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Enhanced Stability Criteria of Network-Based Load Frequency Control of Power Systems with Time-Varying Delays

Wenxi Feng ^{1,2}, Yanshan Xie ^{1,*}, Fei Luo ¹, Xianyong Zhang ³ and Wenyong Duan ⁴

- ³ School of Automation, Guangdong Polytechnic Normal University, Guangzhou 510665, China; zhangfriendjun@163.com
- ⁴ School of Electrical Engineering, Yancheng Institute of Technology, Yancheng 224051, China; dwy1985@ycit.edu.cn
- * Correspondence: 201911002882@mail.scut.edu.cn

Abstract: The stability problem for load frequency control (LFC) of power systems with two timevarying communication delays is studied in this paper. The one-area and two-area LFC systems are considered, respectively, which are modeled as corresponding linear systems with additive timevarying delays. An improved stability criterion is proposed via a modified Lyapunov-Krasovskii functional (LKF) approach. Firstly, an augmented LKF consisting of delay-dependent matrices and some single-integral items containing time-varying delay information in two different delay subintervals is constructed, which makes full use of the coupling information between the system states and time-varying delays. Secondly, the novel negative definite inequality equivalent transformation lemma is used to transform the nonlinear inequality to the linear matrix inequality (LMI) equivalently, which can be easily solved by the MATLAB LMI-Toolbox. Finally, some numerical examples are presented to show the improvement of the proposed approach.

Keywords: load frequency control; Lyapunov stability theory; stability analysis; time-varying delays

1. Introduction

In order to maintain the power grid frequency (an important index of power quality) fixed or within a small allowable range, a load frequency control (LFC) strategy is a common technique equipped in the power systems [1–7]. With the development and expansion of the power grid, a dedicated independent communication network has been unable to meet the operation of the power grid, although the small transmission delay in the dedicated independent communication network can be ignored [8]. At present, the LFC scheme receives sensor signals and outputs control signals through an open communication network with a mass of data and extensive information exchange. However, random delays and data packets will be introduced into the LFC scheme through the open communication network, which are not negligible and important [9]. These network factors result in the LFC system performance degradation and even instability. Literature [10] pointed out that even if there is a time delay less than 100 ms in the process of information measurement and control output, the transient excitation controller of a generator cannot achieve the control goal. Thus, it is necessary to study the influence of time-varying delays on the performance of the LFC system in an open communication network.

The network controlled LFC system involves the transmitting data between controller and plant. Therefore, there are two main cases of time-varying delays: on the one hand, only the communication time-varying delay from the control center to the governor is considered [11–18], where delay-dependent stability analysis and controller design are investigated by using single delay to model all time delays arising in communication



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College of Automation Science and Technology, South China University of Technology, Guangzhou 510640, China; aujiamingxu@mail.scut.edu.cn (W.F.); aufeilluo@scut.edu.cn (F.L.)
 Guangzhou Institute of Standardization, Guangzhou 510050, China

channels. In fact, the time delays arising in the feedback measurement channel and those in the forward control channel may have different properties. It has more useful guidelines with considering the different properties. Thus, on the other hand, not only the communication time-varying delays from the control center to the governor but also from the sensor to the LFC center are considered simultaneously [19–21], where general delay-dependent stability analysis is studied by using additive time-varying delays to model two different time delays arising in communication channels. In this way, the stability problems of the LFC system with two transmission delays can be investigated by using the method of stability analysis for general linear time-delayed systems.

The main analytical purpose is obtaining the stability condition and controller design method via Lyapunov stability theory application. The stability criterion based on Lyapunov stability theory is a sufficient condition, and inevitably has certain conservativeness. There are two main reasons for the conservativeness: the construction of LKF and techniques for estimating the upper bound of the derivative of LKF. Thus, there are many methods and techniques given to address these two aspects. For the construction of LKF, a LKF with the delay decomposition method [22-24], LKF with multiple integral items [25–29] and LKF with some augmented vectors [30,31] are proposed. On the other aspect, the Jensen inequality [32], B-L inequality [33] and relaxed integral inequality techniques [34–38] are used to estimate the upper bound of the derivative of LKF. In order to reduce the conservativeness of the LKF construction, a lot of coupling information between the system state variables and time delays is introduced into the LKF, which leads to some nonlinear terms in the final results. This makes the solution complex and even unsolvable. Recently, a novel negative definite inequality equivalent transformation lemma was proposed in [39], which improved the degree of freedom for solving the linear matrix inequality (LMI) in the main theorem without conservativeness. Thus, according to the development of stability methods for linear time-delayed systems, there is still space to further reduce the conservativeness of stability criteria for the LFC system.

Inspired by the above analysis, the contributions of this paper can be summarized as follows:

- As mentioned above, only one-area LFC system with two different time-varying delays is considered. It is general and important to investigate the stability of two-area or multi-area LFC system with two or more time-varying delays. This paper studies one- and two-area LFC system with two time-varying delays.
- The main improvements of the LKF are summarized as: (a) introducing four delaydependent non-integral terms to the LKF, such as $S_i(t)$, (i = 1, 2, 3, 4); (b) introducing some integral components to the single-integral terms under different time-varying delay subintervals, such as $\int_{t-h_{1t}}^{s} x(\theta) d\theta$, $\int_{t-h_1}^{t-h_{1t}} x(\theta) d\theta$, $\int_{t-h_t}^{s} x(\theta) d\theta$, $\int_{t-h}^{t-h_t} x(\theta) d\theta$, and so on. These improvements make the LKF contain more information (the time-varying delays and the coupling information between the state variables and the time-varying delay) than the literature [11,17,18,21], which reduces the conservativeness caused by the LKF construction.
- To overcome the nonlinear matrix inequality in the stability criterion, the novel negative definite inequality equivalent transformation lemma proposed in [39] is used to transform the nonlinear inequality to the LMI equivalently, which can be easily solved by the MATLAB LMI-Toolbox.

In this paper, the stability problem for LFC power systems with two time-varying communication delays is studied. Both one-area and two-area LFC systems are considered, respectively. An improved stability criterion is proposed via a modified LKF approach. Firstly, an augmented LKF consisting of delay-dependent matrices and some single-integral items containing time-varying delay information in two different delay subintervals is constructed. Secondly, a novel negative definite inequality equivalent transformation lemma is used to transform the nonlinear inequality to the LMI equivalently, which can be easily solved by the MATLAB LMI-Toolbox. Finally, some numerical examples are presented to show the improvement of the proposed approach. Moreover, the stability

results can be applied to the LFC optimization to guarantee the stable operation of power system based on an open interconnect network control.

This paper is organized as follows. Section 2 gives the models of LFC schemes; Section 3 provides a stability assessment for the LFC system. Section 4 shows numerical examples. Conclusions are drawn in Section 5.

Notation 1. Throughout this paper, the notations are standard. \mathbb{R}^n denotes the n-dimensional Euclidean space; $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices; For $P \in \mathbb{R}^{n \times n}$, P > 0 (respectively, P < 0) mean that P is a positive (respectively, negative) definite matrix. diag $\{a_1, a_2, \dots, a_n\}$ denotes an n-order diagonal matrix with diagonal elements a_1, a_2, \dots, a_n . e_i $(i = 1, \dots, m)$ are

block entry matrices. For example, $e_2 = \begin{bmatrix} 0 & I & \underbrace{0 \cdots 0}_{m-2} \end{bmatrix}$. For a real matrix B and two real symmetric matrices A and C of appropriate dimensions, $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ denotes a real symmetric matrix, where * denotes the entries implied by symmetry. Sym $\{A\} = A + A^T$.

2. System Description and Problem Preliminaries

2.1. One-Area LFC System

In this subsection, the model of one-area power system equipped with PI controllers and taking into account the time-varying communication delays is given. The basic diagram of the simplified LFC of one-area power system is shown in Figure 1, where e^{-sd_1} and e^{-sd_2} are time delays, respectively, arising during the measured signal Δf transmitted from sensor to the LCF center and the control signal sent from the control center to the governor.



Figure 1. The basic diagram of the simplified LFC of one-area power system.

According to the LFC system shown in [19] and Figure 1, the common LFC scheme model of one-area can be expressed as follows:

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}\Delta P_c(t), \\ \bar{y}(t) = \bar{C}\bar{x}(t), \end{cases}$$
(1)

where

$$\begin{split} \bar{x}(t) &= \begin{bmatrix} \Delta f(t) \\ \Delta P_m(t) \\ \Delta P_v(t) \end{bmatrix}, \ \bar{y}(t) = ACE(t), \\ \bar{A} &= \begin{bmatrix} -\frac{D}{M} & \frac{1}{M} & 0 \\ 0 & -\frac{1}{T_{ch}} & \frac{1}{T_{ch}} \\ -\frac{1}{RT_g} & 0 & -\frac{1}{T_g} \end{bmatrix}, \ \bar{B} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_g} \end{bmatrix}, \ \bar{C} = [\beta \ 0 \ 0]. \end{split}$$

Here, $\Delta f(t)$, $\Delta P_m(t)$ and $\Delta P_v(t)$ are the frequency deviation, the mechanical output change, and the valve position change, respectively; *M* and *D* are the moment of inertia of the generator and generator damping coefficient, respectively; T_g and T_{ch} are the time constant of the governor and the turbine, respectively; *R* is the speed droop; $\Delta P_C(t)$ is the setpoint; and β is the frequency bias factor. The following PI controller is used as the LFC scheme:

$$u(t) = -K_P A C E(t) - K_I \int A C E(t) dt,$$
(2)

where K_P and K_I are PI gains; and the ACE(t) is the area control error. Due to the existence of time-varying delays ($h_1(t)$ and $h_2(t)$) in feedback and forward channels, respectively, the following is obtained

$$\Delta P_{\rm C}(t) = u(t - h_2(t)), \ ACE(t) = \beta \Delta f(t - h_1(t)).$$
(3)

By defining virtual state and measurement output vector as $y(t) = col\{ACE(t), \int ACE(t)dt\}$ and $x(t) = col\{\Delta f(t), \Delta P_m(t), \Delta P_v(t), \int ACE(t)dt\}$, the closed-loop LFC system can be expressed as the following linear system with two additive time-varying delays:

$$\dot{x}(t) = Ax(t) + A_1 x(t - h_1(t) - h_2(t))$$
(4)

and the system parameters are listed in the following form

$$A = \begin{bmatrix} -\frac{D}{M} & \frac{1}{M} & 0 & 0\\ 0 & -\frac{1}{T_{ch}} & \frac{1}{T_{ch}} & 0\\ -\frac{1}{RT_g} & 0 & -\frac{1}{T_g} & 0\\ \beta & 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ -\frac{K_P\beta}{T_g} & 0 & 0 & -\frac{K_I}{T_g}\\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

2.2. Two-Area LFC System

In this subsection, models of two-area power system equipped with PI controllers and taking into account the time-varying communication delays are given. The basic diagram of the simplified LFC of two-area power system is shown in Figure 2, where ΔP_{12} and T_{12} are the tie-line power transfer and synchronising coefficient of the tie-line. According to the LFC system shown in [20] and Figure 2, the ACE can be expressed as follows:

$$ACE_1(t) = \Delta P_{12}(t) + \beta_1 \Delta f_1(t), \tag{5}$$

$$ACE_2(t) = -\Delta P_{12}(t) + \beta_2 \Delta f_2(t) \tag{6}$$

and the closed-loop LFC system with PI controllers and time-varying communication delays can be described as follows:

$$\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{A}_1\hat{x}(t - h_1(t) - h_2(t)),$$
(7)

where

$$\hat{x}(t) = \operatorname{col}\left\{\Delta f_{1}(t), \Delta P_{m1}(t), \Delta P_{v1}(t), \int ACE_{1}(t)dt, \Delta P_{12}(t), \Delta f_{2}(t), \Delta P_{m2}(t), \Delta P_{v2}(t), \int ACE_{2}(t)dt\right\},$$
$$\hat{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad \hat{A}_{1} = \begin{bmatrix} A_{111} & A_{112} \\ A_{121} & A_{122} \end{bmatrix},$$





As discussed in the Introduction section, the communication channel may encounter time delays, which are usually time-varying and bounded. Then, similar to the previous work, the delays are expressed by time-varying functions satisfying the following conditions:

$$0 \le h_1(t) \le h_1, \ 0 \le h_2(t) \le h_2,$$

$$|\dot{h}_1(t)| \le \mu_1, \ |\dot{h}_2(t)| \le \mu_2, \quad \forall t \ge 0,$$
(8)

where h_1 , h_2 , μ_1 and μ_2 are positive constants. Let $h(t) = h_1(t) + h_2(t)$ and $h = h_1 + h_2$.

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3. Stability Assessment for LFC System

Lemma 1 ([33]). For a positive definite matrix R and differentiable function x in $[a, b] \rightarrow \mathbb{R}^n$, the following holds

$$\int_{a}^{b} \dot{x}^{T}(s) R \dot{x}(s) ds \geq \frac{1}{b-a} \omega^{T} \mathcal{R} \omega,$$

where $\mathcal{R} = diag\{R, 3R, 5R\}$, $\omega = col\{\omega_1, \omega_2, \omega_3\}$ with $\omega_1 = x(b) - x(a)$, $\omega_2 = x(b) + x(a) - \frac{2}{b-a} \int_a^b x(s) ds$, $\omega_3 = \omega_1 - \frac{6}{b-a} \int_a^b x(s) ds + \frac{12}{(b-a)^2} \int_a^b (b-s)x(s) ds$.

Lemma 2 ([30]). For positive definite matrices $R_1, R_2 \in \mathbb{R}^{n \times n}$, vectors $v_1, v_2 \in \mathbb{R}^n$ and a scalar $\alpha \in [0, 1]$, if there exists symmetric matrices $X_1, X_2 \in \mathbb{R}^{n \times n}$ and any matrices $S_1, S_2 \in \mathbb{R}^{n \times n}$ such that

$$\begin{bmatrix} R_1 - X_1 & S_1 \\ * & R_1 \end{bmatrix} \ge 0, \quad \begin{bmatrix} R_2 - X_2 & S_2 \\ * & R_2 \end{bmatrix} \ge 0,$$

the following inequality holds

$$\frac{1}{\alpha}v_1^T R_1 v_1 + \frac{1}{1-\alpha}v_2^T R_2 v_2 \ge v_1^T [R_1 + (1-\alpha)X_1]v_1 + v_2^T [R_2 + \alpha X_2]v_2 + 2v_1^T [\alpha S_1 + (1-\alpha)S_2]v_2.$$

Lemma 3 ([39]). Let symmetric matrices A_0 , A_1 , $A_2 \in \mathbb{R}^{m \times m}$ and a vector $\zeta \in \mathbb{R}^m$. Then, the following inequality

$$\zeta^T (h_t^2 A_2 + h_t A_1 + A_0) \zeta < 0$$

holds for all $h_t \in [0,h]$ if and only if there exists a positive definite matrix $D \in \mathbb{R}^{m \times m}$ and a skew-symmetric matrix $G \in \mathbb{R}^{k \times k}$ such that

$$\begin{bmatrix} A_0 & \frac{1}{2}A_1 \\ * & A_2 \end{bmatrix} < \begin{bmatrix} C \\ J \end{bmatrix}^T \begin{bmatrix} -D & G \\ * & D \end{bmatrix} \begin{bmatrix} C \\ J \end{bmatrix},$$

where $C = \begin{bmatrix} \frac{h}{2}I & 0 \end{bmatrix}$ and $J = \begin{bmatrix} \frac{h}{2}I & -I \end{bmatrix}$.

Theorem 1. Given scalars h_1 , h_2 , μ_1 and μ_2 , the system (7) satisfying condition (8) is asymptotically stable, if there exists positive definite matrices S_{i2} , $Q_j \in \mathbb{R}^{3n \times 3n}$, $Q_k \in \mathbb{R}^{8n \times 8n}$, $R_i \in \mathbb{R}^{n \times n}$, \mathcal{D}_i , $\mathcal{G}_i \in \mathbb{R}^{18n \times 18n}$, symmetric matrices S_{i1} , $X_i \in \mathbb{R}^{3n \times 3n}$, any matrices $Y_i \in \mathbb{R}^{3n \times 3n}$, (i = 1, 2, 3, 4; j = 1, 2; k = 3, 4), such that the following matrix inequalities hold for $\dot{h}_{1t} \triangleq \mu_j \in \{-\mu_1, \mu_1\}$, $\dot{h}_t \triangleq \mu_k \in \{-\mu, \mu\}$.

$$h_1 S_{j1} + S_{j2} > 0, \ h S_{k1} + S_{k2} > 0,$$
 (9)

$$\begin{bmatrix} \bar{R}_i - X_i & Y_i \\ * & \bar{R}_i \end{bmatrix} > 0, \tag{10}$$

$$\begin{bmatrix} \Omega_{10}(\mu_j) & \frac{1}{2}\Omega_{11}(\mu_j) \\ * & \Omega_{21}(\mu_j) \end{bmatrix} - \begin{bmatrix} \mathcal{C} \\ \mathcal{J} \end{bmatrix}^T \begin{bmatrix} -\mathcal{D}_j & \mathcal{G}_j \\ * & \mathcal{D}_j \end{bmatrix} \begin{bmatrix} \mathcal{C} \\ \mathcal{J} \end{bmatrix} < 0,$$
(11)

$$\begin{bmatrix} \Omega_0(\mu_k) & \frac{1}{2}\Omega_1(\mu_k) \\ * & \Omega_2(\mu_k) \end{bmatrix} - \begin{bmatrix} \mathcal{C} \\ \mathcal{J} \end{bmatrix}^I \begin{bmatrix} -\mathcal{D}_k & \mathcal{G}_k \\ * & \mathcal{D}_k \end{bmatrix} \begin{bmatrix} \mathcal{C} \\ \mathcal{J} \end{bmatrix} < 0$$
(12)

with $C = \begin{bmatrix} \frac{h}{2}I0 \end{bmatrix}$, $\mathcal{J} = \begin{bmatrix} \frac{h}{2}I - I \end{bmatrix}$, $\bar{R}_i = \text{diag}\{R_i, 3R_i, 5R_i\}$,

$$\begin{split} &\Omega_{10}(\mu_{j}) = \dot{h}_{11}\Delta_{11}^{T}S_{11}\Delta_{11} - \dot{h}_{11}\Delta_{21}^{T}S_{21}\Delta_{21} - h_{1d}\Delta_{33}^{T}Q_{1}\Delta_{53} \\ &+ h_{1d}\Delta_{63}^{T}Q_{2}\Delta_{63} + h_{1}^{2}h_{1d}e_{7}^{T}(R_{1} - R_{2})e_{7}, \\ &\Omega_{11}(\mu_{j}) = \operatorname{Sym}\left\{\Delta_{13}^{T}S_{11}\Delta_{12} - \Delta_{23}^{T}S_{21}\Delta_{22} + \Delta_{92}^{T}Q_{1}\Lambda_{1} + \Lambda_{12}^{T}Q_{2}\Lambda_{2}\right\} + \dot{h}_{1t}\Delta_{12}^{T}S_{11}\Delta_{12} \\ &- \dot{h}_{1t}\Delta_{22}^{T}S_{21}\Delta_{22} + \Delta_{52}^{T}Q_{1}\Delta_{52} - \Delta_{62}^{T}Q_{2}\Delta_{62} + \Delta_{72}^{T}Q_{3}\Delta_{72} - \Delta_{82}^{T}Q_{4}\Delta_{82} \\ &- h_{1d}\Delta_{54}^{T}Q_{1}\Delta_{54} + h_{1d}\Delta_{64}^{T}Q_{2}\Delta_{64}, \\ &\Omega_{21}(\mu_{j}) = \operatorname{Sym}\left\{\Delta_{13}^{T}(S_{11}\Delta_{11} + S_{12}\Delta_{12}) + h_{1t}\Delta_{11}^{T}S_{11}\Delta_{12} + \Delta_{23}^{T}[(h_{1}S_{21} + S_{22})\Delta_{22} - S_{21}\Delta_{21}] \\ &- \dot{h}_{1t}\Delta_{22}^{T}S_{21}\Delta_{21} + \Delta_{51}Q_{1}\Delta_{52} - \Delta_{61}Q_{1}\Delta_{62} - h_{1d}\Delta_{53}Q_{1}\Delta_{54} + h_{1d}\Delta_{63}Q_{2}\Delta_{64} \\ &+ \Delta_{91}^{T}Q_{1}\Lambda_{1} + \Lambda_{11}^{T}Q_{2}\Lambda_{2}\right\} - h_{1h}h_{1d}e_{7}^{T}(R_{1} - R_{2})e_{7} - \frac{1}{h_{1}}\left[\Gamma_{1}^{T}X_{1}\Gamma_{1} - \Gamma_{2}^{T}X_{2}\Gamma_{2}\right] \\ &+ \operatorname{Sym}\left\{\frac{1}{h_{1}}\left[\Gamma_{1}^{T}(Y_{1} - Y_{2})\Gamma_{2}\right]\right\}, \\ &\Omega_{0}(\mu_{k}) = \operatorname{Sym}\left\{\Delta_{13}^{T}S_{12}\Delta_{11} + \Delta_{23}^{T}(h_{1}S_{21} + S_{22})\Delta_{21} + \Delta_{33}^{T}S_{32}\Delta_{31} + \Delta_{43}^{T}(hS_{41} + S_{42})\Delta_{41} \\ &+ \Delta_{90}^{T}Q_{1}\Lambda_{1} + \Lambda_{10}^{T}Q_{2}\Lambda_{2} + \Lambda_{20}^{T}Q_{3}\Lambda_{3} + \Lambda_{30}^{T}Q_{4}\Lambda_{4}\right\} + h_{t}\Delta_{31}^{T}S_{31}\Delta_{31} \\ &- h_{t}\Delta_{41}^{T}S_{41}\Delta_{41} + \Delta_{51}^{T}Q_{1}\Delta_{51} - \Delta_{61}^{T}Q_{2}\Delta_{61} + \Delta_{71}^{T}Q_{3}\Delta_{71} - \Delta_{81}^{T}Q_{4}\Delta_{81} \\ &- h_{d}\Delta_{73}^{T}Q_{3}\Delta_{73} + h_{d}\Delta_{83}^{T}Q_{4}\Delta_{83} + e_{6}^{T}(h_{1}^{2}R_{2} + h^{2}R_{4})e_{6} + h^{2}h_{d}e_{7}^{T}(R_{3} - R_{4})e_{7} \\ &+ \operatorname{Sym}\left\{\Gamma_{1}^{T}Y_{1}\Gamma_{2} + \Gamma_{3}^{T}Y_{3}\Gamma_{4}\right\} + \operatorname{Sym}\left\{\Delta_{0}^{T}U\Pi_{0}\right\}, \\ &\Omega_{1}(\mu_{k}) = \operatorname{Sym}\left\{\Delta_{33}^{T}S_{31}\Delta_{31} + \Delta_{33}^{T}S_{32}\Delta_{32} + h_{t}\Delta_{31}^{T}S_{31}\Delta_{32} + \Delta_{43}^{T}(hS_{41} + S_{42})\Delta_{42} \\ &- \Delta_{43}^{T}S_{41}\Delta_{41} - h_{t}\Delta_{42}^{T}S_{41}\Delta_{41} + \Lambda_{22}^{T}Q_{3}\Lambda_{3} + \Lambda_{32}^{T}Q_{4}\Lambda_{4}\right\} + \Delta_{72}^{T}Q_{3}\Delta_{72} \\ &- \Delta_{82}^{T}Q_{4}\Delta_{82} - h_{t}\Delta_{74}^{T}Q_{3}\Delta_{74} + h_{d}\Delta_{84}^{T}Q_{4}\Delta_$$

$$\begin{split} \Omega_{2}(\mu_{k}) = & \operatorname{Sym}\left\{\Delta_{33}^{T}S_{31}\Delta_{32} - \Delta_{43}^{T}S_{41}\Delta_{42} + \Lambda_{22}^{T}Q_{3}\Lambda_{3} + \Lambda_{33}^{T}Q_{4}\Lambda_{4}\right\} \\ & + \dot{h}_{t}\Delta_{32}^{T}S_{31}\Delta_{32} - \dot{h}_{t}\Delta_{42}^{T}S_{41}\Delta_{42} + \Delta_{72}^{T}Q_{3}\Delta_{72} \\ & - \Delta_{82}^{T}Q_{4}\Delta_{82} - h_{d}\Delta_{74}^{T}Q_{3}\Delta_{74} + h_{d}\Delta_{84}^{T}Q_{4}\Delta_{84}, \end{split}$$

where the notations of other symbols and matrices can be found in Appendices A and B.

Proof. Refer to Appendix C. \Box

Remark 1. The stability sufficient condition of the LFC system (4) is obtained in Theorem 1, which can guarantee the stability of the LFC system (4) in the range of the maximum allowable time delay. Compared with the literature [11,17,18,21], the Theorem 1 in this paper further reduces the conservativeness via the augmented LKF application. The main improvements of the LKF are summarized as: (a) introducing four delay-dependent non-integral terms to the LKF, such as $S_i(t)$, (i = 1, 2, 3, 4); (b) introducing some integral components to the single-integral terms under different time-varying delay subintervals, such as $\int_{t-h_{1t}}^{s} x(\theta) d\theta$, $\int_{t-h_{1}}^{t-h_{1t}} x(\theta) d\theta$, $\int_{t-h_{t}}^{s} x(\theta) d\theta$, $\int_{t-h_{t}}^{t-h_{t}} x(\theta) d\theta$, and so on. These improvements make the LKF contain more information of the time-varying delay and the coupling information between the state variables and the delay than the literature [11,17,18,21], which reduces the conservativeness caused by the LKF construction.

Remark 2. It is well known that in order to improve the conservativeness of the stability criterion, the modified LKF must combine with some tight inequality technique [36]. Therefore, Lemmas 1 and 2 are used to estimate the derivative of the LKF, where some effective auxiliary functions and free-weight matrices are introduced into the upper bound on the derivative of the LKF. Finally, Theorem 1 based on the modified LKF and two proper inequality techniques is less conservative than those of the recently published literature [11,17,18,21].

Remark 3. However, according to the inequality (A8) in the proof process, the final form of the upper bound on the derivative of the LKF is nonlinear due to $h^2(t)\Omega_2(\dot{h}_t)$, which cannot be solved by MATLAB. The authors of [25] decomposed this nonlinear matrix inequality into three equivalent LMI conditions via the sufficient condition constraint lemma application. It reduced the degree of freedom for solving the matrix inequality in the main theorem, due to the introduction of two additional LMI constraints. Moreover, the sufficient condition constraint lemma is quite conservative [39]. To overcome the nonlinear matrix inequality in the stability criterion, the novel negative definite inequality equivalent transformation lemma (Lemma 3) proposed in [39] is used to transform the inequality (A8) to the LMI (12) equivalently, which can be easily solved by the MATLAB LMI-Toolbox.

4. Results and Discussions

In this section, the effectiveness of the stability criterion proposed in this paper is shown. For different K_P and K_I values, the maximum allowable time-delay upper bound values (MADUB) can be obtained by solving the LMIs in Theorem 1 via Matlab LMI-Toolbox. The LFC system parameters in Table 1 are given in [20]. One and two-area LFC systems will be discussed and comparatively analyzed in the following subsections. In the following subsections, '-' in the tables indicates that the corresponding result is not given.

Table 1. LFC systems parameters with $T_{12} = 0.1986$ pu/rad.

	$T_{ch}(s)$	β	R	$T_g(S)$	D	M(s)
Area 1	0.3	21	0.05	0.1	1	10
Area 2	0.4	21.5	0.05	0.17	1.5	12

4.1. One-Area LFC System

4.1.1. Conservativeness Comparisons

In order to compare with the existing results, Tables 2 and 3 give the MADUB values of the case of fixed K_P and K_I values, $K_P = 0.2$, $K_I = 0.1$, $|\dot{h}_1(t)| \le 0.1$, $|\dot{h}_2(t)| \le 0.8$. The corresponding results cannot be given in [11,17,18] since the time delays from sensor to the LFC center are ignored. Moreover, to show the PI controller gains influence on the MADUB of the LFC system, Table 4 obtains the MADUB values of the case of different K_P , K_I values, $h_1(t) = 0$ and $\mu_2 = 0$; Table 5 gives the MADUB values of the case of different K_P , K_I values, $h_1(t) = 0$ and $\mu_2 = 0.9$. From these tables, it can observe the results of Theorem 1 are similar to those of [11], however, less conservative than those of [14–16,19–21]. Meanwhile, the MADUB increases with K_P increment at a fixed K_I value, and the MADUB decreases with K_I increment at a fixed K_P value.

Table 2. MAUBs h_2 for given h_1 , K_P and K_I under one-area LFC system.

Methods $\setminus h_1$	1.0	1.2	1.5
[20]	4.803	4.603	4.303
[19]	5.882	5.682	5.383
[21]	5.987	5.878	5.559
Th. 1	6.155	6.046	6.003

Methods $\setminus h_2$	2.0	3.0	4.0
[20]	3.803	2.803	1.803
[19]	4.892	3.886	2.885
[21]	4.997	3.979	3.001
Th. 1	5.016	4.031	3.217

Table 3. MAUBs h_1 for given h_2 , K_P and K_I under one-area LFC system.

Table 4. MAUBs h_2 for $h_1(t) = 0$ and $\mu_2 = 0$ under one-area LFC system.

$\mu_2=0$									
K _P	Methods $\setminus K_I$	0.05	0.1	0.15	0.2	0.4	0.6	1	
	[20]	27.93	13.78	9.06	6.69	3.12	1.91	0.89	
	[14]	27.92	13.77	-	6.69	3.12	1.91	0.88	
	[15]	30.92	15.20	9.96	7.34	3.38	2.04	0.92	
0	[16]	30.91	15.20	_	7.34	3.39	2.05	0.93	
	[11]	30.79	15.14	9.92	7.31	3.37	2.31	0.92	
	Th. 1	30.79	15.17	9.95	7.33	3.38	2.03	0.92	
	[20]	27.87	14.06	9.28	6.87	3.22	1.97	0.93	
0.05	[15]	31.88	15.68	10.28	7.58	3.50	2.12	0.97	
0.05	[11]	31.74	15.62	10.24	7.55	3.49	2.12	0.97	
	Th. 1	31.77	15.67	10.27	7.57	3.50	2.12	0.97	
	[14]	27.03	13.69	_	6.94	3.29	2.02	0.96	
0.1	[20]	27.03	13.68	-	6.94	3.29	2.02	0.96	
	[16]	31.61	16.02	_	7.79	3.61	2.19	1.01	
	Th. 1	32.73	16.08	10.31	7.81	3.62	2.19	1.04	

Table 5. MAUBs h_2 for $h_1(t) = 0$ and $\mu_2 = 0.9$ under one-area LFC system.

			$\mu_2 = 0.9$				
K _P	Methods $\setminus K_I$	0.05	0.1	0.2	0.4	0.6	1
	[14]	20.45	9.963	4.59	1.81	1.01	0.48
	[20]	26.37	12.96	6.25	2.85	1.68	0.74
0	[16]	27.26	13.39	6.43	2.91	1.71	0.75
	[17]	27.50	13.73	6.61	3.02	1.80	0.78
	Th. 1	28.03	14.24	7.12	3.20	1.91	0.79
	[14]	17.39	9.16	4.67	1.85	1.05	0.48
	[20]	20.25	11.07	5.93	2.87	1.75	0.74
0.1	[16]	22.00	12.32	6.59	3.11	1.84	0.75
	[17]	29.51	14.52	7.02	3.23	1.94	0.86
	Th. 1	30.07	14.61	7.13	3.24	1.95	0.87

4.1.2. Simulation Verification

Simulation studies are carried out under an increased step load of 0.1 pu occurring at 1 s, time-varying delays and the following assumptions. The simulation results are shown in Figures 3–6, in which the LFC has achieved its objective, and the control system is stable. In Figure 4, the blue curve is close to the critical stability with $K_P = 0$, $K_I = 0.05$, $h_1(t) = 0$ and $h_2(t) = 30.79$.

For Figure 3, fixed $K_P = 0.2$, $K_I = 0.1$: •

1.
$$h_1(t) = \frac{1}{2}sin(\frac{0.2}{1}t) + \frac{1}{2}, h_2(t) = \frac{6.155}{2}sin(\frac{1.6}{6.155}t) + \frac{6.155}{2};$$

- 2. $h_1(t) = \frac{1.5}{2} sin(\frac{0.2}{3}t) + \frac{1.5}{2}, h_2(t) = \frac{6.003}{2} sin(\frac{1.6}{6.003}t) + \frac{6.003}{2};$ 3. $h_1(t) = \frac{5.016}{2} sin(\frac{0.2}{5.016}t) + \frac{5.016}{2}, h_2(t) = sin(0.8t) + 1;$

- 4. $h_1(t) = \frac{3.217}{2} sin(\frac{0.2}{3.217}t) + \frac{3.217}{2}, h_2(t) = 2sin(0.4t) + 2;$
- For Figure 4, different K_P and fixed $h_1(t) = 0$, $K_I = 0.05$:
 - 1. $K_P = 0, h_2(t) = 30.7;$
 - 2. $K_P = 0, h_2(t) = 30.79;$
 - 3. $K_P = 0.1, h_2(t) = 32.73;$
- For Figure 5, different K_P and fixed $h_1(t) = 0$, $K_I = 0.6$:
 - 1. $K_P = 0, K_I = 0.6, h_2(t) = 2.00;$
 - 2. $K_P = 0, K_I = 0.6, h_2(t) = 2.03;$
 - 3. $K_P = 0.1, K_I = 0.6, h_2(t) = 2.20;$
- For Figure 6, different K_P and fixed $h_1(t) = 0$:

1.
$$K_P = 0, K_P = 0.2, h_2(t) = \frac{7.12}{2} sin\left(\frac{1.8}{7.12}t\right) + \frac{7.12}{2};$$

- 2. $K_P = 0.1, K_P = 0.2, h_2(t) = \frac{7.13}{2} sin(\frac{1.8}{7.13}t) + \frac{7.13}{2};$
- 3. $K_P = 0, K_P = 1, h_2(t) = \frac{0.79}{2} sin(\frac{1.8}{0.79}t) + \frac{0.79}{2};$

4.
$$K_P = 0.1, K_P = 1, h_2(t) = \frac{0.87}{2} sin(\frac{1.8}{0.87}t) + \frac{0.87}{2}$$



Figure 3. Frequency deviation and control error responses of one-area deregulated LFC scheme with $K_P = 0.2$, $K_I = 0.1$ and different time-varying delays.



Figure 4. Frequency deviation and control error responses of one-area deregulated LFC scheme with $K_I = 0.05$, $h_1(t) = 0$ and different K_P , $h_2(t)$.



Figure 5. Frequency deviation and control error responses of one-area deregulated LFC scheme with $K_I = 0.6$, $h_1(t) = 0$ and different K_P , $h_2(t)$.





Figure 6. Frequency deviation and control error responses of one-area deregulated LFC scheme with $h_1(t) = 0$ and different K_P , K_I and time-varying $h_2(t)$.

4.2. Two-Area LFC System

0.15 0.1

0.05

0.01

0.005

-0.05 -0.1 -0.15 -0.2 -0.25

4.2.1. Conservativeness Comparison

Tables 6 and 7 give the MADUB values of the case of fixed K_P and K_I values, $K_P = 0.05$, $K_I = 0.4$ and different μ_1 , μ_2 values. The corresponding results cannot be achieved in [11,17,18] since the time delays from sensor to the load frequency control center are ignored. Table 8 gives the MADUB values of the case of different K_P and K_I values, $h_1(t) = 0$ and $\mu_2 = 0$; and Table 9 give the MADUB values of the case of different K_P and K_I values, $h_1(t) = 0$ and $\mu_2 = 0.5$. From these tables, it is obviously that the MADUB increases with K_P increment at a fixed K_I value, and the MADUB decreases with K_I increment at a fixed K_P value. In addition, the results of Theorem 1 are less conservative than those of [11,15,20].

Table 6. MAUBs h_2 for given h_1 , K_P and K_I under two-area LFC system.

μ2			0.2			0.5			0.8	
Methods	$\mu_1 \setminus h_1$	1	1.5	2	1	1.5	2	1	1.5	2
	0.1	2.122	1.774	1.323	2.051	1.637	1.273	2.015	1.582	1.098
Th. 1	0.2	2.018	1.559	1.064	2.003	1.507	1.018	1.997	1.499	1.001
	0.5	1.992	1.529	1.030	1.973	1.481	0.995	1.895	1.398	0.908

Table 7. MAUBs h_1 for given h_2 , K_P and K_I under two-area LFC system.

μ ₂			0.2			0.5			0.8	
Methods	$\mu_1 \setminus h_2$	1	2	3	1	2	3	1	2	3
Th. 1	0.1 0.2 0.5	2.028 2.016 2.009	1.183 1.027 1.012	0.251 0.079 0.053	1.939 1.921 1.907	1.078 1.017 1.007	0.189 0.077 0.037	1.925 1.910 1.897	1.051 1.009 1.007	0.081 0.054 0.036

$\mu_2=0$									
K _P	Methods $\setminus K_I$	0.05	0.1	0.15	0.2	0.4	0.6	1	
	[20]	27.85	13.70	8.97	6.60	3.00	1.75	0.57	
0	[15]	30.81	15.09	9.84	7.21	3.23	1.84	0.59	
0	[11]	30.695	15.042	9.816	7.196	3.225	1.846	0.585	
	Th. 1	30.83	15.11	9.87	7.24	3.23	1.85	0.62	
	[20]	27.83	14.02	9.21	6.78	3.10	1.81	0.62	
0.05	[15]	31.27	15.57	10.16	7.45	3.35	1.92	0.64	
0.05	[11]	31.643	15.526	10.132	7.433	3.334	1.924	0.638	
	Th. 1	31.67	15.55	10.15	7.44	3.34	1.93	0.65	
	[14]	_	13.65	_	6.88	3.17	1.86	_	
0.1	[20]	_	13.65	-	6.88	3.17	1.86	-	
0.1	[16]	_	15.97	-	7.67	3.47	2.03	-	
	[17]	_	16.01	-	7.68	3.47	2.03	-	
	Th. 1	31.67	16.05	10.16	7.71	3.48	2.03	0.65	

Table 8. MAUBs h_2 for $h_1(t) = 0$ and $\mu_2 = 0$ under two-area LFC system.

Table 9. MAUBs h_2 for $h_1(t) = 0$ and $\mu_2 = 0.5$ under two-area LFC system.

$\mu_2 = 0.5$									
K _P	Methods $\setminus K_I$	0.2	0.4	0.6					
	[20]	5.55	2.36	1.18					
	[14]	6.14	2.68	1.40					
0	[16]	6.41	2.81	1.54					
	[17]	6.66	2.95	1.65					
	Th. 1	6.69	2.97	1.65					
	[20]	5.35	2.55	1.30					
	[14]	6.34	2.83	1.51					
0.1	[16]	6.75	2.84	1.53					
	[17]	7.10	3.16	1.78					
	Th. 1	7.13	3.19	1.78					

4.2.2. Simulation Verification

Simulation studies are carried out under an increased step load of 0.1 pu occurring at 1 s, and time-varying delays and the following assumptions:

For Figure 7, fixed $K_P = 0.05$, $K_I = 0.4$: ٠

1.
$$h_1(t) = \frac{1}{2}sin\left(\frac{0.4}{1}t\right) + \frac{1}{2}, h_2(t) = \frac{2.018}{2}sin\left(\frac{0.4}{2.018}t\right) + \frac{2.018}{2}sin\left(\frac{0.4}{2.01$$

2.
$$h_1(t) = \frac{2}{3}sin(\frac{1}{2}t) + \frac{2}{2}, h_2(t) = \frac{0.908}{2}sin(\frac{1.6}{0.008}t) + \frac{0.908}{2};$$

 $h_1(t) = \frac{2}{2}sin\left(\frac{1}{2}t\right) + \frac{2}{2}, h_2(t) = \frac{0.908}{2}sin\left(\frac{1.6}{0.908}t\right) + \frac{0.908}{2};$ $h_1(t) = \frac{2.028}{2}sin\left(\frac{0.2}{2.028}t\right) + \frac{2.028}{2}, h_2(t) = \frac{1}{2}sin\left(\frac{0.4}{1}t\right) + \frac{1}{2};$ 3.

4.
$$h_1(t) = \frac{0.036}{2} sin\left(\frac{1}{0.036}t\right) + \frac{0.036}{2}, h_2(t) = \frac{3}{2} sin\left(\frac{1.6}{3}t\right) + \frac{3}{2};$$

- 4. $h_1(t) = \frac{1}{2} \sin\left(\frac{1}{0.036}t\right) + \frac{1}{2}, h_2(t) = \frac{1}{2} \sin\left(\frac{1}{3}t\right)$ For Figure 8, different K_P and fixed $h_1(t) = 0, K_I = 0.05$: ٠
 - 1. $K_P = 0, h_2(t) = 30.89;$
 - $K_P = 0, h_2(t) = 30.83;$ 2.
 - $K_P = 0.1, h_2(t) = 31.67;$ 3.
- For Figure 9, different K_P and fixed $h_1(t) = 0$, $K_I = 1$:
 - 1. $K_P = 0, h_2(t) = 0.637;$
 - 2. $K_P = 0, h_2(t) = 0.62;$
 - 3. $K_P = 0.1, h_2(t) = 0.65;$

- For Figure 10, different K_P , K_I , time-varying $h_2(t)$ and fixed $h_1(t) = 0$:
 - 1. $K_P = 0, K_I = 0.2, h_2(t) = \frac{6.69}{2} sin\left(\frac{1}{6.69}t\right) + \frac{6.69}{2};$
 - 2. $K_P = 0.1, K_I = 0.2, h_2(t) = \frac{7.13}{2} sin(\frac{1}{7.13}t) + \frac{7.13}{2};$
 - 3. $K_P = 0, K_I = 0.6, h_2(t) = \frac{1.65}{2} sin\left(\frac{1}{1.65}t\right) + \frac{1.65}{2};$
 - 4. $K_P = 0.1, K_I = 0.6, h_2(t) = \frac{1.78}{2} sin\left(\frac{1}{1.78}t\right) + \frac{1.78}{2}.$

The simulation results are shown in Figures 7–10, in which the LFC has achieve its objective, and the control system is stable. It can be seen from Figures 8 and 9 that the red curves are the critical stability with $K_I = 0.05 K_P = 0$, $h_1(t) = 0$, $h_2(t) = 30.89$ and $K_P = 0$, $K_I = 1$, $h_1(t) = 0$, $h_2(t) = 0.637$, respectively. Thus, the black curvess in Figures 8 and 9 are close to the critical stability region, that is, Theorem 1 proposed in this paper is effective in estimating the upper bound of the maximum allowable time delay.



Figure 7. Frequency deviation and control error responses of two-area deregulated LFC scheme with $K_P = 0.05$, $K_I = 0.4$ and time-varying delays.



Figure 8. Frequency deviation and control error responses of two-area deregulated LFC scheme with $K_I = 0.05$, $h_1(t) = 0$ and different K_P , $h_2(t)$.



Figure 9. Frequency deviation and control error responses of two-area deregulated LFC scheme with $K_I = 1$, $h_1(t) = 0$ and different K_P , $h_2(t)$.



Figure 10. Frequency deviation and control error responses of two-area deregulated LFC scheme with $h_1(t) = 0$ and different K_P , K_I , $h_2(t)$.

5. Conclusions

This paper mainly focuses on the stability analysis for load frequency control of power systems with time-varying delays. For the one-area and two-area LFC systems with two communication delays, stability criteria are obtained via Lyapunov stability theory application. Firstly, the one-area LFC system and two-area LFC system are described as linear systems with additive time-varying delays. Secondly, a modified LKF with some delaydependent non-integral terms and augmented integral components in single integral term is constructed. Compared with the LKFs in some previous published literature, it contains more coupling information between time-varying delays and state variables, which reduces the conservativeness of the stability criterion. Thirdly, to overcome the nonlinear coupling in the stability criterion, the novel negative definite inequality equivalent transformation lemma (Lemma 3) is used to transform the nonlinear inequality to the LMI equivalently, which can be easily solved by the MATLAB LMI-Toolbox. Finally, the effectiveness of the proposed method is illustrated by comparisons and discussions in numerical examples. In addition, in order to approach the actual situation, the design and optimization of the controllers have always been a topic of concern, which inspires us to work around it in the future. The stability results can be applied to the LFC design and optimization to guarantee the stable operation of power system based on an open interconnect network control.

However, there are some limitations. Firstly, in order to reduce the conservativeness of the stability criterion, the dimensions of the decision variables for solving LMIs increase with the expanded dimensions of LKF. Secondly, in order to improve the degree of freedom of solving LMIs, the free weight matrices are introduced. Thirdly, some additional decision variables are introduced in LMIs when the nonlinear inequalities are transformed into LMIs by using Lemma 3, which increases the number of decision variables of the LMIs proposed in this paper. In conclusion, the improvement of stability results is in the cost of increasing computational complexity.

The derivation method of the stability criterion presented in this paper can be extended to multi-area LFC system. The relevant theories will be applied into practice, which is one of our further main topics.

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Appendix A

For the sake of simplicity on matrix representation, the notations of several symbols and matrices are defined as

$$\begin{split} h_{1t} &= h_1(t), \ h_t = h(t), \ \bar{h}_{1t} = h_1 - h_1(t), \ \bar{h}_t = h - h(t), \\ h_{1d} &= 1 - \dot{h}_1(t), \ h_d = 1 - \dot{h}(t), \\ \eta_{01}(t) &= \operatorname{col}\{x(t), x(t - h_{1t}), x(t - h_1)\}, \\ \eta_{02}(t) &= \operatorname{col}\{x(t), x(t - h_{1t}), x(t - h_1)\}, \\ \eta_{1}(t) &= \operatorname{col}\{x(t), x(t - h_{1t}), \int_{t-h_{1t}}^{t} x(s) ds\}, \\ \eta_{2}(t) &= \operatorname{col}\{x(t), x(t - h_{1t}), x(t - h_{1}), \int_{t-h_{1}}^{t-h_{1t}} x(s) ds\}, \\ \eta_{3}(t) &= \operatorname{col}\{x(t), x(t - h_{t}), \int_{t-h_{t}}^{t} x(s) ds\}, \\ \eta_{4}(t) &= \operatorname{col}\{x(t), x(t - h_{t}), \int_{t-h_{t}}^{t} x(s) ds\}, \\ \eta_{5}(t, s) &= \operatorname{col}\{\dot{x}(s), x(s), \eta_{01}(t), \int_{s}^{t} x(\theta) d\theta, \int_{t-h_{1t}}^{s} x(\theta) d\theta, \int_{t-h_{1}}^{t-h_{1t}} x(\theta) d\theta\}, \\ \eta_{6}(t, s) &= \operatorname{col}\{\dot{x}(s), x(s), \eta_{02}(t), \int_{s}^{t-h_{1t}} x(\theta) d\theta, \int_{t-h_{1t}}^{t} x(\theta) d\theta, \int_{t-h_{1}}^{s} x(\theta) d\theta\}, \\ \eta_{7}(t, s) &= \operatorname{col}\{\dot{x}(s), x(s), \eta_{02}(t), \int_{s}^{t-h_{1t}} x(\theta) d\theta, \int_{t-h_{1t}}^{t} x(\theta) d\theta, \int_{t-h_{1}}^{s} x(\theta) d\theta\}, \\ \eta_{8}(t, s) &= \operatorname{col}\{\dot{x}(s), x(s), \eta_{02}(t), \int_{s}^{t-h_{1t}} x(\theta) d\theta, \int_{t-h_{1}}^{t} x(\theta) d\theta, \int_{t-h_{1}}^{s} x(\theta) d\theta\}, \\ \rho_{1}(t) &= \int_{t-h_{1}}^{t-h_{1t}} \frac{x(s)}{\bar{h}_{1t}}, \rho_{2}(t) = \int_{t-h_{1}}^{t-h_{1t}} \frac{(t - h_{1t} - s)x(s)}{\bar{h}_{1}^{2}}, \\ \rho_{5}(t) &= \int_{t-h_{1t}}^{t} \frac{x(s)}{h_{t}}, \rho_{6}(t) = \int_{t-h_{1}}^{t-h_{1t}} \frac{(t - s)x(s)}{h_{1}^{2}}, \\ \rho_{7}(t) &= \int_{t-h_{1}}^{t} \frac{x(s)}{h_{t}}, \rho_{8}(t) = \int_{t-h_{1}}^{t} \frac{(t - s)x(s)}{h_{t}^{2}}, \\ \end{array}$$

$$\begin{aligned} \xi(t) = & \operatorname{col}\{x(t), x(t-h_{1t}), x(t-h_{t}), x(t-h_{1}), x(t-h), \\ \dot{x}(t), \dot{x}(t-h_{1t}), \dot{x}(t-h_{t}), \dot{x}(t-h_{1}), \dot{x}(t-h), \\ \rho_{1}(t), \rho_{2}(t), \rho_{3}(t), \rho_{4}(t), \rho_{5}(t), \rho_{6}(t), \rho_{7}(t), \rho_{8}(t) \}. \end{aligned}$$

Appendix **B**

Notations of other symbols and matrices for the Theorem 1 are given as:

 $\Delta_0 = \operatorname{col}\{e_1, e_3, e_6\}, \ \Pi_0 = Ae_1 + A_1e_3 - e_6,$ $\Delta_{11} = \operatorname{col}\{e_1, e_2, e_0\}, \ \Delta_{12} = \operatorname{col}\{e_0, e_0, e_{13}\}, \ \Delta_{13} = \operatorname{col}\{e_6, e_7, e_1 - h_{1d}e_2\},\$ $\Delta_{21} = \operatorname{col}\{e_2, e_4, h_1e_{11}\}, \Delta_{22} = \operatorname{col}\{e_0, e_0, -e_{11}\}, \Delta_{23} = \operatorname{col}\{h_{1d}e_7, e_8, h_{1d}e_2 - e_4\},\$ $\Delta_{31} = \operatorname{col}\{e_1, e_3, h_1e_0\}, \ \Delta_{32} = \operatorname{col}\{e_0, e_0, e_{17}\}, \ \Delta_{33} = \operatorname{col}\{e_6, h_de_3, e_1 - h_de_3\},\$ $\Delta_{41} = \operatorname{col}\{e_3, e_5, he_{13}\}, \Delta_{42} = \operatorname{col}\{e_0, e_0, -e_{13}\}, \Delta_{43} = \operatorname{col}\{h_d e_8, e_{10}, h_d e_3 - e_5\},$ $\Delta_{51} = \operatorname{col}\{e_6, e_1, e_1, e_2, e_4, e_0, e_0, h_1e_{11}\}, \Delta_{52} = \operatorname{col}\{e_0, e_0, e_0, e_0, e_0, e_0, e_{15}, -e_{11}\},\$ $\Delta_{53} = \operatorname{col}\{e_7, e_2, e_1, e_2, e_4, e_0, e_0, h_1e_{11}\}, \Delta_{54} = \operatorname{col}\{e_0, e_0, e_0, e_0, e_0, e_{15}, e_0, -e_{11}\},\$ $\Delta_{61} = \operatorname{col}\{e_9, e_4, e_1, e_2, e_4, h_1e_{11}, e_0, e_0\}, \Delta_{62} = \operatorname{col}\{e_0, e_0, e_0, e_0, e_0, -e_{11}, e_{15}, e_0\},\$ $\Delta_{63} = \operatorname{col}\{e_7, e_2, e_1, e_2, e_4, e_0, e_0, h_1e_{11}\}, \Delta_{64} = \operatorname{col}\{e_0, e_0, e_0, e_0, e_0, e_0, e_{15}, -e_{11}\},\$ $\Delta_{71} = \operatorname{col}\{e_6, e_1, e_1, e_3, e_5, e_0, e_0, he_{13}\}, \Delta_{72} = \operatorname{col}\{e_0, e_0, e_0, e_0, e_0, e_0, e_{17}, -e_{13}\},\$ $\Delta_{73} = \operatorname{col}\{e_8, e_3, e_1, e_3, e_5, e_0, e_0, he_{13}\}, \Delta_{74} = \operatorname{col}\{e_0, e_0, e_0, e_0, e_0, e_{17}, e_0, -e_{13}\},\$ $\Delta_{81} = \operatorname{col}\{e_{10}, e_5, e_1, e_3, e_5, he_{13}, e_0, e_0\}, \Delta_{82} = \operatorname{col}\{e_0, e_0, e_0, e_0, e_0, -e_{13}, e_{17}, e_0\},\$ $\Delta_{83} = \operatorname{col}\{e_8, e_3, e_1, e_3, e_5, e_0, e_0, he_{13}\}, \Delta_{84} = \operatorname{col}\{e_0, e_0, e_0, e_0, e_0, e_0, e_{17}, -e_{13}\},$ $\Delta_{90} = \operatorname{col}\{e_1 - e_2, e_0, e_0, e_0, e_0, e_0, e_0\}, \Delta_{91} = \operatorname{col}\{e_0, e_{15}, e_1, e_2, e_4, e_0, e_0, h_1e_{11}\},$ $\Delta_{92} = \operatorname{col}\{e_0, e_0, e_0, e_0, e_0, e_{15} - e_{16}, e_{16}, -e_{11}\},\$ $\Lambda_1 = \operatorname{col}\{e_0, e_0, e_6, h_{1d}e_7, e_1 - h_{1d}e_7, e_1, -h_{1d}e_2, h_{1d}e_2 - e_4\},\$ $\Lambda_2 = \operatorname{col}\{e_0, e_0, e_6, h_{1d}e_7, e_9, h_{1d}e_2, e_1 - h_{1d}e_2, -e_4\},\$ $\Lambda_3 = \operatorname{col}\{e_0, e_0, e_6, h_d e_8, e_{10}, e_1, -h_d e_3, h_d e_3 - e_5\},\$ $\Lambda_4 = \operatorname{col}\{e_0, e_0, e_6, h_d e_8, e_{10}, h_d e_3, e_1 - h_d e_3, -e_5\},\$ $\Lambda_{10} = \operatorname{col}\{e_2 - e_4, h_1e_1, h_1e_1, h_1e_2, h_1e_4, h_1^2(e_{11} - e_{12}), e_0, h_1^2e_{12}\},\$ $\Lambda_{11} = \operatorname{col}\{e_0, -e_{11}, -e_1, -e_2, -e_4, -2h_1(e_{11} - e_{12}), h_1e_{15}, -2h_1e_{12}\},\$ $\Lambda_{12} = \operatorname{col}\{e_0, e_0, e_0, e_0, e_0, e_{11} - e_{12}, -e_{15}, e_{12}\},\$ $\Gamma_1 = \operatorname{col}\{e_2 - e_4, e_2 + e_4 - 2e_{11}, e_2 - e_4 + 6e_{11} + 12e_{12}\},\$ $\Gamma_2 = \operatorname{col}\{e_1 - e_2, e_1 + e_2 - 2e_{15}, e_1 - e_2 + 6e_{15} + 12e_{16}\},\$ $\Gamma_3 = \operatorname{col}\{e_3 - e_5, e_3 + e_5 - 2e_{13}, e_3 - e_5 + 6e_{13} + 12e_{14}\},\$ $\Gamma_4 = \operatorname{col}\{e_1 - e_3, e_1 + e_3 - 2e_{17}, e_1 - e_3 + 6e_{17} + 12e_{18}\}.$

Appendix C

Proof of Theorem 1. Construct an LKF candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$
(A1)

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with

$$\begin{split} V_{1}(t) &= \eta_{1}^{T}(t)S_{1}(t)\eta_{1}(t) + \eta_{2}^{T}(t)S_{2}(t)\eta_{2}(t) \\ &+ \eta_{3}^{T}(t)S_{3}(t)\eta_{3}(t) + \eta_{4}^{T}(t)S_{4}(t)\eta_{4}(t), \\ V_{2}(t) &= \int_{t-h_{1t}}^{t} \eta_{5}^{T}(t,s)Q_{1}\eta_{5}(t,s)ds + \int_{t-h_{1}}^{t-h_{1t}} \eta_{6}^{T}(t,s)Q_{2}\eta_{6}(t,s)ds \\ &+ \int_{t-h_{t}}^{t} \eta_{7}^{T}(t,s)Q_{3}\eta_{7}(t,s)ds + \int_{t-h_{1}}^{t-h_{t}} \eta_{8}^{T}(t,s)Q_{4}\eta_{8}(t,s)ds, \\ V_{3}(t) &= h_{1}\int_{t-h_{1}}^{t-h_{1t}} (h_{1t}-t+s)\dot{x}^{T}(s)R_{1}\dot{x}(s)ds + h_{1}\int_{t-h_{1t}}^{t} (h_{1}-t+s)\dot{x}^{T}(s)R_{2}\dot{x}(s)ds \\ &+ h\int_{t-h}^{t-h_{t}} (h_{t}-t+s)\dot{x}^{T}(s)R_{3}\dot{x}(s)ds + h\int_{t-h_{t}}^{t} (h-t+s)\dot{x}^{T}(s)R_{4}\dot{x}(s)ds, \end{split}$$

where $S_1(t) = h_{1t}S_{11} + S_{12}$, $S_2(t) = \bar{h}_{1t}S_{21} + S_{22}$, $S_3(t) = h_tS_{31} + S_{32}$ and $S_4(t) = \bar{h}_tS_{41} + S_{42}$.

Calculating the derivative of V(t), we can obtain the following formulas

$$\begin{split} \dot{V}_{1}(t) &= 2\dot{\eta}_{1}^{T}(t)S_{1}(t)\eta_{1}(t) + \eta_{1}^{T}(t)\dot{S}_{1}(t)\eta_{1}(t) \\ &+ 2\dot{\eta}_{2}^{T}(t)S_{2}(t)\eta_{2}(t) + \eta_{2}^{T}(t)\dot{S}_{2}(t)\eta_{2}(t) \\ &+ 2\dot{\eta}_{3}^{T}(t)S_{3}(t)\eta_{3}(t) + \eta_{3}^{T}(t)\dot{S}_{3}(t)\eta_{3}(t) \\ &+ 2\dot{\eta}_{4}^{T}(t)S_{4}(t)\eta_{4}(t) + \eta_{4}^{T}(t)\dot{S}_{4}(t)\eta_{4}(t) \\ &= 2\xi^{T}(t)\Delta_{13}^{T}(h_{1t}S_{11} + S_{12})(\Delta_{11} + h_{1t}\Delta_{12})\xi(t) \\ &+ \xi^{T}(t)(\Delta_{11} + h_{1t}\Delta_{12})^{T}\dot{h}_{1t}S_{11}(\Delta_{11} + h_{1t}\Delta_{12})\xi(t) \\ &+ 2\xi^{T}(t)\Delta_{23}^{T}(\dot{h}_{1t}S_{21} + S_{22})(\Delta_{21} + h_{1t}\Delta_{22})\xi(t) \\ &- \xi^{T}(t)(\Delta_{21} + h_{1t}\Delta_{22})^{T}\dot{h}_{1t}S_{21}(\Delta_{21} + h_{1t}\Delta_{22})\xi(t) \\ &+ 2\xi^{T}(t)\Delta_{33}^{T}(h_{t}S_{31} + S_{32})(\Delta_{31} + h_{t}\Delta_{32})\xi(t) \\ &+ \xi\xi^{T}(t)(\Delta_{41} + h_{t}\Delta_{32})^{T}\dot{h}_{t}S_{31}(\Delta_{31} + h_{t}\Delta_{32})\xi(t) \\ &+ 2\xi^{T}(t)\Delta_{43}^{T}(\dot{h}_{t}S_{41} + S_{42})(\Delta_{41} + h_{t}\Delta_{42})\xi(t) \\ &- \xi^{T}(t)(\Delta_{41} + h_{t}\Delta_{42})^{T}\dot{h}_{t}S_{41}(\Delta_{41} + h_{t}\Delta_{42})\xi(t) \\ &- \xi^{T}(t)(\Delta_{41} + h_{t}\Delta_{42})^{T}\dot{h}_{t}S_{41}(\Delta_{41} + h_{t}\Delta_{42})\xi(t), \end{split}$$

$$(A2)$$

$$\dot{V}_{2}(t) = \eta_{5}^{T}(t,t)Q_{1}\eta_{5}(t,t) - \eta_{6}^{T}(t,t-h_{1})Q_{2}\eta_{6}(t,t-h_{1}) \\ &+ \eta_{7}^{T}(t,t)Q_{3}\eta_{7}(t,t) - \eta_{8}^{T}(t,t-h_{1})Q_{4}\eta_{8}(t,t-h) \\ &- h_{1d}\eta_{5}^{T}(t,t-h_{1})Q_{2}\eta_{6}(t,t-h_{1}) \\ &+ h_{1d}\eta_{6}^{T}(t,t-h_{1})Q_{3}\eta_{7}(t,t-h_{1}) \\ &+ h_{d}\eta_{8}^{T}(t,t-h_{t})Q_{4}\eta_{8}(t,t-h_{t}) \end{split}$$

$$\begin{aligned} &+ 2 \int_{t-h_{1t}}^{t} \eta_{5}^{T}(t,s) Q_{1} \frac{\partial}{\partial t} \eta_{5}(t,s) ds \\ &+ 2 \int_{t-h_{1t}}^{t-h_{1t}} \eta_{6}^{T}(t,s) Q_{2} \frac{\partial}{\partial t} \eta_{6}(t,s) ds \\ &+ 2 \int_{t-h_{1t}}^{t} \eta_{7}^{T}(t,s) Q_{3} \frac{\partial}{\partial t} \eta_{7}(t,s) ds \\ &+ 2 \int_{t-h}^{t-h_{1t}} \eta_{8}^{T}(t,s) Q_{4} \frac{\partial}{\partial t} \eta_{8}(t,s) ds \\ &= \xi^{T}(t) \left[(\Delta_{51} + h_{1t} \Delta_{52})^{T} Q_{1} (\Delta_{51} + h_{1t} \Delta_{52}) \\ &- (\Delta_{61} + h_{1t} \Delta_{62})^{T} Q_{2} (\Delta_{61} + h_{1t} \Delta_{52}) \right] \xi(t) \\ &+ \xi^{T}(t) \left[(\Delta_{71} + h_{t} \Delta_{72})^{T} Q_{3} (\Delta_{71} + h_{t} \Delta_{72}) \\ &- (\Delta_{81} + h_{t} \Delta_{82})^{T} Q_{4} (\Delta_{81} + h_{t} \Delta_{82}) \right] \xi(t) \\ &- h_{1d} \xi^{T}(t) \left[(\Delta_{53} + h_{1t} \Delta_{54})^{T} Q_{1} (\Delta_{53} + h_{1t} \Delta_{54}) \\ &- (\Delta_{63} + h_{1t} \Delta_{64})^{T} Q_{2} (\Delta_{63} + h_{1t} \Delta_{64}) \right] \xi(t) \\ &- h_{d} \xi^{T}(t) \left[(\Delta_{90} + h_{1t} \Delta_{91} + h_{1t}^{2} \Delta_{92})^{T} Q_{1} \Lambda_{1} \\ &+ (\Lambda_{10} + h_{1t} \Lambda_{11} + h_{1t}^{2} \Lambda_{12})^{T} Q_{2} \Lambda_{2} \right] \xi(t) \\ &+ 2 \xi^{T}(t) \left[(\Lambda_{20} + h_{t} \Lambda_{21} + h_{t}^{2} \Lambda_{22})^{T} Q_{3} \Lambda_{3} \\ &+ (\Lambda_{30} + h_{t} \Lambda_{31} + h_{t}^{2} \Lambda_{32})^{T} Q_{4} \Lambda_{4} \right] \xi(t), \end{aligned}$$

$$\dot{V}_{3}(t) = h_{1}^{2} \dot{x}^{T}(t) R_{2} \dot{x}(t) + h^{2} \dot{x}^{T}(t) R_{4} \dot{x}(t) + h_{1} h_{1d} \bar{h}_{1t} \dot{x}^{T}(t - h_{1t}) (R_{1} - R_{2}) \dot{x}(t - h_{1t}) + h h_{d} \bar{h}_{t} \dot{x}^{T}(t - h_{t}) (R_{3} - R_{4}) \dot{x}(t - h_{t}) - h_{1} \left(\int_{t - h_{1}}^{t - h_{1t}} \dot{x}^{T}(s) R_{1} \dot{x}(s) ds + \int_{t - h_{1t}}^{t} \dot{x}^{T}(s) R_{2} \dot{x}(s) ds \right) - h \left(\int_{t - h}^{t - h_{t}} \dot{x}^{T}(s) R_{3} \dot{x}(s) ds + \int_{t - h_{t}}^{t} \dot{x}^{T}(s) R_{4} \dot{x}(s) ds \right).$$
(A4)

According to $R_i > 0$, (i = 1, 2, 3, 4), letting $\alpha = \frac{h_{1t}}{h_1}$ and $\beta = \frac{h_t}{h}$, it follows from Lemmas 1 and 2 that

$$-h_{1}\left(\int_{t-h_{1}}^{t-h_{1t}} \dot{x}^{T}(s)R_{1}\dot{x}(s)ds + \int_{t-h_{1t}}^{t} \dot{x}^{T}(s)R_{2}\dot{x}(s)ds\right)$$

$$\leq -\frac{1}{\alpha}\xi^{T}(t)\Gamma_{1}^{T}\bar{R}_{1}\Gamma_{1}\xi(t) - \frac{1}{1-\alpha}\xi^{T}(t)\Gamma_{2}^{T}\bar{R}_{2}\Gamma_{2}\xi(t)$$

$$\leq -\xi^{T}(t)\left[\Gamma_{1}^{T}(\bar{R}_{1}+\alpha X_{1})\Gamma_{1}+2\Gamma_{1}^{T}[(1-\alpha)Y_{1}+\alpha Y_{2}]\Gamma_{2}\right]$$

$$+\Gamma_{2}^{T}[\bar{R}_{2}+(1-\alpha)X_{2}]\Gamma_{2}]\xi(t), \qquad (A5)$$

$$-h\left(\int_{t-h}^{t-h_{t}} \dot{x}^{T}(s) R_{3} \dot{x}(s) ds + \int_{t-h_{t}}^{t} \dot{x}^{T}(s) R_{4} \dot{x}(s) ds\right)$$

$$\leq -\frac{1}{\beta} \xi^{T}(t) \Gamma_{3}^{T} \bar{R}_{3} \Gamma_{3} \xi(t) - \frac{1}{1-\beta} \xi^{T}(t) \Gamma_{4}^{T} \bar{R}_{4} \Gamma_{4} \xi(t)$$

$$\leq -\xi^{T}(t) \left[\Gamma_{3}^{T} (\bar{R}_{3} + \alpha X_{3}) \Gamma_{3} + 2\Gamma_{3}^{T} [(1-\beta)Y_{3} + \alpha Y_{4}] \Gamma_{4} + \Gamma_{4}^{T} [\bar{R}_{4} + (1-\beta)X_{4}] \Gamma_{4} \right] \xi(t).$$
(A6)

For an appropriately matrix $U \in \mathbb{R}^{3n \times n}$, we can get

$$0 = 2 \Big[x^{T}(t) x^{T}(t-h_{t}) \dot{x}^{T}(t) \Big] U[Ax(t) + A_{1}x(t-h_{t}) - \dot{x}(t)] = 2\xi^{T}(t)\Delta_{0}^{T}U\Pi_{0}\xi(t).$$
(A7)

Finally, from the above derivation (A2)–(A7), we have

$$\dot{V}(t) \leq \xi^{T}(t) \left[\Omega_{0}(\dot{h}_{t}) + h_{t}\Omega_{1}(\dot{h}_{t}) + h_{t}^{2}\Omega_{2}(\dot{h}_{t}) \right] \xi(t) + \xi^{T}(t) \left[\Omega_{10}(\dot{h}_{1t}) + h_{1t}\Omega_{11}(\dot{h}_{1t}) + h_{1t}^{2}\Omega_{21}(\dot{h}_{1t}) \right] \xi(t).$$
(A8)

According to Lemma 3, the matrix inequality (11) together with the matrix inequality (12) imply that $\dot{V}(t) < 0$. Therefore, by Lyapunov stability theorem, it can guarantee that the time-delayed system (4) is asymptotically stable. The proof is completed. \Box

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