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Multi-Criteria Decision-Making under mHF ELECTRE-I and HmF ELECTRE-I

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Abstract: In a few years, hesitant fuzzy sets (HFSs) have had an impact on several different areas of decision science. However, a number of researches have utilized the Elimination and choice translating reality (ELECTRE) methods to determine the multi-criteria decision-making (MCDM) problems with hesitant information. The aim of this research article is to develop new multi-criteria group decision-making (MCGDM) methods, such as the m-polar hesitant fuzzy ELECTRE-I (mHF ELECTRE-I) method and hesitant m-polar fuzzy ELECTRE-I (HmF ELECTRE-I) method. Proposed MCGDM techniques based on the hybrid models, m-polar hesitant fuzzy sets (mHFS-sets) and hesitant m-polar fuzzy sets (HmF-sets), which are the natural generalizations of HFSs and m-polar fuzzy sets (mF sets). These models enable us to deal with multipolar information under hesitancy. We use the proposed methods to deal the complex problems in which the membership degree of an element of given set uses the m different numeric and fuzzy values, to rank all the alternatives and to determine the best alternative. We present two practical examples that illustrate the procedure of the proposed methods. We also discuss the differences and comparative analysis of the proposed methods. Finally, we develop an algorithm that implements our decision-making procedures by using computer programming.

Keywords: HFSs; mF sets; mHF-sets; HmF-sets; MCGDM; mHF ELECTRE-I; HmF ELECTRE-I

1. Introduction

Decision-making [1] can be examined as an ultimate conclusion of some intellectual and psychological measures that leads to the choice of an alternative in several different ones. It is interesting to note that decision-making is a distinctive ability of humans, which is not naturally designed and depended upon absolute expectations, it does not demand specific and entire analysis of data about the set of feasible alternatives. This fact motivated several researchers to apply Zadeh's [2] fuzzy set theory to discuss the vagueness and uncertainty in decision processes. However, the modeling and representative tools of fuzzy sets are defined and limited, on the other hand two or more origins of vagueness can appear together. Thus certain well known extensions and generalizations have been developed such as, bipolar fuzzy sets (BFSs) [3], hesitant fuzzy sets (HFSs) [4] and mF sets [5]. In typical decision-making problems such as, to select a place to visit, to decide, which candidate is suitable for election or choosing the best car to buy. Chen et al. [5] introduced the approach of mF sets, which is the generalization of BFSs, in which they presented that BFSs or 2-polar fuzzy sets are cryptomorphic mathematical approaches. This new concept motivated the researchers to introduce several novel concepts related to mF sets and its hybrid models, including [6–12].

In a number of decision-making problems, the decision-makers or evaluation experts are requested to assign the membership degrees or evaluation performance values of objects, which create hesitancy.

To deal this hesitant situation, Torra and Narukawa [13] introduced the notion of HFSs to compose the extensions of fuzzy sets that is advancing expeditiously with its expansions, functions and utilizations to several fields [14]. This notion is reasonable for the conception of directions where decision-makers face hesitancy in contributing their estimations and judgments against objects, as well as by combining the assumptions of distinct experts into an individual input. Certainly in most of the decision-making cases, experts are generally hesitant or doubtful, which prevents them from cropping exclusive assessments [15]. Farhadinia [16] initiated the concept of lexicographical ordering of HFSs and its application to multi-criteria decision-making (MCDM). Alcantud and Torra [17] introduced the decomposition theorems and extension principles for HFSs. Xia and Xu [18] developed some aggregation operators and presented applications to handle MCDM problems under hesitant fuzzy environment. Xia et al. [19] also introduced some other hesitant fuzzy aggregation approaches and presented its influence in group decision-making. Chen et al. [20] generalized the concept of HFSs and induced the notion of interval-valued hesitant fuzzy sets (IVHFSs). Xu [21] compiled all the theory, results and operators of HFSs in a book named as hesitant fuzzy sets theory. Further, Pei and Yi [22] studied a note on operations of HFSs. After the concept of HFSs, there has been a lot of research and applications including clustering analysis [23], feature selection [24] and decision support system [25] based on HFSs. Group decision-making problems are solved by several researchers by using the concept of HFSs as well as its aggregation operators in [26–30].

Elimination and choice translating reality (ELECTRE) is one of the MCDM methods in which the decision-maker desires to hold different criterions and there may be a robust collection associated with the nature of evaluation surrounded by a number of the standards. The ELECTRE approach was first introduced by Benayoun et al. [31]. After that, the modified concept of ELECTRE known as ELECTRE-I was introduced by Roy [32]. Further, this approach was expended into a variety of alternative variants. Nowadays, the foremost wide used versions are referred as ELECTRE-II, ELECTRE-III and ELECTRE-IV. In the literature, most of these methods have been combined with fuzzy sets by several researchers. For the supplier selection problem, Sevkli [33] analyzed the classical and fuzzy ELECTRE methods. For the choice and evaluation of academic staff, Rouyendegh and Erkan [34] used the concept of fuzzy ELECTRE. Hatami-Marbini et al. [35] applied the method of fuzzy group ELECTRE for the interpretation of haphazard waste reprocessing of plants. Kheirkhah and Dehghani [36] applied the fuzzy group ELECTRE method for the assessment of quality of public transportation facilities. Hatami-Marbini and Tavana [37] expended the method of ELECTRE-I and introduced the method of fuzzy ELECTRE-I with numerical examples to illustrate the effectiveness of their proposed method. Asghari et al. [38] used the fuzzy ELECTRE-I method for the analysis of mobile payment models. Further, fuzzy ELECTRE-I technique was applied in evaluating catering firm alternatives by Aytac et al. [39] and an environmental effect evaluation approach based on fuzzy ELECTRE-I was composed by Kaya and Kahraman [40]. Liao et al. [41,42] discussed two new approaches based on ELECTRE II to solve the MCDM problems with hesitant fuzzy linguistic term sets and to deal with probabilistic linguistic term sets and its application to edge computing. Further, Liao et al. [43] introduced an integrated method for cognitive complex multiple experts MCDM based on ELECTRE III with weighted Borda rule. Lupo [44] used the ELECTRE-III approach to calculate the service quality of three international airports. Akram et al. [45] introduced novel approach in decision-making with mF ELECTRE-I. With the passage of time, a number of extensions for fuzzy ELECTRE-I have been proposed by several researchers including [46–55]. For other notations, terminologies and applications, the readers are referred to [56–60].

Let us now refer to the issue of making choices under hesitancy. In real-world systems, we frequently observe activities and tasks in which it is compulsory to make decisions under hesitant situations. Unless they are extremely naive or clear, the practitioner has to resort to decision-making techniques for the corresponding environment. The purpose of our article is very direct. The methods proposed in the existing literature are unable to provide any information about choices when data appear in multipolar form and hesitancy is allowed in relation to them. In order to deal with such a complexity, we introduce the novel approaches of ELECTRE-I for MCGDM problems based on the concepts of mHF-sets and HmF-sets. Commonly, decision-making is thought of as an intellectual process based on distinct reasoning and rational actions that leads to choose the most reasonable alternative from a set of feasible options in a decision situation. The mHF ELECTRE-I and HmF ELECTRE-I methods are, therefore, capable of dealing with problems when they incorporate multipolar information in terms of hesitancy. Our novel concepts increase the relevance of hesitation in the mF approach and also stand out as an effective, favorable and widely used MCGDM methods.

In Section 2, we review some basic concepts and hybrid models (namely, mHF-sets and HmF-sets), with examples. In Section 3, we propose mHF ELECTRE-I approach and apply it on real life example. In Section 4, we propose the HmF ELECTRE-I approach and describe its potential application. We also present our proposed methods as an algorithm. In Section 5, we discuss the differences and comparative analysis of our proposed decision-making approaches. In Section 6, we present the conclusion and future directions. Finally, in Appendix A we show the computer programming code of our proposed approaches.

2. The Concept of *m*-Polar Hesitant Fuzzy Sets and Hesitant *m*-Polar Fuzzy Sets

In this section we briefly review some basic concepts and novel hybrid models, which are the combination of mF sets and HFSs.

Definition 1. *[5] An mF set on a universe Z is a function* $R = (p_1 \circ R(r), p_2 \circ R(r), \cdots, p_m \circ R(r)) : Z \to [0,1]^m$*, where the i-th projection mapping is defined as* $p_i \circ R : [0,1]^m \to [0,1]$*. In particular,* $\mathbf{0} = (0,0,\cdots,0)$ *is the smallest element in* $[0,1]^m$ *and* $\mathbf{1} = (1,1,\cdots,1)$ *is the largest element in* $[0,1]^m$*.*

Definition 2. *[4] Let A be a reference set, then a HFS on A is defined in terms of a function h that when applied to Z returns a subset of* $[0,1]$*, i.e., an element from* $\mathcal{P}([0,1])$*.*

The next concept is designed to deal with a hesitant situation separately for each degree of membership in an mF set:

Definition 3. *[7] Let A be a reference set, an mHF-set on A is a function* $\hbar_m$ *that returns a subset of values in* $[0,1]^m$ *:*

$$\hbar_m : A \to (\mathcal{P}\{[0,1]^m\}).$$

The mathematical representation of an mHF-set is as follows:

$$H = \{\langle a, \hbar_m(a)\rangle | \forall a \in A\},$$

where $\hbar_m(a) = \Big(\{\zeta_h|\zeta_h \in p_1 \circ \hbar_m(a)\}, \{\zeta_h|\zeta_h \in p_2 \circ \hbar_m(a)\}, \cdots, \{\zeta_h|\zeta_h \in p_m \circ \hbar_m(a)\}\Big)$*. This notation shows that* $\hbar_m(a)$ *is a tuple of m different sets of possible membership degrees of the elements* $a \in A$ *to set H, where* $\hbar_m(a)$ *is called an m-polar hesitant fuzzy element (mHFE).*

The following real life example illustrates the above concept and shows its usefulness.

Example 1. *Let* $A = \{a_1, a_2, a_3\}$ *be a reference set of candidates appearing for selection of job and* $\hbar_m(a)$ *represent the* 3*HF characterization of its evaluating criteria as*

- *C.V evaluation*
- *Interview evaluation*
- *Knowledge evaluation*

These are three main evaluation features or criteria required for the selection a candidate for job. Each candidate $a \in A$ *has the following ratings classified by* 3*HF set according to the evaluating criteria and has* *the respective* 3*-polar hesitant fuzzy elements (3HFEs) representation as*

$$\hbar_m(a_1) = \Big(\{0.34, 0.54\}, \{0.43, 0.45, 0.51\}, \{0.40, 0.58, 0.65\}\Big),$$
$$\hbar_m(a_2) = \Big(\{0.29, 0.38, 0.57\}, \{0.21, 0.30, 0.70\}, \{0.18, 0.42\}\Big),$$
$$\hbar_m(a_3) = \Big(\{0.43, 0.55\}, \{0.47, 0.60\}, \{0.34, 0.46, 0.70, 0.75\}\Big),$$

The 3*HFE* $\hbar_m(a_1) = \Big(\{0.34, 0.54\}, \{0.43, 0.45, 0.51\}, \{0.40, 0.58, 0.65\}\Big)$ *shows, the candidate* a_1 *has the following ratings according to his evaluation criteria as* $(\frac{\{0.34,0.54\}}{C.V\ evaluation}, \frac{\{0.43,0.45,0.51\}}{Interview\ evaluation}, \frac{\{0.40,0.58,0.65\}}{Knowledge\ evaluation})$ *and* $\frac{\{0.34,0.54\}}{C.V\ evaluation}$ *shows candidate* a_1 *has two hesitant values {0.34, 0.54} according to his C.V evaluation, similarly he has the hesitant ratings according to other evaluation criteria. Remaining candidates are evaluated in a same sense and the* 3*HF-set H is given as*

$$\begin{aligned} H = \Big\{ & \Big\langle a_1, \Big(\{0.34, 0.54\}, \{0.43, 0.45, 0.51\}, \{0.40, 0.58, 0.65\}\Big)\Big\rangle, \\ & \Big\langle a_2, \Big(\{0.29, 0.38, 0.57\}, \{0.21, 0.30, 0.70\}, \{0.18, 0.42\}\Big)\Big\rangle, \\ & \Big\langle a_3, \Big(\{0.43, 0.55\}, \{0.47, 0.60\}, \{0.34, 0.46, 0.70, 0.75\}\Big)\Big\rangle\Big\}. \end{aligned}$$

The 3*HF-set H shows the complete information about the evaluation of candidates for a job.*

From Example 1, it is easy to understand the concept of the approach described in Definition 3, in which we deal with the multipolar information under hesitant situation of each degree of membership of 3F set separately.

The next concept is designed to deal the hesitant situations motivated by multipolar information.

Definition 4. [8] *Let A be a reference set, a hesitant m-polar fuzzy set (HmF-set) on A is a function* $\wp_h$ *that returns a subset of values in* $[0,1]^m$:

$$\wp_h : A \to \mathcal{P}([0,1]^m).$$

Mathematical representation of a HmF-set is as follows:

$$M = \{\langle a, \wp_h(a)\rangle | \forall a \in A\},$$

where $\wp_h(a)$ *is a set of some different values in* $[0,1]^m$ *representing the possible m membership degrees of the element* $a \in A$ *to set M, where* $\wp_h(a)$ *is called a hesitant m-polar fuzzy element (HmFE).*

Note that $\wp_h(a)$ is a set of some different values in $[0,1]^m$ and written as

$$\wp_h(a) = \Big\{(p_1 \circ m_h(a), p_2 \circ m_h(a), \cdots, p_m \circ m_h(a))\Big\}, \text{ for all } a \in A,$$

where, $m_h(a) = (p_1 \circ m_h(a), p_2 \circ m_h(a), \cdots, p_m \circ m_h(a))$.

The following real life example illustrates the above concept and shows its usefulness.

Example 2. *Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of an image blocks considered as a reference set and $m_h(a)$ represent the 3F classification of its physical properties as*

- *Color*
- *Shape and size*
- *Texture*

These are different features for the formation of an image block necessary to compose an image. Each block $a \in A$ is classified by 3F set according to its physical properties and represented in respective hesitant 3-polar fuzzy elements (H3FEs) as

$\wp_h(a_1) = \{(0.24, 0.44, 0.50), (0.34, 0.42, 0.61)\}$,
$\wp_h(a_2) = \{(0.54, 0.26, 0.33), (0.65, 0.75, 0.24), (0.34, 0.46, 0.64)\}$,
$\wp_h(a_3) = \{(0.51, 0.22, 0.24), (0.16, 0.34, 0.45), (0.78, 0.57, 0.39)\}$,
$\wp_h(a_4) = \{(0.41, 0.38, 0.57), (0.45, 0.27, 0.79)\}$

The H3FE $\wp_h(a_1) = \{(0.24, 0.44, 0.50), (0.34, 0.42, 0.61)\}$ shows, the a_1 block of an image has the following characteristics as $(\frac{0.24}{Color}, \frac{0.44}{Shape\ and\ size}, \frac{0.50}{Texture})$ and $(\frac{0.34}{Color}, \frac{0.42}{Shape\ and\ size}, \frac{0.61}{Texture})$ is the hesitation part of H3FE $\wp_h(a_1)$, similarly other image blocks are characterized in remaining H3FEs and the H3F-set M is given as

$$\begin{aligned} M = \Big\{ & \Big\langle a_1, \{(0.24, 0.44, 0.50), (0.34, 0.42, 0.61)\} \Big\rangle, \\ & \Big\langle a_2, \{(0.54, 0.26, 0.33), (0.65, 0.75, 0.24), (0.34, 0.46, 0.64)\} \Big\rangle, \\ & \Big\langle a_3, \{(0.51, 0.22, 0.24), (0.16, 0.34, 0.45), (0.78, 0.57, 0.39)\} \Big\rangle, \\ & \Big\langle a_4, \{(0.41, 0.38, 0.57), (0.45, 0.27, 0.79)\} \Big\rangle \Big\}. \end{aligned}$$

The H3F-set M shows the complete formation an image by the characterization and classification of its blocks.

From Example 2, it is easy to understand that in the approach described in Definition 4, in which we deal the multipolar information under hesitant situation of m tuple degrees of membership of mF sets. This approach is bound by the condition of an m tuple, its each degree of membership cannot be handled individually or separately.

3. The *m*-Polar Hesitant Fuzzy ELECTRE-I Approach

In this section, we introduce an mHF ELECTRE-I approach for MCGDM problems, which is based on the concept of mHF-set. We also apply this approach on real life examples in Section 3.1, to show its importance and feasibility. In this approach, we choose $A = \{a_1, a_2, \cdots, a_n\}$ the set of alternatives. According to this approach, we take $\{C_l | l = 1, 2, \cdots, k\}$ the set of criteria, which are further classified by the m-polar information in terms of hesitancy. The structure of the problem is as follows: the decision-makers are responsible for evaluating the n different alternatives under k criteria, and the suitable ratings of the alternatives are assessed in terms of m different characteristics under r different membership values due to hesitancy.

(i). The degree of each alternative ($a_p \in A,\ p = 1, 2, \cdots, n$) over all criteria ($c_l \in C,\ l = 1, 2, \cdots, k$) is given by mHFEs as

$$\hbar_m^{pl}(a) = \Big(\{\zeta_h | \zeta_h \in p_1 \circ \hbar_m^{pl}(a)\}, \{\zeta_h | \zeta_h \in p_2 \circ \hbar_m^{pl}(a)\}, \cdots, \{\zeta_h | \zeta_h \in p_m \circ \hbar_m^{pl}(a)\} \Big).$$

Tabular representation of an mHF decision matrix is given by Table 1, which describes the ratings of alternatives.

Table 1. Tabular representation of mHF decision matrix.

Alternatives	Criteria			
	c_1	c_2	$\cdots$	c_k
a_1	$\hbar_m^{11}(a_1)$	$\hbar_m^{12}(a_1)$	$\cdots$	$\hbar_m^{1k}(a_1)$
a_2	$\hbar_m^{21}(a_2)$	$\hbar_m^{22}(a_2)$	$\cdots$	$\hbar_m^{2k}(a_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_n	$\hbar_m^{n1}(a_n)$	$\hbar_m^{n2}(a_n)$	$\cdots$	$\hbar_m^{nk}(a_n)$

(ii). Decision-makers have an authority to assign the weights to each criteria of alternatives according to their choice and the importance of each criterium. We suppose that the weights assigned by the decision-makers are

$$w = (w_1, w_2, \cdots, w_k) \in (0, 1].$$

Weights assigned by the decision-makers satisfy the normalized condition, i.e.,

$$\sum_{l=1}^{k} w_l = 1.$$

(iii). The weighted mHF decision matrix is calculated as

$$W = \left[\left(\{\zeta_h' | \zeta_h' \in p_1 \circ e_m^{pl}(a)\}, \{\zeta_h' | \zeta_h' \in p_2 \circ e_m^{pl}(a)\}, \cdots, \{\zeta_h' | \zeta_h' \in p_m \circ e_m^{pl}(a)\}\right)\right]_{n \times k},$$

where $\zeta_h' = w_l \zeta_h$ & $p_i \circ e_m^{pl}(a) = w_l p_i \circ \hbar_m^{pl}(a), \forall a \in A$ and $i \in m$.

(iv). The mHF concordance set is defined as

$$Y_{uv} = \{1 \leq l \leq k | e_m^{ul}(a) \geq e_m^{vl}(a), u \neq v;\ u, v = 1, 2, \cdots, n\},$$

where $e_m^{pl}(a) = \sum_h \zeta_h' \in p_1 \circ e_m^{pl}(a) + \sum_h \zeta_h' \in p_2 \circ e_m^{pl}(a) + \cdots + \sum_h \zeta_h' \in p_m \circ e_m^{pl}(a)$.

(v). The mHF concordance indices are determined as

$$y_{uv} = \sum_{l \in Y_{uv}} w_l,$$

therefore, the mHF concordance matrix is computed as

$$Y = \begin{pmatrix} - & y_{12} & y_{13} & \cdots & y_{1n} \\ y_{21} & - & y_{23} & \cdots & y_{2n} \\ y_{31} & y_{32} & - & \cdots & y_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ y_{n1} & y_{n2} & y_{n3} & \cdots & - \end{pmatrix}.$$

(vi). The mHF discordance set is defined as

$$Y_{uv} = \{1 \leq l \leq k | e_m^{ul}(a) \leq e_m^{vl}(a), u \neq v;\ u, v = 1, 2, \cdots, n\},$$

where $e_m^{pl}(a) = \sum_h \zeta_h' \in p_1 \circ e_m^{pl}(a) + \sum_h \zeta_h' \in p_2 \circ e_m^{pl}(a) + \cdots + \sum_h \zeta_h' \in p_m \circ e_m^{pl}(a)$.

(vii). The mHF discordance indices are determined as

$$z_{uv} = \frac{\max\limits_{l \in Z_{uv}} \sqrt{\frac{1}{rm}\left[\sum\limits_{i=1}^{m}\left\{(\zeta_{h1}^{ul'} - \zeta_{h1}^{vl'})^2 + (\zeta_{h2}^{ul'} - \zeta_{h2}^{vl'})^2 + \cdots + (\zeta_{hr}^{ul'} - \zeta_{hr}^{vl'})^2\right\}\right]}}{\max\limits_{l} \sqrt{\frac{1}{rm}\left[\sum\limits_{i=1}^{m}\left\{(\zeta_{h1}^{ul'} - \zeta_{h1}^{vl'})^2 + (\zeta_{h2}^{ul'} - \zeta_{h2}^{vl'})^2 + \cdots + (\zeta_{hr}^{ul'} - \zeta_{hr}^{vl'})^2\right\}\right]}},$$

where $\zeta_{hq}^{ul'} \in p_i \circ e_m^{pl}(a)$, $\forall i \in m$ and $q = \{1, 2, \cdots, r\}$. Therefore, the mHF discordance matrix is be computed as

$$Z = \begin{pmatrix} - & z_{12} & z_{13} & \cdots & z_{1n} \\ z_{21} & - & z_{23} & \cdots & z_{2n} \\ z_{31} & z_{32} & - & \cdots & z_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ z_{n1} & z_{n2} & z_{n3} & \cdots & - \end{pmatrix}.$$

(viii). For the rankings of alternatives, we compute threshold values known as mHF concordance and discordance levels. The mHF concordance and discordance levels are the average of mHF concordance and discordance indices.

$$\bar{y} = \frac{1}{n(n-1)} \sum_{\substack{u=1 \\ u \neq v}}^{n} \sum_{\substack{v=1 \\ u \neq v}}^{n} y_{uv},$$

$$\bar{z} = \frac{1}{n(n-1)} \sum_{\substack{u=1 \\ u \neq v}}^{n} \sum_{\substack{v=1 \\ u \neq v}}^{n} z_{uv}.$$

(ix). The mHF concordance dominance matrix according to its mHF concordance level is computed as

$$R = \begin{pmatrix} - & r_{12} & r_{13} & \cdots & r_{1n} \\ r_{21} & - & r_{23} & \cdots & r_{2n} \\ r_{31} & r_{32} & - & \cdots & r_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ r_{n1} & r_{n2} & r_{n3} & \cdots & - \end{pmatrix},$$

where,

$$r_{uv} = \begin{cases} 1, & y_{uv} \geq \bar{y}; \\ 0, & y_{uv} < \bar{y}. \end{cases}$$

(x). The mHF discordance dominance matrix according to its mHF discordance level is computed as

$$S = \begin{pmatrix} - & s_{12} & s_{13} & \cdots & s_{1n} \\ s_{21} & - & s_{23} & \cdots & s_{2n} \\ s_{31} & s_{32} & - & \cdots & s_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ s_{n1} & s_{n2} & s_{n3} & \cdots & - \end{pmatrix},$$

where,

$$s_{uv} = \begin{cases} 1, & z_{uv} < \bar{z}; \\ 0, & z_{uv} \geq \bar{z}. \end{cases}$$

(xi). The aggregated mHF dominance matrix is computed as

$$T = \begin{pmatrix} - & t_{12} & t_{13} & \cdots & t_{1n} \\ t_{21} & - & t_{23} & \cdots & t_{2n} \\ t_{31} & t_{32} & - & \cdots & t_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ t_{n1} & t_{n2} & t_{n3} & \cdots & - \end{pmatrix},$$

where, t_{uv} is defined as

$$t_{uv} = r_{uv} s_{uv}.$$

(xii). Finally, rank the alternatives according to the outranking values of matrix T. For each pair of alternatives there exist a directed edge from alternative a_u to a_v if and only if $t_{uv} = 1$. Thus, the following three cases arises.

1. There exists a unique directed edge from a_u to a_v, which shows a_u is preferred over a_v.
2. There exists directed edges from a_u to a_v and a_v to a_u, which shows a_u and a_v are indifferent.
3. There does not exist any edge between a_u and a_v, which shows a_u and a_v are not comparable.

3.1. Selection of a Best Brick for Construction

The brick selection is significant in the sense that it regulates a project's constancy and presentation, and crops in a durable impact. It is crucial to analyze and classify which criteria or properties of brick are convenient to acknowledge in choosing the best brick. Brick having vast variety of size, color, strength, texture and shape are accessible. The designer, owner and engineers have to decide which aspects and attributes of brick are most demanding. This procedure of selection can precept the eminence and accomplishment of any project. Our first model mHF-set discusses the criteria or properties which are acknowledged in the selection of the convenient brick for a project under the hesitant decision of project designers or engineers. Selection of brick is based on several factors and criteria. It does not only depend upon durability importance, but absorption, strength, cost and availability are important to the designers, owner and contractors. The selection process may be challenging and tough since each group is trying to entertain several requirements. Generally, the ultimate selection depends upon the adjustment of all the including parties. To apply the concept of our purposed model in real life situation, we consider $Br = \{\mathrm{Br}_1, \mathrm{Br}_2, \mathrm{Br}_3, \mathrm{Br}_4, \mathrm{Br}_5, \mathrm{Br}_6\}$ the set of six different types of bricks which have to be analyzed and $C = \{c_1, c_2, c_3\}$ the set of three main criteria or properties to select the bricks for the construction. For the evaluation the project dealers including owner, designer, constructor and engineer focus on three main criteria or properties of bricks such as physical properties, mechanical properties and durability, which are further classified into three different sub-criteria as

1. The "Physical Properties" may include
    - "Shape", normally an ideal brick has absolutely rectangular shape. Its edges are sharp, well defined and having even and regular surface.
    - "Size and Color", in construction the practiced size of brick differs from place to place and from country to country, where as the color of bricks may vary from dull red to light red and from buff to purple.
    - "Density", the weight per unit volume or the density of bricks mostly depends on the process of brick molding and type of clay used to prepare it.
2. The "Mechanical Properties" may include

- "Compressive Strength", it is the highest considerable and crucial estate of bricks specifically when they are utilized in load-bearing walls, it depends on the degree of burning and formation of the clay.
- "Flexure Strength", usually bricks are utilized in directions and stages where tilting and twisting loads are feasible in a building. In essence, they maintain satisfactory strength across transverse loads.
- "Slenderness Ratio", in turn it depends upon the effective height, length and thickness of the wall or column.

3. The "Durability" may include

- "Absorption Value", this estate is depicted to the brick porosity. True Porosity is described as the rate of the volume of pores to the gross volume of the sample of the substance.
- "Frost Resistance", when bricks are utilized in cold climates, their decomposition due to this phenomenon of frost action may be a common process. therefore, it is significant that bricks in such areas should be accordingly protected from rain to decrease absorption.
- "Efflorescence", it is a natural distorting and depreciating process of bricks in humid and hot climates.

All these criteria or properties are assessed by a group of project dealers, who are responsible for evaluating the best bricks for construction. Due to their collective decision each criteria is further classified by three sub criteria, which are evaluated by four different hesitant values assigned by project dealers. They are free to choose any membership value from the interval $[0,1]$. Thus, project dealers assign hesitant values as described in Table 2. Obviously, the count of 3HFEs in general is not comparable in all 3HF-sets. In order to gain efficiency and accuracy, we extend the largest membership value as far as the lengths of all 3HFEs become equal because the required company wants to take bricks of class one on an optimistic spirit. For this reason we show an optimistic response and improves the 3HF data by adding the maximal values as mentioned in Table 3.

(i). Tabular representation of 3HF decision matrix is given by Table 2.

Table 2. Tabular representation of 3HF decision matrix.

Bricks	Physical Properties		
	Shape	Size and Color	Density
Br_1	$\{0.55, 0.57, 0.67, 0.69\}$	$\{0.39, 0.46, 0.66\}$	$\{0.57, 0.65, 0.66\}$
Br_2	$\{0.46, 0.58, 0.59\}$	$\{0.77, 0.79, 0.80, 0.91\}$	$\{0.70, 0.75\}$
Br_3	$\{0.51, 0.63, 0.77, 0.80\}$	$\{0.66, 0.72\}$	$\{0.59, 0.60, 0.71, 0.82\}$
Br_4	$\{0.39, 0.41, 0.53\}$	$\{0.61, 0.65, 0.81, 0.83\}$	$\{0.54, 0.65, 0.69\}$
Br_5	$\{0.49, 0.56\}$	$\{0.60, 0.65, 0.71\}$	$\{0.48, 0.57, 0.68, 0.71\}$
Br_6	$\{0.50, 0.61, 0.63\}$	$\{0.70, 0.78\}$	$\{0.47, 0.67\}$
Bricks	**Mechanical Properties**		
	Compressive Strength	**Flexure Strength**	**Slenderness Ratio**
Br_1	$\{0.65, 0.66, 0.69\}$	$\{0.40, 0.61, 0.66, 0.70\}$	$\{0.66, 0.68, 0.70\}$
Br_2	$\{0.49, 0.53, 0.58, 0.60\}$	$\{0.47, 0.59\}$	$\{0.55, 0.62, 0.67, 0.69\}$
Br_3	$\{0.61, 0.73\}$	$\{0.56, 0.58, 0.70\}$	$\{0.61, 0.72\}$
Br_4	$\{0.35, 0.47, 0.53, 0.67\}$	$\{0.50, 0.58, 0.61, 0.71\}$	$\{0.54, 0.64, 0.69\}$
Br_5	$\{0.59, 0.61, 0.63\}$	$\{0.61, 0.68, 0.73\}$	$\{0.60, 0.69\}$
Br_6	$\{0.49, 0.60, 0.63, 0.70\}$	$\{0.62, 0.65, 0.71\}$	$\{0.60, 0.77, 0.79, 0.80\}$

Table 2. *Cont.*

Bricks	Durability		
	Absorption Value	**Frost Resistance**	**Efflorescence**
Br_1	$\{0.25, 0.36, 0.37, 0.40\}$	$\{0.73, 0.74, 0.76\}$	$\{0.45, 0.55, 0.56, 0.58\}$
Br_2	$\{0.46, 0.48, 0.49\}$	$\{0.47, 0.49, 0.51, 0.56\}$	$\{0.55, 0.61, 0.66\}$
Br_3	$\{0.31, 0.33, 0.45, 0.46\}$	$\{0.66, 0.68, 0.70\}$	$\{0.51, 0.72\}$
Br_4	$\{0.29, 0.31\}$	$\{0.60, 0.68\}$	$\{0.60, 0.67, 0.69, 0.73\}$
Br_5	$\{0.49, 0.51, 0.53, 0.56\}$	$\{0.60, 0.68, 0.71, 0.73\}$	$\{0.67, 0.69\}$
Br_6	$\{0.39, 0.41, 0.43\}$	$\{0.56, 0.68, 0.73, 0.83\}$	$\{0.50, 0.56, 0.67, 0.69\}$

Table 3. Tabular representation of optimistic 3HF decision matrix by adding maximal values.

Bricks	Physical Properties		
	Shape	**Size and Color**	**Density**
Br_1	$\{0.55, 0.57, 0.67, 0.69\}$	$\{0.39, 0.46, 0.66, 0.66\}$	$\{0.57, 0.65, 0.66, 0.66\}$
Br_2	$\{0.46, 0.58, 0.59, 0.59\}$	$\{0.77, 0.79, 0.80, 0.91\}$	$\{0.70, 0.75, 0.75, 0.75\}$
Br_3	$\{0.51, 0.63, 0.77, 0.80\}$	$\{0.66, 0.72, 0.72, 0.72\}$	$\{0.59, 0.60, 0.71, 0.82\}$
Br_4	$\{0.39, 0.41, 0.53, 0.53\}$	$\{0.61, 0.65, 0.81, 0.83\}$	$\{0.54, 0.65, 0.69, 0.69\}$
Br_5	$\{0.49, 0.56, 0.56, 0.56\}$	$\{0.60, 0.65, 0.71, 0.71\}$	$\{0.48, 0.57, 0.68, 0.71\}$
Br_6	$\{0.50, 0.61, 0.63, 0.63\}$	$\{0.70, 0.78, 0.78, 0.78\}$	$\{0.47, 0.67, 0.67, 0.67\}$
Bricks	**Mechanical Properties**		
	Compressive Strength	**Flexure Strength**	**Slenderness Ratio**
Br_1	$\{0.65, 0.66, 0.69, 0.69\}$	$\{0.40, 0.61, 0.66, 0.70\}$	$\{0.66, 0.68, 0.70, 0.70\}$
Br_2	$\{0.49, 0.53, 0.58, 0.60\}$	$\{0.47, 0.59, 0.59, 0.59\}$	$\{0.55, 0.62, 0.67, 0.69\}$
Br_3	$\{0.61, 0.73, 0.73, 0.73\}$	$\{0.56, 0.58, 0.70, 0.70\}$	$\{0.61, 0.72, 0.72, 0.72\}$
Br_4	$\{0.35, 0.47, 0.53, 0.67\}$	$\{0.50, 0.58, 0.61, 0.71\}$	$\{0.54, 0.64, 0.69, 0.69\}$
Br_5	$\{0.59, 0.61, 0.63, 0.63\}$	$\{0.61, 0.68, 0.73, 0.73\}$	$\{0.60, 0.69, 0.69, 0.69\}$
Br_6	$\{0.49, 0.60, 0.63, 0.70\}$	$\{0.62, 0.65, 0.71, 0.71\}$	$\{0.60, 0.77, 0.79, 0.80\}$
Bricks	**Durability**		
	Absorption Value	**Frost Resistance**	**Efflorescence**
Br_1	$\{0.25, 0.36, 0.37, 0.40\}$	$\{0.73, 0.74, 0.76, 0.76\}$	$\{0.45, 0.55, 0.56, 0.58\}$
Br_2	$\{0.46, 0.48, 0.49, 0.49\}$	$\{0.47, 0.49, 0.51, 0.56\}$	$\{0.55, 0.61, 0.66, 0.66\}$
Br_3	$\{0.31, 0.33, 0.45, 0.46\}$	$\{0.66, 0.68, 0.70, 0.70\}$	$\{0.51, 0.72, 0.72, 0.72\}$
Br_4	$\{0.29, 0.31, 0.31, 0.31\}$	$\{0.60, 0.68, 0.68, 0.68\}$	$\{0.60, 0.67, 0.69, 0.73\}$
Br_5	$\{0.49, 0.51, 0.53, 0.56\}$	$\{0.60, 0.68, 0.71, 0.73\}$	$\{0.67, 0.69, 0.69, 0.69\}$
Br_6	$\{0.39, 0.41, 0.43, 0.43\}$	$\{0.56, 0.68, 0.73, 0.83\}$	$\{0.50, 0.56, 0.67, 0.69\}$

(ii). The normalized weights assigned to each criteria are given as follows:

$$w_l = (0.234, 0.395, 0.371).$$

(iii). The weighted optimistic 3HF decision matrix is calculated in Table 4.

Table 4. Tabular representation of weighted optimistic 3HF decision matrix.

Bricks	**Physical Properties with Weight 0.234**		
	Shape	**Size and Color**	**Density**
Br_1	$\{0.1287, 0.1334, 0.1568, 0.1615\}$	$\{0.0913, 0.1076, 0.1544, 0.1544\}$	$\{0.1334, 0.1521, 0.1544, 0.1544\}$
Br_2	$\{0.1076, 0.1357, 0.1381, 0.1381\}$	$\{0.1802, 0.1849, 0.1872, 0.2129\}$	$\{0.1638, 0.1755, 0.1755, 0.1755\}$
Br_3	$\{0.1193, 0.1474, 0.1802, 0.1872\}$	$\{0.1544, 0.1685, 0.1685, 0.1685\}$	$\{0.1381, 0.1404, 0.1661, 0.1919\}$
Br_4	$\{0.0913, 0.0959, 0.1240, 0.1240\}$	$\{0.1427, 0.1521, 0.1895, 0.1942\}$	$\{0.1264, 0.1521, 0.1615, 0.1615\}$
Br_5	$\{0.1147, 0.1310, 0.1310, 0.1310\}$	$\{0.1404, 0.1521, 0.1661, 0.1661\}$	$\{0.1123, 0.1334, 0.1591, 0.1661\}$
Br_6	$\{0.1170, 0.1427, 0.1474, 0.1474\}$	$\{0.1638, 0.1825, 0.1825, 0.1825\}$	$\{0.1100, 0.1568, 0.1568, 0.1568\}$
Bricks	**Mechanical Properties with Weight 0.395**		
	Compressive Strength	**Flexure Strength**	**Slenderness Ratio**
Br_1	$\{0.2568, 0.2607, 0.2726, 0.2726\}$	$\{0.1580, 0.2410, 0.2607, 0.2765\}$	$\{0.2607, 0.2686, 0.2765, 0.2765\}$
Br_2	$\{0.1936, 0.2094, 0.2291, 0.2370\}$	$\{0.1857, 0.2331, 0.2331, 0.2331\}$	$\{0.2173, 0.2449, 0.2647, 0.2726\}$
Br_3	$\{0.2410, 0.2884, 0.2884, 0.2884\}$	$\{0.2212, 0.2291, 0.2765, 0.2765\}$	$\{0.2410, 0.2844, 0.2844, 0.2844\}$
Br_4	$\{0.1382, 0.1857, 0.2094, 0.2647\}$	$\{0.1975, 0.2291, 0.2410, 0.2804\}$	$\{0.2133, 0.2528, 0.2726, 0.2726\}$
Br_5	$\{0.2331, 0.2410, 0.2489, 0.2489\}$	$\{0.2410, 0.2686, 0.2884, 0.2884\}$	$\{0.2370, 0.2726, 0.2726, 0.2726\}$
Br_6	$\{0.1936, 0.2370, 0.2489, 0.2765\}$	$\{0.2449, 0.2568, 0.2804, 0.2804\}$	$\{0.2370, 0.3042, 0.3121, 0.3160\}$
Bricks	**Durability with Weight 0.371**		
	Absorption Value	**Frost Resistance**	**Efflorescence**
Br_1	$\{0.0927, 0.1336, 0.1373, 0.1484\}$	$\{0.2708, 0.2745, 0.2820, 0.2820\}$	$\{0.1670, 0.2041, 0.2078, 0.2152\}$
Br_2	$\{0.1707, 0.1781, 0.1818, 0.1818\}$	$\{0.1744, 0.1818, 0.1892, 0.2078\}$	$\{0.2041, 0.2263, 0.2449, 0.2449\}$
Br_3	$\{0.1150, 0.1224, 0.1670, 0.1707\}$	$\{0.2449, 0.2523, 0.2597, 0.2597\}$	$\{0.1892, 0.2671, 0.2671, 0.2671\}$
Br_4	$\{0.1076, 0.1150, 0.1150, 0.1150\}$	$\{0.2226, 0.2523, 0.2523, 0.2523\}$	$\{0.2226, 0.2486, 0.2560, 0.2708\}$
Br_5	$\{0.1818, 0.1892, 0.1966, 0.2078\}$	$\{0.2226, 0.2523, 0.2634, 0.2708\}$	$\{0.2486, 0.2560, 0.2560, 0.2560\}$
Br_6	$\{0.1447, 0.1521, 0.1595, 0.1595\}$	$\{0.2078, 0.2523, 0.2708, 0.3079\}$	$\{0.1855, 0.2078, 0.2486, 0.2560\}$

(iv). A 3HF concordance set is calculated in Table 5.

Table 5. Tabular representation of 3HF concordance set.

v	**1**	**2**	**3**	**4**	**5**	**6**
Y_{1v}	–	$\{2,3\}$	$\{\ \}$	$\{2\}$	$\{\ \}$	$\{\ \}$
Y_{2v}	$\{1\}$	–	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
Y_{3v}	$\{1,2,3\}$	$\{2,3\}$	–	$\{1,2,3\}$	$\{1,2\}$	$\{1,2,3\}$
Y_{4v}	$\{1,3\}$	$\{2,3\}$	$\{\ \}$	–	$\{1\}$	$\{\ \}$
Y_{5v}	$\{1,2,3\}$	$\{2,3\}$	$\{3\}$	$\{2,3\}$	–	$\{3\}$
Y_{6v}	$\{1,2,3\}$	$\{2,3\}$	$\{\ \}$	$\{1,2,3\}$	$\{1,2\}$	–

(v). A 3HF concordance matrix is calculated as follows:

$$Y = \begin{pmatrix} - & 0.7660 & 0.0000 & 0.3950 & 0.0000 & 0.0000 \\ 0.2340 & - & 0.2340 & 0.2340 & 0.2340 & 0.2340 \\ 1.0000 & 0.7660 & - & 1.0000 & 0.6290 & 1.0000 \\ 0.6050 & 0.7660 & 0.0000 & - & 0.2340 & 0.0000 \\ 1.0000 & 0.7660 & 0.3710 & 0.7660 & - & 0.3710 \\ 1.0000 & 0.7660 & 0.0000 & 1.0000 & 0.6290 & - \end{pmatrix}.$$

(vi). A 3HF discordance set is calculated in Table 6.

Table 6. Tabular representation of 3HF discordance set.

v	1	2	3	4	5	6
Z_{1v}	–	{1}	{1,2,3}	{1,3}	{1,2,3}	{1,2,3}
Z_{2v}	{2,3}	–	{2,3}	{2,3}	{2,3}	{2,3}
Z_{3v}	{ }	{1}	–	{ }	{3}	{ }
Z_{4v}	{2}	{1}	{1,2,3}	–	{2,3}	{1,2,3}
Z_{5v}	{ }	{1}	{1,2}	{1}	–	{1,2}
Z_{6v}	{ }	{1}	{1,2,3}	{ }	{3}	–

(vii). A 3HF discordance matrix is calculated as follows:

$$Z = \begin{pmatrix} - & 0.6832 & 1.0000 & 0.7801 & 1.0000 & 1.0000 \\ 1.0000 & - & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.0000 & 0.6075 & - & 0.0000 & 1.0000 & 0.0000 \\ 1.0000 & 0.4833 & 1.0000 & - & 1.0000 & 1.0000 \\ 0.0000 & 0.6978 & 0.7181 & 0.3702 & - & 0.6990 \\ 0.0000 & 0.4596 & 1.0000 & 0.0000 & 1.0000 & - \end{pmatrix}.$$

(viii). A 3HF concordance level $\overline{y} = 0.5000$, and 3HF discordance level $\overline{z} = 0.6833$ are calculated.
(ix). A 3HF concordance dominance matrix is calculated as follows:

$$R = \begin{pmatrix} - & 1 & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 & 0 \\ 1 & 1 & - & 1 & 1 & 1 \\ 1 & 1 & 0 & - & 0 & 0 \\ 1 & 1 & 0 & 1 & - & 0 \\ 1 & 1 & 0 & 1 & 1 & - \end{pmatrix}.$$

(x). A 3HF discordance dominance matrix is calculated as follows:

$$S = \begin{pmatrix} - & 1 & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 & 0 \\ 1 & 1 & - & 1 & 0 & 1 \\ 0 & 1 & 0 & - & 0 & 0 \\ 1 & 0 & 0 & 1 & - & 0 \\ 1 & 1 & 0 & 1 & 0 & - \end{pmatrix}.$$

(xi). An aggregated 3HF dominance matrix is calculated as follows:

$$T = \begin{pmatrix} - & 1 & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 & 0 \\ 1 & 1 & - & 1 & 0 & 1 \\ 0 & 1 & 0 & - & 0 & 0 \\ 1 & 0 & 0 & 1 & - & 0 \\ 1 & 1 & 0 & 1 & 0 & - \end{pmatrix}.$$

(xii). According to outranking values of aggregated 3HF dominance matrix the bricks have the following relation as shown in Figure 1.

Hence, Br_3 is the brick having most out ranking value as compared to others and selected for construction.

We show the comparison of bricks and summarize the whole procedure in Table 7.

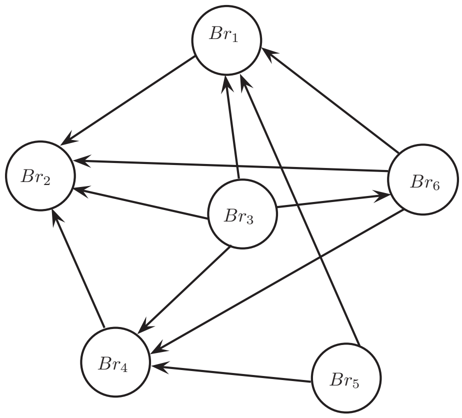

Figure 1. Graphical representation of outranking relation of bricks.

Table 7. Tabular representation of comparison of bricks.

Comparison of CS of Bricks	Y_{uv}	Z_{uv}	y_{uv}	z_{uv}	r_{uv}	s_{uv}	t_{uv}	*Ranking*
(Br_1, Br_2)	$\{2,3\}$	$\{1\}$	0.7660	0.6832	1	1	1	$Br_1 \to Br_2$
(Br_1, Br_3)	$\{\ \}$	$\{1,2,3\}$	0	1	0	0	0	Incomparable
(Br_1, Br_4)	$\{2\}$	$\{1,3\}$	0.3950	0.7801	0	1	0	Incomparable
(Br_1, Br_5)	$\{\ \}$	$\{1,2,3\}$	0	1	0	0	0	Incomparable
(Br_1, Br_6)	$\{\ \}$	$\{1,2,3\}$	0	1	0	0	0	Incomparable
(Br_2, Br_1)	$\{1\}$	$\{2,3\}$	0.2340	1	0	0	0	Incomparable
(Br_2, Br_3)	$\{1\}$	$\{2,3\}$	0.2340	1	0	0	0	Incomparable
(Br_2, Br_4)	$\{1\}$	$\{2,3\}$	0.2340	1	0	0	0	Incomparable
(Br_2, Br_5)	$\{1\}$	$\{2,3\}$	0.2340	1	0	0	0	Incomparable
(Br_2, Br_6)	$\{1\}$	$\{2,3\}$	0.2340	1	0	0	0	Incomparable
(Br_3, Br_1)	$\{1,2,3\}$	$\{\ \}$	1	0	1	1	1	$Br_3 \to Br_1$
(Br_3, Br_2)	$\{2,3\}$	$\{1\}$	0.7660	0.6075	1	1	1	$Br_3 \to Br_2$
(Br_3, Br_4)	$\{1,2,3\}$	$\{\ \}$	1	0	1	1	1	$Br_3 \to Br_4$
(Br_3, Br_5)	$\{1,2\}$	$\{3\}$	0.6290	1	1	0	0	Incomparable
(Br_3, Br_6)	$\{1,2,3\}$	$\{\ \}$	1	0	1	1	1	$Br_3 \to Br_6$
(Br_4, Br_1)	$\{1,3\}$	$\{2\}$	0.6050	1	1	0	0	Incomparable
(Br_4, Br_2)	$\{2,3\}$	$\{1\}$	0.7660	0.4833	1	1	1	$Br_4 \to Br_2$
(Br_4, Br_3)	$\{\ \}$	$\{1,2,3\}$	0	1	0	0	0	Incomparable
(Br_4, Br_5)	$\{1\}$	$\{2,3\}$	0.2340	1	0	0	0	Incomparable
(Br_4, Br_6)	$\{\ \}$	$\{1,2,3\}$	0	1	0	0	0	Incomparable
(Br_5, Br_1)	$\{1,2,3\}$	$\{\ \}$	1	0	1	1	1	$Br_5 \to Br_1$
(Br_5, Br_2)	$\{2,3\}$	$\{1\}$	0.7660	0.6978	1	0	0	Incomparable
(Br_5, Br_3)	$\{3\}$	$\{1,2\}$	0.3710	0.7181	0	0	0	Incomparable
(Br_5, Br_4)	$\{2,3\}$	$\{1\}$	0.7660	0.3702	1	1	1	$Br_5 \to Br_4$
(Br_5, Br_6)	$\{3\}$	$\{1,2\}$	0.3710	0.6990	0	0	0	Incomparable
(Br_6, Br_1)	$\{1,2,3\}$	$\{\ \}$	1	0	1	1	1	$Br_6 \to Br_1$
(Br_6, Br_2)	$\{2,3\}$	$\{1\}$	0.7660	0.4596	1	1	1	$Br_6 \to Br_2$
(Br_6, Br_3)	$\{\ \}$	$\{1,2,3\}$	0	1	0	0	0	Incomparable
(Br_6, Br_4)	$\{1,2,3\}$	$\{\ \}$	1	0	1	1	1	$Br_6 \to Br_4$
(Br_6, Br_5)	$\{1,2\}$	$\{3\}$	0.6290	1	1	0	0	Incomparable

4. The Hesitant *m*-Polar Fuzzy ELECTRE-I Approach

In this section, we propose an H*m*F ELECTRE-I approach for MCGDM, which is flexible and compatible to deal the hesitant situations motivated by the multipolar information as we discuss it in Section 4.1. Our proposed ELECTRE-I approach based on HmF-set deals with MCGDM problems, in which we choose $A = \{a_1, a_2, \cdots, a_n\}$ the set of different alternatives and $\{C_l | l = 1, 2, \cdots, k\}$ the set of HmF criteria which facilitate the management of hesitation, uncertainty and vagueness motivated by multipolar information. In such a case, decision-makers are responsible for evaluating the n different alternatives under k HmF criteria, the suitable ratings of alternatives are according to decision-makers, assessed in term of m different characteristics under r different membership values of hesitancy, where $(q = 1, 2, \cdots, r)$. The following steps for the proposed approach are described as follows:

(i). The degree of each alternative $(a_p \in A,\ p = 1, 2, \cdots, n)$ over all criteria $(c_l \in C,\ l = 1, 2, \cdots, k)$ is given by HmFEs as

$$\wp_h(z) = \Big\{(p_1 \circ m_h(a), p_2 \circ m_h(a), \cdots, p_m \circ m_h(a))\Big\}, \ \forall\, a \in A,$$

and $m_h(a) = (p_1 \circ m_h(a), p_2 \circ m_h(a), \cdots, p_m \circ m_h(a))$ classify the different characteristics of each criteria. Tabular representation of HmF decision matrix is given by Table 8, which describes the ratings of alternatives.

Table 8. Tabular representation of HmF decision matrix.

Alternatives	Hesitant m-Polar Fuzzy Criteria			
	c_1	c_2	$\cdots$	c_k
a_1	$\wp_h^{11}(a_1)$	$\wp_h^{12}(a_1)$	$\cdots$	$\wp_h^{1k}(a_1)$
a_2	$\wp_h^{21}(a_2)$	$\wp_h^{22}(a_2)$	$\cdots$	$\wp_h^{2k}(a_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_n	$\wp_h^{p1}(a_n)$	$\wp_h^{p2}(a_n)$	$\cdots$	$\wp_h^{pk}(a_n)$

For each possible p and l,

$$\wp_h^{pl}(a) = \Big\{(p_1 \circ m_h^{pl}(a), p_2 \circ m_h^{pl}(a), \cdots, p_m \circ m_h^{pl}(a))\Big\}.$$

(ii). Same as described in Section 3.

(iii). The weighted HmF decision matrix is calculated as

$$W = \Big[\Big\{(p_1 \circ e_h^{pl}(a), p_2 \circ e_h^{pl}(a), \cdots, p_m \circ e_h^{pl}(a))\Big\}\Big]_{n\times k},$$

where $p_i \circ e_h^{pl}(a) = w_l p_i \circ m_h^{pl}(a), \forall a \in A$ and $i \in m$.

(iv). The HmF concordance set is defined as

$$Y_{uv} = \{1 \leq l \leq k | e_h^{ul}(a) \geq e_h^{vl}(a), u \neq v;\ u, v = 1, 2, \cdots, n\},$$

where $e_h^{pl}(a) = \sum\limits_{q=1}^{r} \{(p_1 \circ e_h^{pl}(a) + p_2 \circ e_h^{pl}(a) + \cdots + p_m \circ e_h^{pl}(a))_q\}$.

(v). The HmF discordance set is defined as

$$Y_{uv} = \{1 \leq l \leq k | e_h^{ul}(a) \leq e_h^{vl}(a), u \neq v;\ u, v = 1, 2, \cdots, n\},$$

where $e_h^{pl}(a) = \sum\limits_{q=1}^{r} \{(p_1 \circ e_h^{pl}(a) + p_2 \circ e_h^{pl}(a) + \cdots + p_m \circ e_h^{pl}(a))_q\}$.

(vi). Same as described in Section 3.

(vii). The HmF discordance indices are determined as

$$z_{uv} = \frac{\max\limits_{l \in Z_{uv}} \sqrt{\frac{1}{rm}\left[\sum\limits_{q=1}^{r}\left\{\sum\limits_{i=1}^{m}\left((p_i \circ e_h^{ul}(a) - p_i \circ e_h^{vl}(a))^2\right)\right\}_q\right]}}{\max\limits_{l} \sqrt{\frac{1}{rm}\left[\sum\limits_{q=1}^{r}\left\{\sum\limits_{i=1}^{m}\left((p_i \circ e_h^{ul}(a) - p_i \circ e_h^{vl}(a))^2\right)\right\}_q\right]}},$$

therefore, the HmF discordance matrix is be computed as

$$Z = \begin{pmatrix} - & z_{12} & z_{13} & \cdots & z_{1n} \\ z_{21} & - & z_{23} & \cdots & z_{2n} \\ z_{31} & z_{32} & - & \cdots & z_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ z_{n1} & z_{n2} & z_{n3} & \cdots & - \end{pmatrix}.$$

Steps (viii) to (xii) are same as described in Section 3.

4.1. Site Selection for Farming Purposes

Due to an increasing relevance of farming, land is considered to be a very significant element. Therefore, the selection of a site for farming purposes is a basic and fundamental analysis for the farmers. It is the fundamental step to initiate a farm when it has already been decided which crop should be grown. It also incorporates the selection of the suitable geographical location. This is the case with associate enterprising individuals and investors with acceptable dominates. Our second model HmF-set discuss the factors or criteria which must be considered in the selection of the appropriate land site for farming purposes under the hesitant decision of farmers, investors and enter prising individuals. Site selection for farming purposes is based on a number of factors and criteria. To apply the concept of our purposed model in a real life situation, we consider $S = \{\mathrm{S_{f1}}, \mathrm{S_{f2}}, \mathrm{S_{f3}}, \mathrm{S_{f4}}, \mathrm{S_{f5}}\}$ the set of five different sites for farming which have to be analyzed and $C = \{c_1, c_2, c_3, c_4\}$ the set of four main factors or criteria to choose the site. For the evaluation the decision-makers including farmers, investors and enter prising individuals focus on four main criteria or factors of sites such as climatic factor, socioeconomic factor, edaphic factor and other essential factors, which facilitate the hesitation and uncertainty motivated by multipolar information.

1. The "Climatic Factor" may include
    - "Rainfall", which is the most frequent and familiar form of precipitation. The extent, measure and consistency of rainfall differ with area, climate and location types. It induce the influence of certain types of vegetation, growth of crop and its yield.
    - "Humidity", which is the actual measure of water vapor in the air, considered as the percentage of the maximal capacity of water vapor it can dominate at usual temperature. It has different affects on the closing and opening of the stomata, which coordinates deficiency of water from the plant through photosynthesis and transpiration.
    - "Wind Pressure", which is caused by differences in heating and due to the presence of pressure gradient on local and global scale. It compacts and the pressure raises, when the air close to the ground cools and it expands and drops pressure, when it warms.
    - "Temperature", which has a great ascendancy on all growth processes of a plant such as respiration, photosynthesis, etc. At huge temperatures the alteration of photosyntheses is much more rapid and active so that plants tend to develop earlier.
2. The "Socioeconomic Factor" may include

- "Infrastructure", which is the requirement of large scale farming infrastructure to assure the highest yields per acre. Water movement towards the crops as well as away from the crops, is an analytical process to production.
- "Land Tenancy", which includes all models and plans of tenancy and ownership in any form. Land tenancy and land tenure affect the agricultural actions, activities and cropping patterns in many ways. The cultivators proceed the agricultural activities and farm management, by keeping in mind their benefits and occupancy duration on the land.
- "Labor", the availability and possibility of labor is also a major constraint in the use of agricultural land and cropping impressions of a region. It serves as all human maintenance except decision-making and fundamentals. In decision-making process of the farmer, the availability of labor, its quality and quantity at the periods of peak labor demand have a great significance.
- "Marketing facility", the accessibility and approach to the market is a major discussion. The concentration of agriculture and the production of crops descent as the location of cultivation takes away from the marketing centers.

3. The "Edaphic Factor" may include

- "Structure", to execute effectually as a growing medium, soils demand an open structure through the soil profile. For healthy plant growth, an effective soil structure allows water and air into the soil which are crucial. It improves drainage and lower the soil destruction due to excess surface run-off.
- "Fertility", which is the capacity of a soil to assist agricultural growth of plant, to maintain the plant surroundings and result in defend and homogeneous yields of immense quality. It supplies fundamental plant nutrients and water in sufficient amount and proportion for growth and reproduction of plant.
- "Texture",which is an essential soil exclusive that consequences storm water in filtration estimates. The texturing class of a soil is resolved by the ratio of clay, sand and slit.
- "Porosity and Consistency", soil porosity indicates the amount of pores and open spaces between soil particles. The soil compactness is the durability with which soil materials are held together or the resistance of soils to deformation.

4. The "Essential Factor" may include

- "Environment", the different operations of farming should not have a negative impact on the environment. The environment is not suitable or sometimes even harmful when the farming sites are close to an urban area.
- "Government Policies", it is in the interests of distinct governments to make policies that are convenient to attain growth in agriculture. It is possible to use this influence and set up the farm in an area likely to gain from the performance of the policy.
- "Biotic Interactions", which reveal the existence or absence of some beneficial or harmful organisms. The natural population of certain organisms like bees and other pollinators have a great importance in site selection for farming purposes.
- "Economic Agents", this factor is considered as the most important to develop the agricultural business. It includes the benefits, terms of lease or acquisition and cost.

All these criteria or factors are assessed by decision-makers, who are responsible for the selection of site. Due to their collective decision, each factor is further classified by multipolar information and evaluated by three different hesitant values assigned by decision-makers, who are free to choose any membership value from the interval $[0, 1]$. Thus decision-makers assign hesitant values as described in Table 9. Obviously, the count of H4FEs in general is not comparable in all H4F-sets. In order to gain efficiency and accuracy, we extend the smallest membership value such that the lengths of all H4FEs become equal because the required policy wants to select the site with the pessimistic prediction. For this reason we show pessimistic response and improve the H4F data by adding the minimal values as mentioned in Table 10.

(i). Tabular representation of H4F decision matrix is given by Table 9.

Table 9. Tabular representation of H4F decision matrix.

Sites	H4F Factor as Criteria C_1 Climatic Factors
S_{f1}	$\{(0.81, 0.65, 0.45, 0.69), (0.78, 0.66, 0.51, 0.74), (0.77, 0.62, 0.52, 0.67)\}$
S_{f2}	$\{(0.68, 0.59, 0.67, 0.89), (0.61, 0.54, 0.63, 0.70)\}$
S_{f3}	$\{(0.85, 0.79, 0.57, 0.87), (0.74, 0.60, 0.60, 0.80)\}$
S_{f4}	$\{(0.43, 0.84, 0.66, 0.79), (0.49, 0.83, 0.75, 0.82), (0.54, 0.78, 0.71, 0.85)\}$
S_{f5}	$\{(0.67, 0.75, 0.58, 0.75), (0.60, 0.63, 0.62, 0.75)\}$
Sites	**H4F Factor as Criteria C_2 Socio-Economic Factors**
S_{f1}	$\{(0.78, 0.57, 0.69, 0.46), (0.81, 0.69, 0.65, 0.49)\}$
S_{f2}	$\{(0.67, 0.72, 0.51, 0.77), (0.68, 0.74, 0.64, 0.76), (0.66, 0.70, 0.60, 0.74)\}$
S_{f3}	$\{(0.48, 0.68, 0.73, 0.19), (0.46, 0.69, 0.78, 0.21), (0.50, 0.82, 0.85, 0.27)\}$
S_{f4}	$\{(0.49, 0.76, 0.39, 0.79), (0.53, 0.68, 0.41, 0.78)\}$
S_{f5}	$\{(0.46, 0.44, 0.73, 0.79), (0.43, 0.57, 0.88, 0.75), (0.55, 0.53, 0.82, 0.77)\}$
Sites	**H4F Factor as Criteria C_3 Edaphic Factors**
S_{f1}	$\{(0.34, 0.83, 0.69, 0.38), (0.45, 0.86, 0.71, 0.40)\}$
S_{f2}	$\{(0.72, 0.58, 0.77, 0.51), (0.75, 0.53, 0.79, 0.45), (0.80, 0.65, 0.67, 0.50)\}$
S_{f3}	$\{(0.83, 0.54, 0.61, 0.93), (0.76, 0.48, 0.63, 0.82)\}$
S_{f4}	$\{(0.78, 0.47, 0.49, 0.70), (0.73, 0.50, 0.49, 0.88), (0.73, 0.63, 0.50, 0.85)\}$
S_{f5}	$\{(0.63, 0.78, 0.54, 0.66), (0.71, 0.81, 0.49, 0.64), (0.72, 0.82, 0.59, 0.76)\}$
Sites	**H4F Factor as Criteria C_4 Essential Factors**
S_{f1}	$\{(0.54, 0.73, 0.65, 0.77), (0.71, 0.86, 0.71, 0.89), (0.75, 0.89, 0.62, 0.83)\}$
S_{f2}	$\{(0.62, 0.46, 0.71, 0.82), (0.58, 0.53, 0.77, 0.79)\}$
S_{f3}	$\{(0.66, 0.78, 0.89, 0.82), (0.83, 0.58, 0.55, 0.93), (0.63, 0.62, 0.67, 0.91)\}$
S_{f4}	$\{(0.58, 0.58, 0.49, 0.43), (0.55, 0.62, 0.89, 0.29)\}$
S_{f5}	$\{(0.63, 0.67, 0.54, 0.63), (0.29, 0.69, 0.43, 0.75)\}$

Table 10. Tabular representation of pessimistic H4F decision matrix.

Sites	H4F Factor as Criteria C_1 Climatic Factors
S_{f1}	$\{(0.81, 0.65, 0.45, 0.69), (0.78, 0.66, 0.51, 0.74), (0.77, 0.62, 0.52, 0.67)\}$
S_{f2}	$\{(0.68, 0.59, 0.67, 0.89), (0.61, 0.54, 0.63, 0.70), (0.61, 0.54, 0.63, 0.70)\}$
S_{f3}	$\{(0.85, 0.79, 0.57, 0.87), (0.74, 0.60, 0.60, 0.80), (0.74, 0.60, 0.60, 0.80)\}$
S_{f4}	$\{(0.43, 0.84, 0.66, 0.79), (0.49, 0.83, 0.75, 0.82), (0.54, 0.78, 0.71, 0.85)\}$
S_{f5}	$\{(0.67, 0.75, 0.58, 0.75), (0.60, 0.63, 0.62, 0.75), (0.60, 0.63, 0.62, 0.75)\}$
Sites	**H4F Factor as Criteria C_2 Socio-Economic Factors**
S_{f1}	$\{(0.78, 0.57, 0.69, 0.46), (0.78, 0.57, 0.69, 0.46), (0.81, 0.69, 0.65, 0.49)\}$
S_{f2}	$\{(0.67, 0.72, 0.51, 0.77), (0.68, 0.74, 0.64, 0.76), (0.66, 0.70, 0.60, 0.74)\}$
S_{f3}	$\{(0.48, 0.68, 0.73, 0.19), (0.46, 0.69, 0.78, 0.21), (0.50, 0.82, 0.85, 0.27)\}$
S_{f4}	$\{(0.49, 0.76, 0.39, 0.79), (0.53, 0.68, 0.41, 0.78), (0.53, 0.68, 0.41, 0.78)\}$
S_{f5}	$\{(0.46, 0.44, 0.73, 0.79), (0.43, 0.57, 0.88, 0.75), (0.55, 0.53, 0.82, 0.77)\}$
Sites	**H4F Factor as Criteria C_3 Edaphic Factors**
S_{f1}	$\{(0.34, 0.83, 0.69, 0.38), (0.34, 0.83, 0.69, 0.38), (0.45, 0.86, 0.71, 0.40)\}$
S_{f2}	$\{(0.72, 0.58, 0.77, 0.51), (0.75, 0.53, 0.79, 0.45), (0.80, 0.65, 0.67, 0.50)\}$
S_{f3}	$\{(0.83, 0.54, 0.61, 0.93), (0.76, 0.48, 0.63, 0.82), (0.76, 0.48, 0.63, 0.82)\}$
S_{f4}	$\{(0.78, 0.47, 0.49, 0.70), (0.73, 0.50, 0.49, 0.88), (0.73, 0.63, 0.50, 0.85)\}$
S_{f5}	$\{(0.63, 0.78, 0.54, 0.66), (0.71, 0.81, 0.49, 0.64), (0.72, 0.82, 0.59, 0.76)\}$
Sites	**H4F Factor as Criteria C_4 Essential Factors**
S_{f1}	$\{(0.54, 0.73, 0.65, 0.77), (0.71, 0.86, 0.71, 0.89), (0.75, 0.89, 0.62, 0.83)\}$
S_{f2}	$\{(0.62, 0.46, 0.71, 0.82), (0.62, 0.46, 0.71, 0.82), (0.58, 0.53, 0.77, 0.79)\}$
S_{f3}	$\{(0.66, 0.78, 0.89, 0.82), (0.83, 0.58, 0.55, 0.93), (0.63, 0.62, 0.67, 0.91)\}$
S_{f4}	$\{(0.58, 0.58, 0.49, 0.43), (0.58, 0.58, 0.49, 0.43), (0.55, 0.62, 0.89, 0.29)\}$
S_{f5}	$\{(0.63, 0.67, 0.54, 0.63), (0.29, 0.69, 0.43, 0.75), (0.29, 0.69, 0.43, 0.75)\}$

(ii). The normalized weights assigned to each criteria are given as follows:

$$w_l = (0.2501, 0.2458, 0.2633, 0.2408).$$

(iii). The weighted pessimistic H4F decision matrix is calculated in Table 11.

Table 11. Tabular representation of weighted pessimistic H4F decision matrix.

Sites	H4F Factor as Criteria C_1 with Weight 0.2501 Climatic Factors
S_{f1}	$\{(0.2026, 0.1626, 0.1125, 0.1726), (0.1951, 0.1651, 0.1276, 0.1851), (0.1926, 0.1551, 0.1301, 0.1676)\}$
S_{f2}	$\{(0.1701, 0.1476, 0.1676, 0.2226), (0.1526, 0.1351, 0.1576, 0.1751), (0.1526, 0.1351, 0.1576, 0.1751)\}$
S_{f3}	$\{(0.2126, 0.1976, 0.1426, 0.2176), (0.1851, 0.1501, 0.1501, 0.2001), (0.1851, 0.1501, 0.1501, 0.2001)\}$
S_{f4}	$\{(0.1075, 0.2101, 0.1651, 0.1976), (0.1225, 0.2076, 0.1876, 0.2051), (0.1351, 0.1951, 0.1776, 0.2126)\}$
S_{f5}	$\{(0.1676, 0.1876, 0.1451, 0.1876), (0.1501, 0.1576, 0.1551, 0.1876), (0.1501, 0.1576, 0.1551, 0.1876)\}$
Sites	**H4F Factor as Criteria C_2 with Weight 0.2458** **Socio-Economic Factors**
S_{f1}	$\{(0.1917, 0.1401, 0.1696, 0.1131), (0.1917, 0.1401, 0.1696, 0.1131), (0.1991, 0.1696, 0.1598, 0.1204)\}$
S_{f2}	$\{(0.1647, 0.1770, 0.1254, 0.1893), (0.1671, 0.1819, 0.1573, 0.1868), (0.1622, 0.1721, 0.1475, 0.1819)\}$
S_{f3}	$\{(0.1180, 0.1671, 0.1794, 0.0467), (0.1131, 0.1696, 0.1917, 0.0516), (0.1229, 0.2016, 0.2089, 0.0664)\}$
S_{f4}	$\{(0.1204, 0.1868, 0.0959, 0.1942), (0.1303, 0.1671, 0.1008, 0.1917), (0.1303, 0.1671, 0.1008, 0.1917)\}$
S_{f5}	$\{(0.1131, 0.1082, 0.1794, 0.1942), (0.1057, 0.1401, 0.2163, 0.1843), (0.1352, 0.1303, 0.2016, 0.1893)\}$
Sites	**H4F Factor as Criteria C_3 with Weight 0.2633** **Edaphic Factors**
S_{f1}	$\{(0.0895, 0.2185, 0.1817, 0.1001), (0.0895, 0.2185, 0.1817, 0.1001), (0.1185, 0.2264, 0.1869, 0.1053)\}$
S_{f2}	$\{(0.1896, 0.1527, 0.2027, 0.1343), (0.1975, 0.1395, 0.2080, 0.1185), (0.2106, 0.1711, 0.1764, 0.1316)\}$
S_{f3}	$\{(0.2185, 0.1422, 0.1606, 0.2449), (0.2001, 0.1264, 0.1659, 0.2159), (0.2001, 0.1264, 0.1659, 0.2159)\}$
S_{f4}	$\{(0.2054, 0.1238, 0.1290, 0.1843), (0.1922, 0.1316, 0.1290, 0.2317), (0.1922, 0.1659, 0.1316, 0.2238)\}$
S_{f5}	$\{(0.1659, 0.2054, 0.1422, 0.1738), (0.1869, 0.2133, 0.1290, 0.1685), (0.1896, 0.2159, 0.1553, 0.2001)\}$
Sites	**H4F Factor as Criteria C_4 with Weight 0.2408** **Essential Factors**
S_{f1}	$\{(0.1300, 0.1758, 0.1565, 0.1854), (0.1710, 0.2071, 0.1710, 0.2143), (0.1806, 0.2143, 0.1493, 0.1999)\}$
S_{f2}	$\{(0.1493, 0.1108, 0.1710, 0.1975), (0.1493, 0.1108, 0.1710, 0.1975), (0.1397, 0.1276, 0.1854, 0.1902)\}$
S_{f3}	$\{(0.1589, 0.1878, 0.2143, 0.1975), (0.1999, 0.1397, 0.1324, 0.2239), (0.1517, 0.1493, 0.1613, 0.2191)\}$
S_{f4}	$\{(0.1397, 0.1397, 0.1180, 0.1035), (0.1397, 0.1397, 0.1180, 0.1035), (0.1324, 0.1493, 0.2143, 0.0698)\}$
S_{f5}	$\{(0.1517, 0.1613, 0.1300, 0.1517), (0.0698, 0.1662, 0.1035, 0.1806), (0.0698, 0.1662, 0.1035, 0.1806)\}$

(iv). An H4F concordance set is calculated in Table 12.

Table 12. Tabular representation of H4F concordance set.

v	**1**	**2**	**3**	**4**	**5**
Y_{1v}	–	$\{1,4\}$	$\{2,4\}$	$\{2,4\}$	$\{4\}$
Y_{2v}	$\{2,3\}$	–	$\{2\}$	$\{2,4\}$	$\{2,4\}$
Y_{3v}	$\{1,3\}$	$\{1,3,4\}$	–	$\{1,3,4\}$	$\{1,3,4\}$
Y_{4v}	$\{1,3\}$	$\{1,3\}$	$\{2\}$	–	$\{1\}$
Y_{5v}	$\{1,2,3\}$	$\{1,3\}$	$\{2\}$	$\{2,3,4\}$	–

(v). An H4F concordance matrix is calculated as follows:

$$Y = \begin{pmatrix} - & 0.4909 & 0.4866 & 0.4866 & 0.2408 \\ 0.5091 & - & 0.2458 & 0.4866 & 0.4866 \\ 0.5134 & 0.7542 & - & 0.7542 & 0.7542 \\ 0.5134 & 0.5134 & 0.2458 & - & 0.2501 \\ 0.7592 & 0.5134 & 0.2458 & 0.7499 & - \end{pmatrix}.$$

(vi). An H4F discordance set is calculated in Table 13.

Table 13. Tabular representation of 3HF discordance set.

v	**1**	**2**	**3**	**4**	**5**
Z_{1v}	–	$\{2,3\}$	$\{1,3\}$	$\{1,3\}$	$\{1,2,3\}$
Z_{2v}	$\{1,4\}$	–	$\{1,3,4\}$	$\{1,3\}$	$\{1,3\}$
Z_{3v}	$\{2,4\}$	$\{2\}$	–	$\{2\}$	$\{2\}$
Z_{4v}	$\{2,4\}$	$\{2,4\}$	$\{1,3,4\}$	–	$\{2,3,4\}$
Z_{5v}	$\{4\}$	$\{2,4\}$	$\{1,3,4\}$	$\{1\}$	–

(vii). An H4F discordance matrix is calculated as follows:

$$Z = \begin{pmatrix} - & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.7357 & - & 0.7269 & 0.9763 & 0.9502 \\ 0.5634 & 1.0000 & - & 1.0000 & 1.0000 \\ 0.7737 & 1.0000 & 0.8667 & - & 1.0000 \\ 0.9074 & 1.0000 & 0.8059 & 0.5249 & - \end{pmatrix}.$$

(viii). An H4F concordance level $\overline{y} = 0.5000$, and H4HF discordance level $\overline{z} = 1.1995$ are calculated.
(ix). An H4F concordance dominance matrix is calculated as follows:

$$R = \begin{pmatrix} - & 0 & 0 & 0 & 0 \\ 1 & - & 0 & 0 & 0 \\ 1 & 1 & - & 1 & 1 \\ 1 & 1 & 0 & - & 0 \\ 1 & 1 & 0 & 1 & - \end{pmatrix}.$$

(x). An H4F discordance dominance matrix is calculated as follows:

$$S = \begin{pmatrix} - & 1 & 1 & 1 & 1 \\ 1 & - & 1 & 1 & 1 \\ 1 & 1 & - & 1 & 1 \\ 1 & 1 & 1 & - & 1 \\ 1 & 1 & 1 & 1 & - \end{pmatrix}.$$

(xi). An aggregated H4F dominance matrix is calculated as follows:

$$T = \begin{pmatrix} - & 0 & 0 & 0 & 0 \\ 1 & - & 0 & 0 & 0 \\ 1 & 1 & - & 1 & 1 \\ 1 & 1 & 0 & - & 0 \\ 1 & 1 & 0 & 1 & - \end{pmatrix}.$$

(xii). According to outranking values of aggregated H4F dominance matrix the sites for farming have the following relation as shown in Figure 2.

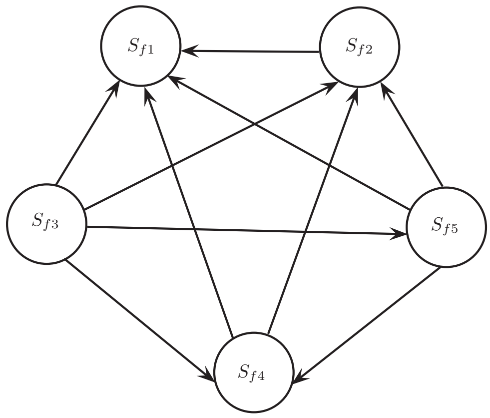

Figure 2. Graphical representation of outranking relation of sites for farming.

Hence, the site S_{f3} is best for farming purposes as compared to others.
We show the comparison of sites for farming and summarize the whole procedure in Table 14.

Table 14. Tabular representation of comparison of sites for farming.

Comparison of CS of Bricks	Y_{uv}	Z_{uv}	y_{uv}	z_{uv}	r_{uv}	s_{uv}	t_{uv}	*Ranking*
(S_{f1}, S_{f2})	$\{1,4\}$	$\{2,3\}$	0.4909	1	0	1	0	Incomparable
(S_{f1}, S_{f3})	$\{2,4\}$	$\{1,3\}$	0.4866	1	0	1	0	Incomparable
(S_{f1}, S_{f4})	$\{2,4\}$	$\{1,3\}$	0.4866	1	0	1	0	Incomparable
(S_{f1}, S_{f5})	$\{4\}$	$\{1,2,3\}$	0.2408	1	0	1	0	Incomparable
(S_{f2}, S_{f1})	$\{2,3\}$	$\{1,4\}$	0.5091	0.7357	1	1	1	$S_{f2} \to S_{f1}$
(S_{f2}, S_{f3})	$\{2\}$	$\{1,3,4\}$	0.2458	0.7269	0	1	0	Incomparable
(S_{f2}, S_{f4})	$\{2,4\}$	$\{1,3\}$	0.4866	0.9763	0	1	0	Incomparable
(S_{f2}, S_{f5})	$\{2,4\}$	$\{1,3\}$	0.4866	0.9502	0	1	0	Incomparable
(S_{f3}, S_{f1})	$\{1,3\}$	$\{2,4\}$	0.5134	0.5634	1	1	1	$S_{f3} \to S_{f1}$
(S_{f3}, S_{f2})	$\{1,3,4\}$	$\{2\}$	0.7542	1	1	1	1	$S_{f3} \to S_{f2}$
(S_{f3}, S_{f4})	$\{1,3,4\}$	$\{2\}$	0.7542	1	1	1	1	$S_{f3} \to S_{f4}$
(S_{f3}, S_{f5})	$\{1,3,4\}$	$\{2\}$	0.7542	1	1	1	1	$S_{f3} \to S_{f5}$
(S_{f4}, S_{f1})	$\{1,3\}$	$\{2,4\}$	0.5134	0.7737	1	1	1	$S_{f4} \to S_{f1}$
(S_{f4}, S_{f2})	$\{1,3\}$	$\{2,4\}$	0.5134	1	1	1	1	$S_{f4} \to S_{f2}$
(S_{f4}, S_{f3})	$\{2\}$	$\{1,3,4\}$	0.2458	0.8667	0	1	0	Incomparable
(S_{f4}, S_{f5})	$\{1\}$	$\{2,3,4\}$	0.2501	1	0	1	0	Incomparable
(S_{f5}, S_{f1})	$\{1,2,3\}$	$\{4\}$	0.7592	0.9074	1	1	1	$S_{f5} \to S_{f1}$
(S_{f5}, S_{f2})	$\{1,3\}$	$\{2,4\}$	0.5134	1	1	1	1	$S_{f5} \to S_{f2}$
(S_{f5}, S_{f3})	$\{2\}$	$\{1,3,4\}$	0.2458	0.8059	0	1	0	Incomparable
(S_{f5}, S_{f4})	$\{2,3,4\}$	$\{1\}$	0.7499	0.5249	1	1	1	$S_{f5} \to S_{f4}$

Finally, we present our proposed methods of decision-making in an Algorithm 1.

Algorithm 1 - The algorithm of proposed approaches for dealing MCGDM problems.

Step 1. Input
n, no. of alternatives against mHF-sets or HmF-sets.
k, no. of criteria.
m, no. of poles.
r, no. of hesitation values.
D_g, mHF or HmF decision matrices.
w_l^g, weights according to decision-makers.
Step 2. Compute an aggregated mHF or HmF decision matrix D.
Step 3. Compute aggregated weights W'.
Step 4. Compute the weighted aggregated mHF or HmF decision matrix W.
Step 5. Compute mHF or HmF concordance set Y_{uv}.
Step 6. Compute mHF or HmF discordance set Z_{uv}.
Step 7. Compute mHF or HmF concordance indices y_{uv} and concordance matrix Y.
Step 8. Compute mHF or HmF discordance indices z_{uv} and discordance matrix Z.
Step 9. Compute mHF or HmF concordance and discordance levels $\bar{y}$ and $\bar{z}$.
Step 10. Compute mHF or HmF concordance dominance matrix R.
Step 11. Compute mHF or HmF discordance dominance matrix S.
Step 12. Compute aggregated mHF or HmF dominance matrix T.
Step 13. Output
The most dominating alternative having maximum value of T.

5. Differences and Comparative Analysis of Proposed Approaches

In this section we discuss the differences and comparative analysis of proposed approaches. Both the proposed hybrid models are the reasonable combination of hesitancy with mF sets. Both the proposed MCGDM approaches have their own fascinating advantages and characteristic and are exposed as more flexible models to be evaluated in multifold ways according to the practical interests and requirements than the existing generalizations of mF sets and HFSs, having multipolar information under hesitancy suggested by taking decision-makers into account.

5.1. Differences of Proposed Approaches

The main differences of proposed models and approaches are given as follows:

1. m-Polar hesitant fuzzy ELECTRE-I approach
   - An mHF ELECTRE-I approach based on the concept mHF-sets, which is the generalization of mF sets under hesitancy.
   - An mHF-set can be reduced to an mF set by reducing the the factor of hesitancy up to one.
   - An mHF ELECTRE-I method is able to deal with problems, when we have multipolar information in terms of hesitancy.
   - This approach deals with the hesitant situation of each degree of membership of mF sets separately.
2. Hesitant m-polar fuzzy ELECTRE-I approach
   - The HmF ELECTRE-I approach based on the concept HmF-sets, which is the natural generalization of HFSs in terms of mF knowledge.
   - The HmF-set can be reduced to a HFS by contracting the multipolar information up to one.
   - The HmF ELECTRE-I method is able to deal with problems, when we have to facilitate the management of hesitation, uncertainty and vagueness motivated by multipolar information.
   - This approach deals the hesitant situation of m tuple degrees of membership of mF sets.

5.2. Comparative Analysis of Proposed Approaches

In this subsection we show the comparative analysis of proposed approaches with existing method, such as mFL ELECTRE-I method (see [6]) and provide theoretical discussion in this regard.

1. Existing method (mFL ELECTRE-I)

   Existing method described in [6] is used to handle the multipolar information in terms of mF linguistic variables in which hesitancy is not allowed. The method is limited up to the mF linguistic variables, we are bound to take the alternatives having mF linguistic variable and cannot apply this method to deal the problems having decision-makers uncertain and hesitant decision. In (Section 3, Subsection 3.1 of [6]), we have applied the mFL ELECTRE-I approach to salary analysis of companies, which is bound only to deal the multipolar information in terms of 4F linguistic variable (salary). Its criteria are the linguistic values of 4F linguistic variable, which are not used to discuss the wide range of problems having multipolar information with different properties or factors under hesitancy.
2. Proposed methods (mHF ELECTRE-I and HmF ELECTRE-I)

   Both the proposed approaches are used to solve the problems having multipolar information under hesitancy. The proposed approaches are not bound to any kind of restriction such as linguistic variables. In these approaches we are free to choose the set of alternatives. In Sections 3.1 and 4.1 we have applied the mHF ELECTRE-I and HmF ELECTRE-I to the selection of a best brick for construction and site selection for farming purposes, which are not bound to deal with the multipolar information under any restriction. These approaches are used to discuss the wide range of problems having multipolar information with different properties or factors under hesitancy.

6. Conclusions

Hesitant structures are generally preferred as compared to clear-cut situations. The hesitation regarding membership degrees can be manipulated using different types of information. However in the current state of affairs, this approach is unable to handle multipolar information. In order to enable the practitioners to avail themselves of multipolar information under hesitancy and to facilitate the management of hesitation, uncertainty and vagueness motivated by multipolar information, we have developed the mHF ELECTRE-I and HmF ELECTRE-I approaches to deal with MCGDM problems, which are the natural generalizations of the ELECTRE-I method. From a basic perspective proposed approaches based on the models (mHF-sets and HmF-sets), which are capable of incorporating knowledge with m different numerical or fuzzy values in a hesitant environment. The proposed methods have fascinating advantages and characteristic of their own and are exposed as being more flexible methods to be evaluated in multifold ways according to the practical interests and requirements in contrast with the existing generalizations of HFSs, which take multipolar information suggested by decision-makers into account. We have illustrated our novel concepts with real life examples. We also have presented the differences and comparative analysis of our proposed approaches. Finally, we have applied our techniques to real life problems, developed an algorithm and presented its computer programming code by using MATLAB (Version: R2014a, Manufacturer: Cleve Moler, Developer: MathWorks, Country: United States of America, Platform: IA-32, x86-64). In the future, we will explore more decision-making methods to be applied to related concepts such as (1) Hesitant m-polar fuzzy rough ELECTRE-I approach, and (2) m-polar hesitant fuzzy rough ELECTRE-I approach.

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Appendix A

We show the computer programming code of our proposed approaches in Table A1 by using MATLAB R2014a.

Table A1. MATLAB computer programming code of proposed approaches for MCGDM.

MATLAB Computer Programming Code

```
clc
n=input('no. of alternatives against mHF-sets or HmF-sets');
k=input('no.of criteria');
m=input('no. of poles');
r=input('no. of hesitation  values');
Rr=(1:n);Cr=1:m*r*k;Cw=1:k;w_g=zeros(1,k);
    D= input('enter the mHF-sets or HmF-sets decision matrix nxkxm');
w=input('enter the weights');
W=zeros(n,m*k);Sm=zeros(n,k);Y_uv=zeros(n,n*k); Z_uv=zeros(n,n*k);
for p=1:n
    l=1;
    for Cr=1:m*r*k
        W(p,Cr)=D(p,Cr.*)w(l,1);
        if mod(Cr,m*r)==0
            l=l+1;
        end
    end
end
W
for p=1:n
    l=1;
    for Cr=1:m*r*k
        Sm(p,l)=Sm(p,l)+W(p,Cr);
        if mod(Cr,m*r)==0
            l=l+1;
        end
    end
end
Q=Sm'
Q=Q(:)';
for p=1:n
    for j=1:k*n
        l=mod(j,k);
        if l==0
            l=k;
        end
        if Sm(p,l)≥ Q(1,j)
            Y_uv(p,j)=1;
        end
        if Sm(p,l)≤ Q(1,j)
            Z_uv(p,j)=1;
        end
    end
end
Y=zeros(n,n);fprintf('\n concordance Set Y_uv =\n')
for p=1:n
    v=0;
    for j=1:k*n
        if mod(j,k)==1
            v=v+1;
        end
        l=mod(j,k);
        if l==0
            l=k;
        end
        if u==v
            if l==1
                fprintf('   -      ')
            end
        elseif p~=v
            if l==1
                fprintf('  {   ')
                c=0;
            end
            if Y_uv(p,j)==1;
```

Table A1. *Cont.*

```
                    c=c+1;
                    fprintf('%d,',l)
                end
                if l==k & c==0
                    fprintf(' ,',l)
                end
                if l==k
                    fprintf('\b}        ')
                end
        end
    end
    fprintf('\n')
end
 fprintf('\n discordance Set Z_uv =\n')
  for u=1:n
        v=0;
        for j=1:k*n
                if mod(j,k)==1
                    v=v+1;
                end
                l=mod(j,k);
                if l==0
                    l=k;
                end
                fprintf('\nY=\n')
                for u=1:n
                      for v=1:n
                          if u==v
                              fprintf('  -     ')
                          else
                              fprintf'%.4f     ',Y(u,v))
                          end
                      end
                      fprintf('  \n  ')
                end
                fprintf('  \n Discordance Set Zuv =\n')
                for u=1:n
                          v=0;
                          for j=1:k*n
                              if mod(j,k)==1
                                  v=v+1;
                              end
                              l=mod(j,k);
                              if l==0
                                  l=k;
                              end
                              if u==v
                                  if l==1
                                      fprintf('  -     ')
                                  end
                              else if u =v
                                  if l==1
                                      fprintf(  '  ')
                                      c=0;
                                  end
                                  if Zuv(u,j)==1;
                                      c=c+1;
                                      fprintf('  %d,  ',l)
                                  end
                                  if l==k & c==0
                                      fprintf('  ,  ',l)
                                  end
                                  if l==k
                                      fprintf('  _    ')
                                  end
                      end
                end
                fprintf('%.4f          ',Y(u,v))
                end
        end
    end
    fprintf('\n          ')
end
z=zeros(n,^2,m*r*k); Cr=1:m*r*k; v=0;
```

Table A1. *Cont.*

```
for u=1:n
    for q=1:n
        v=v+1;
        z(v,Cr)=(W(u,Cr)-W(j,Cr)).^2;
    end
end
A=zeros(n^2,k);g=0; s=0;   C=zeros(n^2,1);B=zeros(n,k);Z1=zeros(n,n);
for p=1:n^2
    x=1;
    for Cr=1:m*r*k
        A(p,x)=A(p,x)+z(p,Cr);
        if mod(Cr,m*)==0
            x=x+1;
    end
    A(p,:)=sqrt(A(p,:)/m*r);
    C(p,1)=max(A(p,:));
    if mod(p,n)==1
        g=g+1;
    end
    for f=1:k
        s=s+1;
        B(g,s)=A(p,f);
    end
    t=mod(p,n);
    if t==0
        t=n;
    end
    Z1(g,t)=C(p,1);
    if mod(p,n)==0
        s=0;
    end
end
  D=zeros(n,n);
for p=1:n
    q=0;
    for j=1:k*n
        if mod(j,k)==1
            q=q+1;
        end
        l=mod(j,k);
        if l==0
            l=k;
        end
        if Z_uv(p,j)==1
            D(p,q)=max(D(p,q),B(p,j));
        end
    end
end
for u=1:n
    for v=1:n
        if u~=v
            Z(u,v)=D(u,v)/Z1(u,v);
        end
    end
end
fprintf('\nZ=\n')
for u=1:n
    for v=1:n
        if u==v
            fprintf('    -         ')
        else
            fprintf('%.4f          ',Z(u,v))
        end
    end
    fprintf('    \n    ')
end
a=sum(Y);     b=sum(a);   a1=sum(Z);  b1=sum(a1);  R=zeros(n,n);S=zeros(n,n);
y_bar=b/(n*(n-1))
z_bar=b1/(n*(n-1))
for u=1:n
    for v=1:n
        if u~=v
            if Y(u,v)≥ y_bar
                R(u,v)=1;
```

Table A1. *Cont.*

```
                end
                if Z(u,v)< z_bar
                    S(u,v)=1;
                end
            end
        end
    end
fprintf('\nR=\n')
for u=1:n
        for v=1:n
            if u==v
                fprintf('-        ')
            else
                fprintf('%d        ',R(u,v))
            end
        end
        fprintf('  \n    ')
end
fprintf('\nS=\n')
for u=1:n
        for v=1:n
            if u==v
                fprintf('-        ')
            else
                fprintf('%d        ',S(u,v))
            end
        end
        fprintf('\n        ')
end
T=R.*S; fprintf('\nT=\n')
for u=1:n
        for v=1:n
            if u==v
                fprintf('-        ')
            else
                fprintf('%d        ',T(u,v))
            end
        end
        fprintf('  \n    ')
end
G=digraphs(T)
plot(G)
```

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