

Review

# Some Key Issues in Hypersonic Propulsion

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**Abstract:** This paper summarizes and discusses some critical aspects of flying hypersonically. The first is the L/D (lift over drag) ratio determining thrust and that in turn depends on the slenderness Küchemann's  $\tau$  parameter. This second parameter is found to depend on the relative importance of wave versus friction drag. Ultimately, all engineering drag is argued to depend on vorticity formed at the expense of the vehicle kinetic energy, thus requiring work by thrust. Different mixing strategies are discussed and shown to depend also on mechanisms forming vorticity when the regime is compressible. Supersonic combustion is briefly analyzed and found, at sufficiently high combustor Mach, to take place locally at constant volume, unlike conventional Brayton cycles.

**Keywords:** scramjet engine; hypersonic propulsion

## 1. Introduction

Hypersonic Propulsion is enjoying a revival of interest not seen since the 1960s, when the US Navy, USAF and the UK MoD were funding research and developments (R&D) in SCRJ engines. The reason is military applications that, using a current buzzword, have been defined 'disruptive', in that they may alter the strategic balance of power. Powered hypersonic missiles can potentially fly at  $M \sim 5\text{--}14$  and altitude  $\sim 20\text{--}40$  km where, because of the Earth curvature, they cannot be detected by radar until too late. Hypersonic gliders, after being boosted to much higher altitude ( $\sim 10^2$  km) fly to their destination unpowered. In both applications range may reach many thousands of km. They realize what E. Sänger [1] pursued until 1943 in Germany using rocket acceleration. Airbreathing engines as well as rocket boosting are the propulsion means.

The purpose of this work is to describe some of the physics behind hypersonic flight, to show it depends on close integration between propulsion system and vehicle and to illustrate how knowledge of this physics helps in designing applications.

As in subsonics, issues associated to hypersonic vehicles vary substantially depending on mission. Two classes of missions, and therefore of vehicles, can be singled out: cruisers and accelerators. The performance of cruisers (including also unpowered gliders) is in terms of range and speed; the Bréguet formula shows range depending on Isp and the L/D ratio. Their flight trajectory is enabled by airbreathing engines, or combined engines, based on the SCRJ cycle. After climbing to their flight corridor their speed  $V$  is practically constant.

Accelerators are either launchers, for instance recoverable first stages of TSTO powered by airbreathing engines, or pure accelerators, their mission being to intercept other hypersonic missiles. While for the time being hypersonic missiles, airbreathing or not, are immune from interception, accelerators powered also by SCRJ are the only way to deal with hostile cruisers. The key feature of accelerators is time-of-flight, or tof, for instance, to reach a specified altitude at a certain speed.

### 1.1. Cruisers

The range,  $R$ , of cruisers is specified by the Bréguet formula [2], see Equation (2), derived from Newton's Second Law when drag,  $D$ , is assumed equal to thrust  $T$ , and lift,  $L$ ,



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equal to weight,  $W$ ; the subscript in and fin in the weight  $W$  denote initial and final weight of the vehicle:

$$R = V Isp \frac{L}{D} \ln \frac{W_{in}}{W_{fin}} \quad (1)$$

$Isp$  tracks fuel consumption per unit thrust of the engine.  $Isp$  is the second parameter of importance of airbreathing engines. After acceleration to the cruising altitude the vehicle loses fuel mass at a nearly constant rate. Thrust determines the mass necessary to accomplish the mission via the specific fuel consumption (or  $Isp$ ). Thrust is a function of  $L/D$ : as a first approximation, assuming  $L = W$  and  $T = D$ ,

$$T = \frac{W}{(L/D)} \quad (2)$$

That is, the thrust necessary to the mission is equal to weight reduced by the  $(L/D)$  factor. This is an advantage compared to rocket propulsion where, to enable long range missions, thrust must be larger than weight. The ratio  $L/D$  is well known to be responsible for range: subsonic gliders may have  $L/D \sim 25\text{--}30$  and airliners are close to 20. Allowance made for climbing and accelerating to the mission altitude, this relationship can be used to size the engine.

The  $L/D$  ratio is the most important parameter driving hypersonic flight range. Generally speaking, as speed increases past  $M = 1$ , the aerodynamic lift coefficient  $C_L$  diminishes while drag increases and, past  $M \sim 3$ ,  $L/D$  drops rapidly: Figure 1 shows aerodynamic test data for the maximum  $L/D$  (as a function of the angle of attack) starting at  $M = 3$ , together with two theoretical curves developed by Küchemann [3–5] in the 1970s. Both are partially satisfactory, in the sense that the theoretical curves fit only parts of the data, and the data themselves are scattered. The flowfield determines  $L/D$  performance far more than the Mach number per se: such plots reveal the range of  $L/D$  potentially available, but their realization in a vehicle depends on actual lift and drag specific to the flow wrapped around and inside a vehicle shape.

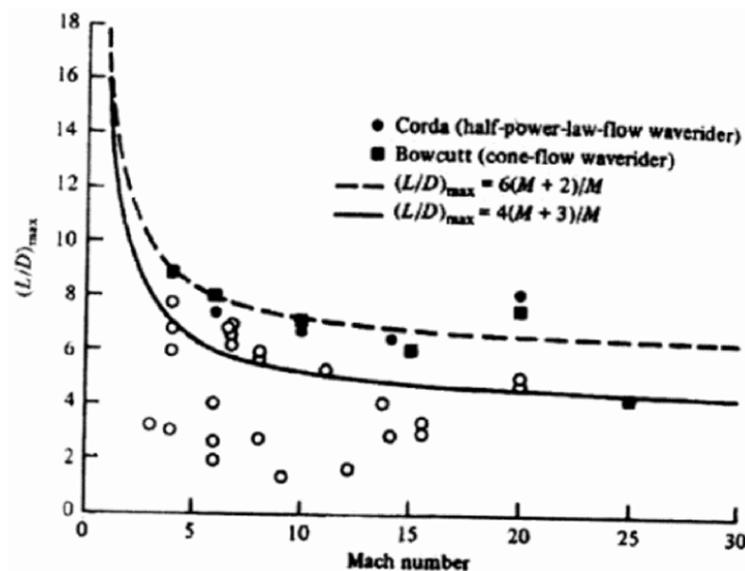


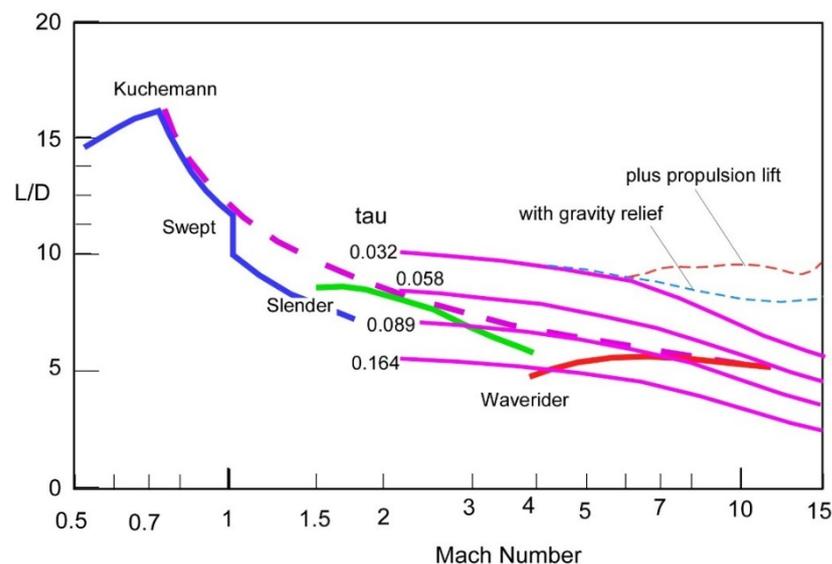
Figure 1.  $L/D$  vs. flight Mach number.

Figure 1 shows that the  $L/D$  ratio is generally a decreasing function of Mach number, but actual values depend on specific  $L$  and  $D$  determined by the vehicle shape. The importance of  $L$  and  $D$  in determining thrust should spur initiatives to investigate the fundamental nature of both. Lift can be shown to be due to circulation (the Kutta–Jukowski theorem), and circulation is linked to vorticity; drag, however, is typically parceled among different contributions (form, friction, shock, induced) each dependent on different vehicle

features. This paper recognizes the importance of the Kuchemann parameter  $\tau$ , the ratio between vehicle volume and (surface)<sup>3/2</sup> [6]. This ratio is in fact proportional to the ratio between shock drag and friction drag, and while still not accounting for fundamental drag physics, at least for a broad range of shapes supplies criteria to optimize hypersonic shapes.

The common factor to all contributions to drag is the formation of vorticity. A body moving in a still medium at supersonic speed must part the still air ahead. That is responsible for the formation of shocks and, in general, vorticity due to stagnation pressure non-uniformity. The simultaneous pressure and velocity field created by the body form vorticity as shown by the Crocco–Vaszyoni theorem in the compressible regime [7]. In terms of energy, inducing vorticity corresponds to rotationally accelerating the medium, and that requires work by the drag force resisting acceleration. All forms of drag end up by creating vorticity, which is eventually converted to heat by thermodynamic dissipation. Vorticity is formed by wall shear (wall friction), by shear and compression inside the thin shocks structure, and more generally by interaction between velocity and pressure field, as is the case of induced drag. Friction is a function of the vehicle surface area, and shocks depend on the volume being displaced by the vehicle, explaining why the Kuchemann  $\tau$  can help in increasing the  $L/D$  ratio.

Slender bodies are characterized by  $\tau \sim 0.1$ , and waveriders (gliders) even a factor two less, precluding carrying much fuel or payload; their drag is mostly due to friction. At the other extreme are rocket powered vehicles that must store both fuel and oxidizer propellants and need  $\tau \sim 0.2$ – $0.3$  or higher: their drag is mostly wave drag. The  $L/D$  ratio so important for range depends on the Kuchemann  $\tau$  and flight Mach number,  $M$ , as shown in the Figure 2. This figure can be interpreted as a sequence of  $L/D$  ratios as a function of Mach but also of shape ( $\tau$ ) for a class of vehicles characterized by a ‘pointy’ body (fuchsia-colored curves). The broken line corresponds to one of the theoretical Kuchemann’s fit. Waveriders can maintain their relatively high  $L/D$  over a broad range of Mach numbers.



**Figure 2.**  $L/D$  vs. the flight Mach number and the shape factor  $\tau$  (courtesy of P.A. Czysz).

Cruiser aerodynamics must ensure high  $L/D$  and high  $I_{sp}$  in order to have thrust a small fraction of the take-off weight, and to limit the fuel fraction of take-off weight. Figure 2 enables preliminary engine and vehicle sizing. Realizing that drag is a function of vorticity simplifies conceptually the task of reducing it and suggests also ways, for instance at Mach numbers  $> 10$ , based on electromagnetics.

### 1.2. Accelerators

This class of applications includes first-stage airbreathing launchers and interceptors. Performance in terms of range and  $I_{sp}$  is replaced by time-of-flight (=tof), and effective

thrust, the difference  $(T-D)$ , or  $(T-D)/T$ , determining acceleration. Short tof requires strong acceleration enabled by  $(T-D)/T \sim 1$ . Effective thrust determines the take-off, or launch, mass consisting mostly of fuel. The other performance quantity is the fuel fraction,  $m_{fuel}/m_{TO}$ , that has been calculated [3] as a function of  $T/D$  as

$$\frac{m_{fuel}}{m_{TO}} = \frac{1 - \frac{E}{2(\frac{T}{D}-1)}}{1 + E \left( 1 + \frac{2}{(\frac{T}{D}-1)} \right)} \quad (3)$$

where total energy normalized using the fuel heat content is

$$E = \frac{\frac{V_{fin}^2}{2} + gh_{fin}}{\eta_p Q} \quad (4)$$

The fuel energy  $Q$  should be the free energy  $F$ , but in Hunt's analysis [8], it has been replaced by drag work  $DV$ . These are relationships enabling preliminary sizing of engine and vehicle for accelerator missions; they may also be used to calculate the fuel fraction for the climb and acceleration segment of cruisers.

## 2. Energy and Propulsion

Hypersonic flight depends on Isp and thrust, therefore, on  $L/D$ . This fact makes close engine-vehicle integration indispensable to reduce drag and weight. Integration, for instance, means that the air compression flowpath must also generate part of lift: the contraction ratio defining compression and temperature necessary to make the engine work is a function of forebody geometry and shape, both responsible for lift. A half-nozzle in 2-D geometries does the same. Together with these intertwined issues, a second critical issue is that of entropy.

Past  $M \sim 3$  vehicle drag work increases and so does entropy, the third thermodynamic quantity of importance in hypersonics after kinetic energy (KE) and enthalpy [9]. Here the tacit assumption is that temperature and pressure limits of conventional ramjets can be overcome only with SCRJ engines or SCRJ-based combined engines. In fact, a hypersonic vehicle can be treated as a thermodynamic machine, its cycle including to a first approximation, the usual Brayton phases, and that is defined by in and out mass and energy fluxes of both air and fuel. Unlike subsonic or mildly supersonic vehicles, entropy becomes critically important in hypersonics, because it is strictly connected with vorticity and drag work [6,10]. Because the KE of the airstream, captured at great cost in terms of speed and drag, must not be wasted, the thickness of the entropy layer formed by the forebody curvature must be reduced as much as possible: thus, the vehicle should be 'pointy' or sharply spatular. By doing so, the volume of the bow and leading edges shock layers shrinks, streamlines are less 'curvy', and temperature and pressure gradients are moderate and entropy layers thinner. Entropy layers are sources of vorticity that causes drag and eventually dissipates and converts the KE of the airstream to useless heat. High temperature materials like ultra-high-temperature-ceramics are indispensable to design sharp leading edges and noses without resort to active cooling; past these high heat transfer zones (e.g., ~1 ft to ~1 m downstream), refractory metals and TBC may be adequate. Reducing entropy formation suggests using friction and shock heating to generate work or extra thrust: for instance, fuel used as a heat sink can produce KE and increase total thrust. In fact, thrust from high-speed fuel injection in the combustor becomes important past  $M \sim 10$ .

Much emphasis in hypersonic should be on conserving the KE of the airstream captured. Energy conservation and management are paramount when airstream KE may easily convert to enthalpy by friction and shocks [9,10]; means to recover internal and external heat fluxes are critical to enable vehicles to be re-usable: in this light cryogenic

and endothermic fuels are very important, but logistics and cost must also be taken in consideration.

### 2.1. Turbulent Mixing

The airstream captured and slowed down by external and internal segments of the flowpath must mix with fuel injected in the combustor and burn within a distance short enough to keep engine friction drag to a minimum. Friction (shear) drag scales with the surface area  $A$  of the combustor:

$$\text{Drag} \sim C_F A \rho V^2 \quad (5)$$

where  $V$  and density peak and where also does the engine drag.  $C_F$  is a function of  $Re$  and  $M$ , of order  $10^{-3}$ . Any fuel or structural element inside the combustion (e.g., struts) raises total area  $A$  and reduces the effective thrust available. For slenderness  $\tau \sim 0.1$  combustor drag may be most of the vehicle drag (see Figure 3).

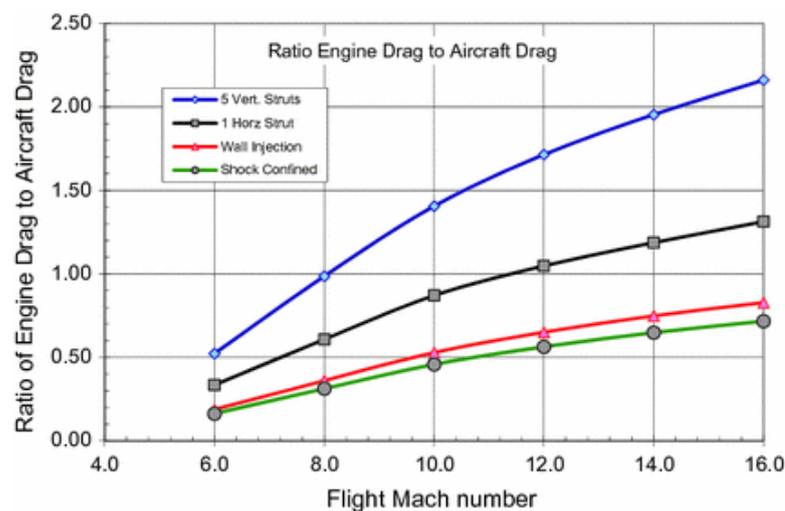


Figure 3. Ratio of engine drag to aircraft drag vs. flight Mach.

Mixing fuel and air depends on vorticity created by interaction among velocity, density and pressure gradients. Interaction consumes a non-negligible fraction of the kinetic energies of the two streams [11]. Basic physics suggests that to penetrate a high momentum airstream the fuel stream(s) needs just as high dynamic pressure and that can be obtained only at the expense of the fuel kinetic energy. Since the air over fuel (A/F) ratio is always  $\gg 1$ , it is fuel that must penetrate air more than the vice-versa. Therefore, the number of importance in injecting fuel into air is the momentum flux ratio,  $J$ , also equal to the ratio between the specific KE of the two streams.

$$J = \frac{\rho_{fuel}}{\rho_{air}} \left( \frac{V_{fuel}}{V_{air}} \right)^2 > 1 \quad (6)$$

$J$  should be  $> 1$  for fast and effective penetration: hydrogen fuel must be injected at supersonic speed to reach  $J > 1$ . Most academic work in SCRJ uses instead choked flow injection. In the  $M > 10$  range the fuel KE not only is necessary to mixing but becomes increasingly needed to compensate for drag. Then the 1-D textbook expression for thrust should also include the change of momentum due to the fuel being injected.

Fast mixing depends on 'turbulence', that is, on sufficient vorticity,  $\omega$ . In crossflow injection, normal or angled, penetration is a function of  $J$ , and both air and fuel KE contribute to mixing: vorticity is formed by stretching but also by baroclinic torque and dilatation, as shown by the Equation (7) reported below [12]. These additional terms (the second

and third on the RHS of Equation (7)) are associated to compressibility and should not be ignored by modelers:

$$\frac{\partial \omega}{\partial t} + (u \cdot \nabla) \omega = \omega \cdot \overline{\nabla u} - \omega (\nabla \cdot u) + \frac{\overline{\nabla \rho \times \nabla p}}{\rho^2} + \nu \overline{\nabla^2 \omega} + \left( -\frac{1}{\rho^2} \overline{\nabla \rho} \times (\nabla \cdot \sigma) + \frac{1}{\rho} \left\{ \overline{\nabla \mu} \times [\overline{\nabla^2 u} + \nabla (\nabla \cdot u)] + 2 \nabla \times (E \overline{\nabla \mu}) \right\} \right) \quad (7)$$

where overbars denote vector and tensors;  $\sigma$  and  $\epsilon$  are the stress and strain tensor, respectively.

All three effects are present in the 3-D interaction between the airstream field and the fuel jet(s) issuing from orifices.

A different mixing strategy, streamwise vortex mixing, relies on vorticity created by the airstream flowing over spatially alternating wedges or lobes ('hypermixers'): fuel is entrained by the pressure field formed by vortex rotation. This strategy requires struts inside the combustor, adding surface area and drag; also, vortices break down at a certain distance from the struts trailing edges, since vortex rotational KE is consumed to form the pressure gradients entraining fuel. Often the result is limited ability to mix to equivalence ratio (E.R.) larger than (roughly) 0.4 or 0.5. This is not a serious problem at high M, when the inlet temperature must not be higher than, say, 1600 K. Note that if the fuel is hydrogen,  $I_{sp}$  may increase by burning rich. Hypermixer results indicate flames are well anchored and at a distance from the strut ensuring reasonably moderate heat transfer, e.g., [13].

Cavity mixing uses the effect of subsonic recirculation coupled to the supersonic air core to generate intense shear. Fuel injection has been tested inside and outside the cavity and both upstream and downstream. Cavity flame anchoring is currently popular, and shapes and fuel injection scheduling have been and keep being tested because this strategy can burn at E.R. up to 0.8 and higher [14]. A downside is the intense heat transfer through the cavity walls.

Crossflow mixing was the first strategy attempted after parallel mixing proved to require excessive combustor length. It does not need struts or other implements inside the combustor. It works well in the high M range, when the Mach number at the combustor entrance is ~3 to 5. Fuel may be injected normal to the walls or at a shallow angle; the AeroRamp injection concept developed at Virginia Tech is such strategy [15]. An advantage is the capability of injecting from all combustor walls independently, realizing the desired fuel injection scheduling.

Predicting mixing distance is based on knowledge of turbulence and scalar transport; in subsonic flows, the theory, based on the Reynolds decomposition, uses the 1941 Kolmogorov article ('K41') and works adequately [16]. In hypersonic flight, M is ~1 to 5 at the combustor entrance, the regime is compressible, and theoretically speaking, K41 is invalid, as it includes only transfer of TKE from large to small scales by stretching and ignores conversion of KE to enthalpy and vice-versa. AS said, two other mechanisms appear in the vorticity equation when density varies, and non-dimensionalization shows they are of the same order of stretching. Compressible turbulence is a theoretical area advancing slowly, if for no other reason because measuring pressure, velocity fluctuation, temperatures and species concentrations in a supersonic stream is difficult and expensive, but more understanding of the physics in this area would have much practical impact. In the 1960s Morkovin [17] hypothesized that compressible turbulence behaved as in subsonics in the low supersonic regime. Thirty years later Batchelor thought this could be true provided  $M < 5$  [18]. This range coincides with that of the air at the combustor entrance, and information from DNS of astrophysical high-Mach turbulence seems to confirm the energy cascade model of K41 [11]. This said, preliminary analyses of compressibility effects on turbulence [12] indicate they may play a role in supersonic combustion, for instance, by creating turbulent KE through the baroclinic and dilatation effects and by increasing kinetic rates. This is a frontier area in turbulence theory.

### 2.2. Combustion

All types of mixers work also as flameholders, since above air temperature  $\sim 1000$  K hydrogen kinetics times are  $\sim 10^{-4}$  to  $10^{-5}$  s as do those of hydrocarbons. The flame regime is both partially premixed and non-premixed, if this classification, valid for simple flames, holds also in SCRJ combustion [19]. Thus, at sufficiently high M heat release is controlled by mixing, not by kinetics. In dual-mode engines, kinetics controls the heat release in the transition from conventional (subsonic) RJ operation to SCRJ at  $M \sim 4.5$  to 6. In this Mach range, pressures and (especially) temperatures drop to below 1 atm and 900 K depending on trajectory, and the combination may result in flame instability or flameout. Variable fuel scheduling is a possible way to control mode transition.

Up to  $M \sim 7-8$  liquid hydrocarbons fuels offer sufficient cooling by cracking and reforming. Partially Oxidized Fuels (POF) also can contribute to solve the many thermal problems without creating excessively complicating engines and infrastructure. Both can potentially reduce entropy rise and drag. Future research on how to control combustion and drag through the control of induced vorticity without also destroying lift, for instance, by electromagnetic means, should be investigated.

The supersonic flame regime is discussed in [19] and on a Borghi diagram is delimited as in the Figure 4 [12].

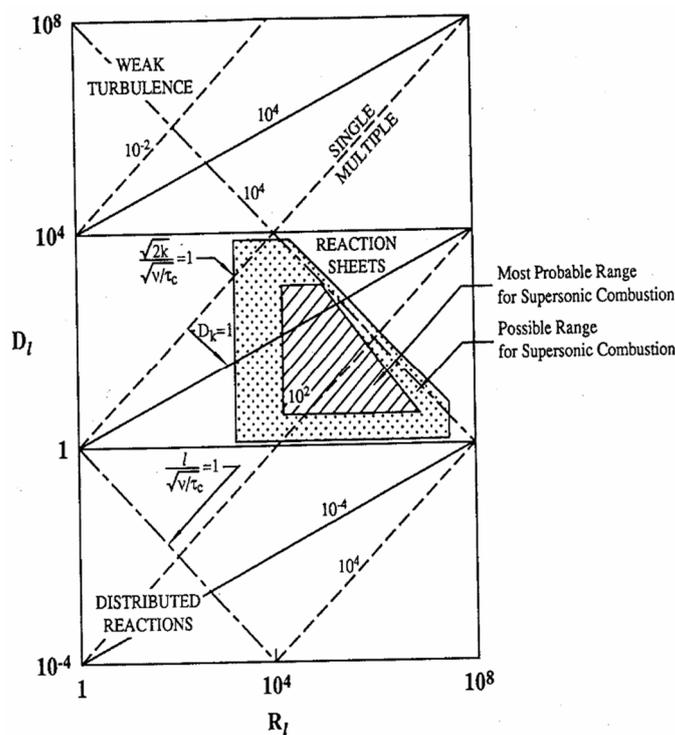


Figure 4. Borghi diagram.

If the chemistry time is shorter than the acoustic time, that is, if

$$\frac{\tau_{chem}}{\tau_{acus}} = \frac{1}{Da Ma} < 1 \tag{8}$$

Combustion occurs at nearly constant volume. This condition is verified in supersonic combustion except near walls, suggesting turbulent transport controlling flame propagation at fine scales, and thus thin flame sheets being responsible for the burn rate.

Finally, the combustion exhaust is expanded through the nozzle. Entropy rises with altitude and limits the work that can be extracted by the expansion; in chemical kinetics terms, endothermic radicals formed by combustion kinetics ‘freeze’ at some point along the nozzle flowpath subtracting free energy from that available if equilibrium conditions held.

Quasi-1-D flow analysis reveals that complete expansion needs a cross section area typically larger than that of the inlet. This is undesirable both for drag and weight considerations, and 3-D or 2-D nozzles are truncated by a nearly horizontal plane so that their bottom is replaced by the external airstream. Doing so reduces expansion thrust but saves weight.

### 3. Conclusions

This paper deals with some critical issues in the physics of powered hypersonic vehicles and purposely avoids delving in the S&T details but not because they are unimportant. Probably the most significant results are the role of vorticity in determining drag and thus the L/D ratio critical to the design of integrated vehicles and the critical need to reduce the entropy formation rate and to recycle the heat dissipated rather than dispersing it to the ambient. Because the ratio air KE/air enthalpy scales  $\sim M^2$  and because at sufficient Mach air KE is also larger than combustion heat release, the Second Principle rules the design of efficient integrated hypersonic vehicles. As temperatures reach locally in the thousand K, management of air KE and technology of materials hold the key to practical applications of powered hypersonics.

Hypersonics and SCRJ engines have matured to the point military applications, mostly gliders but also some SCRJ-powered vehicles, are fielded or in the process of being fielded. Re-usable commercial applications, including TSTO launchers powered by airbreathing engines, seem farther in the future and likely expensive compared to recoverable rocket boosters. Meanwhile, problems remain, for instance in efficient scaling of vehicles, thermal control of structures and about LH<sub>2</sub> logistics.

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