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A Contract Theory Approach to Islamic Financial Securities with an Application to Diminishing Mushārakah

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Abstract: This paper demonstrates how the contract theory framework can and should complement standard financial mathematics for analysing Islamic financial securities (IFSs). It is motivated by the perception that most valuations of IFSs are rather simplistic and are as simple as risk and reward, leading to very simplistic investment strategies, especially by buyers. In fact, there are more dimensions to IFSs and IF in general which can only be properly analysed with more advanced approaches, such as contractual issues which are well-recognised and discussed in the fields of Islamic commercial law and contract theory but not always considered in valuation models. Contract theory can bring together financial mathematics and contractual issues, providing a more sophisticated framework for analysing IFSs. This paper aims to demonstrate this by providing a brief outline of the contract theory approach, followed by a simple demonstration of its use in the analysis of diminishing mushārakah (DM) contracts. The resulting model led to three main conclusions regarding DM contracts: That (i) finance seekers have no ready incentive to spend on asset maintenance, (ii) finance seekers will only spend on asset maintenance if their marginal benefit from the asset's appreciation is greater than the financier's share of the asset, and (iii) if the magnitude of asset appreciation and depreciation is equal, an increase in either will also increase the optimal level of spending on asset maintenance.

Keywords: Islamic finance; diminishing mushārakah; moral hazard; adverse selection; contract theory; investment; valuation



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1. Introduction

Among the difficulties in marketing Islamic financial securities (IFSs) is the simplistic understanding of prospective buyers and sellers with regards to Islamic finance (IF). This leads to simplistic financial strategies such as just comparing rates or going with the most “Islamic” financial providers as illustrated in Berg et al. (2016). This is despite claims that IFSs can be more robust, as discussed by Jobst (2009), or equitable, as discussed by Usmani (1998). Such approaches to IF seems to be accompanied by a lack of rigorous analytical papers on these claims, perhaps reflecting a lack of understanding and appreciation regarding the features of IFSs as well as their associated tradeoffs. In some cases, there might not in fact be any differences with more readily recognised securities. For example, El-Gamal (2009) essentially—and perhaps rightly—argues that the exotically named ‘al-ijārah al-muntahiyah bit-tamlīk (IMBT) or literally “lease ending in transfer of possession” might as well be recognised and treated as a financial lease.

This author hopes to reinvigorate the analysis of IFSs in light of major advances in analytical frameworks with this paper. In particular, this author is interested in the application of contract theory in the manner of Gale and Hellwig (1985) to initiate a further exploration of IFSs. The specific case used is a simple one of asset maintenance in the mushārakah mutanāqīshah or diminishing mushārakah (DM) contract. As an example of how significant this approach is, consider that a simple debt contract (SDC) under basic financial mathematics is simply viewed as:

$$FV = PV(1 + r)$$

In light of the contract theory framework however, [Gale and Hellwig \(1985\)](#) are able to perform a more thorough exploration, revealing further considerations for and against SDCs; merits of SDCs include their incentive-induced robustness towards various sources of moral hazard whereas their short-comings include the tendency for underinvestment or credit-rationing. It is hoped that applying a similar approach to IFSs will reveal similar insights, facilitating a better appreciation and more effective use of them.

Modelling a DM contract using contract theory begins with recognising that it is effectively a co-ownership agreement in which the party seeking finance intends to buy all of the financier's shares in the underlying asset. While such a security can be modelled using a simple amortisation schedule as critiqued by [Asadov et al. \(2018\)](#), it would clearly be a gross simplification. It firstly ignores a key property of the co-ownership arrangement, namely equitability in that risk and reward are both in proportion to a given party's level of ownership. Secondly, it does not take into account the interests of the contracting parties; while not directly relevant to the valuation aspect, understanding the interests of each party can help identify moral hazard scenarios which need to be addressed. As shall be demonstrated, the contract theory approach can yield insights on what constitutes effective use of the DM contract. This includes marketing DM contracts specifically to finance seekers who can productively use the underlying asset as it aligns the interests of both finance seekers and providers, resulting in a robust contractual relationship.

This paper is organised as follows: the next section will contain a literature review on the DM contract, the contract theory framework, and also applications of the latter to IFSs; Section 3 will contain the description of the DM environment; Section 4 will contain the actual analysis of the model and extend it to accommodate economic screening by the financier; Section 5 will contain possible ways to extend the base model, providing directions for future work; Section 6 will contain the paper's summary and conclusions.

2. Literature Review

We begin with a clarification of the key concepts of this paper, namely the DM contract and the field of contract theory.

2.1. Diminishing Mushārah (DM)

DM is based on the mushārah contract, which is basically a contract of partnership encompassing both asset co-ownership and business partnerships. [Usmani \(1998\)](#) provides a brief survey of the various legal opinions concerning the standard mushārah agreement and [Al-Zuhayli \(2007\)](#) provides a more extensive survey treatment. The "diminishing" part of DM comes from the idea that the standard mushārah contract can be used to facilitate financing; a seeker of finance and a financier agree to purchase and co-own the asset with the expectation that the party seeking finance will buyout the financier's share of the underlying asset. One use for this arrangement in practice is for home financing such as with [HSBC Amanah's \(2020\)](#) HomeSmart-i product.

This author has struggled to find papers analytically scrutinising the economic merits of DM. Some that were found are [Bashir et al. \(1993\)](#) on the optimal level of equity to invest into a DM arrangement and several papers on how to implement a similar arrangement into mortgages ([Ebrahim 1996](#); [Ebrahim et al. 2011](#); [Wojakowski et al. 2016](#)). Several other papers explicitly invoke contract theory so they shall be discussed in the next literature review section.

2.2. Contract Theory

[Bolton and Dewatripont \(2004\)](#) explain that contract theory is discussed in the context of information economics, where information asymmetry can significantly affect the parties involved. Two key scenarios are hidden information/adverse selection and hidden action/moral hazard, the first being when information asymmetry occurs before a

transaction and the second being when it occurs after. Contract theory also discusses solutions for these problems, with signalling and/or screening being the main solution for hidden information/adverse selection and incentives being the main solution for hidden action/moral hazard. These solutions can be considered examples of contract design, or how to design an economic arrangement to minimise the effects of information-related problems. The application of contract design is usually the core of the contract theory approach and involves three basic concepts, namely the revelation principle, incentive compatibility, and that there is a basis for all parties to participate in the arrangement. We shall apply this approach to the DM contract in Section 5. A more in-depth exploration of contract theory is provided by Bolton and Dewatripont (2004) whereas its more generalised form, mechanism design, is explored for example in Mas-Colell et al. (1995) and Börgers (2015).

While this author has not found any analytical work applying contract theory to DM contracts, there is a decent amount of work in more general Islamic finance contexts. Examples include Aggarwal and Yousef (2000), Basov and Bhatti (2013), and Azmat et al. (2015). More specialised uses of contract theory include Khan (2015, 2019) and Puspita et al. (2020) who apply the framework for analysing different parts of the Islamic insurance industry.

3. The Model

We assume that the relevant events occur in two periods, an initial period $t = 0$ and the end period $t = T$. In the initial period $t = 0$, a risk-neutral individual seeks DM financing from a risk-neutral financier for an asset with an initial price P_0 . Both parties agree on their initial contributions to purchasing the asset with the individual paying K_{S0} and the rest paid by the financier. Both parties are assumed to have the capital required. The asset is then co-owned by both parties with the individual having a $\frac{K_{S0}}{P_0}$ share of the asset and the rest owned by the financier. In practice, the asset is then leased to the individual for some agreed periodic payment but this cashflow component is not considered for now as it is not important in the two period case.

The main events of the end period are the appreciation or depreciation of the asset as well as the individual's purchase of the financier's share of the asset. It is assumed that the asset's value increases by $\Delta P = A$ with probability $p(c)$ and decreases with the same value of ΔP but with probability $1 - p(c)$, where c represents the asset maintenance cost. This cost is assumed to be borne by the individual as the (constructive) owner of said asset. It is also assumed that the probability of asset value appreciation $p(c)$ is increasing in c at a decreasing rate and $p(0) = 0$. Therefore, the expected change in the asset's value and the expected value of the asset respectively at time T are therefore:

$$E[\Delta P] = p(c)A - (1 - p(c))A$$

$$E[\Delta P] = (2p(c) - 1)A \tag{1}$$

$$E[P_T] = P_0 + (2p(c) - 1)A \tag{2}$$

The net benefit to both parties respectively can therefore be expressed as:

$$u_S = x_T - c - K_{ST} - K_{S0} \tag{3}$$

$$u_f = K_{ST} - (P_0 - K_{S0}) \tag{4}$$

with u_S and u_f referring to the individual's (the finance seeker) and financier's utility functions respectively. K_{ST} is the value of the financier's share of the asset at time T :

$$K_{ST} = \left(1 - \frac{K_{S0}}{P_0}\right)P_T \tag{5}$$

whereas its expected value can be expressed as:

$$E[K_{ST}] = \left(1 - \frac{K_{S0}}{P_0}\right)(P_0 + [2p(c) - 1]A) \tag{6}$$

The individual expects to spend on maintenance and purchase the financier’s share in the asset from accumulated income amounting to x_T at the end period. However, the only one of these observable to the financier is the realisation of P_T which we assume is equal to its expected value. The basic model is then completed by deriving expected rates of return for the seeker r_S and financier r_f respectively:

$$E[r_S] = \frac{x_T - c - E[K_{ST}] - K_{S0}}{K_{S0}} \tag{7}$$

$$E[r_S] = \frac{x_T - c - \left(1 - \frac{K_{S0}}{P_0}\right)(P_0 + [2p(c) - 1]A) - K_{S0}}{K_{S0}}$$

$$E[r_f] = \frac{E[K_{ST}] - K_{f0}}{K_{f0}} \tag{8}$$

$$E[r_f] = [2p(c) - 1] \frac{A}{P_0}$$

Naturally, this setup implies several potential information asymmetry problems. The two main ones being whether or not the individual allocates any resources to asset maintenance and whether or not the individual purchases the remaining shares in the asset. We focus on the first of the two and find that the second can actually be related to the first; the individual is committed to buying out the financier but purposely neglects asset maintenance to lower the final price of the asset. To keep the model simple, we also assume that there is no moral hazard on the financier’s part i.e., that the financier is committed to this transaction.

4. Analysis

4.1. Addressing the Moral Hazard

The individual will only accept the contract if their net benefit is greater than their outside option, which we normalise to zero. This is referred to as the participation constraint in contract theory and is expressed in this case as:

$$u_S = x_T - c - K_{ST} - K_{S0} \geq 0 \tag{9}$$

From here, the individual chooses the level of maintenance expenditure c which maximises their expected utility. This is obtained by first substituting $E[K_{ST}]$ in Equation (6) into the individual’s participation constraint in Equation (9), taking the first derivative of the resulting equation with respect to c , and equating the result to zero. The utility maximising level of c is therefore characterised by the following:

$$p'(c) = -\frac{1}{2\left(1 - \frac{K_{S0}}{P_0}\right)A} \tag{10}$$

or in words, when the first derivative of $p(c)$ is equal to some negative constant. In our model, this is an impossibility as the first derivative of $p(c)$ is assumed to always be positive. The interpretation of this result can be seen in the agent’s participation constraint in Equation (9), namely that a rise in the asset price only increases the individual’s cost burden and so they have a very strong incentive to drive down the asset price neglecting maintenance.

Ideally, the financier has perfect monitoring ability and can compel the individual to commit as many resources as possible to asset maintenance, ensuring that the asset

appreciates with 100% probability. Unfortunately this can be very difficult to do and might not be worth the cost. The go-to contract theory solution for problems such as this then, is incentive alignment, namely for the financier to offer sufficient incentives to the individual. These incentives should have a positive relationship with the interests of the financier, which in this case is capital appreciation from the asset.

One of the benefits of a musharakah contract is that the underlying asset usually does provide the individual with some intrinsic benefit. Possibilities include a desire from the individual to also benefit from capital appreciation of the asset or to utilise the asset to generate profits. The financier can take advantage of the second in a multiperiod setting by leasing the asset to the individual and settle for a lower asset repurchase price. However, in this two period model we assume for brevity that both parties seek to benefit from capital appreciation of the asset. The contract theory framework encourages us to identify points like these, allowing us to gain a deeper understanding of the object of analysis.

Suppose then that the individual’s income depends on the quality of the underlying asset which we assume is perfectly represented by the price of the asset at a given time. Let this be represented by the individual’s income increasing with the price of the asset at a decreasing rate such that the individual’s participation constraint can be expressed as:

$$x_T = \theta P_T \tag{11}$$

$$u_S = x_T(P_T) - c - K_{ST} - K_{S0} \geq 0 \tag{12}$$

and therefore each party’s expected rate of return becomes:

$$E[r_S] = \frac{(\theta - 1 + \frac{K_{S0}}{P_0})(P_0 + [2p(c) - 1]A) - c - K_{S0}}{K_{S0}} \tag{13}$$

$$E[r_f] = [2p(c) - 1] \frac{A}{P_0} \tag{14}$$

Similar to before, we find the level of c chosen by the individual to maximise their expected utility by firstly substituting in $E[K_{ST}]$ from Equation (6) and $E[x_T]$ based on Equation (11) into the new participation constraint, Equation (12). We then take the first derivative of the resulting equation with respect to c and then equate it to zero. This yields:

$$p'(c) = \frac{1}{2(\theta - 1 + \frac{K_{S0}}{P_0})A} \tag{15}$$

which is positive as long as $\theta > 1 - \frac{K_{S0}}{P_0}$. Interestingly, this expression also implies that if the potential asset price change A becomes larger, the more the individual will spend on asset maintenance. This is a sensible result as even though the model indicates that the magnitude of the asset price change is exogenous, the direction of the change is solely determined by the individual’s maintenance expenditure. Therefore, if the individual recognises that there is a sufficiently large benefit from an asset price change, they will be more motivated to ensure that it becomes a benefit to them instead of a loss.

4.2. Discussion of Results

The purpose of the model is to demonstrate simply how important contract theory is to IFSs, especially considering that there will always be an emphasis on their contractual nature. For example, there are many ways to effect a debt security cash flow such as through murābahah (cost plus sale) and ‘ijārah contracts. However, each contract has different legal and therefore financial properties which can be made more pronounced when viewed from the perspective of contract theory.

Applying contract theory to a simplified version of the DM contract allows us to go beyond the standard amortisation schedule lamented by Asadov et al. (2018). From Equation (15), we come to our first result which might not immediately clear.

Proposition 1. *The individual has no ready incentive to spend on asset maintenance.*

This somewhat mirrors a standard contract theory result common to principal-agent settings; as presented for example in Mas-Colell et al. (1995), agents prone to moral hazard should not have incentives which are constant relative to their task. Put simply it makes sense for people to work harder if they are rewarded more for doing so. This leads us to our next result.

Proposition 2. *An individual will only spend a non-zero amount on asset maintenance if (i) they also benefit from the asset's appreciation and (ii) the marginal benefit of asset appreciation θ is greater than the financier's share of the asset.*

In other words the agent is only motivated to maintain the asset if they expect that their financial benefit from the DM contract offsets their financial cost. This can therefore be used as a signal of sorts by prospective DM financiers for choosing which finance seekers to partner with—we shall discuss this more in the next section.

The final main result we wish to present has already been elaborated upon above.

Proposition 3. *Given an equal magnitude of asset appreciation and depreciation A , an increase in A will increase the optimal level of spending on asset maintenance c^* .*

Summarising the implications of the model so far, it is not a good idea for financiers to make DM contracts with just anyone as there are inherent moral hazards. While Islamic commercial law (ICL) has various legal devices for anticipating such problems as discussed by Usmani (1998), Al-Zuhayli (2007), and Jobst (2009), enforcing them can be excessively costly. From a contract theory perspective, it is therefore more prudent to understand the incentive structure of the simple DM contract and only offer it in circumstances when it is robust against inherent moral hazards. An example of such a circumstance is when the prospective seekers of finance can make good enough use out of the underlying asset. We take this idea a step further by considering a two-type economic screening approach; the financier offers one DM contract for finance seekers who are highly productive with the asset they wish to finance and another for those who are not so productive. This allows seekers of finance to self-select and reveal how effectively they can use the asset they wish to finance, therefore also signalling to the financier how prone they are to the moral hazards discussed.

4.3. Screening Individuals

Another layer can be added to the above analysis by considering a financier who wishes to screen prospective individuals on the basis of their ability to benefit from the underlying asset. This is particularly attractive to the financier because the previous section indicates that individuals who can better benefit from the underlying asset facilitate better capital appreciation. This approach is called screening and we provide a simple two-type example based on Bolton and Dewatripont (2004). Basov (2013) provides a treatment of screening for continuous types. The general two-type screening problem involves a proportion β of individuals with a significantly higher marginal benefit θ_H of asset price appreciation than others such that $\theta_H > \theta_L$. The financier must therefore offer two different contracts such that the “high-type” individuals will always choose one whereas the “low-type” individuals will always choose the other. The financier's utility function then becomes:

$$\max_{K_{L0}, K_{H0}} u_F = \beta(K_{H0} + E[K_T(c_H)]) + (1 - \beta)(K_{L0} + E[K_T(c_L)]) - P_0 \quad (16)$$

where K_{H0} and K_{L0} are the initial contributions of the high and low-type finance seekers while $K_T(c_H)$ and $K_T(c_L)$ are the repurchase values of the high and low-type finance seekers respectively. The appropriate participation constraints are:

$$\theta_H E[P_T(c_H)] - c_H - E[K_T(c_H)] - K_{H0} \geq 0 \tag{17}$$

$$\theta_L E[P_T(c_L)] - c_L - E[K_T(c_L)] - K_{L0} \geq 0 \tag{18}$$

and the following incentive compatibility constraints are:

$$\theta_H E[P_T(c_H)] - c_H - E[K_T(c_H)] - K_{H0} \geq \theta_H E[P_T(c_L)] - c_L - E[K_T(c_L)] - K_{L0} \tag{19}$$

$$\theta_L E[P_T(c_L)] - c_L - E[K_T(c_L)] - K_{L0} \geq \theta_L E[P_T(c_H)] - c_H - E[K_T(c_H)] - K_{H0} \tag{20}$$

Note that this setup can accommodate the moral hazard case discussed in the base model simply by setting $\theta_L = 0$. The next step is to scrutinise the constraints for redundancy and whether or not any are binding. Constraint (17) is redundant because it is guaranteed by constraints (18) and (19). Constraint (20) on the other hand is redundant in the sense that it is not feasible for low-type individuals; they will never choose high-type contracts because it ends up costing them more. We then consider that the remaining constraints, (18) and (19), bind because the financier will want to increase the required initial capital for both types as high as possible i.e. until the constraints bind:

$$\theta_L E[P_T(c_L)] - c_L - E[K_T(c_L)] - K_{L0} = 0 \tag{21}$$

$$\theta_H E[P_T(c_H)] - c_H - E[K_T(c_H)] - K_{H0} = \theta_H E[P_T(c_L)] - c_L - E[K_T(c_L)] - K_{L0} \tag{22}$$

The financier’s optimisation problem is therefore characterised by Equations (16), (21) and (22). Substituting in the latter two along with $E[K_T(c_L)]$, $E[P_T(c_L)]$, $E[K_T(c_H)]$, and $E[P_T(c_H)]$ results in the following modified problem for the financier:

$$\max_{K_{L0}, K_{H0}} \beta(2\theta_H A(p(c_H) - p(c_L))) + \theta_L(P_0 + [2p(c_L) - 1]A) - \beta(c_H - c_L) - c_L - P_0 \tag{23}$$

To obtain a closed-form solution, we assume that the probability function has the following form:

$$p(c) = 1 - e^{-c} \tag{24}$$

such that the optimal level of asset maintenance spending c^* is:

$$c^* = \log\left(2\left[\theta - 1 + \frac{K_{S0}}{P_0}\right]A\right) \tag{25}$$

and the corresponding probability of asset appreciation is:

$$p(c^*) = 1 - \frac{1}{2\left(\theta - 1 + \frac{K_{S0}}{P_0}\right)A} \tag{26}$$

Substituting in c^* and $p(c^*)$ for high and low-type finance seekers respectively:

$$\begin{aligned} \max_{K_{L0}, K_{H0}} \beta\left(\theta_H\left[-\frac{1}{\theta_H - 1 + \frac{K_{H0}}{P_0}} + \frac{1}{\theta_L - 1 + \frac{K_{L0}}{P_0}}\right] - \log\frac{\theta_H - 1 + \frac{K_{H0}}{P_0}}{\theta_L - 1 + \frac{K_{L0}}{P_0}}\right) \\ + \theta_L\left(P_0 + A - \frac{1}{\theta_L - 1 + \frac{K_{L0}}{P_0}}\right) - \log\left(2\left[\theta_L - 1 + \frac{K_{L0}}{P_0}\right]A\right) - P_0 \end{aligned} \tag{27}$$

Therefore the optimal level of initial capital that the financier will impose upon the high-type finance seekers can be obtained by differentiating the financier’s maximisation problem with respect to K_{H0} and equating to zero:

$$\frac{\theta_H}{(\theta_H - 1 + \frac{K_{H0}}{P_0})^2} - \frac{1}{(\theta_H - 1 + \frac{K_{H0}}{P_0})} = 0 \tag{28}$$

$$K_{H0}^* = P_0 \tag{29}$$

This result for the high-type finance seekers is significant for reasons that will be discussed later. For now, it is substituted back into Equation (27) and the optimal level of initial capital to be imposed upon the low-type finance seekers is obtained in a similar manner:

$$\beta \left(-\frac{\theta_H}{(\theta_L - 1 + \frac{K_{L0}}{P_0})^2} + \frac{1}{\theta_L - 1 + \frac{K_{L0}}{P_0}} \right) + \frac{\theta_L}{(\theta_L - 1 + \frac{K_{L0}}{P_0})^2} - \frac{1}{\theta_L - 1 + \frac{K_{L0}}{P_0}} = 0 \tag{30}$$

$$K_{L0}^* = \left(1 - \frac{\beta}{1 - \beta} [\theta_H - \theta_L] \right) P_0 \tag{31}$$

The results are as expected for an economic screening approach; high-type finance seekers would be more than happy to bear all of the financing if they could whereas low-type finance seekers would make a markedly lower contribution. Furthermore, we summarise the sources of the the gap between both types of finance seekers in the following proposition:

Proposition 4. *The difference in initial capital that both types would be willing to contribute depends on (i) the proportion of high types to low-types (ii) the difference in marginal benefit of asset appreciation between both types and (iii) the full initial price of the asset itself.*

We complete the screening analysis by comparing the expected rates of return received under both types of finance seekers and ultimately determining how much is received by the financier. Substituting in c^* and $p(c^*)$ into the expected rates of return for both parties from Equations (13) and (14) results in:

$$E[r_S] = \frac{(P_0 + A) \left(\theta - 1 + \frac{K_{S0}}{P_0} \right) - \log \left(2 \left[\theta - 1 + \frac{K_{S0}}{P_0} \right] A \right) - 1}{K_{S0}} - 1 \tag{32}$$

$$E[r_f] = \frac{1}{P_0} \left(A - \frac{1}{\theta - 1 + \frac{K_{S0}}{P_0}} \right) \tag{33}$$

Therefore, the expected rates of return for low-type finance seekers are as follows:

$$E[r_{SL}] = \frac{(P_0 + A) \left(\theta_L - \frac{\beta}{1 - \beta} [\theta_H - \theta_L] \right) - \log \left(2 \left[\theta_L - \frac{\beta}{1 - \beta} (\theta_H - \theta_L) \right] A \right) - 1}{\left(1 - \frac{\beta}{1 - \beta} [\theta_H - \theta_L] \right) P_0} - 1 \tag{34}$$

$$E[r_{fL}] = \frac{1}{P_0} \left(A - \frac{1}{\theta_L - \frac{\beta}{1 - \beta} [\theta_H - \theta_L]} \right) \tag{35}$$

whereas for high-type finance seekers, the expected rates of return are:

$$E[r_{SH}] = \frac{(P_0 + A)\theta_H - \log(2\theta_H A) - 1}{P_0} - 1 \tag{36}$$

$$E[r_{fH}] = \frac{1}{P_0} \left(A - \frac{1}{\theta_H} \right) \tag{37}$$

However, note that the expected returns associated with the high-type finance seekers in Equations (36) and (37) can be misleading. The model suggests that high-type finance seekers would prefer to buy out the financier’s share as soon as possible and so it is very possible for the financier to simply break even. This does approximate real-world conditions but poses a problem for the model. We remedy this by introducing a limit component associated with the high-type finance seeker’s initial capital contribution. We can therefore differentiate between two cases, namely one in which the high-type finance seekers do buy out the financier’s share and another in which they don’t. The above expected rates of return are therefore applicable only when they do not buy out the financier. If they do buy out the financier, the finance seekers do not share any capital appreciation or in flows from the asset with the financier and the financier breaks even with zero return.

Now focusing on the financier, their total expected inflow from DM contracts, based on Equation (16), can be expressed as follows:

$$\lim_{K_{H0} \rightarrow P_0} u_F = \beta \left(1 - \frac{K_{H0}}{P_0} \right) \left(P_0 + A - \frac{1}{\theta_H - 1 + \frac{K_{H0}}{P_0}} \right) + (\beta[\theta_H - \theta_L]) \left(A - \frac{1}{\theta_L - \frac{\beta}{1-\beta}[\theta_H - \theta_L]} \right) + \beta(K_{H0} - P_0) \quad (38)$$

which becomes the following if $K_{H0} = P_0$:

$$u_F = \beta[\theta_H - \theta_L] \left(A - \frac{1}{\theta_L - \frac{\beta}{1-\beta}[\theta_H - \theta_L]} \right) \quad (39)$$

The financier’s total expected rate of return from DM contracts with both types is:

$$r_F = \beta E[r_{fH}] + (1 - \beta) E[r_{fL}] \quad (40)$$

If $K_{H0} < P_0$, it becomes:

$$r_F = \frac{A}{P_0} - \frac{\beta}{\theta_H P_0} - \frac{1 - \beta}{(\theta_L - \frac{\beta}{1-\beta}[\theta_H - \theta_L]) P_0} \quad (41)$$

Otherwise, it is simply equal to $(1 - \beta) E[r_{fL}]$:

$$r_F = \frac{1 - \beta}{P_0} \left(A - \frac{1}{\theta_L - \frac{\beta}{1-\beta}[\theta_H - \theta_L]} \right) \quad (42)$$

We conclude this section with our final proposition and a discussion related to the implications of the screening model especially in relation to the implied returns from the DM contracts.

Proposition 5. *Financiers can be discouraged from entering into a DM contract with finance seekers whose type levels are too high as they have an incentive and possibly the (potential) resources to limit the financier’s share in the gains from the contract.*

This should be clear from our above discussion regarding $K_{H0}^* = P_0$, namely that high-type finance seekers would ideally buy out the financier’s share in the underlying asset as soon as possible. This is more generally reflected in the equations related to how the type levels and type differential $\theta_H - \theta_L$ affect the expected rates of return. Firstly, Equation (34) suggests that the model has an implied constraint $\theta_L > \beta\theta_H$ as otherwise the logarithmic function in the model would take on a negative value. This implied constraint applies to all equations after, suggesting that there is an upper bound on the types and

type differential which is line with the idea that sufficiently high type levels might in fact be bad for the financier. This can especially be seen in Equation (41) representing the financier's total return; the level of θ_H serves only to reduce the financier's rate of return. This result is surprisingly in line with contract theory to the extent that the solution for the standard principal-agent problem such as in [Mas-Colell et al. \(1995\)](#) is to agree on a 50-50 split between the principal and the agent. The economic screening approach suggests that a similar situation seems to be ideal for the financier, namely one in which the type levels and disparity of the prospective finance seekers is not too high. The desired result is that entering into such DM financial markets would allow the financier to secure a sufficient stake in assets but be partnered with finance seekers who can use the assets effectively enough.

5. Future Extensions

The presented model is relatively simplistic especially compared to the ones it takes from, including [Gale and Hellwig \(1985\)](#) and [Dewatripont et al. \(2003\)](#). This was done on purpose to demonstrate the merits of contract theory as a complement to conventional financial mathematics in the valuation of IFSs. More aspects that can be looked into include:

- Alternative arrangements for asset maintenance expenditure
- Different risk attitudes for the parties involved
- Modelling the asset price fluctuation as a Brownian motion
- The aforementioned multiperiod case where the financier's share is repurchased in installments

Some more related issues are how refinancing and contract renegotiation by either party can affect the arrangement as well as the issue of subletting. The last one can be especially concerning; even though subletting can incentivise the finance seeker to maintain the asset, it can also increase the information asymmetry relative to the financier.

Some even more exciting topics for future consideration include the comparison of alternative IFSs. In the case of this paper, a well-recognised alternative to DM financing is the previously mentioned IMBT security. Both can be considered rather iconic because they represent the respective sides of the debt VS equity dichotomy in finance. This makes it reasonable to scrutinise and compare the respective contracting properties of each especially in terms of their tax treatments; [Benninga \(2014\)](#) and [Pinto and Krug Pacheco \(2014\)](#) demonstrate that leases can facilitate tax deductions whereas a common problem with equity-based IFSs is that they are hit with the double stamp duty problem.

A successful comparison of the financial value of different IFSs through the lens of contract theory can pave the way to something still more exciting, namely the possibility of convertible IFSs with convertibility as a real option. This is significant because derivatives are heavily frowned-upon under ICL due to having no intrinsic value. It is, however, permissible to add options (typically referred to individually as *khiyār*) to a contract. More generally, it is also permissible by mutual consent of the relevant parties to terminate the current contract and adopt a new one. In fact, this last form of convertibility is essentially invoked in the transfer of possession under the IMBT contract. It is therefore only natural to apply contract theory ideas to analyse when it is within the interests of all parties involved to exercise such options and how said interests can be influenced.

Lastly, because IFSs can originate from a securitisation process involving various institutions and contractual relationships, it is plausibly vital to apply the contract theory framework to this process. IFSs generally take the form of certificates or *ṣukūk* which are generally the product of a securitisation process called *taṣkīk*, as explained for example in [Ayub \(2007\)](#), and [Ariff et al. \(2012\)](#). [Jobst \(2007, 2009\)](#) evaluate both conventional and Islamic securitisation processes, discussing contract-related issues arising such as differential contract interpretation and moral hazard to incentive misalignment. Applying contract theory to a network of contractual parties is a momentous task but as pointed out by [Jobst \(2009\)](#) more specifically and the literature on the global financial crisis (GFC) more generally, it is indispensable.

6. Summary and Conclusions

This author set out to demonstrate the usefulness of contract theory for analysing IFSs using the DM contract as an example. The main results compel DM financiers to really consider the ability of prospective DM finance seekers to take advantage of the asset they wish to finance. Failure to select “good” finance seekers can in fact result in those with an incentive to sabotage the asset in order to reduce the amount owed to the financier. A finance seeker will only care about asset maintenance if they stand to gain more than their cost of financing. Furthermore, given that the finance seeker’s marginal benefit from capital appreciation outweighs their financial cost - namely the financier’s share of the asset - an increase in the magnitude of capital appreciation/depreciation can increase the seeker’s optimal spending on asset maintenance. Also, an economic screening approach was used to demonstrate that one sign of a “good” finance seeker is their willingness to ask for less financing. This leads to a somewhat ironic result in that, while high-type finance seekers might lead to more robust DM contracts, they might also limit the financier’s ability to profit from capital appreciation of the underlying asset. Surprisingly, this outcome is consistent with a key principal in both ICL (see [Al-Suwailem \(2000\)](#)) and conventional finance theory: No reward without risk.

The model constructed in this paper is rather simple and there is still much ground to cover. Hopefully this paper facilitates more sophisticated approaches to the analysis and utilisation of IFSs.

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Abbreviations

The following abbreviations are used in this manuscript:

| | |
|------|-------------------------------------|
| IFS | Islamic Financial Security |
| IMBT | Al-’Ijarah Al-Muntahiyah Bit-Tamlīk |
| DM | Diminishing Mushārahah |
| SDC | Simple Debt Contract |
| ICL | Islamic Commercial Law |

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