



Article Multi-Objective Constructal Design for Square Heat-Generation Body with "Arrow-Shaped" High-Thermal-Conductivity Channel

Hongwei Zhu^{1,2,3}, Lingen Chen^{1,2,3,*}, Yanlin Ge^{1,2,3}, Shuangshuang Shi^{1,2,3} and Huijun Feng^{1,2,3}

- ¹ Institute of Thermal Science and Power Engineering, Wuhan Institute of Technology, Wuhan 430205, China; 18879837093@139.com (H.Z.); geyali9@hotmail.com (Y.G.); shishuangshuang20@163.com (S.S.); huijunfeng@139.com (H.F.)
- ² Hubei Provincial Engineering Technology Research Center of Green Chemical Equipment, Wuhan 430205, China
- ³ School of Mechanical & Electrical Engineering, Wuhan Institute of Technology, Wuhan 430205, China
- * Correspondence: lingenchen@hotmail.com

Abstract: Based on the square heat-generation body (HGB) with "arrow-shaped" high-thermalconductivity channel (HTCC) model established in the previous literature, we performed multiobjective optimization (MOO) with maximum temperature difference (MTD) minimization and entropy-generation rate (EGR) minimization as optimization objectives for its performance. Pareto frontiers with optimal set were obtained based on NSGA-II. TOPSIS, LINMAP, and Shannon entropy decision methods were used to select the optimal results in Pareto frontiers, and the deviation index was used to compare and analyze advantages and disadvantages of the optimal results for each decision method. At the same time, multi-objective constructal designs of the "arrow-shaped" HTCC were carried out through optimization of single degree of freedom (DOF), two DOF, and three DOF, respectively, and the thermal performance of the square heat-generation body under optimizations of different DOF were compared. The results show that constructal design with the MOO method can achieve the best compromise between the maximum thermal resistance and the irreversible loss of heat transfer of the square heat-generation body, thereby improving the comprehensive thermal performance of the square heat-generation body. The MOO results vary with different DOF, and optimization with increasing DOF can further improve the comprehensive thermal performance of square HGBs.

Keywords: constructal theory; maximum temperature difference; entropy-generation rate; arrow-shaped high-thermal-conductivity channel; multi-objective optimization; generalized thermodynamic optimization

1. Introduction

A crucial link in the design and manufacture of chips is how to effectively avoid damage to components due to excessive local heat generation. The physical size of electronic components is becoming smaller and smaller while their power is increasing. The thermal stability and reliability of electronic devices may be greatly improved using the basic theories of heat transfer optimization to optimize their heat dissipation capacity. Among these is the entropy generation minimization theory [1–7], which provides optimization criteria from the perspective of evaluating and decreasing the irreversibility of heat transfer in processes and systems. Constructal theory [8–18] is a newly developed theory based on thermodynamic optimization research. It addresses this problem by following the idea that structure develops in the direction of better internal flow performance, provides a new solution for traditional thermodynamic optimization problems, and makes many bottlenecks that are difficult to explain and solve in theory and engineering possible to solve through new angles. Introducing these cutting-edge theories into the research on



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Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). constructal design of electronic devices adapts the research to the current development trend in the field of thermal science.

Bejan [8] first studied the thermal performance of electronic components by using constructal theory. Many scholars further applied this theory to the study of constructal designs of various heat-generation structures. In the study of the constructal design problems of the square heat-generation body (HGB), Lorenzini et al. [19,20] performed the constructal design for the square HGB embedded with "I-shaped" [19] and "T-shaped" [20] high-thermal-conductivity channels (HTCCs) by minimizing the maximum temperature difference (MTD). Hajmohammadi and Rezaei [21] studied the constructal design of doublebranched HTCCs in a square HGB, and the research showed that the MTD of the square HGB with double-branched HTCCs is smaller than that of a square HGB with "I-shaped" HTCCs. Hajmohammadi et al. [22] developed a multi-level dendritic HTCC distribution model in a square HGB to achieve the purpose of minimizing the MTD of the HGB. The results showed that under the same number of branches, the same porosity of HTCCs, and the same thermal conductivity, the performance of unequal-length dendritic structures is better than that of equal-length dendritic structures. Fagundes et al. [23] combined an exhaustive search with a genetic algorithm to derive the constructal design of the asymmetric trigeminal HTCC, and obtained optimal constructal design of the trigeminal HTCC when the porosity of HTCC was 0.4. Zhang et al. [24] established the "arrow-shaped" HTCC model in a square HGB, and achieved constructal design under different degrees of freedom (DOF) with minimum MTD. The results showed that the dimensionless minimum MTD of the optimal constructal design obtained under three-DOF optimization was smaller than that of the square HGB with "T-shaped" HTCC. Many scholars also studied the constructal designs of rectangular [25], triangular [26], and disc-shaped [27-29] HGB models. In addition, constructal theory has also been applied to other thermal conduction problems. Konan and Cetkin [30] experimented with constructal design of snowflake-shaped HTCC, and found that the optimal constructal design of the HTCC with minimum MTD is very close to the natural snowflake shape. Hajmohammadi et al. [31] devised a model in which HTCCs are embedded in annular fins to assist heat dissipation, and achieved constructal design of HTCCs with minimum MTD. Li and Feng [32] proposed a quadrilateral HGB model with embedded vein-like HTCCs, and obtained optimal constructal design of this model by with the objective of minimizing the MTD.

The MTD reflects the maximum thermal resistance in the HGB, while the entropygeneration rate (EGR) can reflect the irreversible loss of heat transfer in the HGB. Some scholars have further studied different HGBs based on EGR. Ghodoossi [33] experimented with a rectangular HGB constructal design, and gained the corresponding EGR of the rectangular HGB. You et al. [34], considering that heat-generation rate is non-uniform, obtained optimal constructal design of the triangular HGB with the objective of minimizing the EGR. Feng et al. [35] experimented with a disc-shaped HGB constructal design with the objective of the EGR. Ribeiro and Queiros-Condé [36] obtained optimal constructal design of the "I-shaped" HTCC in a square HGB. Zhu et al. [37] obtained optimal constructal design of the vein-like HTCC in a quadrilateral HGB, and that was different from the optimal constructal design obtained by minimizing the MTD [32].

The above constructal designs were all with single-objective optimizations, but the actual engineering design often needs to meet multiple design requirements. Therefore, it is necessary to reduce the conflict between different objectives through the use of multi-objective optimization (MOO). The non-dominated sorting genetic algorithm II (NSGA-II) with an elite strategy has been successfully applied to many engineering designs [38–49]. In particular, some scholars have demonstrated constructal design based on the NSGA-II. Zhang et al. [50] performed MOO for a trapezoidal HGB with heat conduction and flow, taking the EGR and the pumping power consumption as objectives, obtained the Pareto frontier based on the NSGA-II, and selected the optimal result using the TOPSIS decision method. Feng et al. [51] performed MOO for marine condensers and compared the optimization results of single-objective optimization and three decision methods based on

the deviation index. Feng et al. [52] performed MOO of a marine boiler by taking the EGR and the pumping power consumption as objectives, and compared the optimization results of three decision methods based on the deviation index. Feng et al. [53] performed MOO for a compound heat-dissipation channel, taking the MTD and the pumping power consumption as objectives. Sun et al. [54] performed MOO for a cooling channel with semicircular sidewall ribs, taking the MTD and the pumping power consumption as objectives. Kalkan et al. [55] performed MOO for a microchannel cooling plate.

In this paper, based on the square HGB with "arrow-shaped" HTCC model established in the previous literature [24], MOO with MTD minimization and EGR minimization as optimization objectives was performed, and Pareto frontiers with optimal set were obtained based on NSGA-II. TOPSIS, LINMAP, and Shannon entropy decision methods were used to obtain the optimal results. The deviation index [56] was used to compare and analyze the advantages and disadvantages of the optimal results under each decision method. At the same time, the multi-objective constructal designs of the "arrow-shaped" HTCC were carried out through the optimization of single DOF, two DOF, and three DOF, and the thermal performances of the square heat-generation body under the optimizations of different DOF were compared. Our results indicate that introducing MOO into constructal theory for design can improve the comprehensive thermal conductivity of HTCCs and meet the needs of engineering design.

2. Square HGB Model

2.1. Physical Model

Figure 1 shows the square HGB with "arrow-shaped" HTCC model [24]. The length of the square HGB (the heat-generation rate is $q^{\prime\prime\prime}$, the thermal conductivity is k_0) is L, and an "arrow-shaped" HTCC (thermal conductivity is k_c) is arranged inside it. The heat flow generated in the area of the square HGB converges into the "arrow-shaped" HTCC, and flows out of the heat-generation area through the left side of the "arrow-shaped" HTCC (temperature is T_{\min}). The remaining boundaries of the square HGB are adiabatic. The thermal conductivity ratio of the high- and low-thermal-conductivity materials is defined as $\tilde{k} = k_c/k_0$, and the characteristic sizes of the "arrow-shaped" HTCC are L_1 , H_1 , L_2 and H_2 respectively.



Figure 1. Square HGB with "Arrow-shaped" HTCC model [24].

From Figure 1, the area (A_0) of the "arrow-shaped" HTCC can be expressed as $A_0 = H_1L_1 + H_2L_2$, the area of the entire square is L^2 , and the area fraction (ϕ) of HTCC Pareto frontiers with optimal set of the entire square HGB is defined as:

$$\phi = \frac{H_1 L_1 + H_2 L_2}{L^2} \tag{1}$$

The L_1 , L_2 , H_1 , and H_2 can be dimensionless as: $\tilde{L}_1 = L_1/L$, $\tilde{L}_2 = L_2/L$, $\tilde{H}_1 = H_1/L$, and $\tilde{H}_2 = H_2/L$.

2.2. Mathematical Model

For the square HGB with "arrow-shaped" HTCC, the two-dimensional thermal conductivity differential equations in high- and low-thermal-conductivity materials are [24]:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q^{\prime\prime\prime}}{k_0} = 0$$
⁽²⁾

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{3}$$

Because the square HGB with "arrow-shaped" HTCC model is symmetrical, it is sufficient to analyze the area $y \ge 0$ within the square HGB. For the region of $y \ge 0$, the boundary conditions are:

$$T = T_{\min} x = 0, \ 0 < y < L_1/2$$
 (4)

$$\frac{\partial T}{\partial x} = 0 \begin{cases} x = 0, \quad L_1/2 \le y \le L/2\\ x = L, \quad 0 \le y \le L/2 \end{cases}$$
(5)

$$\frac{\partial T}{\partial y} = 0 \begin{cases} y = 0, & 0 \le x < L \\ y = L/2, & 0 < x < L \end{cases}$$
(6)

The MTD (ΔT) in the square HGB can be dimensionless as:

$$\Delta \widetilde{T} = (T_{\text{max}} - T_{\text{min}}) / (q^{\prime\prime\prime} L^2 / k_0)$$
(7)

where T_{max} is the hot-spot temperature in the square HGB.

With steady-state heat conduction, the EGR in the entire HGB is:

$$\sigma = \sigma_{k_0} + \sigma_{k_c} \tag{8}$$

The expressions of σ_{k_0} and σ_{k_c} are:

$$\sigma_{k_0} = \iint_{A_{k_0}} k_0 \cdot \left[(dT/dx)^2 + (dT/dy)^2 / T^2 \right] dA \tag{9}$$

$$\sigma_{k_c} = \iint_{A_{k_c}} k_c \cdot \left[(dT/dx)^2 + (dT/dy)^2 / T^2 \right] dA \tag{10}$$

where A_{k_0} and A_{k_c} are the areas of the HGB and the HTCC, respectively. k_0 is thermal conductivity of HGB, and k_c is thermal conductivity of HTCC.

The EGR in the square HGB can be dimensionless as:

$$\widetilde{\sigma} = \frac{\sigma T_{\min}}{q''' L^2} \tag{11}$$

where q''' is heat-generation rate per unit volume, *L* is length of square HGB, and T_{min} is the lowest temperature of square HGB.

3. Multi-Objective Constructal Design for "Arrow-Shaped" HTCC

From Figure 1, the "arrow-shaped" HTCC area was determined by four characteristic sizes: L_1 , L_2 , H_1 , and H_2 . Under the given condition of area fraction (ϕ) of the HTCC,

the heat conduction problem of the square HGB has three DOF. The NSGA-II algorithm program was run through MALAB software, and the connection between MATALB and COMSOL was realized through COMSOL LiveLink for MATLAB. In this way, the multiobjective constructal design of the "arrow-shaped" HTCC could be carried out through the optimization of the single DOF, two DOF and three DOF, and the thermal performances of the square HGB under the optimization of different DOF could be compared. In the NSGA-II algorithm, the population size, mutation probability, and evolutionary generation were set to 100, 0.8, and 10, respectively. The TOPSIS decision method was selected based on the distances from each point in the Pareto frontiers to the ideal and non-ideal points. The LINMAP decision method was selected based on the weighted distance from each point in the Pareto frontiers to the ideal point. The Shannon entropy decision method was selected based on the best feasible result of different objectives.

The deviation index is defined as [56]:

$$D = \frac{\sqrt{\sum_{j=1}^{m} (G_{i_{opt}j} - G_{j}^{\text{positive}})^{2}}}{\sqrt{\sum_{j=1}^{m} (G_{i_{opt}j} - G_{j}^{\text{positive}})^{2}} + \sqrt{\sum_{j=1}^{m} (G_{i_{opt}j} - G_{j}^{\text{negative}})^{2}}$$
(12)

where G_j^{positive} is the normalized and weighted value of the jth objective of the ideal point. G_j^{negative} is the normalized and weighted value of the jth objective of the non-ideal point, and i_{opt} is the best feasible result.

3.1. Multi-Objective Constructal Design under Single-DOF Optimization

First, we considered the constructal design of the "arrow-shaped" HTCC when the area of the arrowhead was fixed. Next, taking the width of the rectangular area of the "arrow-shaped" HTCC L_1 as the optimization variable, the multi-objective constructal design of the "arrow-shaped" HTCC in the square HGB could be carried out with the objective of minimizing the MTD and the EGR. According to Ref. [24], the relevant parameters are as follows: $\phi = 0.1$, $\tilde{L}_2 = 0.4$, $\tilde{H}_2 = 0.1$, $\tilde{k} = 200$, $k_0 = 2W/(m \cdot K)$, $q''' = 500W/m^3$.

Figure 2 shows the Pareto fronters of the dimensionless MTD ($\Delta \tilde{T}$) and the dimensionless EGR ($\tilde{\sigma}$) obtained by MOO under single-DOF optimization. Figure 3 shows the distribution of \tilde{L}_1 in the Pareto frontiers under single-DOF optimization. From Figure 2, with the increase of $\Delta \tilde{T}$, $\tilde{\sigma}$ decreased continuously. $\Delta \tilde{T}$ and $\tilde{\sigma}$ cannot reach their minimum values at the same time, so it is necessary to find the best compromise between $\Delta \tilde{T}$ and $\tilde{\sigma}$ to make the comprehensive thermal performance of the square HGB optimal. Having at least one objective function ($\tilde{\sigma}$ or $\Delta \tilde{T}$) in the Pareto frontiers is better than having the other solutions outside the Pareto frontiers. Decreasing $\tilde{\sigma}$ (or $\Delta \tilde{T}$) will inevitably lead to $\Delta \tilde{T}$ (or $\tilde{\sigma}$) increases, so the Pareto frontiers have the least conflict of objectives compared to other solutions. From Figure 3, in the Pareto frontiers under single-DOF optimization, \tilde{L}_1 was distributed between 0.0903 and 0.0981, and the optimal solutions with minimum $\Delta \tilde{T}$ and $\tilde{\sigma}$ of \tilde{L}_1 were on the boundary of the distribution range.

Table 1 shows the optimization results of the square HGB with different objectives under single-DOF optimization. The three decision methods in the table, comprising TOPSIS, LINMAP, and Shannon entropy, are commonly used for solving multi-objective decision-making problems. From Table 1, the optimal result with the Shannon entropy decision method was the same as that obtained with the objective of minimizing the $\Delta \tilde{T}$. The optimal results with the TOPSIS and LINMAP decision methods represent the optimal compromise of $\Delta \tilde{T}$ and $\tilde{\sigma}$. In the single-DOF optimization, the deviation index of the optimal result with the TOPSIS decision method was the smallest, so the optimal result with the TOPSIS decision method was selected as the design scheme for the best compromise between the maximum thermal resistance and the loss of heat transfer irreversibility of the square HGB.



Figure 2. Pareto frontiers of $\Delta \tilde{T}$ and $\tilde{\sigma}$ with MOO under single-DOF optimization.



Figure 3. Distribution of \tilde{L}_1 in the Pareto frontiers under single-DOF optimization.

Optimization	Decision Methods	Design Variables	Optimization Objectives		Deviation Indexes [56]	
Methods		$ ilde{L}_1$	$\Delta \tilde{\mathbf{T}}$	$\tilde{\sigma}$	D	
Multi-objective optimization	LINMAP TOPISIS Shannon Entropy	0.0973 0.0976 0.0981	0.0934 0.0932 0.0929	0.0431 0.0431 0.0432	0.2020 0.1862 0.197	
Single-objective optimizations	$\Delta \widetilde{T} \ \widetilde{\sigma}$	0.0981 0.0903	0.0929 0.0981	0.0432 0.0426	0.1997 0.8003	

3.2. Multi-Objective Constructal Design under Two-DOF Optimization

By further releasing the constraints fixed by the arrowhead shape and taking the width (\tilde{L}_1) of the rectangular area and the width (\tilde{L}_2) of the arrowhead as the optimization variables while keeping the area of the arrowhead ($\tilde{L}_2\tilde{H}_2 = 0.04$) unchanged, the multi-objective constructal design of the "arrow-shaped" HTCC in the square HGB could be carried out with the objective of minimizing the MTD and the EGR.

Figure 4 shows the Pareto frontiers of $\Delta \tilde{T}$ and $\tilde{\sigma}$ obtained by MOO under two- DOF optimization. Figure 5 shows the distributions of \tilde{L}_1 and \tilde{L}_2 in the Pareto frontiers under two-DOF optimization, respectively. From Figure 4, with the increase of $\Delta \tilde{T}$, $\tilde{\sigma}$ decreased continuously. Therefore, it was necessary to find the optimal constructs of \tilde{L}_1 and \tilde{L}_2 . Compared with Figure 2, the minimum values of $\Delta \tilde{T}$ and $\tilde{\sigma}$ on Pareto frontiers were reduced. From Figure 5, in the Pareto frontiers under two-DOF optimization, \tilde{L}_1 was distributed between 0.0911 and 0.0991, \tilde{L}_2 was distributed between 0.4876 and 0.4980, and the optimal solutions with minimum $\Delta \tilde{T}$ and $\tilde{\sigma}$ of \tilde{L}_1 and \tilde{L}_2 were on the boundary of the distribution range.



Figure 4. Pareto frontiers of $\Delta \tilde{T}$ and $\tilde{\sigma}$ with MOO under two-DOF optimization.



Figure 5. Cont.



Figure 5. Distributions of (a) \tilde{L}_1 and (b) \tilde{L}_2 in the Pareto frontiers under two-DOF optimization.

Table 2 shows the optimization results of the square HGB with different objectives under two-DOF optimization. From Table 2, the optimal result with the Shannon entropy decision method was the same as that obtained with the objective of minimizing the $\Delta \tilde{T}$, which has smaller deviation indexes compared to the optimal result with minimum $\tilde{\sigma}$. The optimal results with the TOPSIS and LINMAP decision methods were represent the optimal compromise of $\Delta \tilde{T}$ and $\tilde{\sigma}$. In the two-DOF optimization, the deviation index of the optimal result with the TOPSIS decision method was the smallest, so the optimal result with the TOPSIS decision method was selected as the design scheme of the square HGB.

Optimization	Decision Methods —	Design Variables		Optimization Objectives		Deviation Indexes
Methods		$ ilde{L}_1$	$ ilde{L}_2$	$\Delta ilde{T}$	$\tilde{\sigma}$	D
Multi-objective optimization	LINMAP TOPISIS Shannon Entropy	0.0983 0.0988 0.0991	$0.4954 \\ 0.4948 \\ 0.4876$	0.0907 0.0903 0.0902	0.0415 0.0416 0.0416	0.1867 0.1628 0.1860
Single-objective optimizations	$\Delta \widetilde{T} \ \widetilde{\sigma}$	0.0991 0.0911	0.4876 0.4980	0.0902 0.0959	0.0416 0.0410	0.1860 0.8140

Table 2. Optimization results of the square HGB with different objectives under two-DOF optimization.

3.3. Multi-Objective Constructal Design under Three-DOF Optimization

Considering that the "arrow-shaped" HTCC shape can be changed, taking the width (\tilde{L}_1) of the rectangular area, the width (\tilde{L}_2) of the arrowhead, and the length (\tilde{H}_1) of the arrowhead as the optimization variables, the multi-objective constructal design of the "arrow-shaped" HTCC in the square HGB can be carried out with the objective of minimizing the MTD and the EGR.

Figure 6 shows the Pareto frontiers of ΔT and $\tilde{\sigma}$ obtained by MOO under three- DOF optimization. Figure 7 shows the distributions of \tilde{L}_1 , \tilde{L}_2 , and \tilde{H}_1 in the Pareto optimal frontiers under three-DOF optimization, respectively. From Figure 6, with the increase of $\Delta \tilde{T}$, $\tilde{\sigma}$ decreased continuously. Therefore, it was necessary to find the optimal constructs of \tilde{L}_1 , \tilde{L}_2 , and \tilde{H}_1 . Compared with Figure 4, the minimum values of $\Delta \tilde{T}$ and $\tilde{\sigma}$ on the Pareto frontiers were further reduced. From Figure 7, in the Pareto frontiers with three-DOF optimization, \tilde{L}_1 was distributed between 0.1170 and 0.1198, \tilde{L}_2 was distributed between 0.4794 and 0.4911, and \tilde{H}_1 was distributed between 0.6258 and 0.7019. \tilde{L}_1 , \tilde{L}_2 and \tilde{H}_1 of the "arrow-shaped" HTCC in the square HGB were based on multi-objective

constructal design distributed between the optimal solutions with minimum $\Delta \tilde{T}$ and $\tilde{\sigma}$, and the optimal solutions with minimum $\Delta \tilde{T}$ and $\tilde{\sigma}$ of \tilde{L}_1 , \tilde{L}_2 , and \tilde{H}_1 were on the boundary of the distribution range.



Figure 6. Pareto frontiers of $\Delta \tilde{T}$ and $\tilde{\sigma}$ with MOO under three-DOF optimization.







(b)



Figure 7. Distributions of (a-c) in the Pareto frontiers under three-DOF optimization.

Table 3 shows the optimization results of the square HGB with different objectives under three-DOF optimization. From Table 3, the optimal result with the Shannon entropy decision method was the same as that obtained by taking the objective of minimizing the $\Delta \tilde{T}$. The optimal results with the TOPSIS and LINMAP decision methods represent the optimal compromise of $\Delta \tilde{T}$ and $\tilde{\sigma}$. In three-DOF optimization, the deviation index of the optimal result with the TOPSIS decision method was the smallest, so the optimal result with the TOPSIS decision method was selected as the design scheme of the square HGB.

Table 3. Optimization results of the square HGB with different objectives under three-DOF optimization.

Optimization Methods	Decision Methods	Design Variables			Optimization Objectives		Deviation Indexes
		$ ilde{L}_1$	\tilde{L}_2	$ ilde{H}_1$	$\Delta ilde{T}$	$\tilde{\sigma}$	D
Multi-objective optimization	LINMAP TOPISIS Shannon Entropy	0.1199 0.1195 0.1198	0.4905 0.4914 0.4794	0.6483 0.6399 0.6258	0.0891 0.0886 0.0877	$0.0396 \\ 0.0398 \\ 0.0403$	0.3202 0.3010 0.3575
Single-objective optimizations	$\Delta \widetilde{T} \ \widetilde{\sigma}$	0.1198 0.1170	0.4794 0.4911	0.6258 0.7019	0.0877 0.0929	0.0403 0.0390	0.3575 0.6425

Figure 8 shows the optimal constructs of the "arrow-shaped" HTCC with the TOPSIS decision method based on single-DOF, two-DOF, and three-DOF optimization. From Figure 8, the corresponding $\Delta \tilde{T}$ to the optimal construct under single-DOF, two-DOF, and three-DOF optimizations was 0.0932, 0.0903, and 0.0886, respectively, and the corresponding $\tilde{\sigma}$ was 0.0431, 0.0416, and 0.0398, respectively. Compared with the two-DOF and single-DOF optimizations, the $\Delta \tilde{T}$ of square HGB under three-DOF optimization was reduced by 1.9% and 4.9%, respectively, and the $\tilde{\sigma}$ was reduced by 4.3% and 7.7%, respectively. The results show that $\Delta \tilde{T}$ and $\tilde{\sigma}$ of the square HGB can be further reduced by increasing the DOF optimization of the "arrow-shaped" HTCC, which can improve the comprehensive thermal performance of the square HGB. Compared to what was reported in [24], by taking into account the thermal conductivity of the heat-generation body, the optimization process may not be as obvious as optimizing for only one objective.



(a) $\Delta \tilde{T} = 0.0932$, $\tilde{\sigma} = 0.0431$



(b) $\Delta \tilde{T} = 0.0903$, $\tilde{\sigma} = 0.0416$



(c) $\Delta \tilde{T} = 0.0886$, $\tilde{\sigma} = 0.0398$



4. Conclusions

Based on the square HGB with "arrow-shaped" HTCC model established in the previous literature, in this study, MOO was performed with MTD minimization and EGR minimization as optimization objectives, and the Pareto frontiers with optimal set were obtained based on NSGA-II. TOPSIS, LINMAP, and Shannon entropy decision methods were used to obtain optimal results. The deviation index was used to compare and analyze the advantages and disadvantages of the optimal results under each decision method. At the same time, the multi-objective constructal designs of the "arrow-shaped" HTCC were carried out through the optimization of single DOF, two DOF, and three DOF, and the thermal performances of the square HGB under the optimizations of different DOF were compared. The results show the following:

- 1. In the Pareto frontiers of $\Delta \tilde{T}$ and $\tilde{\sigma}$ obtained by MOO with single-DOF, two-DOF, and three-DOF optimizations, with the increase of $\Delta \tilde{T}$, $\tilde{\sigma}$ decreased continuously. $\Delta \tilde{T}$ and $\tilde{\sigma}$ could not reach their optimal values under single-objective optimization at the same time; however, they could effectively compromise the two objectives and reduce the conflict between them.
- 2. Under the optimization of single DOF, two DOF and three DOF, the deviation index of the optimal result based on the TOPSIS decision method was the smallest, so the optimal result with the TOPSIS decision method was selected as the design scheme for the best compromise between the maximum thermal resistance and the loss of heat transfer irreversibility of the square HGB.
- 3. Compared with the two-DOF and single-DOF optimizations, the $\Delta \tilde{T}$ of square HGB under three-DOF optimization was reduced by 1.9% and 4.9%, respectively, and the $\tilde{\sigma}$ of square HGB under three-DOF optimization was reduced by 4.3% and 7.7%, respectively. The $\Delta \tilde{T}$ and $\tilde{\sigma}$ of the square HGB could be further reduced by increasing the DOF optimization of the "arrow-shaped" HTCC, which could further improve the comprehensive thermal performance of the square HGB. It may be possible to further improve the comprehensive thermal conductivity of the square heat-generation body by establishing HTCCs with more degrees of freedom.
- 4. Constructal theory and NSGA-II are powerful tools for thermal performance improvements of a square HGB with "arrow-shaped" HTCC, and the optimization methods can be applied to many problems.

At this stage, the optimal construct of the HTCC is only studied by means of simulation, and experimental research will be carried out in the future.

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- A_0 Area of high thermal conductivity material (m²)
- *L* Length of square heat-generation body (m)
- H_1 Length of rectangular area of "arrow-shaped" channel (m)
- L_1 Width of rectangular area of "arrow-shaped" channel (m)
- H_2 Length of arrowhead area of "arrow-shaped" channel (m)
- L_2 Width of arrowhead area of "arrow-shaped" channel (m)
- *k* Ratio of thermal conductivity (-)
- k_0 Thermal conductivity of heat-generation body (W/mK)
- k_c Thermal conductivity of high-thermal-conductivity channel (W/mK)
- $q^{\prime\prime\prime}$ Heat generation rate per unit volume (W/m³)
- *T* Temperature (K)
- Greek Symbols
- *α* Porosity of high-thermal-conductivity channel (-)
- σ Entropy-generation rate (W/mK)
- Superscript
 - Dimensionless

Subscripts

- min Minimum
- Abbreviations
- HGB Heat-generation body
- HTCC High-thermal-conductivity channel
- MTD Maximum temperature difference
- EGR Entropy-generation rate
- MOO Multi-objective optimization
- DOF Degree of freedom

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