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Abstract: In this paper, a full-order terminal sliding-mode control method is proposed for the rectifier side and the inverter side of the soft open point (SOP). The rectifier-side DC voltage control system consists of the voltage- and current-loops with controllers which are designed using full-order sliding-mode (FOSM) to enhance the dynamic performances and anti-disturbance. The integral type virtual control signal without chattering is designed to compensate for the unmatched uncertainties including external disturbances and some parameter perturbations. The full-order terminal sliding-mode (FOTSM) controller for the current-loop can force the current response to track its reference in finite time. The inverter side power control system is designed to regulate the power. The FOTSM controller for the power-loop ensures the power-tracking accuracy under a disturbed condition. Finally, the simulations demonstrate the effectiveness of the proposed controllers for the rectifier and inverter sides in the soft open point (SOP).

Keywords: soft open point (SOP); power distribution network; sliding-mode control (SMC); terminal sliding-mode; unmatched uncertainties

1. Introduction

Soft open point is a power electronic device which can flexibly connect different voltage-level feeders and provide flexible and accurate power regulation to networks [1,2]. Thanks to the great real-time power controllability, it has been widely used to replace the normal open points in the electricity distribution networks [3,4].

The back-to-back voltage source converter (VSC) is a typical topology of the SOP, including the rectifier-side, the inverter-side, and the DC-side components [5,6]. The one-side VSC works in u_{dc} -Q mode to ensure the stability of the DC-side voltage [7]. The other VSCs work in *P*-Q mode to realize the power exchange of several AC feeders [8]. The mathematical model of the SOP is a class of nonlinear system with matched and unmatched uncertainties [9]. The uncertainties contain external disturbance and parameter perturbation, which bring a huge challenge to the controller design [10].

At present, model predictive control (MPC) is widely used for the current-loop and power-loop controller design in the power electronic converters, owing to its simple algorithm and excellent dynamic performance [11,12]. To enhance the dynamic performance, a model predictive controller is proposed in [13] to replace the PI controller of the current-loop. In [14], a three-vector-based MPC is proposed to reduce the current ripple in SOP. However, MPC is a type of control method that depends on the model, which has unavoidable sensitivity to the parameter perturbation of the SOP [15]. Thus, the requirements of the current- and power-tracking accuracy may not be satisfied under the condition of parameter variation [16]. In practice, the most popular control methods for the outer-loop



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Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). are PI control [17]; however, the PI controller is also sensitive to the parameter perturbation and will result in large overshoot or extended response time [18]. Therefore, some advanced control methods such as sliding-mode control (SMC) are utilized to enhance the anti-disturbance of the SOP control system [19]. In [20], a linear sliding-mode controller is designed for the SOP. Although the SMC method brings strong robustness to the control system, it has irreducible chattering which may cause an unmodeled dynamic in the control loop. In [21], a quasi-sliding-mode controller is proposed for the outer loop. However, this method attenuates the chattering by sacrificing precision of the DC voltage and power tracking. A disturbance–observer-based sliding-mode controller is proposed in [22]; however, the control method can only weaken the chattering but not eliminate that. Therefore, the chattering-free control performance of SMC needs to be further improved.

The conventional sliding-mode control can only compensate for the matched uncertainties existing in the same channel with the control input [23]. However, the uncertainties may not satisfy the so-called matched condition, which becomes a challenge for sliding-modecontroller design [24]. To compensate for the unmatched uncertainties, many improved SMC methods have been proposed, such as integral-sliding-mode control (ISMC) [25], disturbance–observer-based sliding-mode control (DOBSMC) [26] and backstepping [27]. In [28], the high-frequency-switching gain in ISMC can force the system states to converge to the equilibrium in the case of unmatched uncertainties. However, the integral action brings large overshoot and long setting time to the control system. In [29], the disturbance observer is designed to deal with the parameter variation in the SOP. Nevertheless, these methods can only suffer a class of time-invariant or slow-time-varying uncertainties [30].

According to the above analysis, the main problems in the control system of the SOP can be summarized as follows:

- 1. The influence of the parameter perturbation in the SOP is not considered completely in the existing controller design.
- 2. The uncertainties in the SOP control system cannot be compensated for by the existing control methods such as PI and MPC. Therefore, the PI and MPC controllers cannot satisfy the requirements of power and voltage control precision.
- 3. The conventional sliding-mode controller in the SOP control system cannot compensate for the unmatched uncertainties, and the inherent chattering problem may stimulate the unmodeled dynamic of the system.

To enhance the robustness and dynamic performance of the SOP control system, this paper proposes a novel virtual-control-technique-based full-order sliding-mode (FOSM) controller for the rectifier side to establish the DC-side voltage. In addition, a full-order terminal sliding-mode (FOTSM) controller is designed for the inverter side to ensure the stabilization of the power exchange. The main contributions of the paper can be summarized as follows:

- 1. The precision, rapidness and robustness of the SOP control system are improved by adopting the full-order sliding-mode control method.
- 2. On the premise that the anti-disturbance of the SMC is kept, the chattering in the conventional sliding-mode controller is eliminated by the integral-type control law; thus, the smoothed outputs of the controller can be obtained.

The paper is organized as follows: Section 2 introduces the mathematical model of the soft open point and the outer-loop and the inner-loop subsystem. Sections 3 and 4 present the FOTSM controllers for the rectifier side and the inverter side. Section 5 gives the simulation results. Finally, the conclusion is shown in Section 6.

Notations: Let \mathbb{R}^{n-m} and \mathbb{R}^m be the sets of n - m real matrices and m-dimensional Euclidean space, respectively. For a vector $\mathbf{x} = [x_1, ..., x_n]^T \in \mathbb{R}^n$, $\|\mathbf{x}\|$ denoting the 2.norm of the vector $\|\mathbf{x}\|_2$, and $\|\mathbf{x}\|_2 = (\sum_{i=1}^N (x_i^2))^{1/2}$. In addition, for a square matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$, $\|\mathbf{A}\|$ denoting the 2.norm of the vector $\|\mathbf{A}\|_2$, and $\|\mathbf{A}\|_2 = [\max \{\lambda_i(\mathbf{A}^H \mathbf{A})\}]^{1/2}$.

The back-to-back voltage source converter is a typical topology of the SOP. Each port of the SOP consists of a three-phase voltage source converter which is connected to the common DC bus. With its excellent real-time power controllability, the SOP has been widely used in electricity distribution networks to deal with the problems caused by the access of distributed power generation equipment to the distribution network. The related work about the model and control strategy of the soft open point can refer to [14], and the related work about the sliding-mode control methods can refer to [31].

2. Preliminary

2.1. Dynamic Model of the SOP

The structure of the two-port SOP is displayed in Figure 1 [32], where the rectifier side and the inverter side are interconnected by the capacitance of the DC side.



Figure 1. Structure of the two-port SOP with the rectifier side and the inverter side.

The mathematical model of the rectifier side and the inverter side in the stationary coordinates can be obtained by

$$\begin{bmatrix} e_{1a} \\ e_{1b} \\ e_{1c} \end{bmatrix} = L_1 \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} i_{1a} \\ i_{1b} \\ i_{1c} \end{bmatrix} + R_1 \begin{bmatrix} i_{1a} \\ i_{1b} \\ i_{1c} \end{bmatrix} + \begin{bmatrix} u_{1a} \\ u_{1b} \\ u_{1c} \end{bmatrix}$$
(1)

$$\begin{bmatrix} e_{2a} \\ e_{2b} \\ e_{2c} \end{bmatrix} = \begin{bmatrix} u_{2a} \\ u_{2b} \\ u_{2c} \end{bmatrix} - L_2 \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} i_{2a} \\ i_{2b} \\ i_{2c} \end{bmatrix} - R_2 \begin{bmatrix} i_{2a} \\ i_{2b} \\ i_{2c} \end{bmatrix}$$
(2)

where *e* is the gird voltage, *i* the current, *u* the voltage, the subscript $[\cdot]_{a,b,c}$ represents states in *abc*-axes, $[\cdot]_{1,2}$ represents the input states from the rectifier side and the output states of the inverter side, L_1 and L_2 the inductance of the rectifier side and the inverter side, respectively, R_1 and R_2 the resistance of the rectifier side and the inverter side, respectively.

The switching states are defined as follows

$$S_x = \begin{cases} 1, \text{ the upper switch is on and the lower switch is off} \\ 0, \text{ the lower switch is on and the upper switch is off}, x = a, b, c. \end{cases} (3)$$

Then, the voltages of each converter can be calculated by

$$\begin{bmatrix} u_{ia} \\ u_{ib} \\ u_{ic} \end{bmatrix} = \frac{u_{dc}}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix}$$
(4)

where u_{dc} is the capacitor voltage of the DC link.

The current of the DC side in the *d*-*q* coordinates can be expressed as

$$C\frac{\mathrm{d}u_{dc}}{\mathrm{d}t} = i_{1dc} + i_{2dc} = 1.5(-S_{1d}i_{1d} + S_{1q}i_{1q}) + 1.5(-S_{2d}i_{2d} + S_{2q}i_{2q}) \tag{5}$$

where i_{1dc} and i_{2dc} are the currents in the DC side from the rectifer side and the inverter side, respectively, i_{1d} and i_{1q} the input currents of the rectifier side in *d*- and *q*-axes, respectively, i_{2d} and i_{2q} the output currents of the inverter side in *d*- and *q*-axes, respectively, S_{1d} and S_{1q} the switching states of the rectifier side in *d*- and *q*-axes, S_{2d} and S_{2q} the switching states of the inverter side in *d*- and *q*-axes.

Since the structure of each port of the SOP is the same, in this paper, one port of the VSC is selected to build the mathematical model. The dynamic mathematical model of the SOP in the d-q coordinates can be represented by

$$\frac{\mathrm{d}\mathbf{i}_{1dq}}{\mathrm{d}t} = -\frac{R_1}{L_1}\mathbf{i}_{1dq} + \begin{bmatrix} 0 & \omega_1 \\ -\omega_1 & 0 \end{bmatrix}\mathbf{i}_{1dq} - \frac{1}{L_1}\mathbf{u}_{1dq} + \frac{1}{L_1}\mathbf{e}_{1dq}
= -\frac{R_1}{L_1}\mathbf{i}_{1dq} + \mathbf{H}_1\mathbf{i}_{1dq} - \frac{1}{L_1}\mathbf{u}_{1dq} + \frac{1}{L_1}\mathbf{e}_{1dq}$$
(6)

where $\mathbf{i}_{1dq} = [i_{1d}, i_{1q}]^T$ is the rectifier-side current vector in *d*- and *q*-axes, ω_1 the angular speed of the grid voltage, $\mathbf{u}_{1dq} = [u_{1d}, u_{1q}]^T$ the rectifier-side voltage vector in *d*- and *q*-axes, $\mathbf{e}_{1dq} = [e_{1d}, e_{1q}]^T$ the rectifier-side grid voltage vector in *d*- and *q*-axes.

2.2. Proposed Model of the SOP

2.2.1. Outer-Loop Subsystem

The outer-loop subsystem of the rectifier side in the SOP is the DC-side voltage control subsystem. The parameter variation results in the unmatched uncertainties in the outer-loop subsystem. The voltage controller uses u_{dcref} as the input and i_{1dref} as the output to establish the reference value of i_{1d} , which is designed to stabilize the DC-side voltage. According to the aforementioned mathematical model of the SOP, the tracking error dynamic of the DC-side voltage can be expressed by

$$\dot{e}_{u} = \dot{u}_{dcref} - \dot{u}_{dc} = \dot{u}_{dcref} + \frac{3}{2C}(S_{1d}i_{1d} - S_{1q}i_{1q}) + \frac{3}{2C}(S_{2d}i_{2d} - S_{2q}i_{2q})$$
(7)

Due to $i_{1qref} = 0$ being adopted as the current control strategy, substituting the switching states $S_{1d} = (e_{1d} - R_1 i_{1d})/u_{dc}$ and $S_{2d} = (e_{2d} - R_2 i_{2d})/u_{dc}$ into the above, (7) can be rewritten as

$$\dot{e}_{u} = \dot{u}_{dcref} + \frac{3(e_{1d} - R_{1}i_{1d})}{2Cu_{dc}}i_{1d} + \frac{3(e_{2d} - R_{2}i_{2d})}{2Cu_{dc}}i_{2d}$$
(8)

Considering the variation of the parameters caused by the change of temperature and frequency in the SOP, the resistance, the inductance and the capacitance can be expressed by

$$R_{j} = \hat{R}_{j} + \Delta R_{j}, \ L_{j} = \hat{L}_{j} + \Delta L_{j}, \ C = \hat{C} + \Delta C, \ j = 1, 2$$
 (9)

where $[\cdot]$ and $\Delta[\cdot]$ represent the nominal and the changed value of the parameters. The above parameter perturbations can be assumed to be bounded as follows

$$\left|\Delta R_{j}\right| \leq M_{R_{j}}, \left|\Delta L_{j}\right| \leq M_{L_{j}}, \left|\Delta C\right| \leq M_{C}, j = 1, 2 \tag{10}$$

where M_{R_i} , M_{L_i} and M_C are positive constants

Therefore, considering the parameter perturbation and the external disturbance, the error dynamics of the DC-side voltage can be rewritten as

$$\dot{e}_{u} = \dot{u}_{dcref} + \frac{3[e_{1d} - (\hat{R}_{1} + \Delta R_{1})i_{1d}]}{2(\hat{C} + \Delta C)u_{dc}}i_{1d} + \frac{3[e_{2d} - (\hat{R}_{2} + \Delta R_{2})i_{2d}]}{2(\hat{C} + \Delta C)u_{dc}}i_{2d} + \rho_{1}(t)$$
(11)

where $\rho_1(t)$ is the external disturbance which can be upper-bounded as $|\rho_1(t)| \le M_{\rho_1}(t)$.

The virtual control signal is defined as $\phi = g_1 i_{1dref} = 1.5(T_{1/C}e_{1d} - T_{R_1/C}i_{1d})i_{1dref}/u_{dc}$, where $T_{1/C} = 1/\hat{C}$, $T_{R_1/C} = \hat{R}_1/\hat{C}$ and $g_1 = 1.5(T_{1/C}e_{1d} - T_{R_1/C}i_{1d})/u_{dc}$, the *d*-axis current tracking reference value can be calculated by $i_{1dref} = g_1^{-1}\phi = u_{dc}\phi/1.5(T_{1/C}e_{1d} - T_{R_1/C}i_{1d})$. Define the *d*-axis current tracking error as follows

$$e_{id} = i_{1dref} - i_{1d} = g_1^{-1}\phi - i_{1d} \tag{12}$$

To simplify the equation, let $f_1 = \dot{u}_{dcref} + 1.5(T_{1/C}e_{2d} - T_{R_2/C}i_{2d})i_{2d}/u_{dc}$, where $T_{R_2/C} = \hat{R}_2/\hat{C}$. Then, the tracking error dynamic of the DC-side voltage can be represented by

$$\begin{split} \dot{e}_{u} &= \dot{u}_{dcref} + \frac{3[e_{1d} - (\hat{R}_{1} + \Delta R_{1})i_{1d}]}{2(\hat{C} + \Delta C)u_{dc}}i_{1d} + \frac{3[e_{2d} - (\hat{R}_{2} + \Delta R_{2})i_{2d}]}{2(\hat{C} + \Delta C)u_{dc}}i_{2d} + \rho_{1}(t) \\ &= \dot{u}_{dcref} + \frac{1.5(T_{1/C}e_{2d} - T_{R_{2}/C}i_{2d})i_{2d}}{u_{dc}} + \frac{1.5(T_{1/C}e_{1d} - T_{R_{1}/C}i_{1d})i_{1d}}{u_{dc}} \\ &+ \frac{1.5(\Delta T_{1/C}e_{2d} - \Delta T_{R_{2}/C}i_{2d})i_{2d}}{u_{dc}} + \frac{1.5(\Delta T_{1/C}e_{1d} - \Delta T_{R_{1}/C}i_{1d})(i_{1dref} - e_{id})}{u_{dc}} \\ &= \dot{u}_{dcref} + \frac{1.5(T_{1/C}e_{2d} - T_{R_{2}/C}i_{2d})i_{2d}}{u_{dc}} + \frac{1.5(\Delta T_{1/C}e_{1d} - \Delta T_{R_{1}/C}i_{1d})(i_{1dref} - e_{id})}{u_{dc}} \\ &+ \frac{1.5(\Delta T_{1/C}e_{2d} - \Delta T_{R_{2}/C}i_{2d})i_{2d}}{u_{dc}} + \frac{1.5(\Delta T_{1/C}e_{1d} - \Delta T_{R_{1}/C}i_{1d})(i_{1dref} - e_{id})}{u_{dc}} \\ &+ \frac{1.5(\Delta T_{1/C}e_{2d} - \Delta T_{R_{2}/C}i_{2d})i_{2d}}{u_{dc}} + \frac{1.5(\Delta T_{1/C}e_{1d} - \Delta T_{R_{1}/C}i_{1d})(i_{1dref} - e_{id})}{u_{dc}} \\ &+ \frac{1.5(\Delta T_{1/C}e_{2d} - \Delta T_{R_{2}/C}i_{2d})i_{2d}}{u_{dc}} + \frac{1.5(\Delta T_{1/C}e_{1d} - \Delta T_{R_{1}/C}i_{1d})(i_{1dref} - e_{id})}{u_{dc}} \\ &+ \frac{1.5(\Delta T_{1/C}e_{2d} - \Delta T_{R_{2}/C}i_{2d})i_{2d}}{u_{dc}} + \frac{1.5(\Delta T_{1/C}e_{1d} - \Delta T_{R_{1}/C}i_{1d})(i_{1dref} - e_{id})}{u_{dc}} \\ &+ \frac{1.5(\Delta T_{1/C}e_{2d} - \Delta T_{R_{2}/C}i_{2d})i_{2d}}{u_{dc}} + \frac{1.5(\Delta T_{1/C}e_{1d} - \Delta T_{R_{1}/C}i_{1d})(i_{1dref} - e_{id})}{u_{dc}} \\ &+ \frac{1.5(\Delta T_{1/C}e_{2d} - \Delta T_{R_{2}/C}i_{2d})i_{2d}}{u_{dc}} \\ &= f_{1} + \phi + \Delta g_{1}g_{1}^{-1}\phi - g_{1}e_{id} + d_{1}(t) + \rho_{1}(t) \\ &= f_{1} + \phi + \Delta g_{1}g_{1}^{-1}\phi - g_{1}e_{id} + d_{1}(t) \\ \end{split}$$

where $\hat{d}_1(t) = d_1(t) + \rho_1(t)$ is defined as the lumped unmatched uncertainty in the rectifier-side system of the SOP, $d_1(t) = 1.5(\Delta T_{1/C}e_{2d} - \Delta T_{R_2/C}i_{2d})i_{2d}/u_{dc} + 1.5(\Delta T_{1/C}e_{1d} - \Delta T_{R_1/C}i_{1d})e_{id}/u_{dc}$.

To analysis the lumped unmatched uncertainty $\hat{d}_1(t)$, the following uncertainties are defined as

$$\delta_u = \Delta g_1 / g_1 \tag{14a}$$

$$\delta_{L_j} = \Delta T_{1/L_j} / T_{1/L_j} \tag{14b}$$

$$\Delta g_1 = -1.5(\Delta F_{1/C} e_{1d} - \Delta F_{R_1/C} i_{1d}) / u_{dc} \tag{14c}$$

$$\Delta T_{1/C} = -\Delta C / (\hat{C}(\hat{C} + \Delta C))$$
(14d)

$$\Delta T_{1/L_j} = -\Delta L_J / (\hat{L}_j (\hat{L}_j + \Delta L_j))$$
(14e)

$$\Delta T_{R_j/C} = -(\hat{C}\Delta R_j - \hat{R}_j \Delta C) / (\hat{C}(\hat{C} + \Delta C))$$
(14f)

$$\Delta T_{R_j/L_j} = -(\hat{L}_j \Delta R_j - \hat{R}_j \Delta L_j) / (\hat{L}_j (\hat{L}_j + \Delta L_j)), \ j = 1, 2$$
(14g)

According to (10), the aforementioned uncertainties (14) can be upper-bounded as

$$\begin{aligned} |\delta_{u}| &\leq M_{\delta_{u}} < 1, \left| \delta_{L_{j}} \right| \leq M_{\delta_{L_{j}}} < 1, \left| \Delta T_{1/C} \right| \leq M_{1/C}, \left| i_{jd} \right| \leq i_{jdmax}, \left| u_{jd} \right| \leq u_{jdmax}, \\ |\Delta g_{1}| &\leq 1.5 (M_{1/C} e_{1dmax} - M_{R_{1}/C} i_{1dmax}) / u_{dc} = M_{g_{1}}, \\ \left| \Delta T_{1/L_{j}} \right| &\leq M_{1/L_{j'}} \left| \Delta T_{R_{j}/C} \right| \leq M_{R_{j}/C}, \left| \Delta T_{R_{j}/L_{j}} \right| \leq M_{R_{j}/L_{j}}, j = 1, 2 \end{aligned}$$
(15)

where M_{δ_u} , $M_{\delta_{Lj}}$, $M_{1/C}$, M_{g_1} , M_{1/L_j} , $M_{R_j/C}$ and M_{R_j/L_j} are positive constants. Hence, the lumped unmatched uncertainty $\hat{d}_1(t)$ satisfies

$$\hat{d}_1(t) = d_1(t) + \rho_1(t) = 1.5(\Delta T_{1/C}e_{2d} - \Delta T_{R_2/C}i_{2d})i_{2d}/u_{dc} + M_{\rho_1}(t)$$
(16)

where $d_1(t)$ satisfies $|d_1(t)| \leq 1.5(M_{1/C}e_{2dmax} - M_{R_1/C}i_{2dmax})i_{2dmax}/u_{dc} = M_{d_1}(t)$. The lumped unmatched uncertainty and its derivative are bounded by $|\hat{d}_1(t)| = M_{d_1}(t) + M_{\rho_1}(t) \leq M_{\hat{d}_1}(t)$ and $|\hat{d}_1(t)| \leq M_{\hat{d}_1}(t)$.

2.2.2. Inner-Loop Subsystem

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The inner-loop subsystem of the rectifier side in the SOP is the current-control subsystem. To ensure independent dynamic characteristics of the *d-q* currents, a feedforward compensation designed as $\tilde{u}_1 = -u_{1dq} + e_{1dq} + \hat{L}_1 H_1 i_{1dq}$ is used to achieve dynamic decoupling and avoid the interaction effect of the coupled voltage.

There is no longer external disturbance but only parameter variation in the inner-loop subsystem, which belongs to the matched uncertainties. The current controllers use i_{1dref} as the input and \tilde{u}_1 as the output to generate the SVPWM modulation signals. According to the aforementioned mathematical model of the SOP, the inner-loop subsystem can be obtained by

$$\frac{\mathrm{d}\mathbf{i}_{1dq}}{\mathrm{d}t} = -\frac{R_1}{L_1}\mathbf{i}_{1dq} + \frac{1}{L_1}(-\mathbf{u}_{1dq} + \mathbf{e}_{1dq} + L_1H_1\mathbf{i}_{1dq}) \tag{17}$$

Considering the parameter variation, the inner-loop subsystem can be rewritten as

$$\frac{d\mathbf{i}_{1dq}}{dt} = -\frac{R_1}{L_1}\mathbf{i}_{1dq} + \frac{1}{L_1}(-\mathbf{u}_{1dq} + \mathbf{e}_{1dq} + L_1\mathbf{H}_1\mathbf{i}_{1dq})
= -\frac{\hat{R}_1 + \Delta R_1}{\hat{L}_1 + \Delta L_1}\mathbf{i}_{1dq} + \frac{1}{\hat{L}_1 + \Delta L_1}(-\mathbf{u}_{1dq} + \mathbf{e}_{1dq} + (\hat{L}_1 + \Delta L_1)\mathbf{H}_1\mathbf{i}_{1dq})
= -T_{R_1/L_1}\mathbf{i}_{1dq} + (T_{1/L_1} + \Delta T_{1/L_1})(-\mathbf{u}_{1dq} + \mathbf{e}_{1dq} + \hat{L}_1\mathbf{H}_1\mathbf{i}_{1dq}) + \hat{\mathbf{d}}_2(t)$$
(18)

where $T_{1/L_1} = 1/\hat{L}_1$, $T_{R_1/L_1} = \hat{R}_1/\hat{L}_1$, and the lumped matched uncertainty $\hat{d}_2(t) = -\Delta T_{R_1/L_1} \mathbf{i}_{1dq} + (T_{1/L_1} + \Delta T_{1/L_1})\Delta L_1 \mathbf{H}_1 \mathbf{i}_{1dq}$ and satisfies $\|\hat{d}_2(t)\| \le M_{R_1/L_1} \|\mathbf{i}_{1dq}\| + (T_{1/L_1} + M_{1/L_1})M_{L_1}\mathbf{H}_1 \|\mathbf{i}_{1dq}\|$.

Introducing the feedforward compensation $\tilde{u}_1 = -u_{1dq} + e_{1dq} + \hat{L}_1 H_1 i_{1dq}$ into (18) yields

$$\frac{\mathrm{d}\mathbf{i}_{1dq}}{\mathrm{d}t} = -T_{R_1/L_1}\mathbf{i}_{1dq} + T_{1/L_1}\mathbf{\tilde{u}}_1 + \Delta T_{1/L_1}\mathbf{\tilde{u}}_1 + \mathbf{\hat{d}}_2(t)$$
(19)

where the lumped matched uncertainty and its derivative can be upper-bounded as $\|\hat{d}_2(t)\| \leq M_{\hat{d}_2}(t)$ and $\|\dot{d}_2(t)\| \leq M_{\hat{d}_2}$.

Lemma 1 ([33]). Consider a system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, $\mathbf{f}(0) = 0$, $\mathbf{x} \in \mathbf{R}^n$, if there exists a positive definite continuous function $V(\mathbf{x})$: $\mathbf{U} \to \mathbf{R}$, real numbers c > 0 and $0 < \alpha < 1$, and an open neighborhood $\mathbf{U}_0 \subset \mathbf{U}$ of the origin such that $\dot{V} + cV^{\alpha}(\mathbf{x}) \leq 0$, $\mathbf{x} \in \mathbf{U}_0\{0\}$, and then $V(\mathbf{x})$ can approach zero in a finite-time, $t_r \leq V^{1-\alpha}(\mathbf{x}(0))/(c(1-\alpha))$.

3. Full-Order Sliding-Mode Control for the Rectifier Side

Let $T_{1/L_1}\tilde{u}_1 = u_{i1}$ and combine the outer-loop voltage subsystem and the decoupled inner-loop current subsystem, the mathematical model of the SOP can be summarized as a second-order system with matched and unmatched uncertainties in the following form.

$$\begin{cases} \dot{e}_{u} = f_{1} + \phi + \delta_{u}\phi - g_{1}e_{id} + \hat{d}_{1}(t) \\ \dot{i}_{1dq} = -T_{R_{1}/L_{1}}\dot{i}_{1dq} + u_{i1} + \delta_{L_{1}}u_{i1} + \hat{d}_{2}(t) \end{cases}$$
(20)

where $\phi = g_1 i_{1dref}$ is the virtual control signal of the outer-loop subsystem.

S

In this section, a novel virtual-control-technique-based full-order sliding-mode controller is proposed for the rectifier side of the SOP. The virtual control signal ϕ is designed to deal with the unmatched uncertainties $\hat{d}_1(t)$ and force the DC-side voltage tracking error e_u to converge to zero. In addition, the actual control signal u_{i1} is designed to compensate for the matched uncertainties $\hat{d}_2(t)$ and make the rectifier-side input current vector i_{1dq} track its reference $i_{1dqref} = g_1^{-1}\phi$ accurately.

3.1. Voltage Controller Design for the Outer-Loop

A full-order sliding-mode manifold $s_1 \in R^1$ is designed for the outer-loop subsystem with the form as follows

$$\dot{e}_1 = \dot{e}_u + c_1 e_u \tag{21}$$

where c_1 is a positive constant.

Theorem 1. If the sliding manifold is selected as (21) and the virtual control law ϕ is designed as follows, the outer-loop voltage-tracking error dynamics can be regulated to reach the ideal manifold in a finite time $t_{r1} \leq (|s_1(0)|/\eta_1)$, thereafter remaining on it and converging to an equilibrium point asymptotically after the current tracking error e_{id} and its derivative \dot{e}_{id} converge to zero.

$$\phi = \phi_{eq} + \phi_n \tag{22}$$

$$\phi_{eq} = -f_1 - c_1 e_u \tag{23}$$

$$\phi_n = -\int_0^t k_1 \operatorname{sgn}(s_1) \mathrm{d}\tau \tag{24}$$

$$k_1 = \frac{M_{\delta_u} M_{eq}(t) + M_{\dot{d}_1}(t) + \eta_1}{1 - M_{\delta_u}}$$
(25)

where M_{δ_u} and $M_{\hat{d}_1}(t)$ are defined by (14a) and (16), $M_{deq}(t)$ is the upper bound of the derivative of the equivalent virtual control law (23), and η_1 is a positive constant.

Proof of Theorem 1. Substituting the error dynamic of the DC-side voltage (13) into the sliding manifold (21), yields

$$s_1 = \dot{e_u} + c_1 e_u = f_1 + \phi + \delta_u \phi - g_1 e_{id} + \dot{d_1}(t) + c_1 e_u \tag{26}$$

According to the designed virtual control law (22), the derivative of the sliding manifold can be represented by

$$\dot{s}_1 = \dot{\phi}_n + \delta_u \dot{\phi} + \hat{d}_1(t) - g_1 \dot{e}_{id} - \dot{g}_1 e_{id}$$
(27)

defined by a Lyapunov function as $V_1 = 0.5s_1^2$. The derivative of V_1 can be obtained by

$$\dot{V}_1 = s_1 \dot{s}_1 = s_1 \dot{\phi}_n + s_1 \delta_u \dot{\phi}_{eq} + s_1 \delta_u \dot{\phi}_n + s_1 \hat{d}_1(t) - s_1 g_1 \dot{e}_{id} - s_1 \dot{g}_1 e_{id}$$
(28)

Substituting the integral-type switching-control law (24) into the above, gives

$$\begin{aligned} \dot{V}_{1} &\leq -k_{1}|s_{1}| + k_{1}|s_{1}||\delta_{u}| + |s_{1}||\delta_{u}| \left|\dot{\phi}_{eq}\right| + |s_{1}| \left|\dot{d}_{1}(t)\right| - s_{1}g_{1}\dot{e}_{id} - s_{1}\dot{g}_{1}e_{id} \\ &\leq -k_{1}|s_{1}|(1 - M_{\delta_{u}}) + M_{\delta_{u}}|s_{1}| \left|\dot{\phi}_{eq}\right| + |s_{1}|M_{\dot{d}_{1}}(t) - s_{1}g_{1}\dot{e}_{id} - s_{1}\dot{g}_{1}e_{id} \end{aligned}$$

$$(29)$$

Then, the derivative of the Lyapunov function V_1 satisfies

$$\dot{V}_{1} \leq -|s_{1}|[(1-M_{\delta_{u}})k_{1}-M_{\delta_{u}}M_{eq}(t)-M_{\dot{d}_{1}}(t)]-s_{1}g_{1}\dot{e}_{id}-s_{1}\dot{g}_{1}e_{id}$$
(30)

where $M_{eq}(t)$ is the upper bound of $\dot{\phi}_{eq}$.

If the switching gain
$$k_1$$
 satisfies $k_1 = (M_{\delta_u} M_{eq}(t) + M_{\hat{d}_1}(t) + \eta_1)/(1 - M_{\delta_u})$, it yields

$$\dot{V}_1 = s_1 \dot{s}_1 \le -\eta_1 |s_1| - s_1 g_1 \dot{e}_{id} - s_1 \dot{g}_1 e_{id} \tag{31}$$

According to Theorem 2, the inner-loop tracking error e_{id} and its derivative \dot{e}_{id} can be forced to converge to zero in finite time. Then, the above becomes

$$\dot{V}_1 \le -\eta_1 |s_1| \le -\sqrt{2}\eta_1 V_1^{1/2} < 0 \tag{32}$$

According to Lemma 1, if the current-tracking error e_{id} and its derivative \dot{e}_{id} can converge to zero in finite time, the DC-side voltage tracking error dynamic will be forced to reach the ideal sliding manifold (21) in a finite time $t_{r1} \leq (|s_1(0)|/\eta_1)$ under the designed virtual control law (22). In addition, the system trajectory will converge to zero asymptotically along the sliding manifold. It is obvious that the integral-type control law can compensate for the unmatched uncertainties completely and make the control signal smooth. This completes the proof. \Box

The calculation flowchart of the FOSMC is depicted in Figure 2.



Figure 2. Calculation flowchart of the full-order sliding-mode control method.

3.2. Current Controller Design for the Inner-Loop

The current tracking error is defined as $e_{i1} = i_{idqref} - i_{1dq} = [i_{1dref} - i_{1d}, 0 - i_{1q}]^T$. According to the current subsystem (), the error dynamic of the rectifier-side current can be obtained by

$$\dot{\boldsymbol{e}}_{i1} = \dot{\boldsymbol{i}}_{idqref} + T_{R_1/L_1} \dot{\boldsymbol{i}}_{1dq} - \boldsymbol{u}_{i1} - \delta_{L_1} \boldsymbol{u}_{i1} - \dot{\boldsymbol{d}}_2(t)$$
(33)

A full-order terminal sliding-mode manifold $s_2 \in R^2$ is designed for the inner-loop subsystem to have the form of [33]

$$s_2 = \dot{e}_{i1} + C_2 e_{i1}^{q/p} \tag{34}$$

where $e_{i1} = [i_{1dref} - i_{1d}, 0 - i_{1q}]^T$; $C_2 = diag(c_{21}, c_{22})$ is the positive diagonal matrix, c_{21} and c_{22} are positive constants and q and p are odds and satisfy 0 < q/p < 1.

Theorem 2. Based on the designed sliding surface (34), the virtual control law (22) and the following actual control law, the inner-loop current-tracking error dynamics can be regulated to reach the sliding surface $s_2(t) = 0$ in finite time $t_{r2} \leq (||s_2(0)||/\eta_2)$. Then, the inner-loop tracking error e_{i1} and its derivative can converge to zero along the ideal sliding manifold in finite time $t_{s2} \leq \max\{p/[(c_{21}(p-q))|e_{i1d}(t_{r1d})|^{1-q/p}], p/[(c_{22}(p-q))|e_{i1q}(t_{r1q})|^{1-q/p}]\}$ [31].

$$u_{i1} = u_{i1eq} + u_{i1n} \tag{35}$$

$$\boldsymbol{u}_{i1eq} = \begin{bmatrix} -g_1^{-1}[c_1(f_1 + g_1i_{1d}) + k_1\mathrm{sgn}(s_1)] - g_1^{-2}\dot{g}_1\boldsymbol{\phi} \\ 0 \end{bmatrix} + T_{R_1/L_1}\dot{\boldsymbol{i}}_{1dq} + C_2\boldsymbol{e}_{i1}^{q/p}$$
(36)

$$\boldsymbol{u}_{i1n} = \int_0^t k_2 \mathrm{sgn}(\boldsymbol{s}_2) \mathrm{d}\tau \tag{37}$$

$$k_{2} = \frac{M_{\delta_{L1}}M_{i1eq}(t) + M_{\dot{d}_{2}(t)} + |g_{1}|^{-1}(M_{f1} + c_{1}M_{\dot{d}_{1}}(t)) + \left|g_{1}^{-2}\dot{g}_{1}\right|(M_{f2} + c_{1}M_{\dot{d}_{1}}(t)) + \eta_{2}}{1 - M_{\delta_{L1}}}$$
(38)

where the known $M_{\delta_{L1}}$, $M_{\hat{d}_1}(t)$, $M_{\hat{d}_1}(t)$ and $M_{\hat{d}_1}(t)$ are defined by (15), (16) and (19), respectively; $M_{i1dq}(t)$ represents the upper bound of the equivalent actual control signal u_{i1eq} , which satisfies $\|u_{i1eq}\| \leq M_{i1dq}(t)$; M_{f1} and M_{f2} are assumed as the upper bound of the known function f_1 and its derivative, which satisfy $|\dot{f}_1| \leq M_{f1}$, $|\ddot{f}_1| \leq M_{f2}$.

Proof of Theorem 2. Substituting the current tracking error dynamic (33) into the sliding manifold (34), yields

$$s_{2} = \dot{i}_{idqref} + T_{R_{1}/L_{1}}\dot{i}_{1dq} - u_{i1} - \delta_{L_{1}}u_{i1} - \hat{d}_{2}(t) + C_{2}e_{i1}^{q/p}$$

$$= \begin{bmatrix} \dot{\phi} \\ 0 \end{bmatrix} + T_{R_{1}/L_{1}}\dot{i}_{1dq} - u_{i1} - \delta_{L_{1}}u_{i1} - \hat{d}_{2}(t) + C_{2}e_{i1}^{q/p}$$
(39)

Considering the designed virtual control law (22), the above becomes

$$s_{2} = \begin{bmatrix} g_{1}^{-1}[-\dot{f}_{1} - c_{1}(f_{1} + g_{1}i_{1d} + \hat{d}_{1}(t)) - k_{1}\mathrm{sgn}(s_{1})] - g_{1}^{-2}\dot{g}_{1}\phi \\ 0 \end{bmatrix} + T_{R_{1}/L_{1}}\dot{i}_{1dq} - u_{i1} - \delta_{L_{1}}u_{i1} - \hat{d}_{2}(t) + C_{2}e_{i1}^{q/p}.$$

$$(40)$$

Substituting the actual control law (35) into the above, then we obtain

$$s_{2} = \begin{bmatrix} -g_{1}^{-1}\dot{f}_{1} - g_{1}^{-1}c_{1}\hat{d}_{1}(t) \\ 0 \end{bmatrix} - u_{i1n} - \delta_{L_{1}}u_{i1} - \hat{d}_{2}(t)$$
(41)

and the derivative of the sliding manifold (34) can be calculated by

$$\dot{s}_{2} = \begin{bmatrix} -g_{1}^{-1}\ddot{f}_{1} + g_{1}^{-2}\dot{g}_{1}\dot{f}_{1} - g_{1}^{-1}c_{1}\dot{d}_{1}(t) + g_{1}^{-2}\dot{g}_{1}c_{1}\dot{d}_{1}(t) \\ 0 \end{bmatrix} - \dot{u}_{i1n} - \delta_{L_{1}}\dot{u}_{i1} - \dot{\dot{d}}_{2}(t) \quad (42)$$

We define a Lyapunov function as $V_2 = 0.5 s_2^T s_2$, and its derivative is expressed as

$$\dot{V}_{2} = \mathbf{s}_{2}^{T} \dot{\mathbf{s}}_{2} \leq \begin{pmatrix} -\mathbf{s}_{2}^{T} \dot{\mathbf{u}}_{i1n} - \mathbf{s}_{2}^{T} \delta_{L_{1}} \dot{\mathbf{u}}_{i1n} - \mathbf{s}_{2}^{T} \delta_{L_{1}} \dot{\mathbf{u}}_{i1eq} - \mathbf{s}_{2}^{T} \dot{\mathbf{d}}_{2}(t) - \mathbf{s}_{2}^{T} \mathbf{g}_{1}^{-1} \dot{f}_{1} \\ + \mathbf{s}_{2}^{T} \mathbf{g}_{1}^{-2} \dot{g}_{1} \dot{f}_{1} - \mathbf{s}_{2}^{T} \mathbf{g}_{1}^{-1} c_{1} \dot{d}_{1}(t) + \mathbf{s}_{2}^{T} \mathbf{g}_{1}^{-2} \dot{g}_{1} c_{1} \dot{d}_{1}(t) \end{pmatrix}$$
(43)

Substituting (37) into the above gives

assume $|\dot{f}_1| \leq M_{f1}$, $|\ddot{f}_1| \leq M_{f2}$ and $||\dot{u}_{i1eq}|| \leq M_{i1eq}(t)$, considering the upper bounds of the lumped uncertainties (16) and (19), the above becomes

$$\dot{V}_{2} \leq \|\boldsymbol{s}_{2}\| \begin{bmatrix} -k_{2}(1-M_{\delta_{L1}}) + M_{\delta_{L1}}M_{i1eq}(t) + M_{\dot{d}_{2}}(t) + \left|g_{1}^{-1}\right|M_{f2} \\ + \left|g_{1}^{-2}\dot{g}_{1}\right|M_{f1} + c_{1}|g_{1}|^{-1}M_{\dot{d}_{1}}(t) + c_{1}\left|g_{1}^{-2}\dot{g}_{1}\right|M_{\hat{d}_{1}}(t) \end{bmatrix}$$

$$(45)$$

If the switching gain k_2 satisfies $k_2 = (M_{\delta_{L1}}M_{i1eq}(t) + M_{\hat{d}_2}(t) + |g_1|^{-1}(M_{f1} + c_1M_{\hat{d}_1}(t)) + |g_1^{-2}\dot{g}_1|(M_{f2} + c_1M_{\hat{d}_1}(t)) + \eta_2)/(1 - M_{\delta_{L1}})$, the derivative of the Lyapunov function \dot{V}_2 satisfies

$$\dot{V}_2 \le -\eta_2 \|s_2\| \le -\sqrt{2}\eta_2 V_2^{1/2}.$$
(46)

According to Lemma 1, the above means that the error dynamic can approach the ideal sliding manifold $s_2 = 0$ in a finite time $t_{r2} \le (||\mathbf{s}_2(0)|| / \eta_2)$. Thereafter, the tracking error of the rectifier-side input current and its derivative will converge to the equilibrium point in a finite time $t_{s2} \le \max\{p/[(c_{21}(p-q))|e_{i1d}(t_{r1d})|^{1-q/p}], p/[(c_{22}(p-q))|e_{i1q}(t_{r1q})|^{1-q/p}]\}$. This completes the proof. \Box

The calculation flowchart of the FOTSMC is depicted in Figure 3.



Figure 3. Calculation flowchart of the full-order terminal sliding-mode control method.

4. Full-Order Sliding-Mode Control for the Inverter Side

The controller for the inverter side is designed to control the output active power and reactive power of the load power grid. The output active power P_2 and the reactive power Q_2 of the inverter side in the *d*-*q* coordinates can be expressed by

$$P_2 = e_{2d}i_{2d} + e_{2q}i_{2q}, \ Q_2 = e_{2q}i_{2d} - e_{2d}i_{2q} \tag{47}$$

where e_{2d} and e_{2q} are the inverter side grid voltages in the *d*- and *q*-axes, i_{2d} and i_{2q} the output currents of the inverter side in the *d*- and *q*-axes.

Aligning the voltage vector of the AC system with the *d*-axis of the rotating coordinate system, $e_q = 0$ can be obtained, and e_d is a constant, then

$$P_2 = 1.5e_d i_{2d}, \ Q_2 = -1.5e_d i_{2q} \tag{48}$$

which illustrates that the power output from the inverter side to the grid can be controlled independently by controlling the inverter-side output currents i_{2d} and i_{2q} . Therefore, the design of the current controllers can refer to Theorem 2.

The tracking error of the output power can be defined as $e_o = [P_{2ref} - P_2, Q_{2ref} - Q_2]^T$. According to the inner-loop subsystem (20), the error dynamic can be represented by

$$\dot{\boldsymbol{e}}_{o} = \begin{bmatrix} \dot{P}_{2ref} \\ \dot{Q}_{2ref} \end{bmatrix} - \begin{bmatrix} 1.5e_{2d}\dot{i}_{2d} \\ -1.5e_{2d}\dot{i}_{2q} \end{bmatrix} = \boldsymbol{E}\frac{\mathrm{d}\boldsymbol{i}_{2dq}}{\mathrm{d}t} = \boldsymbol{E}[T_{R_{2}/L_{2}}\boldsymbol{i}_{2dq} - \boldsymbol{u}_{i2} - \delta_{L_{2}}\boldsymbol{u}_{i2} - \hat{\boldsymbol{d}}_{3}(t)] \quad (49)$$

where $E = diag(1.5e_{2d}, -1.5e_{2d})$, u_{i2} the decoupled control signal with the form as following

$$u_{i2} = T_{1/L_2} \tilde{u}_2 = -u_{2dq} + e_{2dq} + L_2 H_2 i_{2dq}, H_2 = [0, \omega_2; -\omega_2, 0]$$
(50)

where $\rho_2(t)$ is the external disturbance which can be upper-bounded as $\|\rho_2(t)\| \leq M_{\rho_2}(t)$, and the lumped uncertainties $\hat{d}_3(t) = -\Delta T_{R_2/L_2} i_{2dq} + (T_{1/L_2} + \Delta T_{1/L_2}) \Delta L_2 H_2 i_{2dq} + \rho_2(t)$, which satisfy $\|\hat{d}_3(t)\| \leq M_{R_2/L_2} \|i_{2dq}\| + (T_{1/L_2} + M_{1/L_2}) M_{L_2} H_2 \|i_{2dq}\| + M_{\rho_2}(t)$. The lumped uncertainty and its derivative can be upper-bounded as $\|\hat{d}_3(t)\| \leq M_{\hat{d}_3}(t)$ and $\|\hat{d}_3(t)\| \leq M_{\hat{d}_3}$.

A full-order terminal sliding manifold is designed as follows

$$s_3 = \dot{e}_0 + C_3 e_0^{q/p} \tag{51}$$

where $C_3 = diag(c_{31}, c_{32})$ is the positive diagonal matrix, c_{31} and x_{32} are positive constants q and p are odds and satisfy 0 < q/p < 1.

Theorem 3. If the sliding manifold is selected as (50), the output-power-tracking error dynamics can approach the sliding manifold in finite time. The system trajectory will remain on sliding manifold and converge to zero under the designed control law as follows.

$$u_{i2} = u_{i2eq} + u_{i2n} \tag{52}$$

$$u_{i2eq} = T_{R_1/L_1} i_{1dq} + C_3 E^{-1} e_0^{q/p}$$
(53)

$$\boldsymbol{u}_{i2n} = \int_0^t k_3 \mathrm{sgn}(s_3) \mathrm{d}\tau \tag{54}$$

$$k_3 = \frac{M_{\delta_{L2}}M_{i2eq}(t) + M_{\dot{d}_3} + \eta_3}{1 - M_{\delta_{L2}}}$$
(55)

where the known $M_{\delta_{12}}$ and $M_{\hat{d}_3}(t)$ are defined by (14b) and (49), respectively, and $M_{i2dq}(t)$ represents the upper bound of the equivalent actual control signal u_{i2eq} , which satisfies $||u_{i2eq}| \leq M_{i2dq}(t)$.

Proof of Theorem 3. According to the output-power-tracking error dynamic (48), the sliding manifold (50) can be rewritten as

$$s_3 = E(T_{R_2/L_2}i_{2dq} - u_{i2} - \delta_{L_2}u_{i2} - \hat{d}_3(t)) + C_3 e_o^{q/p}$$
(56)

Substituting the control law (51) into the above, yields

$$s_3 = E(-u_{i2n} - \delta_{L_2} u_{i2} - \hat{d}_3(t))$$
(57)

Thus, the derivative of the sliding manifold (50) can be calculated by

$$\dot{s}_3 = E(-\dot{u}_{i2n} - \delta_{L_2}\dot{u}_{i2} - \dot{d}_3(t))$$
(58)

We define a Lyapunov function as $V_3 = 0.5s_3^T s_2$, and the derivative is

$$\dot{V}_3 = \mathbf{s}_3^T \dot{\mathbf{s}}_3 \le \mathbf{s}_3^T \mathbf{E} (-\dot{\mathbf{u}}_{i2n} - \delta_{L_2} \dot{\mathbf{u}}_{i2} - \hat{\mathbf{d}}_3(t))$$
(59)

Substituting (53) into the above gives

$$\dot{V}_{3} \leq \|\boldsymbol{s}_{3}\|[-k_{3}(1-M_{\delta_{L2}})+M_{\delta_{L2}}\|\boldsymbol{\dot{u}}_{i2eq}\|+\|\boldsymbol{\dot{d}}_{3}(t)\|]$$
(60)

assuming $\|\dot{u}_{i2eq}\| \le M_{i2eq}(t)$. Then, it can be obtained from the designed switching gain (54) that

$$\dot{V}_{3} \le \|\boldsymbol{s}_{3}\| [-k_{3}(1 - M_{\delta_{L2}}) + M_{\delta_{L2}}M_{i2eq}(t) + M_{\dot{d}_{3}}] \le -\sqrt{2}\eta_{3}V_{3}^{1/2}$$
(61)

which illustrates that the output-power-tracking error dynamic can be forced to reach the sliding manifold in a finite time. Thereafter, the system trajectory will converge to zero along the sliding manifold in finite time. This completes the proof. \Box

According to Theorems 1–3, the block diagram of the control scheme of the SOP is shown in Figure 4. The phase-locked loop (PLL) is used to obtain the angular speed of the grid voltage which is omitted here for space due to its simplicity.



Figure 4. Block diagram of the control scheme of the SOP.

5. Simulations

5.1. Empirical Research Methodology

To demonstrate the effectiveness of the proposed method under a serious condition, a model predictive control (MPC), a linear sliding-mode control (LSMC) and a full-order sliding-mode control (FOSMC) were used to design the controllers for performance comparison in MATLAB/Simulink. The parameters of the SOP are listed in Table 1 [14], and the controllers design parameters of the rectifier side and the inverter side are listed in Tables 2 and 3, respectively.

Symbol	Mean	Value
e_{1abc}	Grid-phase voltage	220 V
e_{2abc}	Load-grid-phase voltage	220 V
$F_{1,2}$	Voltage frequency	50 Hz
u_{dc}	DC-side voltage	850 V
С	DC-side capacitor	5000 µf
R_1	Grid resistance	0.01 Ω
R_2	Load-grid resistance	0.01 Ω
L_1	Grid inductances	20 mH
L_2	Load-grid inductance	20 mH

Table 1. The parameters of the SOP.

Table 2. Controller design parameters for the rectifier side.

Control	Outer-Loop Controller	Inner-Loop Controller
MPC	PI controller $K_p = 50, K_i = 0.1$	Model predictive controller
LSM	$k_1 = 4000$	$k_2 = 6000$
FOSM	$c_1 = 155, k_1 = 15,000$	$c_{21} = c_{22} = 8000, k_2 = 15,000, q/p = 3/5$

Table 3. Controller design parameters for the inverter side.

Control	Controller
MPC	Model predictive controller
LSM	k = 5000
FOTSM	$c_{31} = c_{32} = 1000, \ k_3 = 6000, \ q/p = 3/5$

5.2. Simulation Results

5.2.1. Start-up Response in the Case of Parameter Perturbation

To characterize the parameter perturbation, the resistances and inductances in the rectifier and inverter sides are set to 300% of the original value, and the capacitance in the DC side is set to 150% of the original value. The simulation starts at 0 s and runs for 0.5 s in total. The reference value of the DC-side voltage u_{dc} and the load grid current i_{2d} are 850 V and 40 A, respectively. The DC-side voltage response under the the MPC, the LSM and the FOTSM controllers is depicted in Figure 5. It can be seen from Figure 5 that the settling time of the DC-side voltage under the MPC, the LSM and the FOTSM is 0.096 s, 0.061 s and 0.032 s, respectively, which means that the proposed FTSM controller has the most rapid dynamic response. The DC-side voltage response under the MPC controller cannot be forced to reach the reference value and has a steady-state error of 2.5 V. The MPC controller cannot satisfy the control requirements in the SOP. Figure 6 shows the *d*-axis current response in the rectifier side. The voltage response under the MPC controller fails to track the reference value in the case of parameter perturbation due to the controller's characteristics of model dependance. This also results in the steady-state error in the DC-side voltage response. In Figure 5, it is obvious that the DC-side voltage response under the LSM and the FOSM tracks the reference value. This demonstrates that the two controllers can compensate for the uncertainties caused by the parameter perturbation. It can be seen from Figure 6 that the output of the LSM controller contains chattering, and the output of the FOSM controller is smooth thanks to the integral-type control law. The load grid active and reactive power responses are shown in Figures 7 and 8. In the case of parameter perturbation, the MPC controllers cannot force the active and reactive power-tracking errors to converge to zero. It is evident that the ripple of the active and reactive power under the FOSM controller is smaller than the LSM controller in Figure 7 because of the integral-type control law and the adaptive switching gain.



Figure 5. The DC-side voltage responses under the MPC, the LSM and the FOSM.



Figure 6. Therectifier side *d*-axis current responses under the MPC, the LSM and the FOSM.



Figure 7. The load grid active power responses under the MPC, the LSM and the FOTSM.



Figure 8. Theload grid reactive power responses under the MPC, the LSM and the FOTSM.

5.2.2. Load-Adding Response in the Case of External Disturbance

To attest the excellent dynamic performance and robustness of the proposed control method, the load grid current i_{2d} is changed from 40 A to 80 A at 0.2 s, and the external disturbance $\rho_1(t) = 5 \sin (100t)$ and $\rho_2(t) = diag(100 \sin (100t), 100 \sin (100t))$ are added at 0.4 s. The parameters of the resistances, inductances and capacitance are set to the original value. The simulation starts at 0 s and runs for 0.6 s in total. From Figure 9, the setting time of the DC-side voltage response under the MPC, the LSM and the FOSM are 0.094 s, 0.0017 s and 0.0026 s, respectively. Moreover, the DC-side voltage drop under the three control methods are 7.2 V, 2.7 V and 2.6 V, which means that the proposed FOSM controller has a better dynamic performance in anti-disturbance and rapidness. After the external disturbance $\rho_1(t)$ is added at 0.4 s, it is obvious that the outer-loop PI controller cannot suffer the external disturbance in the form of sinusoid. The FOSM controller can deal with the added external disturbance $\rho_1(t)$ and force the DC-side voltage-tracking error to converge to zero owing to the integral-type virtual control law. The LSM controller can also eliminate the external disturbance; however, the irreducible chattering exists in the control signal as shown in Figure 10. Figures 11 and 12 show the load grid active and reactive power response under the three control methods. It can be seen that the MPC controller fails to deal with the sinusoidal disturbance $\rho_2(t)$, which results in the tracking error after the external disturbance is added at 0.4 s. The LSM and FOTSM controllers compensate for the external disturbance completely and force the tracking error to converge to zero. However, the chattering still exists in the control law of the LSM and leads to a larger rippler than the FOTSM in the active and reactive power responses as shown in Figures 10 and 11.



Figure 9. The DC-side voltage responses under the MPC, the LSM and the FOTSM.



Figure 10. Therectifier-side *d*-axis current responses under the MPC, the LSM and the FOTSM.



Figure 11. The load grid active responses under the MPC, the LSM and the FOTSM.



Figure 12. The load grid reactive power responses under the MPC, the LSM and the FOTSM.

6. Discussion

In this paper, a full-order sliding-mode control algorithm is designed in u_{dc} -Q and P-Q working modes, and a corresponding simulation verification is carried out. However, the dynamic process of switching between different working modes is not taken into account. Combined with the dynamic process of the SOP, it will be the aim of a future work to verify the control effect of the proposed method in an electricity distribution network.

7. Conclusions

In this paper, a full-order terminal sliding-mode control theory is proposed to enhance the rapidness and robustness of the rectifier-side and the inverter-side controllers in a soft open point converter. The matched and unmatched uncertainties in the control system including external disturbances and parameter perturbation are considered in detail. The main contributions of the paper can be summarized as: (1) The proposed virtualcontrol-techinique-based full-order sliding-mode control method can improve the dynamic performance and anti-disturbance of the SOP control system with matched and unmatched uncertainties. (2) The integral-type control law eliminates the chattering and guarantees the current references and smooth responses . Finally, the simulations have demonstrated the effectiveness of the proposed full-order sliding-mode controllers.

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Abbreviations

The following abbreviations are used in this manuscript:

SOP	Soft open point
VSC	Voltage source converter
AC	Alternating current
DC	Direct current
LSM	Linear sliding-mode
FOTSM	Full-order terminal sliding-mode
FOSM	Full-order sliding-mode
PI	Practical proportional-integral
MPC	Model predictive control
SMC	Sliding-mode control
ISMC	Integral sliding-mode control
DOBSMC	Disturbance observer-based sliding-mode control
SVPWM	Signals of space vector pulse width modulation
PLL	Phase look loop

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