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The Convergence Evolution in Europe from a Complex Networks Perspective

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Abstract: The evolution of the convergence among the European countries, including both Eurozone as well as non-Eurozone economies, is investigated in this paper. To do so, we construct correlation-based networks and study them by employing the Threshold Weighted-Minimum Dominating Set (TW–MDS) algorithm and analyzing standard quantitative performance graph theory metrics. Each country is represented by a network node, while the edges represent the cross-correlations calculated for a specific macroeconomic variable, for a given time window. To study the intertemporal evolution of the network's interconnections, we examine its structure in three consecutive time intervals: 1999–2004, 2005–2010 and 2011–2019. The empirical findings provide a mixed pattern. The European countries exhibit a common behavior over time for some macroeconomic variables, but not for all of them.

Keywords: complex networks; convergence; correlation; European business cycles; graph theory; synchronization; EU integration; EU deepening; EU enlargement

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1. Introduction

The European Coal and Steel Community (ECSC) organization, founded in 1951, was the foundation of what is called today the European Union (EU). The aim of the ECSC was to prevent competition over natural resources and further the progress of economic and political unity within Europe. Through the period following the 1950s, the Community has been developing, going through phases of enlargement in terms of new country memberships and of economic growth. This process finally led to the European Union formation; in 1993, the Treaty on European Union became effective, marking a significant milestone in the EU's path towards unification. Alongside the introduction of the European citizenship concept, the formation of common foreign and security policy and the initiation of cooperation between the EU countries in the field of justice, the Treaty founded the passage to a single currency era, which was due to commence on the 1 of January, 1999. The Treaty also provided for the establishment of the European System of Central Banks along with the European Central Bank (Treaty on European Union, Council of the European Communities 1992). The responsibility of the latter was to preserve price stability and put into effect a single monetary policy. However, a one-for-all monetary policy requires common economic cycles within the area it is applied on, so as to carry a beneficial effect for the economies. Mundell (1961), McKinnon (1963) and Kenen (1969) already by the 1960s had laid the decisive features for two or more sovereign countries to relinquish national currency and monetary policy in favor of a union possessing a single monetary policy, introducing the Optimum Currency Area (OCA) concept. Although the three authors are cited as the pioneers of the OCA theory, some principles of it had been formulated by the late 1940s and the early 1950s (Cesarano 2006). In the years following the 1960s, it is argued that specific factors were disregarded in the OCA discussion; among them, the differentiations related to economic development among the sovereign economies (Dellas and Tavlas 2009). This topic is of great importance, given that economies on different courses cannot be fitted within the same monetary policy (Rogoff 1985; Frankel and Rose 1998). An overheated economy should be subject to a tight monetary policy in order for inflation to be confined, whereas an economy operating in the opposite direction requires an expansionary monetary policy, so as for economic activity to be boosted. Were the two economies under the same policy regime, the authorities would have faced a delicate dilemma regarding the proposed monetary policy.

That being said, the business cycles synchronization of different countries/regions forming a single currency area is of great importance. Given that the topic has been extensively studied in the past, this study tries to differ; the research is not restricted to business cycle synchronization (in terms of Gross Domestic Product evolution) through time, although the GDP is the most representative gauge of the aggregate economic activity. Rather, it is extended to examine a broader set of macroeconomic variables and thus to shed light to the convergence pattern topic on a multi-dimensional basis. Hence, the present study attempts to illuminate the convergence process within Europe from several different angles. The objective is to examine (a) whether there is evidence of economic convergence among the European countries and how it evolves (grows or declines) over time and (b) how the convergence process was affected by the global financial crisis of 2007–2008. The study covers a period that spans from 1999 with introduction of the common currency to 2019, just before the massive Severe Acute Respiratory Syndrome Coronavirus-2 (SARS-CoV-2) outbreak. The dataset is composed of thirty European sovereign states. In this context, the study aims to detect (a) how European economies evolve over time on a multivariate basis and (b) how they relate with each other. In other words, we examine whether the behavior of each of the European countries is related intertemporally and synchronized to the others. If this is the case, it might be an indication that structural disparities among the countries throughout Europe decrease over time, providing evidence that a convergence process is on track.

The remaining part of this paper is organized as follows. Section 2 includes the literature review. Section 3 contains the data description along with the proposed methodology. The results of the study are reported in Section 4. The paper concludes in Section 5.

2. Literature Review

There is a wide range of literature on the topic of business cycle synchronization from different aspects (Clarida et al. 1999; Crucini et al. 2011; Johnson 2014; Cesa-Bianchi et al. 2016; Di Giovanni et al. 2016; Spatafora and Sommer 2007; Stock and Watson 2005; Ambler et al. 2004; Antonakakis et al. 2016). Concerning Europe per se, De Haan et al. (2008) find that there are periods during which the business cycles within the Euro area either converge or diverge. Next to that, there is also some evidence showing that during the 1990s convergence increases. Giannone et al. (2009) find that the cross-correlations among the Eurozone countries' business cycles before the common currency introduction do not differ from those of the post-Euro era. Gayer (2007) examines the national cycles of the countries that adopted the common currency and finds a high-level cycle synchronization to be present after 1999. However, the level he came across is not higher than the one observed in the 1990–1996 period. On the contrary, Campos et al. (2017) argue that the synchronization of European countries' business cycles rose remarkably in the period following the common currency launch, especially within the Euro area. Gogas's (2013) research is conducted on a set of countries of the European Union and concludes that the business cycle synchronization amongst the Euro area countries is more intense in the period after 1999. Papadimitriou et al. (2016) provide evidence of an increased synchronization level among 22 European economies in the period following the common currency introduction. In De Grauwe and Ji's (2016) work, the business cycles correlations within the Euro area are found to be quite high. Franks et al. (2018) examine the synchronization among countries that adopted the Euro and find evidence for business cycles convergence. However, (a) after the 2008 crisis, their amplitude varies among countries and (b) so far there is no evidence for per capita income convergence, as a result of the common currency adoption, for the early EA members. Belke et al. (2016) carried out their study on the business cycle synchronization topic within the EMU area after the 2008 financial crisis. They conclude that core countries, namely Germany, France, Austria, Finland and the Netherlands, experience a higher level of synchronization among each other. In contrast, Portugal, Italy, Ireland, Greece and Spain experience a reduced synchronization, both among each other as well as in comparison to the core countries. As in the case of Giannone et al. (2009), the authors also identify different amplitudes with respect to the cycle's synchronization. Ferroni and Klaus (2015) study the business cycles of the four largest economies within the Euro Area, namely Germany, France, Italy and Spain. Their findings show that the cycle's fluctuations among the four countries were similar before the 2008 crisis. However, in the aftermath of the financial crisis, Spain's cycle does not go along with the Euro Area cycles. Stanisic (2013) focuses on the convergence pattern among specific Central and Eastern European countries and the Euro Area. His findings show that within the period following the EU accession, business cycle convergence among them is confirmed.

Our study is conducted utilizing Complex Networks metrics and tools. Other studies have also been carried out within the Complex Networks framework. Yet, their topic concerns mainly the evolution of business cycles, either international (Antonakakis et al. 2016; Caraiani 2013; Xi et al. 2014) or European ones (Papadimitriou et al. 2016; Matesanz Gomez et al. 2017; Matesanz and Ortega 2016). Besides GDP relations, trade relationships both in Europe and internationally are also examined within the same framework (Krings et al. 2014; Fagiolo et al. 2010; Kali and Reyes 2007; Maluck and Donner 2015).

Following the paths of Papadimitriou et al. (2016) and Antonakakis et al. (2016), we examine the convergence among European countries. The present research mainly differs in three aspects: (a) The method uses an extension of the Threshold–Minimum Dominating Set (T-MDS) algorithm used in Papadimitriou et al. (2016) and Antonakakis et al. (2016). The new version has already been employed by Papadimitriou et al. (2020) to examine price synchronization in the cryptocurrencies market over time. In the new model, a weight parameter is inserted within the T-MDS process to ensure that we can perform a consistent temporal analysis on networks of consecutive periods (the new approach is analyzed in the Data and Methodology section). (b) We use a large set of macroeconomic variables. The research in both Papadimitriou et al. (2016) and Antonakakis et al. (2016), as well as in the big majority of the literature on the synchronization topic, touches the business cycles synchronization subject only and does not perform any analysis on other macroeconomic variables. (c) The countries collection is larger than in the cases of Papadimitriou et al. (2016) and Antonakakis et al. (2016); the set counts 30 European countries.

Our goal is to carry out research that sheds light on as many aspects as possible of the synchronization concept. Thus, our effort does not focus solely on business cycle convergence, a topic that has already been heavily studied. Rather the opposite; we perform our study taking into consideration a collection of macroeconomic variables that we believe serve the purpose of our study. We consider that the present research will contribute to and complement the relative literature by investigating the convergence process topic from several points of view. We deem these different standpoints to be essential in order for the complete picture to be drawn.

3. Data and Methodology

The data for the study were obtained from the World Bank, the Eurostat and the IMF, and concern eight macroeconomic variables: the Gross Domestic Product in constant 2010 USD, the Harmonized Index of Consumer Prices, the Unemployment Rate, the External Balance on Goods and Services (% of GDP), the Imports of Goods and Services (% of GDP), the Exports of Goods and Services (% of GDP), the General Government Debt (% of GDP) and the Debt in absolute values[1]. The dataset covers thirty European countries (Table 1), on the eight variables, in annual frequency.

Table 1. The 30 European countries in the dataset, ranked by their GDP (constant 2010 USD) as of 2019 (descending order).

1	Germany	EA	16	Czech Republic	EU
2	France	EA	17	Portugal	EA
3	United Kingdom	EU until 2020	18	Romania	EU
4	Italy	EA	19	Hungary	EU
5	Spain	EA	20	Slovak Republic	EA
6	Netherlands	EA	21	Luxembourg	EA
7	Poland	EU	22	Croatia	EU
8	Sweden	EU	23	Bulgaria	EU
9	Belgium	EA	24	Slovenia	EA
10	Norway	EEAA	25	Lithuania	EA
11	Austria	EA	26	Latvia	EA
12	Ireland	EA	27	Cyprus	EA
13	Denmark	EU	28	Estonia	EA
14	Finland	EA	29	Iceland	EEAA
15	Greece	EA	30	Malta	EA

The research deals with the evolution of European convergence across time. Thus, we investigate the networks for three different consecutive time periods: (a) the first period concerns 1999–2004 and describes the first years of the common currency, (b) the second period covers the years of the recent financial crisis (2005–2010) and (c) the third period refers to the years after the crisis (2011–2019).

For each of the eight variables, three networks are constructed (one for every time period of the study). Each node corresponds to a country. The value on each edge describes the temporal similarity through the Pearson's correlation coefficient $r_{i,j}$, calculated on the time series of the variable under examination using:

$$r_{i,j} \triangleq \frac{COV(Z_i, Z_j)}{\sqrt{VAR\ (Z_i)\ VAR(Z_j)}} \tag{1}$$

where Z_i, Z_j are pairs of the observations of the variable under examination for the countries i and j. The correlation coefficient value ranges in $[-1, 1]$. Values close to the upper and the lower bound indicate a strong relationship; when close to -1, it is about a strong negative relation, while close to +1, it is about a strong positive relation. Values close to zero reflect no linear association between the countries i and j.

At this point, it is important to highlight some key-concepts of our approach.

Graph: A graph $G = (V, E)$ embodies a set of nodes V and a set of edges E, where the nodes are connected through the edges. In the present configuration, a parameter defined as "strength"[2] is applied to each edge $e_{i,j} \in E$ connecting the nodes i and j. This parameter measures the temporal similarity between the associated nodes. In addition, the collection of the nodes linked to node i, is defined as the neighborhood B_i of node i, i.e., $B_i = \{j \in V : \exists e_{ij} \in E\}$.

Isolated node: A node $i \in V$ which does not have any connection to the rest of the network, is called isolated.

Interconnected node: A node $i \in V$, for which there is at least one edge that connects it to another node in the network, is called interconnected.

The network of the current research in its initial setup is complete (there is an edge linking every couple of nodes).

Dominating Set: A Dominating Set $DS \subseteq V$ is a subset of the network interconnected nodes, which carries the following property: every node i that does not count as a DS member is connected to at least one DS node through an edge i.e., $\{\forall\ i\ \notin DS,\ \exists\ j\ \in DS\ : e_{ij}\ \in E\}$.

For each of the n nodes of the network, a binary parameter x_i, $i = 1, 2, \ldots\ n$ is considered to symbolize its DS membership status, such that:

$$x_i = \begin{cases} 0, & if\ i\ \notin DS \\ 1, & if\ i\ \in DS \end{cases} \tag{2}$$

The Formula (3) displays mathematically the DS concept:

$$x_i + \sum_{j \in B_i} x_j \geq 1,\ \ i = 1,\ 2,\ \ldots n \tag{3}$$

What can be clearly derived from (3) is that a node i has to be either a DS node or adjacent (i.e., connected) to at least one node of the DS. However, this is not a mutually exclusive relationship, as nodes verifying both situations may be present. On the contrary, it is not possible for an interconnected node to be neither a DS node nor linked to at least one such node.

Minimum Dominating Set: The Minimum Dominating Set (MDS) is the DS with the smallest cardinality.

$$\min\left(\sum_{i=1}^{n} x_i\right) \tag{4}$$

The identification of the MDS is formed through (4) under the constraints of (3).

Nevertheless, the MDS carries a built-in feature that renders itself not suitable as-is for the purpose of the present research. The network under examination is a correlation-based one. In such a network, the edges may represent both high and low similarity. Low correlation value edges denote low temporal similarity and thus should not be taken into consideration in the MDS calculation process. To get over this downside, the MDS algorithm is modified according to Papadimitriou et al. (2014). The modification involves an additional phase that precedes the MDS identification: the imposition of a threshold on the edges' strength (correlation values). This action aims to eliminate all edges with a value that is lower than the applied threshold level. The modification described leads to a network with two features: (a) it is not complete and (b) isolated nodes may be developed as a direct consequence of the removed edges.

Threshold–Minimum Dominating Set: The Threshold–Minimum Dominating Set (T–MDS) algorithm incorporates two steps.

Step 1: Imposition of a threshold level on the edges' "strength", resulting to the removal of the edges that depict a correlation lower than the applied threshold level.

Step 2: Identification of the MDS nodes in the remaining network.

Both the dominant nodes as well as the isolated ones compose the T–MDS. Each of the dominant nodes represents a neighborhood; each neighborhood is constituted of nodes interconnected to the dominant one, though a neighborhood's nodes may be also linked among each other and/or simultaneously be part of other neighborhoods as well. The nodes of a neighborhood exhibit similar behavior to the dominant node, since they are highly correlated. Contrariwise, none of the isolated nodes is linked to any other node. An isolated node demonstrates an individually separate (idiosyncratic) behavior and thus represents just itself.

Unfortunately, the optimization algorithm described above may have more than one solution and algorithmically we cannot control which case will be produced (Figure 1). This inconsistency renders the T-MDS method inadequate for temporal analysis.

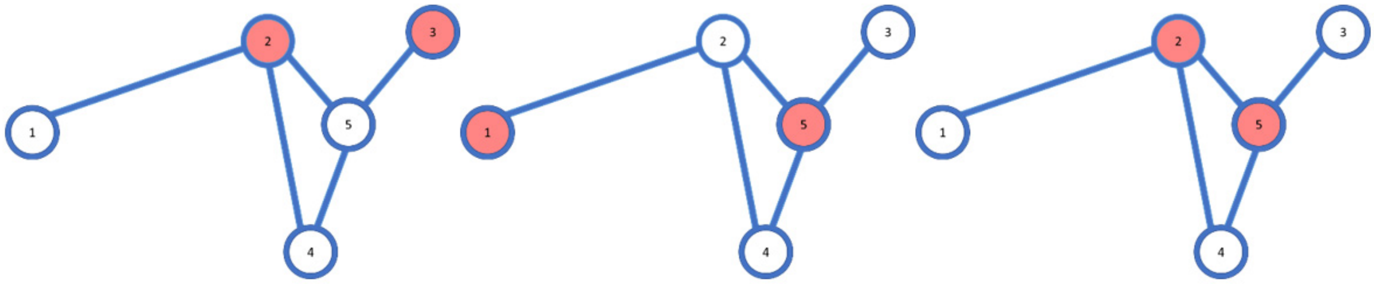

Figure 1. A five node network with three possible Dominating Sets.

In order for this drawback to be dealt with, a parameter was added within the T–MDS algorithm. The new form is the Threshold Weighted–Minimum Dominating Set (TW–MDS). In this configuration a weight w_i is attributed to every node in (4):

$$\min\left(\sum_{i=1}^{n} w_i x_i\right) \tag{5}$$

The TW–MDS identification is acquired through (5) subject to (3). Taking into consideration the node weights, this modification minimizes the probability that several solutions may be produced by the optimization algorithm. Sufficiency is now ensured for the temporal analysis of the network, due to consistency concerning MDS generation throughout the different time periods. Within the frame of the present research, the weight that is attributed to each of the countries is inversely proportionate to its GDP; the greater the GDP of a country, the more this country is boosted to be a dominant node.

Depending on the dominant set and how this varies across time, a critical piece of information may come to light. An expanding dominant set, over time, provides evidence of a diverging process among the behaviors of the network entities. A dominant set that expands is the consequence of a network that becomes less connected as time passes and thus more dominant nodes are required to describe it. Conversely, when the dominant set shrinks over time, there is a strong indication of convergence: in general, this is a denser network that can be described by fewer dominant nodes. The same is true for the set of isolated nodes, for similar reasons.

Beside the TW–MDS method, standard quantitative performance metrics were used in our analysis. The first such metric is the average node degree. In a finite undirected graph G, the sum of all node degrees is equal to twice the number of the edges:

$$\sum_{i=1}^{n} k_i = 2\,|E| \tag{6}$$

where k_i indicates the degree of node i (i.e., the number of the edges incident to node i) and E the set of the edges connecting a set of nodes V. The degree of a node k_i counts the interconnectedness of a node i within the graph G. This result (6) was proved by Euler (1741) and is known in Graph Theory as the *handshaking lemma*.

The average node degree k_{avg} is given by the formula:

$$K_{avg} = \frac{1}{n}\,2\,|E| \tag{7}$$

where n denotes the number of the graph's nodes. The higher the average node degree is, the more interconnected among each other the nodes are.

The density d_G is the second metric employed. It demonstrates how connected a network is and is given by the formula:

$$d_G = \frac{2\,|E|}{\mathrm{v}(\mathrm{v}-1)} \tag{8}$$

where E is the set of the edges connecting a set of nodes V, and v is the number of the network nodes. A density value of 1 describes a complete network, whereas a value of 0 points to a disconnected network where each node is isolated.

4. Empirical Results

This section contains the descriptive statistical metrics with respect to the eight macroeconomic variables examined for the purpose of the present study (Table 2). We can see that there are asymmetries and a wide dispersion of these variables among the countries we studied. Also, most of these variables do not seem to follow a normal distribution as is evidenced from the skewness and kurtosis statistics that deviate significantly from the zero and three values of the normal distribution. Further, the empirical evidence on the convergence process among the European countries is presented and discussed.

Table 2. The descriptive statistical analysis of the eight macroeconomic variables examined.

	Mean	Median	Min	Max	St. Dev.	Q1	Q3	Skewness	Kurtosis
GDP (USD, Billions)	579.59	228.28	6.69	3959.4	889.79	49.23	487.60	2.02	3.02
HICP	88.30	91.47	19.23	111.50	14.06	79.18	100.00	−1.03	1.23
Unemployment (% of total labour force)	8.33	7.26	1.80	27.47	4.38	5.25	10.25	1.40	2.31
External Balance (% of GDP)	1.89	1.10	−20.69	36.01	8.53	−2.21	5.48	1.20	3.43
Imports (% of GDP)	55.68	46.90	21.42	187.17	29.20	35.56	68.90	1.87	4.17
Exports (% of GDP)	57.57	47.30	18.54	221.20	34.24	35.48	70.07	2.11	5.47
Debt (% of GDP)	56.97	50.39	3.77	184.76	32.89	35.30	73.83	1.05	1.51
Debt (USD, Billions)	422.72	107.43	0.71	2916.04	737.86	17.16	359.37	2.18	3.47

For each of the time intervals, a network is built on every single variable; the nodes of the network represent the countries under examination, while the values assigned to the edges represent the temporal similarity among them, as measured by the Pearson's correlation coefficient $r_{i,j}$. The TW–MDS algorithm is applied on all three networks for a 0.8 threshold level[3].

I. Gross Domestic Product (constant 2010 USD)

The first variable on which the network is examined is the Gross Domestic Product in constant 2010 USD. Table 3 displays the average node degree[4], the density[5], the number of the isolated and the interconnected nodes, and the number of the dominant nodes.

Table 3. Network metrics for 0.8 threshold level on the GDP (constant 2010 US$) variable.

Metrics	1999–2004	2005–2010	2011–2019
Density	0.97	0.50	0.82
Avg Degree	28	14.4	23.87
Isolated Nodes	0	0	1
Interconnected Nodes	30	30	29
Dominant Nodes	1	2	2

The results in Table 3, show that within the 1999–2004 period the network exhibits the highest density and average node degree values. The metrics decline during the 2005–2010 period and increase again in the last period. No isolated nodes are observed, except for the last period, where just one node, Greece, is isolated from the network. Greece, during this period, dealt with the sovereign debt crisis and this result is somewhat expected. The network collective behavior can be described by one country in the 1999–2004 period and by two countries in both the succeeding periods.

It is clear that the countries forming the network started with a high degree of connectedness in the first period that significantly declined during the financial crisis, and it bounced back over the last period. It is worth noting that although the connectedness in the 2005–2010 interval is the lowest observed (density = 0.5), no isolated countries are detected in the network. This is an interesting finding; even in the period of the lowest synchronization level, the ties among the countries are preserved in such a way that no country is left isolated, attesting the resilience of the network as a whole. Yet, the decreasing density provides evidence of disintegration of the network. The crisis affected different groups of countries in diverse ways. This is not expected. During an external crisis, we would expect that all countries would exhibit a similar pattern: decreased GDP, increased unemployment, etc. Thus, the convergence (similarity) between them would not decrease; it may be even expected to increase as all countries are impacted negatively by the same external shock. The evidence from the TW–MDS uncovers the inherent structural differences that are present in these European countries that lead to the divergent behavior manifested by the disintegration of the network in the period of the crisis. From 2011 onwards, the convergence process is back on track. However, it fails to reach pre-crisis levels.

II. Harmonized Index of Consumer Prices (HICP)

The empirical results on the Harmonized Index of Consumer Prices are displayed in Table 4.

Table 4. Network metrics for 0.8 threshold level on the HICP variable.

Metrics	1999–2004	2005–2010	2011–2019
Density	0.98	0.97	0.78
Avg Degree	28.27	28.13	22.67
Isolated Nodes	0	0	3
Interconnected Nodes	30	30	27
Dominant Nodes	1	1	1

Table 4 describes a stable status of high similarity (dense network, one dominant node) for the first two periods and although the network metrics remain at a high level within the last time-interval, a decrease is true, compared to those of the two preceding periods.

The networks in both the 1999–2004 and 2005–2010 periods are highly interconnected. There are no isolated countries, while the presence of just one dominant node in each period denotes a unique behavior for all of their elements. Contrasting the two preceding periods, three isolated countries (Greece, Cyprus and Bulgaria) show up within the 2011–2019 period. Yet, the structure of the remaining interconnected part of the network is such that the network behavior can be described by just one country.

The 2007–2008 crisis does not seem to affect the convergence with respect to the HICP. The countries show a high level of similarity before and during the financial crisis. Rather, the consequences of the financial crisis are imprinted within the succeeding 2011–2019 period. We have to note here that two of the isolated countries, Greece and Cyprus, were dealing with a significant sovereign debt and banking crisis, respectively, during this period that may explain their distinct behavior. Nonetheless, although three countries are identified to be isolated, the interconnected part of the network seems to maintain a considerable level of connectedness (as shown by the one dominant node).

III. Unemployment

The empirical results with respect to the Unemployment variable are displayed in Table 5.

Table 5. Network metrics for 0.8 threshold level on the Unemployment variable.

Metrics	1999–2004	2005–2010	2011–2019
Density	0.14	0.32	0.46
Avg Degree	4.13	9.33	13.2
Isolated Nodes	3	1	2
Interconnected Nodes	27	29	28
Dominant Nodes	7	4	2

The network metrics displayed in Table 5 indicate an increasing convergence over time. The density and the average degree values within the 2005–2010 period are much greater than the ones of the 1999–2004 period, while in the third interval the metrics values are boosted further along. Isolated countries are present in all three time periods, the first period being the one containing the most. With regard to the dominant countries, there is a clear pattern; moving forward from one period to another, the number of the dominant nodes declines, indicating once more the convergence in the network. A large Dominant Set points to a sparse network that requires more countries to describe its behavior. The opposite holds for a small DS, where just a few countries are sufficient to describe its behavior, an explicit indication of high similarity. It seems that the vision and plan for the creation of a single labor market, a prerequisite for an OCA within Europe, is taking shape and produces results.

IV. External Balance on Goods and Services (% of GDP)

The empirical results on the External Balance on Goods and Services (% of GDP) variable are presented in Table 6.

Table 6. Network metrics for 0.8 threshold level on the External Balance on Goods and Services (% of GDP) variable.

Metrics	1999–2004	2005–2010	2011–2019
Density	0.05	0.14	0.11
Avg Degree	1.6	4.2	3.13
Isolated Nodes	7	3	13
Interconnected Nodes	23	27	17
Dominant Nodes	8	7	3

The values of density and average degree in Table 6 are the lowest observed, in comparison to all variables on which the network is examined. The highest metrics values of Table 6 are observed within the 2005–2010 period. However, the upward movement from the first to the second period fails to reach a truly high spot in the values scale and remains situated at a low level. Subsequently, the metrics values fall back again to a lower level (slightly higher than the ones of the 1999–2004 period) in the 2011–2019 period. In addition, the number of the isolated nodes is high, especially in the last period (almost half of the network is isolated), meaning that each of the isolated countries displays a distinct behavior. The results of Table 6 are clear; they describe a sparse network, that even in the case of the 2005–2010 period (where the metrics reach the highest values), fails to develop dense ties among its nodes. The network constructed on the External Balance variable is the most fragmented compared to the networks based on all other variables. Not only does it hold a considerable number of isolated nodes in every time period, but also its interconnected part includes many dominant nodes. According to these results, it is evident that European countries started in the first period with a great dissimilarity between them and this pattern did not change significantly over time. We observe no synchronization; the network remains widely fragmented. This is a direct conclusion derived from the extensive network fragmentation taking place in all three time periods, a phenomenon that seems to worsen after the 2007–2008 crisis.

V. Imports and Exports of Goods and Services (% of GDP)

The results displayed in Table 7 refer to the network based on the Imports of Goods and Services (% of GDP) variable.

Table 7. Network metrics for 0.8 threshold level on the Imports of Goods and Services (% of GDP) variable.

Metrics	1999–2004	2005–2010	2011–2019
Density	0.16	0.36	0.16
Avg Degree	4.6	10.33	4.6
Isolated Nodes	3	2	5
Interconnected Nodes	27	28	25
Dominant Nodes	6	4	5

The metrics show that the denser network (i.e., the one displaying the most similar behavior among its nodes) is that during the financial crisis: maximum avg degree, fewest isolated nodes, fewest dominant nodes. The metrics before and after the crisis period are quite similar. A straight inference is that the countries before and after the crisis follow different paths in what concerns the volume of their imports. However, during the 2005–2010 period, their behavior carries a few common characteristics and reflects the common reduction and subsequently the common increase in their imports volume as a result of the global financial crisis. Nonetheless, the denser network in this period may not be interpreted as evidence of synchronization in the sense of a common market. It merely reflects the general contraction of GDP and thus of imports internationally as a result of the crisis.

The results on the Exports of Goods and Services (% of GDP) variable are displayed in Table 8.

Table 8. Network metrics for 0.8 threshold level on the Exports of Goods and Services (% of GDP) variable.

Metrics	1999–2004	2005–2010	2011–2019
Density	0.19	0.28	0.21
Avg Degree	5.6	8.2	6.07
Isolated Nodes	3	1	4
Interconnected Nodes	27	29	26
Dominant Nodes	5	5	6

The network metrics of Table 8 are similar to the ones obtained from the Imports variable. The network is denser in the period of the crisis (maximum density, maximum avg degree, fewer isolated nodes). Before and after the crisis, the results are similar. The results provide evidence similar to the Imports variable.

VI. General Government Debt (% of GDP)

The results with respect to the Debt-to-GDP ratio are presented in Table 9.

Table 9. Network metrics for 0.8 threshold level on the General Government Debt (% of GDP) variable.

Metrics	1999–2004	2005–2010	2011–2019
Density	0.15	0.61	0.18
Avg Degree	4.4	17.6	5.2
Isolated Nodes	1	1	3
Interconnected Nodes	29	29	27
Dominant Nodes	7	3	6

Focusing on the results of Table 9, a significant increase in both the density and the average degree values is observed from the first to the second period. This increase indicates a quite denser network. Within the 1999–2004 period, the countries follow rather distinct courses. This pattern is reversed within the 2005–2010 period, where it seems that the crisis acts as a catalyst for the countries' behavior and pushes towards convergence; almost all Debt-to-GDP ratios follow an upward movement from 2007 onwards. However, these co-movements do not continue in the third period as well, within which the countries return to their distinct courses. The metrics now follow the opposite direction and fall significantly to a level that is slightly higher than the one of the 1999–2004 period. The Dominant Set magnitude points to the same pattern; during the crisis period (2005–2010), three dominant nodes, in contrast to seven (1999–2004) and six (2011–2019), are enough to describe the behavior of the twenty-nine interconnected nodes.

VII. Debt (absolute values)

The results concerning the Debt in absolute values can be found in Table 10.

Table 10. Network metrics for 0.8 threshold level on the Debt (absolute values) variable.

Metrics	1999–2004	2005–2010	2011–2019
Density	0.29	0.61	0.31
Avg Degree	8.3	17.8	8.9
Isolated Nodes	2	2	2
Interconnected Nodes	28	28	28
Dominant Nodes	5	2	4

The pattern in Table 10 is similar to the one in Table 9 (Debt-to-GDP ratio). The network metrics follow an upward course from the first to the second period, and then, as in the case of the Debt-to-GDP ratio variable, the metrics decline and end up at a level that is close to that of the first period. As in the preceding case, the DS magnitude shrinks remarkably, moving from the 1999–2004 period to the 2005–2010 one; two dominant nodes opposed to five (1999–2004) and four (2010–2019), are sufficient to describe the behavior of the network, confirming the relatively common behavior which is captured by the density and average degree.

During the 2005–2010 period, the Debt in absolute values increases for the majority of the countries after 2007. Although this fact does not take place to the same degree for all countries, it is indicative of—in a way—synchronized behaviors, implying that the financial turmoil was a driver of the synchronized debts in the specific period, within the frame of the coordinated joint government efforts to support the economy. Though, this synchronization has a negative shade, as it concerns a rise in the countries debts. During the last period (2011–2019), what is noticed is a divergence of the courses the countries move on, denoting that the crisis acted for a limited period of time in favor of the convergence. In the years following, the governments did not manage to restrain the debt expansion phenomenon in a universal way, so as for the convergence to move further on. This is reflected in the study's results that show the obvious; some countries applied a stricter practice than others did.

5. Conclusions

The current study deals with the convergence process of a group of European countries, using a set of tools from Graph Theory. The findings, as expected, are mixed: no constant pattern was observed in the macroeconomic variables studied. In the case of the unemployment variable, the synchronization level displays a continuous rise through time, providing evidence of European convergence. In others—such as the GDP, HICP, External Balance or the Debt-to-GDP ratio—more complex patterns are identified.

Most of the macroeconomic variables studied seem to be affected by the financial crisis in the 2005–2010 period. This is true for the GDP, the External Balance, the Imports and Exports, the General Government Debt as a percentage of the GDP and the Debt in absolute values. In the case of the GDP, during the crisis period, the density of the network decreased, revealing a diverging pattern for the EU countries. However, from 2011 and onwards, we observe a return close to the pre-crisis levels. The opposite pattern can be observed for the External Balance, the Imports and Exports, the General Government Debt as a percentage of the GDP and the Debt in absolute values. There are signs of convergence during the crisis that are reversed in the period after that (it is somewhat expected that during the crisis a negative impact to all the variables would take place which in turn would lead the countries to converge). A rough analysis of these findings is that each country faced the crisis in a different way, tracing different paths. These paths slowed the convergence of GDP evolution, one of the main targets of EU strategic policies. The same can be seen by studying the number of the dominant nodes: during the crisis the number of the dominant nodes is the smallest observed, revealing less fragmented networks.

The case of the unemployment variable is unique since it is the only variable where the convergence process follows a stable upward angle. The results reveal a trend of an increased convergence over time, which strengthens further from the crisis onwards.

Overall, the results highlight two important facts: First that, in general, the GDP-based convergence is on track and strong in times of normal economic activity. Second, that when there is an economic crisis, this convergence pattern is in general reversed. The European countries behave as fragmented idiosyncratic groups. It is possible that this behavior highlights (a) the inherent deep differences of these economies[6] that need more time—and possibly effort—to truly converge in all dimensions and under any external shocks and (b) that the common market and convergence process within Europe was well-designed to function in times of normality, yet without taking into account periods of severe distress such as the ones we have observed recently: the global financial crisis, the sovereign debt crises of Greece, Portugal, Italy and Spain, the banking crisis in Cyprus, the COVID-19 pandemic and the recent energy crisis driven by the Russian invasion in Ukraine.

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Notes

1 The Debt in absolute values was calculated by multiplying the Debt-to-GDP ratio by the GDP (constant US$) values.

2 In Graph Theory, this measurable factor is usually termed "weight".The "strength" term is used instead, in order to avoid any confusion with the weights that are used in the TW–MDS methodology.

[3] The 0.7 and 0.9 threshold levels were also tested. However, the specific results were omitted from our analysis. The 0.7 level was too low and the 0.9 level was too high to give consistent results. There is a trade-off we must consider when choosing the threshold level; a low level allows edges corresponding to low similarity to survive; a high threshold may eliminate important edges from the network.

[4] For the average node degree calculation, all nodes are taken into consideration. That being said, the average node degree is not calculated just on the interconnected part of the network, rather on the total network which may contain isolated nodes as well. The metric is treated this way, so as for a comparison among different time intervals and/or variables to be more direct and efficient.

[5] In order for the density to be calculated, all network edges that survive after the threshold imposition step are taken into consideration, irrespective of whether it is about a *connected* or *disconnected* network. The metric is treated this way, so as for a comparison among different time intervals and/or variables to be more direct and efficient.

[6] Kaufmann et al. (1999) conclude that there is a robust *"causal link from better governance to better development results"*. In the same line, Edison (2003), Rodrik et al. (2004) and Acemoglu (2009) provide evidence of the significance of the institutions regarding the different economic performance observed across a series of countries. Thus, established practices and especially the effective implementation of policies within the context of governance are critical for the economic development a country experiences. In their paper, Anagnostou et al. (2016) study the convergence among the EU countries within the frame of governance across the 1996–2012 period. They find that while a tendency towards convergence exists, noteworthy divisions among the states also take place.

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