

Article

# Stochastic Unit Commitment Based on Multi-Scenario Tree Method Considering Uncertainty

Kyu-Hyung Jo and Mun-Kyeom Kim \*

Department of Energy System Engineering, Chung-Ang University, 84 Heukseok-ro, Dongjak-gu, Seoul 156-756, Korea; primelife@cau.ac.kr

\* Correspondence: mkim@cau.ac.kr; Tel.: +82-2-820-5271

Received: 13 February 2018; Accepted: 21 March 2018; Published: 24 March 2018



**Abstract:** With the increasing penetration of renewable energy, it is difficult to schedule unit commitment (UC) in a power system because of the uncertainty associated with various factors. In this paper, a new solution procedure based on a multi-scenario tree method (MSTM) is presented and applied to the proposed stochastic UC problem. In this process, the initial input data of load and wind power are modeled as different levels using the mean absolute percentage error (MAPE). The load and wind scenarios are generated using Monte Carlo simulation (MCS) that considers forecasting errors. These multiple scenarios are applied in the MSTM for solving the stochastic UC problem, including not only the load and wind power uncertainties, but also sudden outages of the thermal unit. When the UC problem has been formulated, the simulation is conducted for 24-h period by using the short-term UC model, and the operating costs and additional reserve requirements are thus obtained. The effectiveness of the proposed solution approach is demonstrated through a case study based on a modified IEEE-118 bus test system.

**Keywords:** unit commitment; multi-scenario tree method; reserve requirement; uncertainty; operating cost

---

## 1. Introduction

### 1.1. Background

The development of renewable source to overcome the energy crisis has become an area of focus in the power industry. Wind energy is one of the most widely used renewable energies because of the feasibility of current related technologies. However, wind power involves nature of fluctuation and unpredictability. The fluctuating output generated by a wind turbine is uncontrollable as compared to that of a thermal generator. Owing to the nature of these uncertainties, the penetration of wind power has an impact on the safe and reliable operation of power systems [1]. Thus, to integrate the energy generated at a wind farm into a power system, it is necessary to guarantee sufficient reserve requirements to allow the schedule to be modified according to the operating conditions of the power system.

Typically, the unit commitment (UC) problem is an optimization problem that determines the optimal generators' start and stop conditions, and it is an important consideration in power generation planning. It constitutes a process of optimizing the fuel costs by calculating an optimal start and stop state of the generator such that the time-varying load in a state is satisfied while considering the constraints according to the system operation. Thus, an analysis should be performed based on accurate factors, which can lead to significant economic aspects. In addition, the UC problem could be rendered more complex and unpredictable by the uncertainties of various factors.

### 1.2. Literature Review and Motivation

Much research has been conducted that deals with solving UC problems, such as mixed-integer linear programming [2], bacterial foraging [3], discrete differential evolution approach [4], binary particle swarm optimization [5], second-order cone programming [6], memetic algorithms [7], and genetic algorithms [8]. However, these methods took only the thermal generation into account and did not consider the effect of renewable sources on the system operation.

Recently, UC models were introduced that consider wind power when a thermal unit is involved [9–17]. A short-term generation scheduling model of wind power integration was presented as part of a liberalized electricity market model in [9]. The authors used the stochastic UC model to compute the reserve requirement and compared the pre-computed reserve requirement with that simulated using wind power realization. Ruiz et al. [10] proposed a method of managing the uncertainty of the UC problem through a proposed stochastic formulation. The same stochastic framework was used in [11] to extend the model to account for the uncertainty and volatility of wind power generation. In another study, the authors implemented a long-term solution for a security-constrained UC [12]. In [13], the authors presented a study in which the solutions optimistically and pessimistically obtained from scenario-based stochastic UC were compared. A risk-based stochastic UC model that considers the loss-of-load risk caused by wind power uncertainty was implemented in the study presented in [14]. In [15], the authors extended robust optimization, which was developed as a practical solution based on a combination of Benders decomposition algorithm and the outer approximation technique. The implementation of a robust optimization-based UC model that accounts for the worst of wind power fluctuation scenarios was described in [16]. The authors proposed a bilinear chance-constrained UC models that was developed to solve the resulting large-scaled linear counterpart [17]. Most of the models presented in the literature were focused on integrated modeling; however, they were limited to consider the load and wind power uncertainties. Many UC solutions have also been proposed to solve the various uncertainty problems currently available to calculate optimal reserve requirement and minimum operating costs. The optimal UC of the spinning reserve was described in [18]. It is associated with a load forecasting by tightly integrating the probabilistic reserve assessment with a conventional Lagrangian relaxation approach. In [19], the authors presented a methodology to determine the optimal level of reserve requirement and to calculate the cost within a daily time framework. However, other uncertainties such as the load and forced generator outages were temporally ignored in these stochastic studies to focus on wind power uncertainties.

### 1.3. Contribution and Paper Organization

In this paper, we propose a solution procedure for integrating the uncertainty in the stochastic UC problem using the multi-scenario tree method (MSTM). Initial input data are created, including the forecasted load and wind data for each day-ahead hour. We used a Monte Carlo simulation (MCS) to generate load and wind scenarios that considered the uncertainties, which are modeled at different levels using the mean absolute percentage error (MAPE). The proposed stochastic UC problem is optimized by the modified priority list method [2]. It is repeated until an optimal solution is obtained considering various constraints.

The main contributions of this paper with respect to the previous ones can be briefly summarized as follows:

- The MSTM is developed to control the load and wind power uncertainties. The MSTM helps to satisfy the power system balance and minimize the total operating costs.
- The application of the MSTM based on multiple scenarios to increase the accuracy of stochastic models when simulating the effect of the prediction errors is described. This represents a possible future realization of the random process.

- The proposed solution procedure of the stochastic UC problem can obtain an approximate value for the probability variable with different accuracy requirements, thereby solving the optimal UC problem with the best unit state combination, considering the forecasting error factors.

The remainder of this paper is organized as follows: the implementation of a stochastic approach with load and wind uncertainties is described in Section 2. A mathematical formulation that considers thermal unit and uncertainty constraints is presented in Section 3. Section 4 presents the proposed solution procedure for the stochastic UC problem based on the MSTM. The simulation results for case studies are provided in Section 5, along with an analysis of the operating costs and additional reserve requirements according to the uncertainty. The conclusions are given in Section 6.

## 2. Implementation of Stochastic Approach

### 2.1. Forecast Uncertainty Modeling

The forecast uncertainty is a significant factor in the operating cost of UC because of the reserve that is required to protect a power system from sudden load increases and wind speed variability. It also plays an important role in determining the amount of power to be scheduled during UC in the energy market. Thus, accurate forecasting must be required to minimize the operating cost with little spinning reserve, while ensuring the reliability and security of a power system. Generally, the forecasting errors of the sources can be modeled as different levels of the MAPE, which is the ratio between the actual values and the absolute forecasting errors:

$$\text{MAPE} = \frac{100}{n} \sum_{t=1}^n \frac{|P_t^{\text{For}} - P_t^{\text{Act}}|}{|P_t^{\text{Act}}|}. \quad (1)$$

In this study, the day-ahead forecasting error has been adopted based on our previous research [20], and then the selected MAPE range could be generated for the load and wind power uncertainties [21]. Because of the cyclical nature of the load and the maturity of load forecasting methods, these are commonly assumed to follow the normal distributions, the parameters of which can be estimated using historical analyses of different types of data and their corresponding standard deviations over a period of time. To realize a reliable and economical electricity supply in the UC problem, load forecasting with the MAPE needs to be examined in determining the range of 2–4%, which is used for each scenario based on an MCS. On the other hand, the wind speed forecasting mainly focuses on the immediate short-term period of seconds to minutes, the short-term period of hours up to two days, and the medium-term period of 2–7 days. Owing to the uncertainty and intermittent behavior of wind speed, in our study, we assessed the influence of wind generation using MCS sampling based on the Weibull distribution. The MCS can be generated with a large number of random computational scenarios according to the probability density function of an input variable, such as the wind speed. Then, for wind power scenario generation with randomized forecasting error, the MAPE can have a value of 15–20% of the installed capacity, which is the common range of the wind forecasting error.

### 2.2. Multi-Scenario Tree Method

Scenario generation is a procedure of selecting a set of parameters, measurements, expected input, and disturbance to obtain the optimal solution in a power system. This procedure can be applied to solve a problem involving random variables, where their normal and probability distributions are based on the random sampling of scenarios created using MCS. The operation cost of each of these scenarios is then calculated using a short-term UC model. In each scenario, the hourly random load and wind power are considered, which are based on the forecast values. The values of each parameter are determined by the probability of the scenario, which is one divided by the number of net load and reserve scenarios. Each of these scenarios represents a possible future realization of the random process. In our study, 10 scenarios were generated for a 24-h period based on a random load

and wind output using MCS according to the forecast error to determine the simulation; a different number of created scenarios can be set for each case. Figure 1 shows the overall structure of the MSTM. Each functional box represents a separate module or sub-module. The detailed descriptions of these modules are as follows:

- Initial Input: this module includes the forecasted load and wind data for each hour of day-ahead.
- Scenario Generation: this module utilizes MCS to generate the random variables of the load and wind scenarios, which are structures branching from the load and wind forecasting error information.
- Forced Outages: this module generates the reserve required by the forced outages of a thermal generator to realize a reliable and economical electrical supply.
- Reserve Scenarios: this module uses the outputs of the load and wind scenarios and forced outages module, together with the forecasting errors, to calculate the additional reserve requirement.
- Net Load Scenarios: this module represents each load and wind scenario for 24 h to generate net load scenarios. Depending on the source considered, net load is calculated as follows:
  - Load Only: Basic reserve + Load + Load error reserve,
  - Wind Only: Basic reserve + Wind + Wind error reserve,
  - Load and Wind both considered: Basic reserve + Load scenario + Wind scenario + Load error reserve + Wind error reserve,
- Output Module: this module collates and converts the information from the other modules into input data.

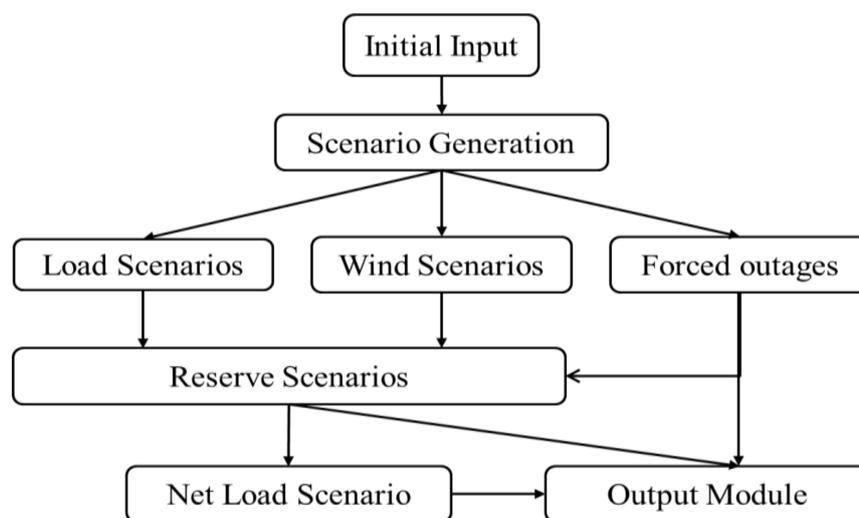


Figure 1. Structure of multi-scenario tree method (MSTM).

### 3. Proposed Stochastic UC Problem

Different mathematical formulations of the UC problem have been used to represent the commitment of sufficient resources, one deterministic and two stochastic [22]. The conventional systems usually treat the deterministic UC for the short term, which assumes no stochastic factor in the generation and load profiles, to ensure the dispatch of the power system by fixing the commitment action. Alternatively, the stochastic UC problem is related to the uncertain factors, such as the load and wind forecasting errors and generator outages. In this problem, the decision variables are used for day-ahead UC, which include turning on/off thermal generators to satisfy UC constraints, such as the power balance, and output capacities based on the forecast and reserve requirement. Then,

the uncertainties in the load and wind forecasting error produce a stochastic UC problem that requires some additional reserve, not only to guarantee stable operation, but also to minimize the expected operating cost of a power system with uncertain sources.

### 3.1. Formulation

#### 3.1.1. Objective Functions

The objective function of this process involves the on/off states and power levels for a set of distributed power generators at a time interval for each scenario. The overall objective function of the stochastic UC problem can be stated as follows:

$$\text{Min} \sum_{g \in G} \sum_{s \in S} \sum_{t \in T} \pi_s \{ (FC_g p_{gst}) u_{gst} + S_g s_{gst} \}. \quad (2)$$

Since the operating cost of wind power is negligible, the total operating cost is the sum of the fuel cost and the start-up/shut-down cost of all the thermal generators in a given set of online generating units to meet the load in the power system. The fuel cost function of a thermal unit is typically a quadratic function [23]:

$$(FC_g p_{gst}) = a_i + b_i \times p_{gt} + c_i \times p_{gt}^2. \quad (3)$$

In Label (2), the start-up/shut-down cost of thermal generator  $g$  in scenario  $s$  at time  $t$ ,  $S_g s_{gst}$ , is given by the following linear function of time in the optimization problem:

$$S_g s_{gst} = SU_g (1 - v_{gs,t-1}) v_{gst} + SD_g (1 - x_{gst}) x_{gs,t-1}. \quad (4)$$

#### 3.1.2. System Constraints

To satisfy the power balance in each scenario, the total generated power at every time slot should be equal to the load for every feasible combination as follows:

$$\sum_{g \in G} p_{gst} = D_{st}, \quad s \in S, t \in T. \quad (5)$$

The power system is maintained with a sufficient amount of reserve at any time to cover a possible mismatch between the power output and the load as shown in Labels (6)–(9):

$$\sum_{g \in G} \bar{p}_{gst} \geq D_{st} + R_{st}, \quad s \in S, t \in T, \quad (6)$$

$$\sum_{g \in G} US_{gst} \geq R_{st}, \quad s \in S, t \in T, \quad (7)$$

$$\sum_{g \in G} R_{gst} \geq \underline{R}_{st}, \quad s \in S, t \in T, \quad (8)$$

$$0 \leq R_{gst} \leq \bar{R}_g u_{gst}. \quad (9)$$

A certain amount of system spinning reserve must always be maintained in such a way that the loss of one or more units does not cause an excessive drop in system frequency. The down-spinning reserve contributes to a sudden decrease in load by ramping-down the capacity of the thermal units when the load is suddenly raised. Conversely, the up-spinning reserve is supported by ramping-up the thermal units.

### 3.1.3. Thermal Unit Constraints

Each generator must satisfy the minimum and maximum limits, which can be accomplished through the following interval formulation:

$$\underline{P}_g u_{gst} \leq p_{gst} u_{gst} + R_{gst} \leq \bar{P}_g u_{gst}, \quad g \in G, s \in S, t \in T. \quad (10)$$

Additional important constraints of the generators are the minimum on and off time limit of each unit  $i$ . The corresponding constraints are given by

$$u_{gst} - u_{gs,t-1} \leq 1 - u_{gst}, \quad t \geq 2, s \in S, \tau = t + 1, \dots, \min\{t + UT_g - 1, T\}, \quad (11)$$

$$u_{gs,t-1} - u_{gst} \leq 1 - u_{gst}, \quad t \geq 2, s \in S, \tau = t + 1, \dots, \min\{t + DT_g - 1, T\}. \quad (12)$$

To avoid damaging a turbine, the power output of a generator cannot be changed by more than a certain amount over the optimization period of time  $t$ . For each generator, the operation ramp rate constraints can be formulated as shown in Labels (13) and (14):

$$p_{gst} - p_{gs,t-1} + R_{gst} \leq RU_g u_{gs,t-1} + SUR_g (1 - u_{gs,t-1}), \quad g \in G, s \in S, t \in T, \quad (13)$$

$$p_{gs,t-1} - p_{gst} \leq RD_g u_{gst} + SDR_g (1 - u_{gst}), \quad g \in G, s \in S, t \in T. \quad (14)$$

Here, the start-up and shut-down cost constraints are imposed as follows:

$$SU_{gst} u_{gst} \geq SU_g (u_{gst} - u_{gs,t-1}), \quad g \in G, s \in S, t \in T. \quad (15)$$

### 3.2. Uncertainty Constraints

The active power output of a wind turbine can be represented as a function of the wind speed, which is a nonlinear relationship:

$$W_{gst} = 0 \quad \text{for } V_{gst} \leq V_c \text{ and } V_{gst} > V_o, \quad (16)$$

$$W_{gst} = W_{gst}^{rated} \times \frac{[V_{gst} - V_c]}{[V_r - V_{gst}]} \quad \text{for } V_{gst} \geq V_c \text{ and } V_{gst} \leq V_r, \quad (17)$$

$$W_{gst} = W_{gst}^{rated} \quad \text{for } V_{gst} \geq V_r \text{ and } V_{gst} \leq V_o. \quad (18)$$

In Labels (16)–(18), the most frequently used quadratic model for expressing the wind turbine power is a constant rated output of wind power between the rated and the cut-out wind speed. The forecasting errors of the load and wind power being used as compared to the predicted value are given as below:

$$D_{st} = D_{forecasted,st} \times (1 - \text{load forecasting error}), \quad (19)$$

$$W_{gst} = \sum_{g \in G_w} W_{forecasted,gst} \times (1 - \text{wind forecasting error}). \quad (20)$$

The forecasting errors of the load and wind power can compensate for the constraints shown in Labels (19) and (20) using the spinning reserve of the thermal units, which leads to a higher cost. Because this may significantly affect the cost of an integrated wind power system, the related constraints are considered limitations of the wind power effect. The reserve requirement for the forecasting error constraints of the load and wind power can be expressed as follows:

$$W_{st} \geq D_{error,st} \times D_{forecasted,st} + \sum_{g \in G_w} W_{error,gst} \times \sum_{g \in G_w} W_{forecasted,gst}, \quad s \in S, t \in T, \quad (21)$$

$$R_{total,st} = R_{st} + WLR_{st}, \quad s \in S, t \in T. \quad (22)$$

Because of the load, and wind power uncertainties and their natural variability, a sufficient reserve is needed for the total amount of generated power available to maintain the stable operation of a power system. To compensate for the fluctuations of wind power and unpredicted load, an additional reserve requirement is assumed as a percentage of the total actual wind power and load in the power system. Accordingly, the additional reserve requirement for the load and wind power penetration system can be reformulated as follows:

$$\sum_{g \in G} \bar{p}_{gst} + \sum_{g \in G_w} W_{gst} \geq D_{st} + R_{st} + WLR_{st}, \quad s \in S, t \in T, \quad (23)$$

$$R_{total,st} \geq \sum_{g \in G} R_{gst} + WLR_{st}, \quad s \in S, t \in T. \quad (24)$$

When the expected wind power is higher than the actual wind power, an upregulation load arises. Conversely, a downregulation load exists if the expected wind power is lower than the actual one. The proposed reserve requirement constraints are applied to influence these two components in the optimal stochastic UC problem, which includes different levels of forecasting errors. First, the reserve is determined by using the percentage of the total loads. In other words, the load forecasting errors are assumed to be different percentages of the total load at each hour to compensate for the forced outage of a generator. Second, we allow for the compensation of the wind forecasting errors, which represent the possible mismatch between the actual and forecasted values. The mismatch error can be supported by an additional reserve requirement with the wind power uncertainty, which is based on the MSTM. Moreover, to simplify the notation, we will use the net load concept in formulating the UC problem of a power system with load and wind uncertainties:

$$D_{net,st} = D_{st} - \sum_{g \in G_w} W_{gst} + WLR_{st} + R_{st}, \quad s \in S, t \in T. \quad (25)$$

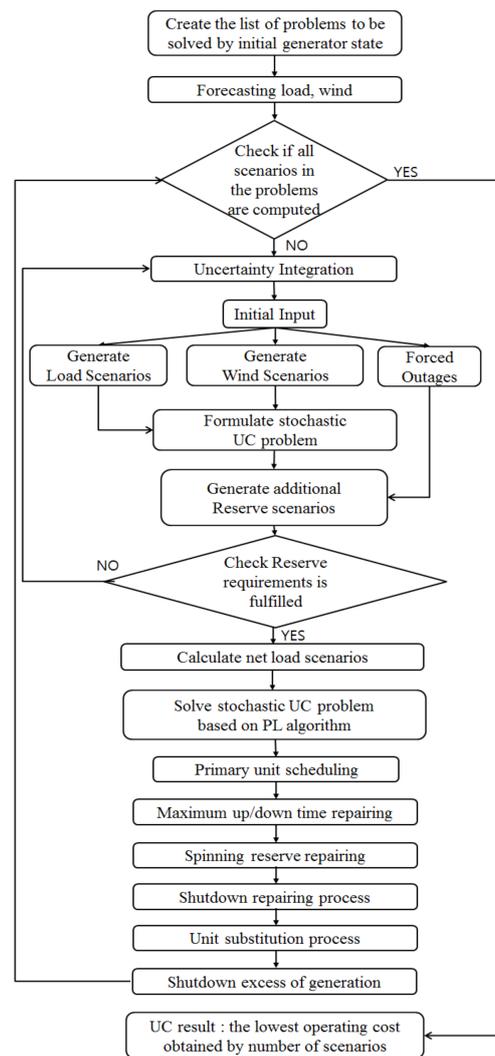
Because of the intermittent nature of the load and wind power, scheduling decisions need to be made in advance, in order to meet the net load.

#### 4. Solution Procedure of Stochastic UC Problem

Because of the competitiveness of the power market, utilities desire to obtain optimized UC schedules and reserve levels to overcome economic and reliability issues [24]. However, in the case of rapid load or wind power changes, the conventional stochastic UC problem has an important drawback: it is not able to consider the additional reserve requirement. To solve this, in the stochastic UC problem, the MSTM is utilized to precisely determine the reserve requirement and operating cost. Figure 2 summarizes the proposed solution process for the stochastic UC problem. The process can obtain an approximate value for the probability variable with different accuracy requirements, thereby solving the optimal UC problem, considering the forecasting error factors. To optimize the UC problem, we apply the priority list method, one of the most frequently used algorithms [2]. This method prioritizes each unit according to certain criteria and is the simplest means of solving a UC problem. In general, two states for the unit are encoded: if it is on, "1", and if it is off, "0". The designed priority list algorithm is comprised of primary unit scheduling, maximum uptime/downtime repair, spinning reserve repair, shutdown repair process, unit substitution process, and shut down of excess generation.

The procedure is implemented sequentially.

- Step 1 Determine the uncertain parameters when forecasting the load and wind power based on an initial configuration using the available system information.
- Step 2 Obtain the initial input data for the load and wind power using the MAPE. The data, in general, evolve over time according to a multivariate stochastic process, which represents branching trees comprising probability scenarios.
- Step 3 Generate each load and wind power scenario considering random forecasting errors of the MAPE using MCS.
- Step 4 Formulate the stochastic UC problem:
  - (i) The additional reserve requirement in Labels (21),  $WLR_{st}$ , is calculated without including forced outages, and, then, the scheduled reserve in the UC problem is put together to give the load and wind forecasting error for 24 h.
  - (ii) When the set of additional reserve requirements has been computed using the MSTM, the total amount of reserve required can be calculated with the basic reserve for forced outages from Labels (22) for each scenario.
- Step 5 Check the power system balance, including the reserve requirements in Labels (23) and (24). If it is satisfied, go to Step 6; otherwise, return to Step 2.
- Step 6 Compute the net load scenario from Labels (25). The two random scenarios can be merged into a single scenario for which a better UC solution becomes available.
- Step 7 Solve the hourly UC problem. This procedure for each stochastic value with forecasting error is repeated to obtain the lowest operating cost for a number of scenarios sequentially.
  - (i) Primary unit scheduling: Determine the order in which each generator is committed according to the average production cost.
  - (ii) Maximum uptime/downtime repair: Performs repair operations to meet minimum up/down constraints in primary unit scheduling.
  - (iii) Spinning reserve repair: The estimated spinning reserve in the primary unit scheduling process is reduced because of capacity generation. This is the process of repairing the reduced spinning reserve.
  - (iv) Shutdown repair process: This is the process of adjusting units so that they have sufficient time to be effectively decommitted. In this process, the shutdown ramp rate constraint should also be considered.
  - (v) Unit substitution process: After the minimum uptime/downtime repair process is performed, this process substitutes units to achieve cost-effective scheduling.
  - (vi) Shutdown excess generation: This is a procedure for obtaining efficient unit scheduling considering the increased generation cost caused by minimum uptime/downtime repair and spinning reserve repair.



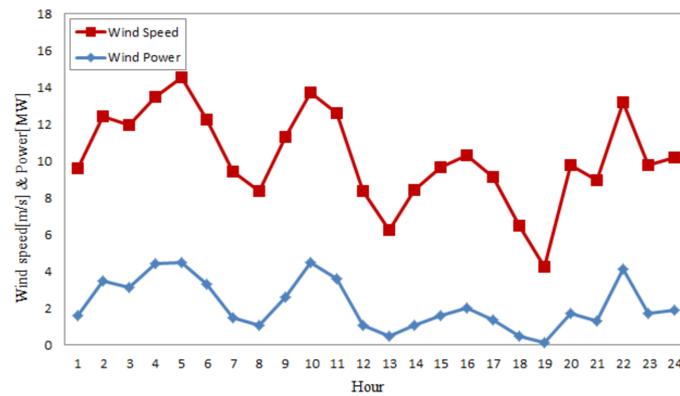
**Figure 2.** Solution process of the proposed stochastic unit commitment (UC) problem.

## 5. Numerical Studies

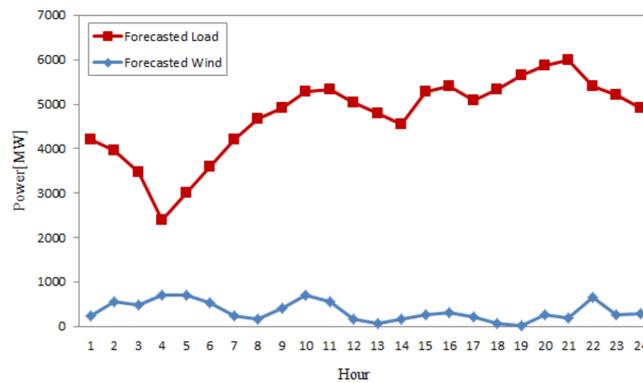
### 5.1. Dataset for Test System

This paper presents the results of the stochastic UC problem with MSTM based on the two stochastic parameters, load and wind power. A modified IEEE-118 bus test system was used and the results with different reserve requirements and operating costs for each case were compared. The system was comprised of 54 conventional thermal units and one large wind farm at Bus 3, which was comprised of the parallel operation of several units [25]. The wind farm was assumed to comprise 160 wind turbines rated at 5 MW each, which represented approximately 10% of the total system generation capacity. The simulation results were analyzed using the MATLAB/Simulink software (MATLAB R2013b, The Mathwork Inc., Natick, MA, USA, 2013) [26].

Figure 3 shows the initial input data of 24 h for the simulation. Here, the data are assumed to be independent random processes with the different forecasting methods based on historical values. The time horizon of the scheduling for this simulation was one day with 24 1-h intervals. The experiment, the results of which are shown in Figure 3a, was conducted by applying a series of ascending and descending slopes for the wind power output from the forecast wind speed. The forecasted load and wind used to solve the proposed UC problem are depicted in Figure 3b.



(a) Wind power and speed



(b) Forecasted load and wind farm

**Figure 3.** Initial input data of 24 h. (a) Wind power and speed (b) Forecasted load and wind farm.

Table 1 summarizes each of the cases, which are described as follows.

Base Case: This case did not consider uncertainties. The deterministic problem using the mean values of the load and wind power was solved in each period.

Case 1 Considering only the uncertainty of the load, input data were generated for 10 random scenarios with a load forecasting error of 2–4% deviance from the predicted values in the day-ahead scheduling.

Case 2 This case considered only a wind forecasting error of 15–20% to solve the stochastic UC problem based on each of the different scenarios.

Case 3 Combining the load and wind forecasting errors, the scenarios were generated to simulate the load and wind fluctuations.

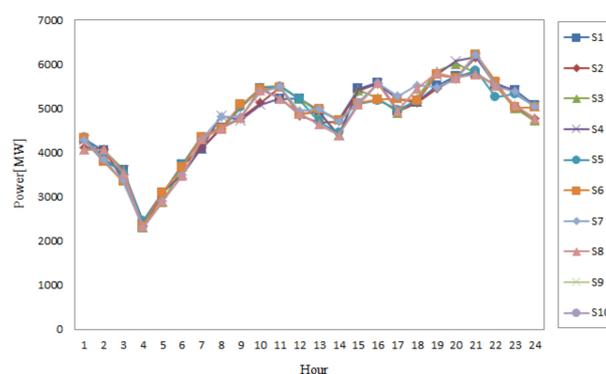
**Table 1.** Simulation cases for unit commitment (UC) strategies.

Case	Descriptions	Basic Reserve
Base	No load forecasting error No wind power	10%
1	Load forecasting error of 2–4% No wind power	10%
2	No load forecasting error Wind forecasting error of 15–20%	10%
3	Load forecasting error of 2–4% Wind forecasting error of 15–20%	10%

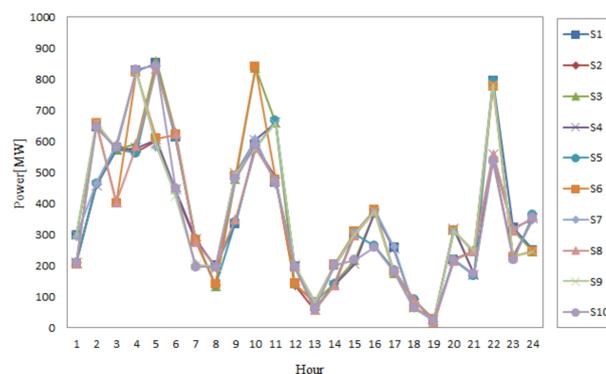
As shown in Table 1, the base reserve requirement in the UC formulation is assumed to be 10% of the system load for guaranteeing the power system stability against any forced generator outages in all cases.

### 5.2. Simulation Results of UC Performance

In this study, stochastic cases were implemented to examine the performance of the proposed UC procedure. The average time taken for Case 3 with the best solution was 2.4 s. Figure 4 shows the results of input data for the load and wind in Cases 1, 2, and 3. Here, we generated input data for 10 scenarios in each stochastic case using the MSTM, which is applied to find a suitable solution to compensate for the reserve generated by the uncertainty. It can be observed that the scenario generation for the load and wind power varied with the MAPE, which is computed as a randomized forecasting error.



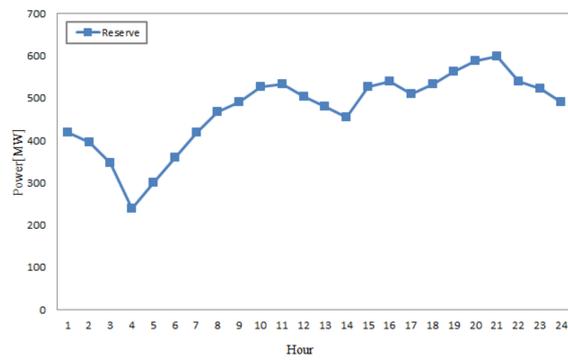
(a) Load scenarios



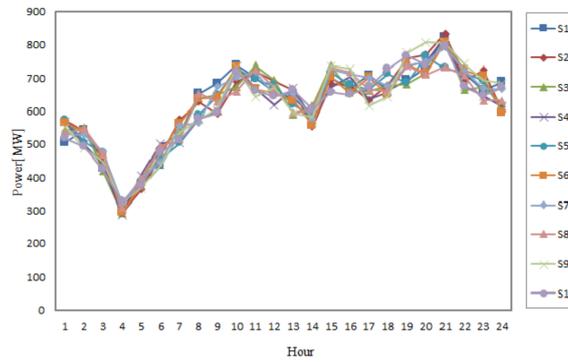
(b) Wind scenarios

**Figure 4.** Scenario generation for 24 h. (a) Load scenarios (b) Wind scenarios.

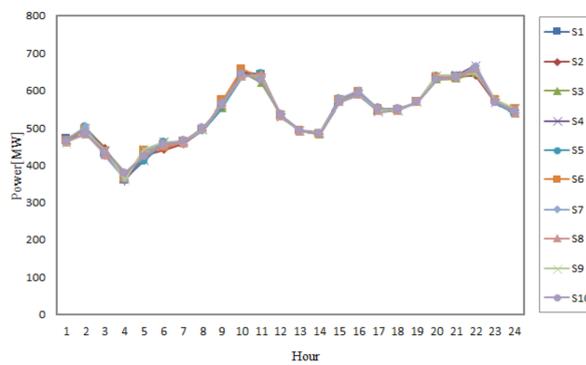
Figure 5 illustrates the amount of reserve by scenario for each case. Figure 5a shows the level of the reserve in the Base Case, which considered only the deterministic reserve to prevent the forced outage of a generator. Figure 5b,c show the total reserve requirement for each scenario, including the additional reserve caused by only the load or the wind forecasting error, respectively. A comparison of these two cases confirms that the variation in the reserve due to the wind forecasting error in Case 2 is smaller than that due to the load uncertainty in Case 1. Meanwhile, Figure 5d shows the simulation results when considering both the load and wind uncertainties. The total reserve of Case 3 is higher than those of the other cases because the reserve caused by the load and wind forecasting errors can be compensated to obtain an additional reserve.



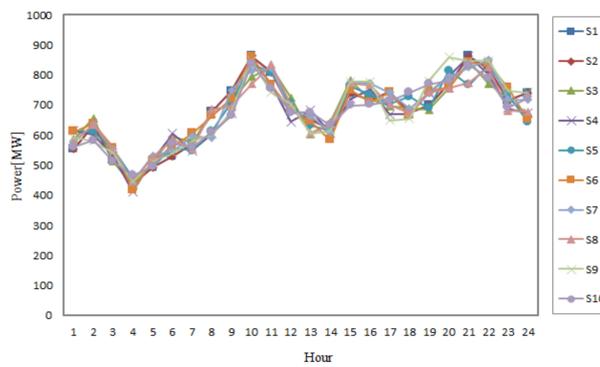
(a) Base Case



(b) Case 1



(c) Case 2



(d) Case 3

Figure 5. Reserve scenario generation for 24 h. (a) Base Case (b) Case 1 (c) Case 2 (d) Case 3.

Based on these reserve requirements, the stochastic UC problem is solved for the proposed test system. Table 2 depicts the results of 10 scenarios for Cases 1, 2 and 3. Here, the total operating cost of the Base Case is \$3,496,305, which is used as a basis for comparison with the stochastic cases. In the stochastic cases, on the other hand, the order of the operating costs from the largest to the smallest is Case 1 > Case 3 > Case 2. As shown in Table 2, Cases 2 and 3 have higher levels of reserve requirement than Case 1, but lower overall operating costs. The reason is that penetration of wind power contributes to the reduction of the total operating cost, even though the reserve requirement may be increased, because the wind power is replacing the thermal unit with reducing fuel costs and uncertainties of the load and wind power compensate each other. Therefore, when considering the level of reserve requirement and total operating cost simultaneously, Case 3 shows better performance than the other cases, and then scenario 7 obtains the best compromised solution.

**Table 2.** Results of each scenario in stochastic cases.

Scenario	Case 1		Case 2		Case 3	
	Reserve (MW)	Operating Cost (\$)	Reserve (MW)	Operating Cost (\$)	Reserve (MW)	Operating Cost (\$)
S1	16,959	3,614,894	15,779	3,248,309	17,855	3,335,855
S2	17,032	3,614,894	15,686	3,226,984	17,815	3,311,563
S3	16,885	3,583,911	15,754	3,229,614	17,723	3,310,324
S4	17,889	3,682,022	15,970	3,270,765	18,080	3,372,785
S5	16,856	3,565,803	15,577	3,137,272	18,110	3,385,998
S6	17,561	3,645,439	15,654	3,154,512	18,124	3,394,307
S7	16,763	3,539,429	15,325	3,109,556	17,578	3,303,129
S8	16,922	3,607,255	15,679	3,182,966	18,317	3,453,657
S9	17,738	3,675,453	15,361	3,129,913	18,136	3,400,132
S10	17,339	3,640,091	15,902	3,248,621	18,278	3,420,737
Average	17,194	3,616,731	15,669	3,193,851	17,972	3,367,849

Table 3 shows the reserve requirement and total operating costs with respect to the wind penetration. The results show that Case 3, which considers both load and wind forecasting errors, presents the best performance when the penetration level of wind power is highest. When the level of penetration of wind power increases, an additional reserve requirement is secured to compensate for the forecasting error of this increased penetration level of wind power. As shown in Table 3, the operating cost per reserve also shows a proportional decrease, although the reserve requirement increases. The reason is that the operation of the thermal units is replaced by wind power and thus the fuel cost is reduced, which leads to a reduction in the operating costs.

**Table 3.** Comparison with penetration level of wind power.

Penetration Level of Wind Power	Case 2			Case 3		
	Reserve (MW)	Total Operating Cost (\$)	Operating Cost per Reserve (\$/MW)	Reserve (MW)	Total Operating Cost (\$)	Operating Cost per Reserve (\$/MW)
5%	17,454	3,224,845	184.76	20,078	3,492,843	173.96
10%	17,977	3,160,753	175.82	20,986	3,367,859	160.48
15%	18,667	2,936,810	157.32	21,753	3,173,760	145.89
20%	18,906	2,736,799	144.75	21,986	3,039,813	138.26
25%	18,931	2,524,389	133.34	22,119	2,856,692	129.15

Table 4 shows the unit state combination of the best solution (S7) in Case 3 for each thermal unit along the time horizon. The computational time taken for Case 3 with the best solution was 24.3 s. Basically, the minimum uptime/downtime limits are satisfied for the overall units, and 32 out of 54 units, which have a higher commitment than the other units, keep their states at “on” the entire time. This means that the uncertainty caused by load and wind power can be compensated for by 32 units maintaining their “on” state. To reliably capture the additional reserve caused by uncertainty,

these results offer a probabilistic perspective of the role of each generation unit in the solution of the stochastic UC problem.

**Table 4.** Unit state combination of the best solution in Case 3.

Unit	Time (h)																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
26	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
27	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
28	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
29	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
35	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
36	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
37	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
39	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
43	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
44	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
45	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
48	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
51	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
52	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
53	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0

### 6. Conclusions

In this paper, an optimal solution procedure was proposed for solving the stochastic UC problem considering the load and wind power uncertainties, which are calculated randomly within the range determined by MAPE. The MSTM was applied to guarantee the stability of a power system against the uncertainty of load and wind power sources. This method was also implemented to determine the amount of reserve requirement with a net load scenario based on random load and wind forecasting errors using MCS. To demonstrate the effectiveness of the proposed stochastic UC problem, various cases were studied, and the effects of forecasting errors with the reserve requirement were compared for the modified IEEE-118 bus test system. The numerical scenarios indicated that the proposed

solution procedure coordinated the uncertainties of load and wind power and forced generator outage. The forecasting error for uncertain sources significantly affected the UC problem; however, it was possible to find a more adequate reserve requirement to minimize the total operating cost. It was also proven that the proposed approach can help relieve the strict reserve requirement aggravated by the high penetration of wind power. Therefore, based on the results of these computational simulations, the proposed stochastic UC approach should be a suitable alternative for determining the benefits of a better reserve requirement in the day-ahead scheduling problem.

**Acknowledgments:** This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of education (2017R1D1A1B03029308).

**Author Contributions:** Kyu-Hyung Jo proposed the main idea of this paper and Mun-Kyeom Kim coordinated the proposed approach in the manuscript.

**Conflicts of Interest:** The authors declare no conflict of interest.

## Nomenclature

### Constants

$\pi_s$	probability of occurrence scenario $s$
$FC_g$	generator fuel cost function of generator $g$
$S_g$	start-up and shut down cost of generator $g$
$a_i, b_i, c_i$	fitted parameters of fuel cost coefficients for each unit
$SU_g$	start-up cost of generator $g$
$SD_g$	shut-down cost of generator $g$
$\underline{P}_g, \overline{P}_g$	minimum and maximum capacity of generator $g$
$\overline{R}_g$	maximum ramping rate of generator $g$
$UT_i, DT_i$	minimum required down and up times for thermal generator $g$
$RU_g$	ramping-up rate of generator $g$
$SUR_g$	start-up ramping rate of generator $g$
$RD_g$	ramping-down rate of generator $g$
$SDR_g$	shut-down ramping rate of generator $g$
$V_c$	cut-in wind speed in m/s
$V_o$	cut-out wind speed in m/s
$V_r$	rated wind speed in m/s

### Variables

$p_t^{For}$	forecasted value at $t$ -hour
$p_t^{Act}$	actual value at $t$ -hour
$n$	population size
$p_{gst}$	power output of generator $g$ in scenario $s$ at time $t$
$u_{gst}$	commitment of generator $g$ in scenario $s$ at time $t$
$s_{gst}$	start-up and shut down cost of generator $g$ in scenario $s$ at time $t$
$x_{gst}$	shut-down of generator $g$ in scenario $s$ at time $t$
$W_{gst}$	wind power of generator $g$ in scenario $s$ at time $t$
$D_{st}$	load in scenario $s$ at time $t$
$\overline{p}_{gst}$	maximum capacity of generator $g$ in scenario $s$ at time $t$
$R_{st}$	basic reserve requirement in scenario $s$ at time $t$
$R_{gst}$	basic reserve requirement of generator $g$ in scenario $s$ at time $t$
$US_{gst}$	up-spinning reserve of thermal generator $g$ in scenario $s$ at time $t$
$\underline{R}_{st}, \overline{R}_{st}$	minimum and maximum basic reserve requirement in scenario $s$ at time $t$
$SU_{gst}$	start-up cost for thermal generator $g$ in scenario $s$ at time $t$
$W_{gst}$	wind power of generator $g$ in scenario $s$ at time $t$
$v_{gst}$	wind speed of wind turbine generator $g$ in scenario $s$ at time $t$ in m/s

$WLR_{st}$	additional reserve requirement in scenario $s$ at time $t$
$W_{gst}^{rated}$	rated output of wind power
$D_{forecasted,st}$	forecasted load in scenario $s$ at time $t$
$W_{forecasted,gst}$	forecasted wind power of generator $g$ in scenario $s$ at time $t$
$W_{error,gst}$	wind power forecasting error of generator $g$ in scenario $s$ at time $t$
$D_{error,st}$	load forecasting error in scenario $s$ at time $t$
$R_{total,st}$	total reserve requirement in scenario $s$ at time $t$
$D_{net,st}$	net load in scenario $s$ at time $t$
Indices	
$g$	Index of generator
$s$	Index of scenario
$t$	Index of time
$st$	Index of scenario $s$ at time $t$
$gst$	Index of generator $g$ in scenario $s$ at time $t$
$i$	Index of thermal generator

## References

- Kim, H.Y.; Kim, M.K. Optimal generation rescheduling for meshed AC/HIS grids with multi-terminal voltage source converter high voltage direct current and battery energy storage system. *Energy* **2016**, *119*, 309–321. [[CrossRef](#)]
- Ma, H.; Wang, B.; Gao, W.; Liu, D.; Sun, Y.; Liu, Z. Optimal Scheduling of a Regional Integrated Energy System with Energy Storage Systems for Service Regulation. *Energies* **2018**, *11*, 195.
- Ji, B.; Yuan, X.; Chen, Z.; Tian, H. Improved gravitational search algorithm for unit commitment considering uncertainty of wind power. *Energy* **2014**, *67*, 52–62. [[CrossRef](#)]
- Tolba, M.A.; Rezk, H.; Tulsy, V.; Diab, A.A.Z.; Abdelaziz, A.Y.; Vanin, A. Impact of Optimum Allocation of Renewable Distributed Generations on Distribution Networks Based on Different Optimization Algorithms. *Energies* **2018**, *11*, 245.
- Zhou, J.; Xu, Y.; Zheng, Y.; Zhang, Y. Optimization of Guide Vane Closing Schemes of Pumped Storage Hydro Unit Using an Enhanced Multi-Objective Gravitational Search Algorithm. *Energies* **2015**, *109*, 765–780.
- Bai, Y.; Zhong, H.; Xia, Q.; Kang, C.; Xie, L. A decomposition method for network-constrained unit commitment with AC power flow constraints. *Energy* **2015**, *88*, 595–603. [[CrossRef](#)]
- Jorge, V.; Smith, A. A seeded memetic algorithm for large unit commitment problems. *J. Heurist.* **2002**, *8*, 173–195.
- Roque, L.A.; Fontes, D.B.; Fontes, F.A. A Metaheuristic Approach to the Multi-Objective Unit Commitment Problem Combining Economic and Environmental Criteria. *Energies* **2017**, *10*, 2029.
- Wind Power Integration in Liberalised Electricity Markets (Wilmar) Project. Available online: <http://www.wilmar.risoe.dk> (accessed on 2 October 2017).
- Ruiz, P.A.; Philbrick, C.R.; Zak, E.; Cheung, K.W.; Sauer, P.W. Uncertainty management in the unit commitment problem. *IEEE Trans. Power Syst.* **2009**, *24*, 642–651. [[CrossRef](#)]
- Ruiz, P.A.; Philbrick, C.R.; Sauer, P.W. Wind power day-ahead uncertainty management through stochastic unit commitment policies. In Proceedings of the pOwer Systems Conference and Exhibition, Seattle, WA, USA, 15–18 March 2009.
- Wu, L.; Shahidehpour, M.; Li, T. Stochastic security-constrained unit commitment. *IEEE Trans. Power Syst.* **2007**, *22*, 800–811. [[CrossRef](#)]
- Wu, L.; Shahidehpour, M.; Li, Z. Comparison of scenario-based and interval optimization approaches to stochastic SCUC. *IEEE Trans. Power Syst.* **2012**, *27*, 913–921. [[CrossRef](#)]
- Zhang, Y.; Wang, J.; Ding, T.; Wang, X. Conditional value at risk-based stochastic unit commitment considering the uncertainty of wind power generation. *IET Gener. Trans. Distrib.* **2018**, *12*, 482–489. [[CrossRef](#)]
- Bertsimas, D.; Litvinov, E.; Sun, X.A.; Zhao, J.; Zheng, T. Adaptive robust optimization for the security constrained unit commitment problem. *IEEE Trans. Power Syst.* **2013**, *28*, 52–63. [[CrossRef](#)]

16. Jiang, R.; Wang, J.; Guan, Y. Robust unit commitment with wind power and pumped storage hydro. *IEEE Trans. Power Syst.* **2012**, *27*, 800–810. [[CrossRef](#)]
17. Wang, Q.; Guan, Y.; Wang, J. A Chance-Constrained Two-Stage Stochastic Program for Unit Commitment With Uncertain Wind Power Output IEEE Trans. Power Syst. Power Systems. *IEEE Trans. Power Syst.* **2012**, *27*, 206–215. [[CrossRef](#)]
18. Gooi, H.B.; Mendes, D.P.; Bell, K.R.W.; Kirschen, D.S. Optimal Scheduling of Spinning Reserve. *IEEE Trans. Power Syst.* **1999**, *14*, 1485–1492. [[CrossRef](#)]
19. Morales, J.M.; Conejo, A.J.; Perez-Ruiz, J. Economic Valuation of Reserves in Power Systems with High Penetration of Wind Power. *IEEE Trans. Power Syst.* **2009**, *24*, 900–910. [[CrossRef](#)]
20. Kim, M.-K. Short-term price forecasting of Nordic power market by combination Levenberg-Marquardt and Cuckoo search algorithms. *IET Gener. Trans. Distrib.* **2015**, *9*, 1553–1653. [[CrossRef](#)]
21. Jordan, G.; Piwko, R. *The Value of Wind Power Forecasting*; National Renewable Energy Lab. (NREL): Golden, CO, USA, 2011.
22. Constantinescu, E.M.; Zavala, V.M.; Rocklin, M.; Lee, S.; Anitescu, M. *Unit Commitment with Wind Power Generation: Integrating Wind Forecast Uncertainty and Stochastic Programming*; ANL/MCS-TM-309; Argonne National Lab. (ANL): Argonne, IL, USA, 2009.
23. Safari, A.; Shayanfar, H.A.; Jahani, R. Optimal Unit Commitment of Power System using Fast Messy Genetic Algorithm. *Int. J. Tech. Phys. Probl. Eng.* **2010**, *2*, 22–27.
24. Pappala, V.S.; Rohrig, I.E.K.; Dobschinski, J. A Stochastic Model for the Optimal Operation of a Wind-Thermal Power System. *IEEE Trans. Power Syst.* **2009**, *24*, 940–950. [[CrossRef](#)]
25. Illinois Institute of Technology Power Group Test Case. Available online: <http://www.motor.ece.iit.edu/Data/JEASIEEE118.doc> (accessed on 7 July 2017).
26. Kazemi, M.; Siano, P.; Sarno, D.; Goudarzi, A. Evaluating the impact of sub-hourly unit commitment method on spinning reserve in presence of intermittent generators. *Energy* **2016**, *113*, 338–354. [[CrossRef](#)]



© 2018 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).