), contingency analysis, the effect of ZIB, channel limitation, and small-signal stability of the network.

(b) Mixed-Integer Linear Programming (MILP)

MILP adds one additional condition in ILP that at least one decision variable does not have discrete value, which means that at least one variable is not an integer. Work done in OPP using MILP is discussed below.

Aminifar et al. [67] proposed OPP with consideration of the predefined probability of observability. The placement of PMUs includes the outage probability and stochastic nature of the equipment. MILP was practiced for optimization projected. Due to financial and physical constraints, the placement of PMUs was staged into multi-year planning. When a probabilistic constraint was accounted for in the model, the solution provides acceptable outage protection in a probabilistic manner. In [68], an analytic framework for OPP was presented with consideration of cost/benefit investigation, long-term economic facts, and current technical problems.

Esmaili et al. [69] used MILP for the new redundant observability scheme in the OPP problem. The method proposed has a new objective function that was used to improve the observability redundancy while utilizing the same PMU number as in the prevailing method. Nikkhah et al. [70] proposed contingency constraint OPP on n-k redundancy criteria via robust optimization. Network observability, underneath any contingency state that contains damage to k = 2 PMUs, was ensured by the developed security criterion. Aminifar et al. [71] discussed OPP for AC/DC systems for observability using MILP. Phasors are no longer valid for HVDC, so the integration of HVDC in HVAC would disturb the measurement of PMU indirectly. The objective was to minimize the installation cost of PMU while taking into account the observability in the AC/DC transmission system and the variable cost of PMU. Results dictate that the DC line effect was negative in the OPP problem, i.e., the minimum PMU number was increased. Ruben et al. [72] used MILP to solve two objectives: minimizing the cost of PMUs and gross error detection for CNO. The model was flexible to change weights of objectives as per the budget of the customer.

Zhu et al. [73] proposed optimal placement of PMUs and communication links and implemented the concept of zero injection buses to minimize the overall cost of installation of the wide-area monitoring system. The main contribution of this work is that it incorporated data transmission bandwidth under consideration.

(c) Equivalent Integer Linear Programming (EILP)

A theory of EILP claims that all integer programming problems are equivalent to infinitely many other integer programming problems. The equivalence means that the solution to any one problem in the equivalence class can find the solution to every other problem in the class.

Azizi et al. [74] proposed OPP using EILP. Such placement has a completely linear state estimation, which eliminates the issues in SCADA-based state estimation. EILP models can easily include additional constraints like $N-1$ contingency, channel limitation, and the ZIBs. A summary of LP based OPP is presented in Table 2.

Table 2. A summary of LP based OPP.

Ref.	Technique	Objective Function	Constraints		
			Observability	**Contingency**	**Installation Scheme**
[42]	BILP	Min. no. of PMUs, Min. no. of Data links	Pseudo observability	$N-1$, measurements and communication	3-stage installation of PMUs
[43]	BILP	Min. no. of PMUs	CNO and INO	–	Effect of ZIB and CMs
[44]	BILP	Min. no. of PMUs and flow measurements	CNO and observability in case of faults	–	Effect of CMs

Table 2. *Cont.*

Ref.	Technique	Objective Function	Constraints		
			Observability	Contingency	Installation Scheme
[45]	BILP	Min. no. of PMUs, Max. System observability	CNO	$N-1$	Multistage installation of PMUs
[46]	BILP	Min. no. of PMUs	CNO,	$N-1$ and measurements	Effect of CMs, flow measurements, Channel limitation, and measured injection buses
[47]	BILP	Min. no. of PMUs	CNO	Measurements	Effect of CMs, converting CMs to non-CMs
[48]	BILP	Min. Substation disruption, Max. bus observability using DULRs	–	$N-1$	Handling relays synchrophasors, critical buses, prohibited substations, existing PMUs, unknown transformer tap ratio, ZIB, and channel limitation
[49]	BILP	Min. no. of PMUs	CNO	$N-1$	Effect of CMs and ZIB
[50]	BILP	Min. no. of PMUs, Max. System observability	–	$N-1$	Effect of ZIB
[51]	BILP	Min. no. of PMUs	CNO	–	Effect of CMs and old power flows
[52]	BILP	Min. Substation disruption	–	$N-1$	Handling Channel capacity, prohibited substation, preinstalled PMUs, ZIB, relays synchrophasors, Redundancy in critical measurements, estimating unknown tap setting and tap ratios of transformer
[53]	BILP	Min. no. of PMUs	CNO	$N-1$	Redundancy in critical measurements
[54]	BILP	Min. no. of PMUs	CNO	$N-1$	–
[55]	BILP	Min. no. of PMUs for branch monitoring	CNO	$N-1$ and transformer	Improved redundancy
[56]	BILP	Min. no. of PMUs	CNO	$N-1$	Effect of ZIB and controlled islanding
[58]	BILP	Min. no. of PMUs	CNO	$N-1$	Effect of line switching
[59]	BILP	Min. no. of PMUs, communication links, and operation cost	–	$N \geq 1$	Limitation on communication infrastructure
[36]	BILP	Min. no. of PMUs, Max. system observability	–	$N-1$	Effect of CMs and ZIB
[37]	BILP	Min. no. of PMUs	CNO with maximum redundancy	$N-1$	Effect of channel limitation and ZIB
[39]	BILP	Min. no. of PMUs	CNO	–	Effect of post-disturbance coherency of the buses

Table 2. *Cont.*

Ref.	Technique	Objective Function	Constraints		
			Observability	Contingency	Installation Scheme
[40]	BILP	Min. no. of PMUs, Max. measurement redundancy	CNO with two adjacent injection measurements	$N-1$	Effect of CMs and PMUs multistage installation
[61]	BILP	Min. no. of PMUs	CNO	–	Effect of CMs and cyberattack
[62]	BILP	Min. no. of PMUs, Max. measurement redundancy,	CNO	$N \geq 1$	Effect of Channel limitation, CMs and ZIB
[63]	BILP	Min. no. of PMUs	CNO	–	Effect of Channel limitation and backup faults protection
[64]	BILP	Min. no. of PMUs	CNO	–	based on three attributes DOV, the ANDOV, and BOI
[65]	BILP	Min. no. of PMUs	CNO	$N-1$	Effect of channel limitation and ZIB
[66]	BILP	Min. no. of PMUs	CNO	$N-1$	Effect of channel limitation, ZIB, network impedences, and small signal stability of network
[67]	MILP	Min. no. of PMUs, max system observability	–	Probability of power system components outage	Financial and physical constraints, PMUs Multistage installation
[69]	MILP	Min. no. of PMUs, Max. redundancy	CNO	$N \geq 1$	–
[70]	MILP	Min. no. of PMUs	CNO	$N-1$	Effect of ZIB
[71]	MILP	Min. no. of PMUs	CNO	$N-1$	Variable cost PMUs and ZIB
[72]	MILP	Min. no. of PMUs	CNO	–	Budgeted PMUs allocation
[73]	MILP	Min. no. of PMUs, Min. no. of communication channel,	CNO	$N-1$	Effect of ZIB and data transmission bandwidth
[74]	EILP	Min. no. of PMUs	–	$N-1$	Effect of communication infrastructure, CMs and ZIB

3.1.3. Non-Linear Programming (NLP) Based OPP

NLP is an optimization technique where either the objective function is non-linear or one or more constraints are non-linear. Manousakis et al. [75] proposed OPP as a quadratic minimization problem employing continuous decision variables subjected to the non-linearity in boundaries of observability. The unconstrained nonlinear weighted least square approach was used to find the optimum solution. Chakrabarti et al. [76] proposed OPP and maintained the measurement redundancy for CNO using integer quadratic programming. Existing measurement technology can be incorporated into the proposed formulation. CNO was also possible in the case of $N-1$ contingency. Qi et al. [77] proposed OPP and calculated empirical observability Gramian around the power system operating region to quantify system state observability under specified placement. The determinant of empirical observability Gramian was maximized by the formulated problem and unraveled by a nomad solver.

Lofberg et al. [78] used a binary semi-definite programming model (BSDP) to formulate OPP that was solved using BILP. The model considered any number and kind of existing PMUs, SCADA measurements, or AC or DC measurements. The global optimal solution was also guaranteed by the model and gives the minimum number of the phasor measurement unit as compared to other techniques. Manousakis et al. [79] used BSDP to solve OPP and considered zero injection buses, synchrophasor channel limits, and CMs in the power network. Li et al. [80] used integer semi-definite programming for OPP and gathered data from PMU and SCADA for enhanced hybrid state estimation. For comparison of PMU placement impact, a useful metric was introduced that accounts for three main requirements in state estimation of network, i.e., convergence, observability, and performance.

Shi et al. [81] proposed optimal PMU placement and considered minimization of mean squared error between the measurement output and the system. Effect of ZIB, N-1 contingency, and limitation of communication channel per PMU were formulated. The problem of binary nonlinear optimization taxonomic category is solved by an efficient proposed algorithm that is scalable and gives at least a local optimal solution. The algorithm is also tested on benchmark grids for the validity of the algorithm.

3.2. Exhaustive Search

Exhaustive search optimization checks every possible solution. In our opinion, this method is recommended to solve OPP, as OPP is an offline problem, and we can easily trade off time and save the cost of extra PMUs. The only problem is, the solution of OPP for large power networks will need more time and high computation power, which is acceptable in offline planning. PMU placement using different exhaustive techniques are discussed below:

3.2.1. Graph Theory-Based OPP

Graph theory is the study of relationships in terms of nodes and vertices and is used to model pairwise relations between objects. Ghosh et al. [82] used graph theory and analytical hierarchy process for multi-criteria decision-making in OPP and considered CNO, maximum measurement redundancy, ZIBs, and $N-1$ contingency. The proposed formulation gives improved results and better measurement redundancy. Xie et al. [83] used the graph theory method to find OPP and the minimum set of critical measurements for CNO. The decentralized monitoring system was the key contribution of the article.

3.2.2. Binary Search-Based OPP

This is also known as half interval search that divides the interval into two halves and finds the value in a sorted array. It finds value in any of two intervals by comparing the targeted value with the middle value. Chakrabarti et al. [84] used binary search method for OPP and considered CNO and $N-1$ contingency. The technique is used to overcome the limitations of the genetic algorithm as well as integer programming.

4. OPP Using Non-Conventional Optimization

Non-conventional techniques are classified into heuristic, metaheuristic, and machine learning techniques. Heuristics are problem-dependent with a pre-defined set of rules to explore the search space. Metaheuristics are problem-independent and maneuver and alter subordinate heuristics to produce a good quality solution with efficacy that can be applied to a broad range of problems.

4.1. Heuristic Techniques

A heuristic technique solves a problem using a practical or shortcut method to give solutions that can or cannot be optimal in a limited timeframe. PMU placement using heuristic techniques is discussed below:

4.1.1. Unified Algorithm Based OPP

The unification of existing adaptive-learning-rate optimization algorithms, such as adaptive mean square gradient (AMSGrad), adaptive moment estimation (Adam), AMSGrad with weighted gradient and dynamic bound of learning rate (AMSGWDC), Adam with weighted gradient and dynamic bound of learning rate (GWDC), and adapting step-sizes by the belief in observed gradients (AdaBelief), etc., is called a unified algorithm. It chooses any of the above-given techniques and combines them (as the name suggests, unified) to find the optimal solution. Lu et al. [85] used a unified PMU placement model that incorporates zero injection observation reliability. PMU number was reduced and the reliability of ZI observation was increased as a result of the proposed method, and complete system observability was ensured.

4.1.2. Fuzzy Decision Based OPP

When data are incomplete or vague, fuzzy decision-making is used to solve single or multi-criteria problems. Aghaei et al. [86] proposed OPP with a multi-objective probabilistic model. Optimization of two objective functions that minimize the number of PMUs and maximize system redundancy was done simultaneously with $N - 1$ contingency and ZIBs.

4.1.3. Monte Carlo Simulation Based OPP

When the dataset is very large or complicated, random samples are taken and compared to find the optimal results using the Monte Carlo technique. Esmaili et al. [87] proposed OPP using a channel-oriented method employing the explicit cost of synchrophasor and their channels as the objective function. Channels are assigned only if it was economically justified. For the dependability of the power system, synchrophasors and their channels are applied to two or more reliable buses and branches. Additionally, to observe delicate areas of the power system and to prevent voltage collapse, the synchrophasors and their channels are allotted in order to observe the buses that are defenseless to voltage stability status. Also, Monte Carlo simulation was practiced to recognize the contingencies that are incorporated in the problem. Lu et al. [88] proposed OPP using vulnerability index and its derivatives. First, the dataset required by the partitioning was generated using the Monte Carlo simulation. Second, a genetic algorithm was used to partition the network. Subsequently, quadratic programming was practiced to crack the optimal phasor measurement unit assignment problem. Finally, a dynamic vulnerability assessment was performed by placing a phasor measurement unit on a respective bus location.

4.2. Metaheuristic Techniques

Metaheuristic techniques guide through the search space to find near-optimal solutions ranging from local search to complex learning processes. Metaheuristic techniques used in OPP are discussed below:

4.2.1. Chemical Reaction Based OPP

It is an algorithm based on the principles of chemical reactions to transform reactants to a product through a sequence of reactions, Wen et al. [89] proposed multistage OPP with budget constraints. When problem size is big, the problem of relocating the existing PMUs or purchasing and installing new PMUs is very complex to solve by the mathematical program design method and is unable to unravel within the timescale. Therefore, chemical reaction optimization was introduced for OPP.

4.2.2. Cuckoo Search Based OPP

This is a nature-inspired algorithm that is based on the brood parasitism of the cuckoo species with Lévy flights random walks. The modified binary cuckoo optimization algorithm (MBCOA) was practiced to crack the synchrophasor placement problem in [90]. The process was classified as a topological approach. It was found that MBCOA gives the better result as minimum iterations as compared to other methods.

4.2.3. Differential Evolution Based OPP

This works on the principles of the evolutionary process and can quickly explore a very large search space. Dubey et al. [16] proposed OPP using multi-objective differential evolution that incorporates the effect of communication infrastructure, PDC, optical fiber, existing CMs, and $N - 1$ contingency. The results are also provided for pre-existing PMUs and optical fiber paths.

4.2.4. Gravitational Search-Based OPP

This is a nature-inspired algorithm based on the principles of Newton's law of gravity and motion. Singh et al. [91] proposed OPP using gravitational search algorithm for maximum system observability. The Dijkstra algorithm (DA) was used to find the minimum path between PMU and PDC. Pratap et al. [92] proposed OPP using binary gravitational search algorithm for complete network observability. The multi-objective problem was formulated for minimizing the number of PMUs and improving observability redundancy. Results have reduced or equal PMUs in the network and improved or equal observability.

4.2.5. Genetic Algorithm (GA) Based OPP

Genetic algorithm is a metaheuristic optimization technique inspired by Darwin's theory [93] that mimics biological crossover of genes to produce new offspring and the fittest survive [94]. Castro et al. [95] proposed OPP with system security issues of single PMU loss criteria. Consideration of synchrophasor pre-allocation was also discussed in a realistic situation. Satish Kumar et al. [96] proposed OPP using ILP and genetic algorithm for comprehensive observability of the power system. The test of observability was carried out by root vector rather than on the triangularization of the matrix to reduce the computational efforts and time. Marin et al. [97] proposed OPP using a genetic algorithm for CNO. The association between the PMU number and current phasors measured on each PMU was found. Results showed that this method finds the minimum phasors measured via PMU for the insertion of minimum PMUs. Kumar et al. [98] proposed OPP for component reliability and increasing the system observability. A genetic algorithm was practiced to crack multiple optimal solutions for CNO. For selecting the most suitable solution within multiple solutions, a reliability index was proposed. By the use of the proposed reliability index, the observability criteria are used to find the buses for PMU placement. The analytical hierarchical process was used to find the multi-phasing of synchrophasor placement. Appasani et al. [99] proposed co-optimal settlement of PMU and communication infrastructure (CI) to curtail propagation delay for the wide-area monitoring system. Links reliability and geographical topological variations are also considered and in terms of cost and reliability, the microwave-based CI provides better results that were verified by evaluating the eastern power grid of India.

(a) Binary Genetic Algorithm (BGA)

The population generated in BGA is in binary form. Each gene in a chromosome is represented by 0 or 1 bit. In the interval [0,1], each gene in a chromosome is converted into a normalized continuous (real) form in the generated population.

Almasabi et al. [100] proposed multistage OPP using BGA. Most important buses can be preferred first and helped in handling the application-based OPP.

(b) Non-Dominated Sorting Genetic Algorithm (NSGA)

NSGA is a genetic algorithm for multi-objective optimization. This algorithm is very effective but is criticized for its computational complexity and lack of elitism. Milosevic et al. [101] proposed OPP for Pareto-optimal solutions. The benefit of this procedure is that it delivers a complete Pareto-optimal front as an alternative to the single-point key and has an application where multi-objective large search space optimization is required.

(c) Immunity Genetic Algorithm (IGA)

IGA is inspired by the defense process of the immune system and engages a genetic algorithm (GA) to concisely filter out initial antibody repertoires for the immune system. Aminifar et al. [102] propose OPP using IGA. Algorithm efficiency was improved by including an immune operator in the canonical genetic algorithm. The use of native and previous knowledge related to the problem was the core theme, which was determined from topological observability and inattention as a vaccine. Injecting this vaccine increased convergence speed remarkably. Asgari et al. [103] proposed OPP for immobile and flexible PMU assignment scenarios to have CNO with $N-1$ contingency. The $N-1$ index was included in the objective function as a new term to have OPP with the effect of measurement channel and redundancy. Minimizing PMUs was the primary objective, and minimizing voltage and current measurements are the sub-objectives. The two scenarios are: PMU rearrangement after network expansion and no PMU rearrangement after network expansion and just installing new PMUs. The results show that the second scenario is practically implementable.

(d) Cellular Genetic Algorithm (CGA)

CGA combines evolutionary algorithm and genetic algorithm, in which an individual mates with its closest neighbor and is not allowed to mate randomly. It includes (selection, variation, replacement).

Miljanic et al. [104] proposed OPP using CGA with consideration of channel availability, basic observability, metering configuration robustness, and $N-1$ contingency. The solutions indicate PMU channel availability helps to save money in optimal metering configurations. A summary of GA based OPP is presented in Table 3.

4.2.6. Particle Swarm Optimization (PSO) Based OPP

PSO is a technique that is inspired by bird flocks and schooling fish. It is a population-based technique. In this technique, initialization of the population of random solutions gives the optimal solution by updating the generations [105,106]. Binary particle swarm optimization (BPSO) is a form of PSO applied to binary domains but uses the concepts of velocity and momentum from continuous PSO. Saleh et al. [107] proposed OPP for state estimation having CNO. An improved particle swarm optimization algorithm was practiced along with the weighted least square method intended for state estimation. Rather et al. [108] proposed OPP for CNO by considering a realistic cost-effective model. The hidden or unaccounted practical cost was involved in the installation of PMU. This hidden but significant cost was incorporated as an integral part of the total cost. Pal et al. [109] proposed OPP by minimizing substation number. A large portion of the substation installation cost was the deployment cost and not the device cost. Particle swarm optimization was used to observe all voltage levels with a minimum number of substations where installation was made subject to the practical constraints. Rahman et al. [110] proposed OPP using modifications in binary particle swarm optimization. It integrates v-shaped sigmoid functions and mutations strategy for OPP with CNO, ZIB, $N-1$ contingency, and PMU channel limits. OPP was done based on uppermost measurement redundancy. Maji et al. [111] proposed effective exponential binary particle swarm optimization for OPP. The algorithm contains a nonlinear inertia weight coefficient, which helps in improving the searching capability of the system. A summary of PSO based OPP is presented in Table 4.

Table 3. A summary of GA based OPP.

Ref.	Technique	Objective Function	Constraints		
			Observability	Contingency	Installation Scheme
[95]	GA	Min. no. of PMUs, Max. measurement Redundancy	CNO	$N-1$	Pre-installed PMUs
[88]	GA	Min. no. of PMU, Max. vulnerability index	CNO	$N-1$	–
[96]	GA	Min. no. of PMUs	CNO	–	–
[97]	GA	Min. no. of PMUs based on current phasor measurements	CNO	–	–
[98]	GA	Min. no. of PMUs	CNO	–	Multistage installation
[99]	GA	Min. no. of PMUs, Min. Propagation delay	CNO	–	–
[100]	BGA	Min. no. of PMUs, Min. cost of communication infrastructure	CNO	$N-1$	Multistage installation
[101]	NS-GA	Min. no. of PMUs, Max. measurement Redundancy	CNO	$N-1$	–
[102]	IGA	Min. no. of PMUs, Min. no. of unobserved buses	CNO	–	–
[104]	CGA	Min. no. of PMUs	CNO	$N-1$	Channel and communication limitation

Table 4. A summary of PSO based OPP.

Ref.	Technique	Objective Function	Constraints		
			Observability	Contingency	Installation Scheme
[107]	BPSO	Min. no. of PMUs	Numerical CNO	–	–
[108]	BPSO	Min. total realistic cost	CNO	$N \geq 1$	–
[109]	BPSO	min. no. of substations	CNO	–	Prioritize critical elements. Prohibited substation installation and digital relays as relays synchrophasors
[110]	BPSO	Min. no. of PMUs, Max. Measurement redundancy, Max. System observability	CNO	$N-1$	channel limitation
[111]	BPSO	Min. no. of PMUs	CNO	$N-1$	Multistage

4.2.7. Cellular Learning Automata (CLA) Based OPP

To analyze physical systems cellular automata (CA) models are used, which are discrete spatially extended dynamical systems. Mazhari et al. [112] proposed multi-objective OPP with maximum measurement redundancy, CMs, ZIBs, and $N-1$ contingency with CNO. CLA with new local rules was used to find OPP.

4.2.8. Artificial Bee Colony Based (ABC) OPP

The ABC algorithm is based on the intelligent foraging behavior of a group of honeybees. ArulJeyaraj et al. [113] proposed multi-objective OPP for CNO and maximizing voltage stability. A fuzzified binary ABC algorithm was used due to the conflicting nature of objective functions. Weak buses found using fast voltage stability index were prioritized in PMU placement to prevent outage.

4.2.9. Ant Colony (AC) Based OPP

The AC algorithm is based on the intelligent behavior of a group of ants to find the optimal path using graphs. It has an advantage over simulated annealing and genetic algorithm when the graph may change dynamically. Mouwafi et al. [114] proposed a two-stage OPP. The first stage was solved using ant colony optimization to find minimum number and position of PMUs for CNO along with channel limitation and $N-1$ contingency. In the second stage, a reduction strategy was proposed to find minimum PMUs measuring channel capacity with CNO.

4.2.10. Binary Imperialistic Competition Algorithm (BICA) Based OPP

The imperialist competitive algorithm (ICA) was introduced by Atashpaz-Gargari and Lucas [115] after inspiration from socio-political behaviors. Mohammadi et al. [116] used BICA for OPP and considered pre-installed synchrophasors and optical fiber links in the power system. Mahari et al. [117] also used BICA for OPP with CNO.

4.2.11. Tabu Search (TS) Based OPP

TS is used to optimize a multi-parameter model that can yield better results. However, the implementation is not insignificant and is capable of solving many of the problems once it is created. Korres et al. [118] proposed OPP using a recursive tabu search (RTS) scheme for CNO. It investigates the two TS restrictions, i.e., tabu length and tabu search iterations, along with three different TS initialization schemes and provided numerical observability.

4.2.12. Simulated Annealing Based OPP

Simulated annealing develops trial structural models to investigate energy hypersurfaces, to cross obstructions, and to look for regions with low energy structures allows a high degree of latitude in the growth of initial starting points. Nuqui et al. [119] proposed a tree search method and simulated annealing for OPP, given that unobservable regions diminish gradually with CNO. In addition to OPP, the authors also identified the location for new facilities.

4.3. Machine Learning (ML) Based OPP

The learning and adaptation power of computer systems without explicit instructions through training from data is called machine learning. ML-based optimization runs on real-time data streaming to give optimal solutions. For optimization, different machine learning algorithms, i.e., fitting logistic regression models and training artificial neural networks, are used.

Ghosh et al. [120] used Bayesian networks to find overall PMU reliability by including different necessary submodules and components of PMUs. The status of PMU was assessed based on its components. Hence, reliability improvement of PMU through reliability allocation to its submodules and components has been adopted. Full observability reliability (OR), loss of data expectation (LODE), and loss of situational awareness (LOSA) were also reported, validating improved PMU reliability. The description of the problem to identify the single line outage was described in [121]. The voltage phasor of each bus node was changed due to the change in the network topology. An ideal case of the PMU at each bus was considered and then regularized techniques for optimization help to place PMUs on a subset of buses in which as many outage events can be discriminated with the help of a subset of buses. The test results show that the classifier performs as well when 25%

of buses are furnished with the PMUs compared to the ideal case where entire buses are furnished with PMUs.

5. Discussion and Future Work

5.1. Testbed Systems

For OPP results, verification literature include IEEE 14-bus, 30-bus, 57-bus, and 118-bus systems as the most commonly used testbed systems. In the OPP problem, the Polish system and energy corporation systems were the largest systems used, with 2746 and 2285 buses, respectively, to date.

5.2. Optimization Solvers

To solve the problem of PMU placement, different optimization solvers are used in the literature. Optimization solvers help to improve decision-making around planning, allocating, and scheduling scarce resources. MATLAB, TOMLAB, IBM, ILOG, VPLEX, GAMS, and GUROBI are the popular solvers that are used in the available literature.

5.3. Pros and Cons

In this paragraph, we will discuss the pros and cons of commonly used techniques in OPP. Different optimization techniques have their benefits and drawbacks. In literature, the linear programming technique is widely used because OPP for CNO is a linear problem. Some practical constraints make this problem nonlinear, and these are solved with other optimization techniques. ILP is computationally fast but faces difficulty with non-linear constraints. The problem of non-linear constraints in OPP was solved by converting them to linear constraints using the linearization method. Particle swarm optimization has easy implementation with more efficient control and fewer parameter adjustment problems. The problem with PSO is that the computational time increases with the increased size of the solution. A genetic algorithm provides the best Pareto optimal solution instead of a single solution. The problem with GA is the long execution time. The greedy search algorithm is good at providing the local optimal value with less computation, but it provides a global solution only in very few cases. Simulated annealing is good at giving complete observability as well as valuable dynamic data of the power system. The problem with SA is very high computation time, which is why a reduction in time calculation is always necessary. The artificial neural network provides many solutions that depend on the computational model but faces the problem with the complexity of network structure. As per discussion and investigation, ILP is more suitable than conventional techniques and metaheuristic (PSO and GA) is more suitable as compared to heuristic and ML technique to solve OPP.

5.4. Future Scope

Following are the factors that need further investigation in the PMU placement problem.

5.4.1. Application Based Planning

Currently, the researchers are not confined to complete network observability only in the OPP problem, but they are trying to find the optimum location of PMUs from the application point of view. The areas that need more attention are OPP with controlled islanding, fault tolerance, small-signal analysis, and voltage stability from the application point of view.

5.4.2. Node-Breaker Model

Almost all the work done in PMU placement was tested and verified on the bus-bus model or bus-branch model on the IEEE test-beds and very few on practical systems. The actual power system is depicted by the node-breaker model in reality. Placement of PMUs on bus-bus or bus-branch models does not satisfy the requirements of a practical power system in complete monitoring and control in case of contingencies. In the node-breaker model, there is no need for network reduction, which helps in combined state estimation and network topology without approximation [122–126]. The transition from existing models to the node-breaker model is an unexplored area in PMU placement that will satisfy all the requirements of a practical power system, but the issue is it needs proprietary information about the power system.

5.4.3. Hybrid Optimization Algorithm

A hybrid algorithm combines two or more algorithms that solve the same problem, either choosing one (depending on the data) or switching between them over the course of the algorithm. In OPP, a combination of ML/AI/classical techniques is an unexplored area for future work. In our opinion, exhaustive techniques also need more exploration, as OPP is an offline problem, and we can easily trade off time and save the cost of extra PMUs as discussed in Section 3.2.

5.4.4. Performance Evaluators

In OPP, different techniques provide us with multiple solutions based on CNO and different constraints, and there are few indexes to evaluate the performance of the solution. This area needs further attention to introduce different performance indices to evaluate the solution set.

5.4.5. μPMU Placement

In OPP, the distribution system is only investigated by a few authors. The distribution system is completely different from the transmission network, in terms of less deviation of magnitudes and angles of electrical quantities. The distribution system also faces the problems of having no proper standardization, distributed generation islanding [127], reverse power flow, and false tripping as compared to the transmission system. Micro phasor measurement units (μPMUs) are new in the market, specially designed for distribution systems. If the resolution of μPMU is increased for the distribution network, then it will have much more cost than the PMU for transmission [3,128–131]. The μPMUs have a much wider range of applications in terms of operation, control, reliability, and future planning of the system as discussed in [9,132,133]. Lack of proper communication infrastructure and standardization are big hurdles in emerging μPMU technology for WAMS.

6. Conclusions

The critical issue in power system monitoring is to address the transition from the conventional power system to the modern power system and ensure the full observability of the system, or at least the maximum possible observability using PMUs. Therefore, installing the minimum number of PMUs for complete system observability is the root formulation in the PMU placement problem. This paper presents a careful summary of different optimization techniques used along with objective functions used, i.e., cost minimization of PMUs or maximize network observability, and constraints used, i.e., complete network observability, effects of conventional measurements, contingencies, zero injection buses, communication infrastructure, controlled islanding, and improved redundancy, PMUs (variable cost, channel limitation, multistage installation), and pre-installed PMUs for the optimal PMU placement. For OPP results verification, the literature includes IEEE 14-bus, 30-bus, 57-bus, and 118-bus systems as the most commonly used testbed systems. MATLAB, TOMLAB, IBM, ILOG, VPLEX, GAMS, and GUROBI are the popular solvers that are used to solve OPP. The most commonly used technique to solve this problem is integer linear programming. The pros and cons of different techniques used in OPP are discussed in the discussion section. It also provides a bird's-eye view of the OPP problem as a foundation for the research community and highlights the following crucial research gaps. Realistic OPP needs to consider application-based planning, the node-breaker model because it better represents practical power systems, and the use of hybrid optimization algorithms for optimal results. Moreover, robust performance evaluation indices must be developed to compare different solutions offered by different techniques. The identified research gaps will motivate the research community to find practical solutions for power systems of the future.

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