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New Insight of Maximum Transferred Power by Matching Capacitance of a Wireless Power Transfer System

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Abstract: Most research on wireless power transfer (WPT) has been focused on how to achieve a high-efficiency power transfer. Our study found that under the impedance matching for achieving maximum WPT efficiency, the power transferred to the load cannot reach the maximum when a WPT system is supplied by an AC voltage source with constant amplitude. However, the load power or the voltage across the load is essential for a low-power electric device such as the implanted medical device where the transfer efficiency is not the priority to be considered. The paper presents a method for achieving maximum power on the load by matching capacitance in a WPT system with given two-coupled-coils. Three sets of matching capacitances for extreme load power were deduced based on the circuit model considering the coil's resistance, and all these three matching make the WPT system operate at the resonant state. Two sets can make the system achieve the global maximum of load power. One set can make the system achieve the local maximum of load power and reach the power transfer efficiency next to 1. Experimental results verified the theoretical calculations. The results can contribute to the compensation design of a practical WPT system for transferring the maximum power to the load.

Keywords: matching capacitance; maximum load power; wireless power transfer; power transfer efficiency

1. Introduction

The wireless power transfer (WPT) technique by magnetic coupling coils has been applied in many fields, such as portable electronic devices [1–4], electric vehicles [5–7], and implanted medical devices [8–11]. For mid-range air gap between transmitting and receiving coil (the distance between two coils is usually less than eight times the diameter of the coil), the power transfer efficiency is vitally important and must be given priority in applications of high-power or continuous-operation electric devices such as power supply for household appliance, vehicle charging and microwave power transmission. Accordingly, the research on WPT technique has been mostly focused on matching impedance and controlling the system to operate at a resonant state to transfer considerable energy at high efficiency [12–15]. However, under the impedance matching condition for the maximum WPT efficiency, this study found that the power transferred to the load cannot reach the maximum when the WPT system is supplied by an AC voltage source with constant amplitude. In actual applications, the load power or the voltage across the load is essential for low power electric devices, rechargeable devices, etc. Therefore, the paper presents a new insight of maximum-transferred power by matching capacitance of WPT system.

The ultimate goal of most WPT research was usually to achieve high power-transfer-efficiency [16–18], where the power transferred to the load was often involved and expressed. However, whether the transferred power reaches the maximum was seldom concerned with. There are also some references take the WPT power as the research objective. For example, reference [19] and [20] optimized the load condition to achieve the high load power. Reference [21] studied how the output power depends upon the type of the source: current-source or voltage-source. In order to transfer the maximum power to the load, there are different goals at stages of design and control of a WPT system. At the initial design stage of a WPT system with given two-coupled-coils, the top priority is to match capacitances to ensure that the system operates at resonant state and transfers the maximum power to load. Reference [22] determined the optimal tuning capacitor values of transcutaneous energy transfer system for achieving maximum power transfer. However, it only gave two types of capacitance matching methods and did not address the efficiency difference between these two capacitance matching. In this paper, three methods of matching capacitance for achieving extreme power transfer are presented and the WPT efficiencies are also given. Reference [23] calculated the input impedance and frequency for the maximum transferred power based on the equivalent circuit model without considering the coil's resistance. Wherein, the matching capacitors cannot make the system operate at resonant state, thus the transferred power cannot reach the maximum. Reference [24] used full-wave electromagnetic modeling of WPT links for matching impedances to achieve maximum power on the load, where matching capacitance was not given in explicit expression and multiple sets of solutions were not presented.

After two coils of a WPT system are compensated by matching impedance, the resonant stability of the system needs to be ensured for achieving high transferred power. These methods, such as tracking operating frequency of the primary converter, adjusting mutual inductance, etc., are usually adopted. References [25–32] built the special symmetrical circuit model where two coils are the same, and took the load power extreme as the goal to study frequency splitting. The phenomenon of frequency splitting is defined as: the peak of the load power for a WPT system is divided into several peaks due to the change of mutual inductance between coupled coils. When the extreme load power is divided into several peaks, the corresponding frequencies will be changed from one to multiple points.

Different from the analysis beginning with peaks of input impedance [23] and full-wave electromagnetic modeling [24] of WPT links, this paper directly derives the analytical expression of load power from a circuit model considering the resistance of two-coupled-coils. This model is closer to the actual system, and theoretical derivation is simpler and more direct. Second, analytical solutions of matching capacitance are deduced for transferring the maximum power to load, which contributes to the design of a WPT system with given two-coupled-coils supplied by an AC voltage source. Then, the effects of resonant frequency on the load power and transfer efficiency are analyzed by calculations. Next, practical conditions to determine the maximum load power are presented. Finally, the experimental WPT system is built up to verify the theoretical results of the maximum transfer power.

2. Expressions of the Power Transferred to the Load

The schematic diagram of the WPT system with two-coupled-coils is shown in Figure 1. The WPT system consists of a transmitter and a receiver, which are magnetically coupled through mutual coils. In actual applications, rectifier, filter and switched-mode controller are normally used to drive the load that may be inductive or capacitive. However, it is usual to represent this load as an equivalent resistance R_L. To derive the analytical solutions for the voltage across the load and the transferred power, a simplified equivalent circuit model of the WPT system is presented as shown in Figure 1b. The output voltage of AC source is $u_{\text{in}}(t) = \sqrt{2}U_{\text{in}}\cos(\omega\, t)$, where U_{in} is the RMS (root-mean-square) voltage and $\omega = 2\pi f$ is the angular frequency. The power source excites a transmitting coil with resistance R_t and inductance L_t. The receiving coil with resistance R_r and inductance L_r is coupled with the transmitting coil. The subscripts "t" and "r" stand for the transmitter and the receiver respectively.

M is the mutual inductance between two coils. The matching capacitances C_t and C_r are respectively connected with the transmitting coil and the receiving coil in series.

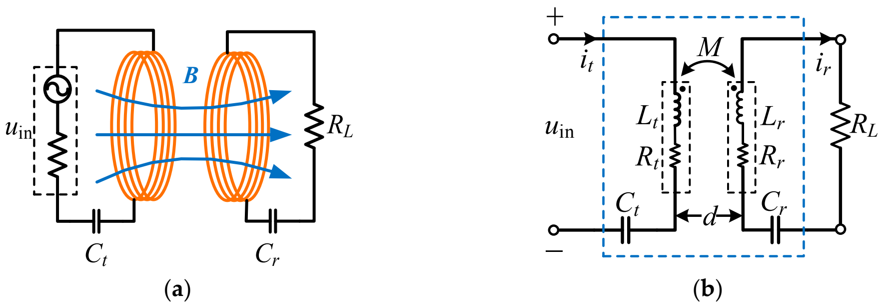

Figure 1. Equivalent circuit model for the WPT system compensated by a single capacitor on both primary and secondary side. (**a**) Two-coupled-coils for the WPT system. (**b**) Simplified equivalent circuit model.

The input impedance Z_t on the primary side is expressed in the phasor form as,

$$Z_t = R_t + \mathrm{j}A_t + \frac{\omega^2 M^2}{R_L + R_r + \mathrm{j}A_r}, \tag{1}$$

where $A_t = \omega L_t - \frac{1}{\omega C_t}$ and $A_r = \omega L_r - \frac{1}{\omega C_r}$.

According to the Thévenin's Theorem, the equivalent circuit connected to the load is an open-circuit voltage source U_0 connected with an equivalent impedance Z_{eq} in series. The open-circuit voltage is given as

$$U_{\mathrm{o}} = \frac{\mathrm{j}\omega M U_{\mathrm{in}}}{R_t + \mathrm{j}A_t},$$

and the short- circuit current of load side is given as

$$I_{\mathrm{d}} = \frac{\mathrm{j}\omega M U_{\mathrm{in}}}{(R_r + \mathrm{j}A_r)(R_t + \mathrm{j}A_t) + \omega^2 M^2}.$$

Then, the equivalent impedance is given by

$$Z_{\mathrm{eq}} = \frac{U_{\mathrm{o}}}{I_{\mathrm{d}}} = (R_r + \mathrm{j}A_r) + \frac{\omega^2 M^2}{R_t + \mathrm{j}A_t}. \tag{2}$$

Further, the voltage across the resistive load U_{R_L} can be obtained as follows,

$$U_{R_L} = U_{\mathrm{o}} \frac{R_L}{Z_{\mathrm{eq}} + R_L} = \frac{\mathrm{j}\omega M U_{\mathrm{in}} R_L}{[R_t(R_L + R_r) - A_t A_r + \omega^2 M^2] + \mathrm{j}[(R_L + R_r)A_t + R_t A_r]}. \tag{3}$$

The average power on the resistive load in one cycle is

$$P = \left|U_{R_L}\right|^2 / R_L = g \cdot v^{-1}, \tag{4}$$

where

$$g = (\omega M U_{\mathrm{in}})^2 R_L, \tag{5}$$

$$v = \left[R_t(R_L + R_r) + \omega^2 M^2 - A_t A_r\right]^2 + [(R_L + R_r)A_t + R_t A_r]^2. \tag{6}$$

After analysis of (4), we know that the voltage across the resistive load and the power transferred to the resistive load approach the extreme values simultaneously, so we only need to study either of them. Hereafter the power transferred to the load will be studied only.

3. Maximum Power Transferred to the Load

For given two-coupled-coils and a load, namely, $R_L,\ R_t,\ L_t,\ R_r,\ L_r$ are known, the load power of (4) is the function of ω, M, C_t, C_r under a certain magnitude of power source voltage U_{in}. When $\frac{\partial P}{\partial \omega} = \frac{\partial P}{\partial M} = \frac{\partial P}{\partial C_t} = \frac{\partial P}{\partial C_r} = 0$, the load power of the system reaches the global extreme value. While for an actual system, it almost never needs to optimize the above four parameters simultaneously, and it is also difficult to solve these four equations. Therefore, we discuss the maximum power transferred to the load by matching impedance, i.e., matching capacitance for the system when the power source frequency and the mutual inductance maintain constant, which is required during the initial design of a WPT system.

3.1. Matching Capacitance for Maximum Power Transferred to Load

Suppose the load power is only related to the capacitances on the sides of transmitter and receiver, while other circuit parameters keep constant, i.e., $P(C_t, C_r)$. If the load power (4) reaches the extreme value, then its denominator (6) gets the extreme value as its numerator (5) in the fraction of (4) is not the function of capacitance. It is required that

$$\frac{\partial v}{\partial C_t} = 0 \text{ and } \frac{\partial v}{\partial C_r} = 0. \tag{7}$$

After the theoretical derivation of (7), three sets of matching capacitance can be obtained as follows,

$$\text{I} \qquad \begin{cases} \frac{1}{\omega C_t} = \omega L_t \\ \frac{1}{\omega C_r} = \omega L_r \end{cases}, \tag{8a}$$

$$\text{II}(+) \text{ and III}(-) \qquad \begin{cases} \frac{1}{\omega C_t} = \omega L_t \pm R_t\sqrt{\Delta} \\ \frac{1}{\omega C_r} = \omega L_r \pm (R_L + R_r)\sqrt{\Delta} \end{cases}. \tag{8b}$$

where $\Delta = \frac{\omega^2 M^2}{R_t(R_L+R_r)} - 1$. The solution II has a plus sign and the solution III has a minus sign. According to (8b), if there exist two sets of solutions for matching capacitance ($C_t > 0$ and $C_r > 0$) which are different from solution I, then it is required that: ① $\Delta > 0$, i.e., $\omega^2 M^2 > (R_L + R_r)R_t$, ②$\omega L_t \pm R_t\sqrt{\Delta} > 0$, and ③ $\omega L_r \pm (R_L + R_r)\sqrt{\Delta} > 0$.

By analysis of (8b), we can also know that: ① When $\omega = \omega_0 = \sqrt{R_t(R_L + R_r)}/M$ or $R_L = \omega^2 M^2/R_t - R_r$, three sets of solutions become one set of solution I. Only if $\omega > \omega_0$, there exists three sets of solutions. We call the frequency ω_0 as the cutoff frequency. ② The higher the frequency is, the larger the difference between these three sets of solutions is.

At a certain frequency, the set I of matching capacitances are only determined by the self-inductances of transmitting and receiving coil, while the set II and III of matching capacitances are also related to the load resistance, the resistances of two coils, and the mutual inductance, besides the self-inductances of two coils. Therefore, it is easier to match for the set I of capacitance, and the system is more stable due to fewer influence factors.

When the capacitors in the circuitry are matched according to (8a) and (8b), the equivalent impedances on the primary side are

$$Z_t^{\mathrm{I}} = R_t + \frac{\omega^2 M^2}{R_L + R_r} \text{ and } Z_t^{\mathrm{II,III}} = 2R_t;$$

and the equivalent impedances on the secondary side are

$$Z_{\mathrm{eq}}^{\mathrm{I}} = R_r + \omega^2 M^2/R_t \text{ and } Z_{\mathrm{eq}}^{\mathrm{II,III}} = 2R_r + R_L.$$

It is obvious that the impedances of Z_t and Z_{eq} for all these three solutions are purely resistive, and the imaginary parts of the impedances on both sides of transmitter and receiver equal zero.

Therefore, there are three methods of matching capacitance to make the system operate under the resonant status. Thus, the power source voltage U_{in} has the same phase as the primary current I_t.

3.2. Transferred Power under Three Sets of Matching Capacitances

Substituting the matching capacitance of (8a) and (8b) into (4), the corresponding load powers become

$$P^{\text{I}} = \frac{U_{\text{in}}^2 \omega^2 M^2 R_L}{\left[R_t(R_L + R_r) + \omega^2 M^2\right]^2}, \tag{9a}$$

and

$$P^{\text{II,III}} = \frac{U_{\text{in}}{}^2}{4R_t(1 + R_r/R_L)}. \tag{9b}$$

Comparing (9a) and (9b), due to $\left[R_t(R_L + R_r) + \omega^2 M^2\right]^2 \geq 4R_t(R_L + R_r) \cdot \omega^2 M^2$, there is $P^{\text{I}} \leq P^{\text{II, III}}$. The equality is valid only if $\omega^2 M^2 = R_t(R_L + R_r)$. In this case, $P^{\text{I}} = P^{\text{II,III}}$, and there is only one set of solutions in fact. Otherwise, there is $P^{\text{I}} < P^{\text{II, III}}$ if there exist three different solutions. According to above theoretical analysis, the matching capacitance of (8a) enables the load power to achieve the local extreme value; and the matching capacitance of (8b) enables the system to achieve the global maximum load power, which is only related to the supply voltage and three resistors of R_t, R_r, R_L. We give a calculation example to show the difference of load power in the following.

The load powers at different matching capacitances when U_{in} = 1 V, f = 200 kHz, M = 2 μH, L_t = 6 μH, L_r = 50 μH, R_t = 0.1 Ω, R_r = 0.1 Ω, R_L = 9.9 Ω are shown in Figure 2. We can see that there is one local extremum of load power 1.168 W under the set I of matching capacitances (105.54 nF, 12.67 nF), while there are two maxima of load power 2.475 W under the sets II and III of matching capacitances (102.41 nF, 9.265 nF) and (108.87 nF, 20.01nF).

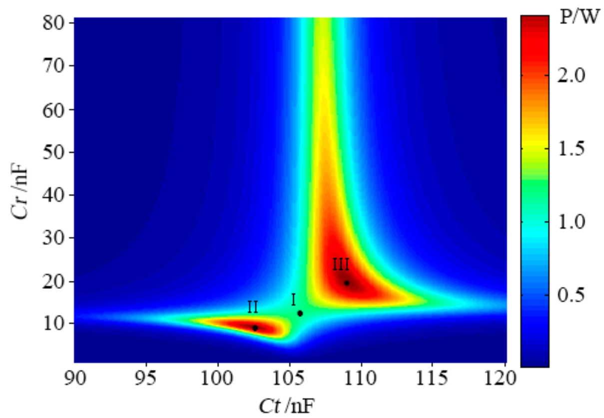

Figure 2. Load power of a wireless power transfer system with different matching capacitances when U_{in} = 1 V, f = 200 kHz, M = 2 μH, L_t = 6 μH, L_r = 50 μH, R_t = 0.1 Ω, R_r = 0.1 Ω, R_L = 9.9 Ω.

3.3. Power Transfer Efficiency of Two-Coupled-Coils

The average output power of a power supply within a cycle is $P_{\text{in}} = U_{\text{in}}^2/Z_t$, and the average power consumed by the load R_L within a cycle is $P = \left|U_{R_L}\right|^2/R_L = \left|I_{R_L}\right|^2 R_L$, where I_{R_L} is the phasor expression of $i_{R_L}(t)$. The power transfer efficiency of two-coupled-coils is defined as the ratio of load power to the output power of power source. Therefore, the magnetic coupling resonance (MCR) WPT efficiency η of two-coupled-coils can be written as

$$\eta = \frac{P}{P_{\text{in}}} = \frac{\left|I_{R_L}\right|^2}{U_{\text{in}}{}^2} R_L Z_t. \tag{10}$$

The WPT efficiencies corresponding to the three sets of matching capacitances can be respectively written as:

$$\eta_{\mathrm{I}} = \frac{\beta}{(1+\beta)[1+(1+\beta)/G]}, \tag{11a}$$

$$\eta_{\mathrm{II,III}} = \frac{\beta}{2(1+\beta)}. \tag{11b}$$

where the load coefficient $\beta = R_L/R_r$, and $G = \omega^2 M^2/(R_t R_r)$. From (11b), it is obvious that $\eta_{\mathrm{II,III}} < 0.5$. We can also know from (11a) that η_{I} can approach 1 when $G \to \infty$ and $\beta \to \infty$.

The power supply and controller normally control both the primary voltage and the frequency to transfer the maximum power to the load. Assuming the voltage of the power supply can be adjusted to voltage U, if the receiving load power under the set I of matching capacitances can catch up with that under the set II and III of matching capacitances, the voltage of power supply should be increased by

$$\frac{U}{U_{\mathrm{in}}} = \frac{\omega^2 M^2 + R_t(R_L + R_r)}{2\omega M\sqrt{R_t(R_L + R_r)}}$$

However, as the power transfer efficiency of set I is higher, the current of power supply is still lower than that under the set II and III of matching capacitances, being the time of $\sqrt{R_t(R_L + R_r)}/\omega M$, less than 1. In summary, in order to make the set I of system keep the same load power as set II and III of systems, the voltage of power source needs to be increased, but the current of supply is still less than that of set II and III, which helps the system to decrease the current through a rectifier.

3.4. Effect of Frequency on Power and Efficiency by Three Sets of Matching Capacitances

For a WPT system with given load and coils, we know from (9b) and (11b) that the maximum load power employing the set II and III of matching capacitances is only related to the voltage magnitude of power source, and is independent of frequency. Similarly, when the load coefficient is kept constant, the transfer efficiency remains constant and is also independent of frequency. Therefore, the matching method is easy to achieve constant load power and transfer efficiency. Herein, according to the experimental parameters in Section 4, we give a calculation example to discuss the effect of frequency on power and efficiency using three sets of matching capacitances.

Variations of load power and transfer efficiency with the frequency when U_{in} = 2.83 V ($U_{\text{peak-peak}}$ = 8 V), M = 1.783 μH, R_t = 0.58 Ω, R_r = 1.03 Ω, R_L = 4.90 Ω are shown in Figure 3. Figure 3a shows that: (1) below the cutoff frequency f_0 = 165.54 kHz, the compensation capacitance cannot be matched to achieve maximum load power; (2) at the point of frequency f_0, the WPT system has only one set of matching capacitances making the load power reach the maximum; (3) when the frequency is higher than the frequency f_0, the load power keeps a constant maximum at the set II and III of matching capacitances; while at the set I of matching capacitances, the load power decreases sharply as the frequency increases. From Figure 3b, we can see that the transfer efficiency keeps constant at the set II and III of matching capacitances, while at the set I of matching capacitances it increases as the frequency increases and ultimately approaches 1. In short, using the set II and III of matching capacitances, the maximum load power can be achieved, when the load power and the transfer efficiency are independent of frequency.

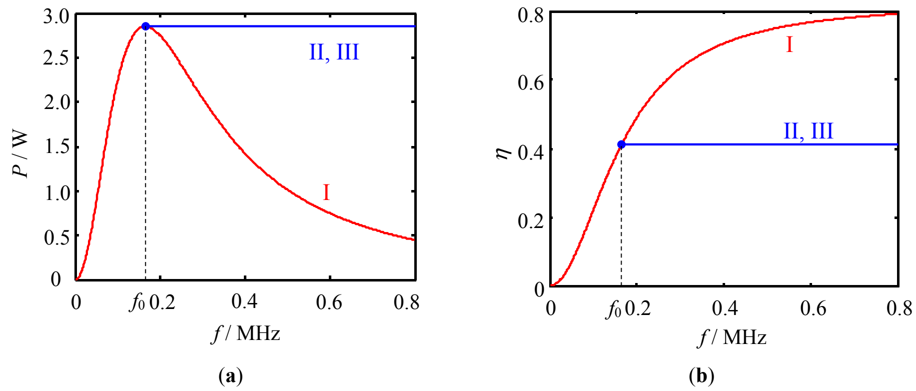

Figure 3. Power (**a**) and efficiency (**b**) vary with the frequency at three sets of capacitance matching when U_{in} = 2.83 V, M = 1.783 μH, R_t = 0.58 Ω, R_r = 1.03 Ω, R_L = 4.90 Ω.

3.5. Practical Conditions to Judge the System Status

From the above derivation, we know that using the three sets of matching capacitances, the WPT system operates under the resonant status. However, according to the resonant condition that the frequency of power supply equals the coupling resonant frequencies of the transmitter and the receiver, it is difficult to judge whether the system transfers the maximum power in practical applications. Therefore, we deduce the practical resonance conditions of the MCR-WPT system according to the expressions of matching capacitance. The criteria to determine the resonance of MCR-WPT system with two-coupled-coils are as follows. (i) On the side of the transmitter, the current has the same phase as the output voltage of the power source; (ii) On the side of the receiver, the phase angle difference between the voltage across the load resistance and the voltage of power source is $\theta = \angle\left(U_{R_L}/U_{\mathrm{in}}\right)$, where $\theta_{\mathrm{I}} = \pi/2$, $\theta_{\mathrm{II}} = \pi - \arctan\left(1/\sqrt{\Delta}\right)$, $\theta_{\mathrm{III}} = \arctan\left(1/\sqrt{\Delta}\right)$.

4. Experiments and Results

To verify the theoretical method for maximum load power using capacitance matching, we built the experimental system as shown in Figure 4.

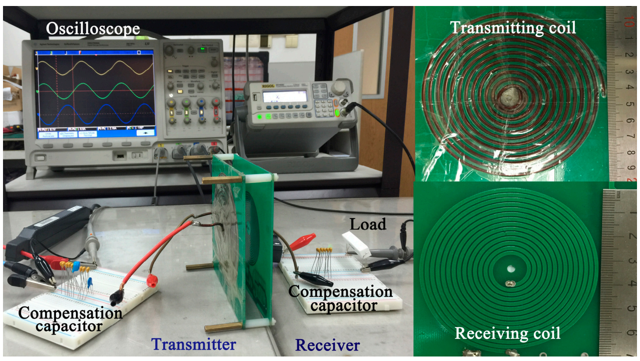

Figure 4. Experimental system for magnetic coupling resonance (MCR)-WPT consisting of a printed circuit boards (PCB) receiving coil and a spiral transmitting coil made of enamel insulated wire, wirelessly powers the receiver.

Two-coupled-coils include a receiving coil made of printed circuit boards (PCB) coil and a spiral transmitting coil made of enamel insulated wire. The two coils' parameters are shown in Table 1. The load resistance is 4.90 Ω. In the following, the experiments are conducted at different cases to study: (1) the WPT power and efficiency under set I and set II(+) of matching capacitances, (2) the effect of the distance between two coils on the maximum WPT power, (3) the effect of the frequency on the maximum WPT power.

Table 1. Parameters of two coils and experimental results of power and efficiency of the WPT system at different impedance matching, frequency and distance.

Matching Methods				I	II(+)	II(+)	II(+)
	f/kHz			300	300	300	200
	d/mm			20	20	10	20
	M/μH			1.783	1.783	2.840	1.783
Transmitter	Resistance R_t/Ω			0.580	0.580	0.580	0.580
	Self-inductance L_t/μH			6.31	6.31	6.31	6.31
	Compensation capacitance C_t/nF	[1] Theo.		44.60	41.54	39.40	95.62
		[2] Act.		43.69	41.94	39.79	100.50
	Peak-peak voltage u_{in}/V			8.0	8.0	8.0	8.0
	Peak-peak current i_t/A			2.75	8.6	7.8	7.2
	Output power of supply/W			2.75	8.6	7.8	7.2
Receiver	Resistance R_r/Ω			1.03	1.03	1.03	1.03
	Self-inductance L_r/μH			7.60	7.60	7.60	7.60
	Compensation capacitance C_r/nF	Theo.		37.03	23.47	17.46	58.64
		Act.		35.98	21.22	16.62	57.14
	Peak-peak current i_r/A			1.56	2.13	2.09	2.06
	Load power P/W	Theo.		2.04	2.85	2.85	2.85
		Act.		1.49	2.78	2.68	2.60
Phase angle difference θ between i_r and i_t		Theo.		90°	147°	160°	124°
		Act.		90°	150°	161°	124°
Power transfer efficiency η		Theo.		0.63	0.41	0.41	0.41
		Act.		0.54	0.32	0.34	0.36

[1] Theo. = Theoretical value; [2] Act. = Actual value.

When the distance between two coils d is 20 mm, their mutual inductance M is 1.783 μH. At the frequency of power supply f = 300 kHz, according to the expressions of matching capacitance (8a) and (8b), we obtained that for the set I of matching capacitances, the primary and secondary compensation capacitances in theory should be 44.60 nF and 37.03 nF respectively. In the actual experiments, they are respectively 43.69 nF and 35.98 nF after the introduction of the stray inductance and capacitance. Similarly, for the set II of matching capacitances, the primary and secondary capacitance in theory should be 41.54 nF and 23.47 nF respectively, while in experiments they are actually 41.94 nF and 21.22 nF respectively. The errors may come from the resistance of connection wire and the equivalent series resistance of compensation capacitance, except for the stray inductance and capacitance. Therefore, the capacitance matching of set II is a little more difficult than that of set I because the set II of matching capacitances are related to much more parameters, and are especially sensitive to the resistance of transmitting coil R_t.

The voltage and current waveforms of the WPT system under different impedance matching, frequency and air gap, are shown in Figure 5. As shown from Figure 5a, for the system matched by the set I of capacitances, the power source voltage has the same phase with the primary current, and the secondary voltage leads the primary voltage by 90.0°. It can be seen from Figure 5b that, at the same frequency and air gap, for the system matched by the set II of capacitances, similar to the set I, the primary voltage also has the same phase as the current. The secondary voltage leads the primary voltage by 150°. Under the above two kinds of capacitance matching, both systems achieved the magnetic coupling resonance status. When the system is supplied by the same voltage source of 8 V (peak-peak value), the peak-peak current flowing through the resistive load under set II of matching is 2.13 A, higher than 1.56 A of set I. The peak-peak current of power source under set II of matching is 8.6 A, also higher than 2.75 A of set I. The load power under the set II of matching capacitances is

2.78 W, approximately twice of the load power under the set I of matching capacitances. On the aspect of the power transfer efficiency, it is 0.54 under the set I, higher than 0.32 under the set II.

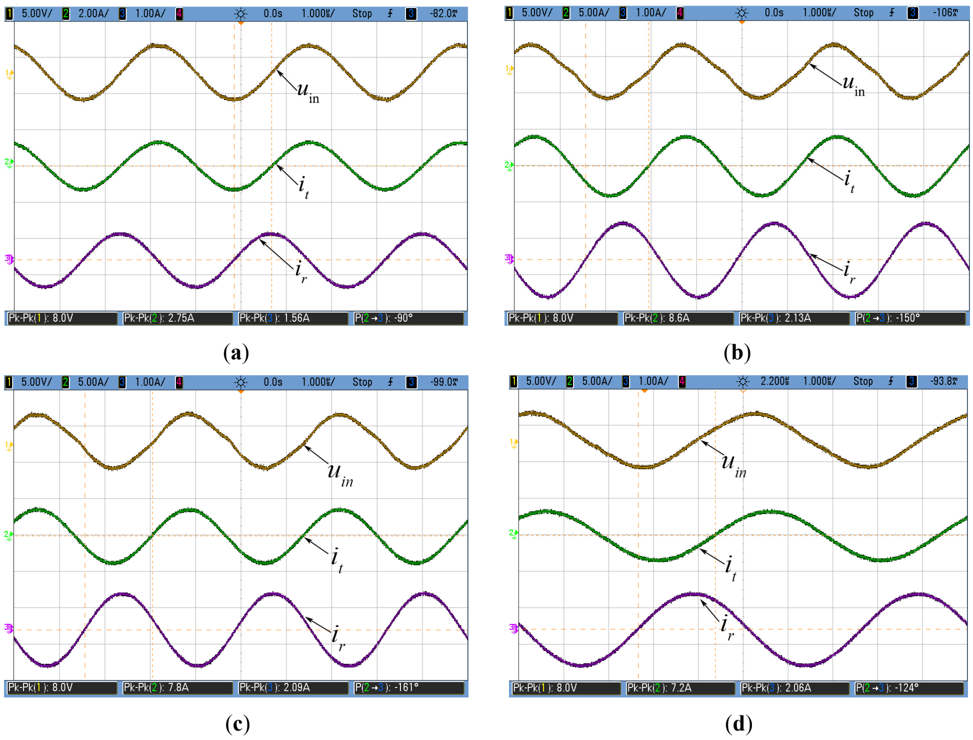

Figure 5. Voltage and current waveforms of the WPT system (**a**) under the set I of capacitance matching when f = 300 kHz and d = 20 mm; under the set II(+) of capacitance matching (**b**) when f = 300 kHz and d = 20 mm, (**c**) when f = 300 kHz and d = 10 mm, and (**d**) when f = 200 kHz and d = 20 mm.

The frequency of 300 kHz remains unchanged, while the distance between two coils is changed from 20 mm to 10 mm, i.e., the mutual inductance of two coils is increased to 2.840 μH. Under the set II of matching capacitances, the theoretical primary and secondary compensation capacitors are 39.40 nF and 17.46 nF, and their experimental values are changed to 39.79 nF and 16.62 nF respectively. As shown in Figure 5c, the phase angle difference has also changed, the theoretical value becomes 160° and the measured value becomes 161°. However, as shown from (9b) and (11b), the maximum WPT power and efficiency under the set II of matching are independent of the mutual inductance, and the theoretical value remains unchanged at 2.85 W and 0.41 respectively. The experimental values are 2.68 W and 0.34 respectively, in good agreement with the theoretical values.

The distance between two coils of 20mm remains unchanged, while the frequency of 300 kHz is changed to 200 kHz. Similarly, under the set II of matching capacitances, the theoretical primary and secondary compensation capacitors are respectively 95.62 nF and 58.64 nF, and experimental values are 100.50 nF and 57.14 nF. Figure 5d shows the phase angle difference. Both the theoretical and measured values are 124 °. Experimental results show that the maximum WPT power under the set II of matching capacitances is 2.60 W, independent of the frequency; and the maximum efficiency is 0.36, also independent of frequency.

In summary, the above measurements and calculations verified that the analytical expressions of matching capacitance for the WPT system can predict successfully the maximum transferred power and the transfer efficiency. Under the AC voltage source excitation, the set II of capacitance matching can reach the maximum load power, which is obviously far higher than that of set I. Furthermore, for given two-coupled-coils, the maximum power has no relationship with the frequency and the air gap under the set II of matching capacitances. Its cost is large power consumption and high current flowing through the power supply. As compared with that of the set I of capacitance matching, it is a little harder to match capacitances and easier to detune. The system under set I of the matching capacitances has higher power transfer efficiency.

5. Conclusions

The equivalent circuit model for a WPT system is established in this paper. The methods for transferring the maximum power to the load are studied through theoretical calculations and experiments. The conclusions and contributions of the paper are as follows.

(1) For a WPT system, matching capacitances for maximum load power and maximum transfer efficiency are different. This paper presents a method for achieving maximum power on the load by matching capacitances, which can contribute to the WPT compensation design for low power electric devices, rechargeable devices, etc.
(2) Three sets of matching capacitances were deduced for achieving extreme load power, and all these three matchings make the WPT system operate at the magnetic coupling resonance state. Two sets can make the system achieve the global maximum of load power, which can be used in the applications where the load needs the maximum power or voltage. One set can make the system achieve the local maximum of load power and reach the power transfer efficiency next to 1. Experimental results verified the theoretical calculations.
(3) The practical condition to determine the maximum power transfer was presented, which provided an easy access to judge the WPT system transferring the maximum power to the load.

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